Research Article

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Surface waves on a coated incompressible elastic half-space

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Abstract: Recently, Khajiyeva *et al.* (2018) studied the dynamics of surface wave propagation in a pre-stressed incompressible half-space when accompanied by both vertical and tangential loads. In this study, however, we extend the work of Khajiyeva *et al.* to include the presence of a coating layer above the half-space, upon which the loads are acted. Moreover, we prescribe sufficient perfect continuity conditions between the two layers, resulting in an inhomogeneous composite structure. Furthermore, related effective boundary conditions within the long-wave assumption are acquired for the model through the application of the asymptotic approximation method. Finally, approximate uncoupled pseudo-differential equations are derived on the surface, thereby admitting all the results of Khajiyeva *et al.* as limiting cases of concern.

Keywords: coated half-space, incompressible, pre-stress, surface waves

1 Introduction

Propagation of waves in assorted structural arrangements has been expansively investigated since long ago owing to its immense happening in a variety of science and technological applications. In this regard, one can easily find some pertinent areas of real applications that involve the production and examination of elastic waves, such areas include seismology, materials science, earthquake, fluid dynamics, elastodynamics, biomechanics, geophysics, and geology [1–4] to state a few. More so, there exist other fields from modern-day engineering that hugely enjoy massive contributions from the governing phenomena, including, for instance, structural engineering, transportation engineering, aeronautical engineering, and metallurgical engineering, just to mention a few [5–8]. Structure-wise, various studies have been reported with regard to the propagation of different wave forms in disparate structural bodies and shapes, comprising singled-layered media [9–11], composite structures [12–15], and the generalized multilayered bodies [16–22]. Besides, the case of the propagation of waves in singled-layered media is actually the classical consideration of different situations. One would read the book of Achenbach [23] for dissimilar scenarios about the propagation of waves in different elastic solids. Dutta [24] examined the longitudinal vibration of elastic disturbance associated with linear excitation by parameters. Bhattacharyya and Bera [25] made use of the legendary Adomian's method to examine the propagation of waves in single elastic bars with linear and random material constituents. Moreover, Ahmad and Zaman [26] examined the case of a propagating wave in an isotropic nonhomogeneous single rod *via* exact and asymptotic methods. We also recall the latest work of Alzaidi *et al.* [27] for the study of the influence of non-integer temporal variation on the propagation of waves on an inhomogeneous finite elastic substrate.

Equally, for composites and multilayered structures, including coated media, there exist ample related examinations in both the past and recent works of the literature, with regard to the propagation and dispersion of elastic waves in such media. Moreover, such structural formations are realized in the present-day designs and inventions; take a look at the wings of an aeroplane, photovoltaic solar panels, metamaterials, multilayered petitioning timbers, nano-indentation tests, and laminated glazing glasses to refer to a few, see ref. [28] and references therein. Thus, upon putting more weights behind the coated media, since their applications are numerous, as we further recall some of their significance in seismic protection, rail transportation, and highway engineering, refer refs [29–31]. More so, their fundamental impacts are to protect material surfaces from external agents such as chemicals, thereby causing the whole structure to deteriorate [32]; read also about their

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relevance in the design and construction of biomaterials, such as implantations [33,34].

Nevertheless, stating some of the latest findings with regard to the dynamics of waves in coated media, Mubaraki *et al.* [35] examined the influence of certain external excitations, including rotation, and the gravitational and magnetic field forces on the vibration of surface waves on a viscoelastic-coated half-plane. Mubaraki *et al.* [36] determined the approximate explicit equations for the propagation of elastic waves on an elastic half-plane endowed with a thin-coating strip; read also ref. [37] for the study of Rayleigh waves' effect on the propagation of surface waves on such media amidst the influence of prescribed stresses and gravity. In addition, a doubly coated inhomogeneous half-space was equally examined in ref. [38] with regard to the implication of rotation in the media, and on the other hand, the impact of the material inhomogeneity in the half-space; adequate approximate effective boundary conditions were constructed, which successfully approximated the derived secular equation with high precision. Additionally, the recent works of Manna *et al.* [39,40] on the propagation of Love-type wave on coated extended medium within the setting of orthotropic and anisotropic materials, respectively, are quite relevant in this arena; read also the study of Selim and Althobaiti [41] about the vibration of waves in supported walled carbon nanotubes by utilizing the wave-based method.

