Research Article

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Dynamic properties of the attachment oscillator arising in the nanophysics

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Abstract: The attachment oscillator, which plays an important role in the nanophysics such as nano/microelectromechanical systems, molecular devices, and nanofibers, is studied in this work. With the help of the semi-inverse method, the variational principle is established, and the Hamiltonian of the system is correspondingly constructed based on the obtained variational principle. Then, according to the principle of energy conservation, the energy balance theory is implemented to seek for the amplitude–frequency relationship. As predicted, the obtained solution has a good agreement with the existing results, which shows that the presented method is simple but effective, and is expected to provide a new idea for the study of the nonlinear oscillator arising in the nanophysics.

Keywords: attachment oscillator, semi-inverse method, variational principle, Hamiltonian, energy balance theory

1 Introduction

Many complex phenomena arising in nature, such as thermodynamics [1–3], optics [4–10], water waves [11–13], electronic circuit [14,15], and so on [16–19], can be modelled by nonlinear partial differential equations. As an interesting phenomenon, nonlinear vibration occurs everywhere in our daily life and how to determine the amplitude–frequency relationship has always been the focus of research. The amplitude–frequency relationship can help us better understand the nature of vibration. Up to now, some effective methods have been proposed to inquire into the nonlinear vibration such as homotopy perturbation method (HPM) [20–26], variational method [27,28], variational iteration method [29,30], Hamiltonian-based method [31–33], Gamma function method [34], He's frequency formula [35–37], average residual method [38], and others [39]. In this study, we will pay attention to the attachment oscillator which reads as [40]:

$$\varphi'' + \varphi + \lambda_1\varphi^3 + \frac{\lambda_2}{\varphi^3} = 0, \tag{1.1}$$

with the following conditions:

$$\varphi(0) = \Pi, \ \varphi'(0) = 0, \tag{1.2}$$

Eq. (1.1) is used to express the molecular oscillation induced by geometrical potential during electrospinning to produce the nanofibrous membranes and plays an important role in the nanotechnology, especially in nano/microelectromechanical systems and molecular devices.

For $\lambda_2 = 0$, Eq. (1.1) becomes the classic Duffing oscillator as:

$$\varphi'' + \varphi + \lambda_1\varphi^3 = 0. \tag{1.3}$$

Eq. (1.1) has been studied by the residual theory in ref. [40]. In ref. [41], the HPM is employed to find the frequency–amplitude formulation. Here in this work, we apply the energy balance theory (EBT) to study it. The rest of this article is organized as follows. In Section 2, the variational principle is established *via* the semi-inverse method and Hamiltonian is constructed. In Section 3, the EBT is adopted to find the amplitude–frequency relation. Finally, the conclusion is reached in Section 4.

2 Variational principle and Hamiltonian

The objective of this section is to construct the variational principle and Hamiltonian of the system.

By means of the semi-inverse method [42–50], the variational principle of Eq. (1.1) can be found as:

$$J(\varphi) = \int \left\{\frac{1}{2}(\varphi')^2 - \frac{1}{2}\varphi^2 - \frac{1}{4}\lambda_1\varphi^4 + \frac{1}{2}\lambda_2\varphi^{-2}\right\}\mathrm{d}t, \tag{2.1}$$

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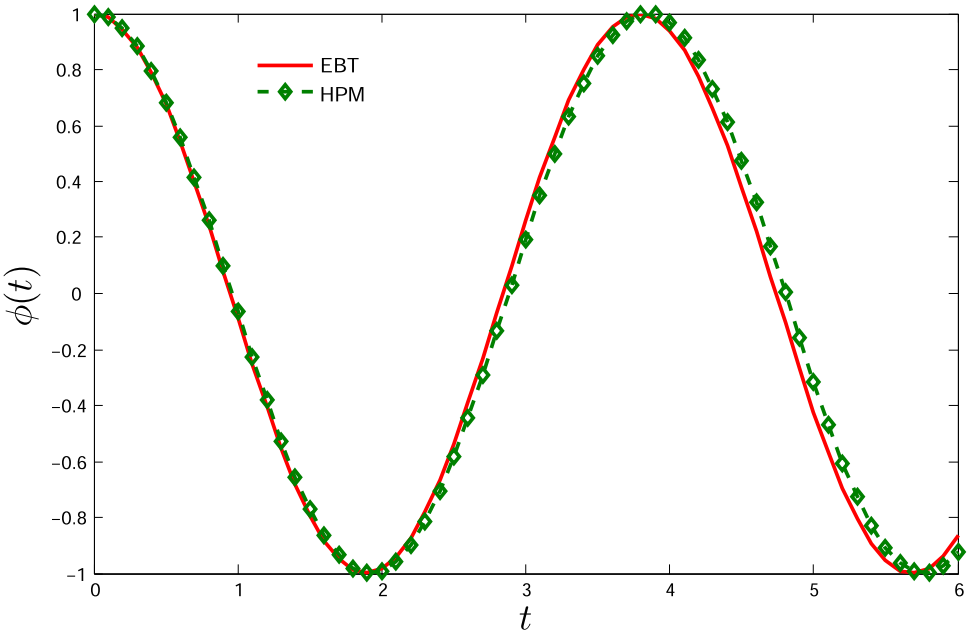

Figure 1: Comparison of results of the two methods for $\Pi = 1$, $\lambda_1 = 1$, and $\lambda_2 = 0.5$.

which can be rewritten as:

$$J(\varphi) = \int \left\{ \frac{1}{2}(\varphi')^2 - \frac{1}{2}\varphi^2 - \frac{1}{4}\lambda_1\varphi^4 + \frac{1}{2}\lambda_2\varphi^{-2} \right\} \mathrm{dt} = \int \{\Re - \aleph\} \mathrm{dt}, \tag{2.2}$$

where 2.2 $\Re$ represents the kinetic energy and $\aleph$ indicates the potential energy. They are obtained as follows [51]:

$$\Re = \frac{1}{2}(\varphi')^2, \tag{2.3}$$

$$\aleph = \frac{1}{2}\varphi^2 + \frac{1}{4}\lambda_1\varphi^4 - \frac{1}{2}\lambda_2\varphi^{-2}. \tag{2.4}$$

Thus, the Hamiltonian of the system can be obtained as [52,53]:

$$H = \Re + \aleph = \frac{1}{2}(\varphi')^2 + \frac{1}{2}\varphi^2 + \frac{1}{4}\lambda_1\varphi^4 - \frac{1}{2}\lambda_2\varphi^{-2}. \tag{2.5}$$

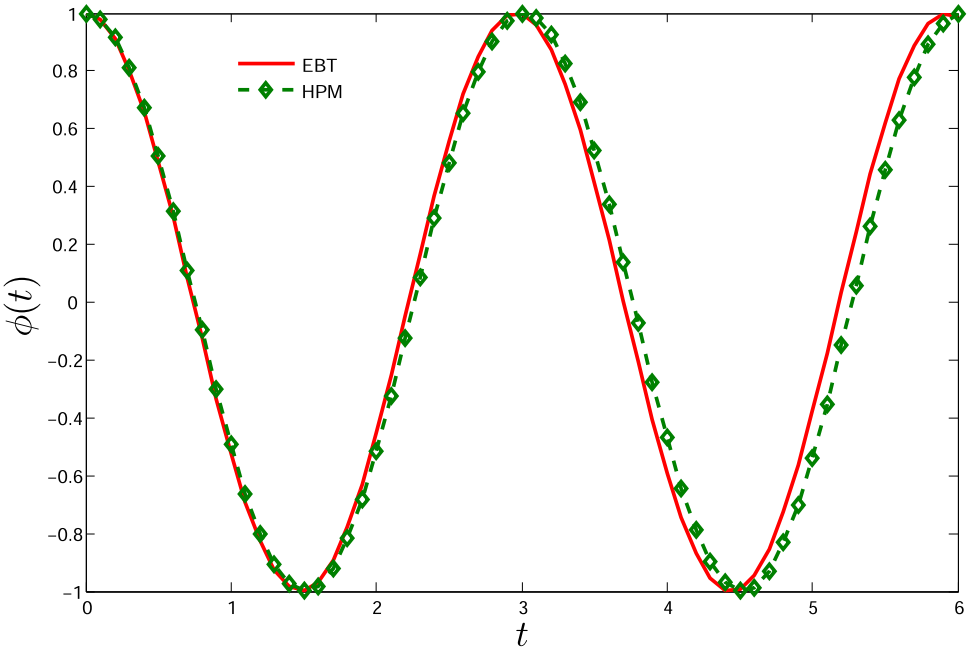

Figure 2: Comparison of results of the two methods for $\Pi = 1$, $\lambda_1 = 2$, and $\lambda_2 = 1$.