However, the present study aims at extending the recent work of Khajiyeva *et al.* [42] for the determination of approximate equations of motion with regard to the propagation of surface waves in a pre-stressed incompressible half-space, by incorporating a single coating layer with sufficient perfect continuity conditions in-between. Additionally, the present inhomogeneous composite structure is further assumed to be exerted by vertical and tangential loads from the other end of the coating. Furthermore, the related effective boundary conditions within the long-wave will be acquired for the model through the application of the asymptotic approximation method [37]. More so, the resulting approximate uncoupled pseudo-differential equations will be derived on the surface, thereby generalizing the results of Khajiyeva *et al.* [42], as all their results serve as limiting cases of the current study; for more relevant and recent considerations on the propagation of waves on coated and other related media, comprising the methodology of hyperbolic-elliptic types of Rayleigh waves, the consideration of surface waves in a coated half-space with Dirichlet-type boundary conditions, the extension to arbitrary anisotropy, and others; read refs [43–50] and references therein. Besides, the present study will be useful in the construction of medical biomaterials, which improves the quality of lives, in particular, in addition to its usefulness in addition to its usefulness in the modeling and analysis of coated media in modern technological inventions; see the frontier references cited in refs [1–8] and in refs [33,34] as an instance.

Finally, we organize the present article as follows: Section 2 gives the formulation of the governing problem. Section 3 derived the related appropriate effective boundary conditions; while Section 4 derives the asymptotic approximation for the propagation of surface waves by acquiring the resulting pseudo-differential equations. Finally, Section 5 gives some concluding notes and future perspectives.

2 Problem formulation

Consider a homogeneous, incompressible elastic coating of constant thickness h, subject to vertical and tangential loads, overlying a homogeneous elastic body $\mathscr{B}$, which when a natural unstressed occupies the configuration $\mathscr{B}_u$. Purely homogeneous state deformations are imposed to produce a pre-stress equilibrium upon $\mathscr{B}_e$. The position vectors of a representative particle are denoted by X and $x(X)$ in $\mathscr{B}_u$ and $\mathscr{B}_e$, respectively. The deformation between them can be described by the mapping φ, so that

$$x = \varphi(X), \quad X \in \mathscr{B}_u, \tag{1}$$

where φ is assumed to be one-to-one mapping and at least twice continuously differentiable. We further consider an infinitesimal time-dependent motion $u_i(X_A, t)$, which is then superimposed upon $\mathscr{B}_e$, resulting in $\mathscr{B}_t$ that occupies new time-dependent deformations. The position vectors of representative particle are denoted by X_A, $x_i(X_A)$, and $\tilde{x}_i(X_A, t)$ in $\mathscr{B}_u$, $\mathscr{B}_e$, and $\mathscr{B}_t$, respectively. The position vector $\tilde{x}_i(X_A, t)$ may therefore be expressible in the form

$$\tilde{x}_i(X_A, t) = x_i(X_A) + u_i(X_A, t). \tag{2}$$

A Cartesian coordinate system Ox_1x_2 is chosen to be coincident with the deformed principal axes in $\mathscr{B}_e$, the Cartesian axes, Ox_1 lies on the upper surface of the coating and Ox_2 is located between the coating and half-plane (Figure 1).

In addition, the linearized measures of incremental tractions, associated with a surface having outward unit normals along Ox_1 and Ox_2 in $\mathscr{B}_e$, are obtained in the component forms as follows:

$$\begin{aligned}
\tau_{11} &= (A_{1111} + \bar{p})u_{1,1} + A_{1122}u_{2,2} - p_t,\\
\tau_{12} &= (A_{1221} + \bar{p})u_{1,2} + A_{1212}u_{2,1},\\
\tau_{21} &= A_{2121}u_{1,2} + (A_{1221} + \bar{p})u_{2,1},\\
\tau_{22} &= A_{1122}u_{1,1} + (A_{2222} + \bar{p})u_{2,2} - p_t,
\end{aligned} \tag{3}$$

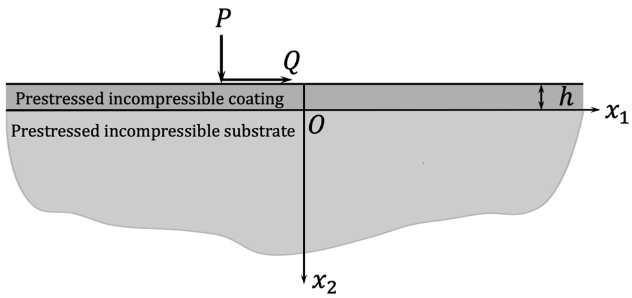

Figure 1: The tangential and vertical loads acting on a half-plane coated by a pre-stressed incompressible elastic layer.

where $\bar{p}$ is a static pressure in the configuration $\mathscr{B}_e$, that is, taken of the form

$$\bar{p} = A_{2121} - A_{1221} - \sigma_2,$$

with σ_2 denoting the principal Cauchy stresses in $\mathscr{B}_e$, and p_t is the associated time-dependent incremental component of pressure arising due to the incompressibility conditions, given by

$$u_{1,1} + u_{2,2} = 0. \tag{4}$$

Here and below, the comma indicates the differentiation with respect to the appropriate variables.

Then, the two coupled non-trivial equations of motion are thus given by

$$\begin{aligned} A_{1111}u_{1,11} + A_{2121}u_{1,22} + \delta u_{2,12} - p_{t,1} &= \rho_e u_{1,tt}, \\ A_{1212}u_{2,11} + A_{2222}u_{2,22} + \delta u_{1,12} - p_{t,2} &= \rho_e u_{2,tt}, \end{aligned} \tag{5}$$

in which

$$\delta = A_{1122} + A_{1221},$$

and ρ_e is the material density per unit volume of $\mathscr{B}_e$, while A_{ijkl} are components of the fourth-order elasticity tensor. We assume the material parameters of the coating ($-h \le x_2 \le 0$) are ρ_e^0, A_{iijj}^0, A_{ijij}^0, δ^0, and $\bar{p}^0$. More so, the boundary conditions at the surface of the coating ($x_2 = -h$) are the prescribed tangential and vertical loads given as follows:

$$\tau_{21} = -P(x_1, t), \quad \text{and} \quad \tau_{22} = -Q(x_1, t), \tag{6}$$

while perfect continuity conditions involving all the displacements and stresses τ_{2i} for $i = 1, 2$ are assumed along the interface ($x_2 = 0$). Accordingly, we set the following boundary conditions:

$$u_i = v_i, \tag{7}$$

where $v_i = v_i(x_1, t)$, $i = 1, 2$ are displacements in the substrate at $x_2 = 0$.

3 Effective boundary conditions

The derivation of effective boundary conditions goes by first introducing a small asymptotic parameter ε within the long-wave assumption as follows:

$$\varepsilon = \frac{h}{\lambda} \ll 1, \tag{8}$$

where h is the thickness of the coating, while λ is the wave length. Next, we make use of the following scaling:

$$\xi = \frac{x_1}{\lambda}, \quad \eta = \frac{x_2 + h}{h}, \quad \tau = \frac{c_0}{\lambda}t, \tag{9}$$

along with the following dimensionless quantities:

$$\begin{aligned} &u_i^* = \frac{1}{\lambda}u_i, \quad v_i^* = \frac{1}{\lambda}v_i, \quad p_t^* = \frac{p_t}{\gamma^0}, \\ &p^* = \frac{\lambda}{h\gamma^0}P, \quad q^* = \frac{\lambda}{h\gamma^0}Q, \end{aligned} \tag{10}$$

where $\gamma^0 = A_{2121}^0$, and c_0 is the wave speed, defined as $c_0 = \sqrt{\gamma^0/\rho_e^0}$.

In addition, the equations of motion expressed in Eq. (5) can now be rewritten as:

$$\begin{aligned} \gamma^0 u^*_{1,\eta\eta} + \varepsilon\delta^0 u^*_{2,\xi\eta} + \varepsilon^2(A^0_{1111}u^*_{1,\xi\xi} - \gamma^0(p^*_{t,\xi} + u^*_{1,\tau\tau})) = 0, \\ A^0_{2222}u^*_{2,\eta\eta} + \varepsilon(\delta^0 u^*_{1,\xi\eta} - \gamma^0 p^*_{t,\eta}) + \varepsilon^2(A^0_{1212}u^*_{2,\xi\xi} - \gamma^0 u^*_{2,\tau\tau}) = 0, \end{aligned} \tag{11}$$

along with the transformed incompressibility equation as follows:

$$u^*_{2,\eta} + \varepsilon u^*_{1,\xi} = 0, \tag{12}$$

where $\delta^0 = A^0_{1122} + A^0_{1221}$.