3 Application of the EBT

In this section, the EBT will be used to seek for the amplitude–frequency relation.

Here we can suppose the solution of Eq. (1.1) as:

$$\varphi(t) = \Pi \cos(\Omega t), \tag{3.1}$$

where Π represents the amplitude and Ω represents the frequency.

Based on the conditions given by Eq. (1.2), we can determine the Hamiltonian constant of the system as:

$$H_0 = \Re + \aleph = \frac{1}{2}\Pi^2 + \frac{1}{4}\lambda_1\Pi^4 - \frac{1}{2}\lambda_2\Pi^{-2}. \tag{3.2}$$

On the basis of the EBT, the energy of the system remains constant throughout the whole process of the vibration, so substituting Eq. (3.1) into Eq. (2.5), there should be:

$$\begin{aligned} H = \Re + \aleph = {} & \frac{1}{2}[-\Omega\Pi \sin(\Omega t)]^2 + \frac{1}{2}[\Pi \cos(\Omega t)]^2 \\ & + \frac{1}{4}\lambda_1[\Pi \cos(\Omega t)]^4 - \frac{1}{2}\lambda_2[\Pi \cos(\Omega t)]^{-2} = H_0, \end{aligned} \tag{3.3}$$

which is

$$\begin{aligned} & \frac{1}{2}[-\Omega\Pi \sin(\Omega t)]^2 + \frac{1}{2}[\Pi \cos(\Omega t)]^2 \\ & + \frac{1}{4}\lambda_1[\Pi \cos(\Omega t)]^4 - \frac{1}{2}\lambda_2[\Pi \cos(\Omega t)]^{-2} \\ & = \frac{1}{2}\Pi^2 + \frac{1}{4}\lambda_1\Pi^4 - \frac{1}{2}\lambda_2\Pi^{-2}. \end{aligned} \tag{3.4}$$

We can set [54]:

$$\Omega t = \frac{\pi}{4}. \tag{3.5}$$

Thus, Eq. (3.4) becomes

$$\begin{aligned} & \frac{1}{4}\Omega^2\Pi^2 + \frac{1}{4}\Pi^2 + \frac{1}{16}\lambda_1\Pi^4 - \lambda_2\Pi^{-2} \\ & = \frac{1}{2}\Pi^2 + \frac{1}{4}\lambda_1\Pi^4 - \frac{1}{2}\lambda_2\Pi^{-2}. \end{aligned} \tag{3.6}$$

On solving it, we can obtain the amplitude–frequency relationship as:

$$\Omega = \sqrt{1 + \frac{3}{4}\lambda_1\Pi^2 + \frac{2\lambda_2}{\Pi^4}}, \tag{3.7}$$

which has a good agreement with results given in ref. [41] by using the HPM.

By using $\Pi = 1$, $\lambda_1 = 1$, and $\lambda_2 = 0.5$, we compare the results of the EBT and HPM in Figure 1 with the help of MATLAB, from which, we can find a good agreement between the two methods. The results reveal the correctness and effectiveness of our method.

If we select $\Pi = 1$, $\lambda_1 = 2$, and $\lambda_2 = 1$, the comparison of results of the two methods is presented in Figure 2. A good agreement is also reached in this case. Thus, we can confirm that the EBT is correct and effective.

4 Conclusion

In this article, the attachment oscillator is studied by using the EBT, which is based on the variational principle and Hamiltonian theory. The frequency–amplitude relation is obtained and a comparative analysis between the proposed method and the existing results is presented. The results show that the presented method is simple but effective and is expected to provide a new idea for the nonlinear oscillator theory arising in the nanophysics.

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Data availability statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

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