Furthermore, the prescribed boundary conditions in Eqs. (6) and (7) are transformed based on the above development to the following dimensionless forms:

$$\begin{aligned} &\gamma^0 u^*_{1,\eta} + \varepsilon(A^0_{1221} + \bar{p}^0)u^*_{2,\xi} = -\varepsilon^2\gamma^0 p^*, \quad \text{at } \eta = 0, \\ &(A^0_{2222} + \bar{p}^0)u^*_{2,\eta} + \varepsilon(A^0_{1122}u^*_{1,\xi} - \gamma^0 p^*_t) = -\varepsilon^2\gamma^0 q^*, \\ &\qquad \text{at } \eta = 0, \\ &u_i^* = v_i^*, \text{ at } \eta = 1. \end{aligned} \tag{13}$$

Next, the components of displacements u_i^* and the pressure p_t^* may be written by asymptotic series in terms of the small parameter ε as

$$u_i^* = u_i^{(0)} + \varepsilon u_i^{(1)} + \ldots, \text{ and } p_t^* = p_t^{(0)} + \varepsilon p_t^{(1)} + \ldots. \tag{14}$$

Therefore, at $O(1)$, we obtain

$$\begin{aligned} &u^{(0)}_{i,\eta\eta} = 0, \qquad u^*_{2,\eta} = 0, \\ &u^{(0)}_{i,\eta} = 0, \quad \text{at} \quad \eta = 0, \\ &u^{(0)}_i = v^*_i, \quad \text{at} \quad \eta = 1, \end{aligned} \tag{15}$$

such that the above system admits the following solution:

$$u_i^{(0)} = v_i^*. \tag{16}$$

Going further to order $O(\varepsilon)$, we obtain

$$\begin{aligned} &\gamma^0 u_{1,\eta\eta}^{(1)} + \delta^0 u_{2,\xi\eta}^{(0)} = 0, \\ &A_{2222}^0 u_{2,\eta\eta}^{(1)} + \delta^0 u_{1,\xi\eta}^{(0)} - \gamma^0 p_{t,\eta}^{(0)} = 0, \\ &u_{2,\eta}^{(1)} + u_{1,\xi}^{(0)} = 0, \end{aligned} \tag{17}$$

associated with

$$\begin{aligned} &\gamma^0 u_{1,\eta}^{(1)} + (A_{1221}^0 + \bar{p}^0) u_{2,\xi}^{(0)} = 0, \ \text{at} \ \eta = 0, \\ &(A_{2222}^0 + \bar{p}^0) u_{2,\eta}^{(1)} + (A_{1122}^0 u_{1,\xi}^{(0)} - \gamma^0 p_t^{(0)}) = 0, \ \text{at} \ \eta = 0, \\ &u_i^{(1)} = 0, \ \text{at} \ \eta = 1. \end{aligned} \tag{18}$$

More so, the displacement components $u_1^{(1)}$ and $u_2^{(1)}$ are determined by solving Eqs. $(17)_1$ and $(17)_3$, together with Eqs. (16), $(18)_1$, and $(18)_3$ to obtain

$$u_1^{(1)} = \frac{1}{\gamma^0}(1-\eta)(A_{1221}^0 + \bar{p}^0) v_{2,\xi}^* \tag{19}$$

and

$$u_2^{(1)} = (1-\eta) v_{1,\xi}^*. \tag{20}$$

Also, the pressure $p_t^{(0)}$ is obtained from Eqs. $(17)_2$ and (19) along with the boundary condition given in Eq. $(18)_2$ as

$$p_t^{(0)} = \frac{1}{\gamma^0}(A_{1122}^0 - (A_{2222}^0 + \bar{p}^0)) v_{1,\xi}^*. \tag{21}$$

The next order $O(\varepsilon^2)$ gives

$$\begin{aligned} &\gamma^0 u_{1,\eta\eta}^{(2)} + \delta^0 u_{2,\xi\eta}^{(1)} + A_{1111}^0 u_{1,\xi\xi}^{(0)} - \gamma^0 (p_{t,\xi}^{(0)} + u_{1,\tau\tau}^{(0)}) = 0, \\ &A_{2222}^0 u_{2,\eta\eta}^{(2)} + \delta^0 u_{1,\xi\eta}^{(1)} - \gamma^0 p_{t,\eta}^{(1)} + A_{1212}^0 u_{2,\xi\xi}^{(0)} - \gamma^0 u_{2,\tau\tau}^{(0)} = 0, \\ &u_{2,\eta}^{(2)} + u_{1,\xi}^{(1)} = 0, \end{aligned} \tag{22}$$

subject to

$$\begin{aligned} &\gamma^0 A_{2121}^0 u_{1,\eta}^{(2)} + (A_{1221}^0 + \bar{p}^0) u_{2,\xi}^{(1)} = -\gamma^0 p^*, \ \text{at} \ \eta = 0, \\ &(A_{2222}^0 + \bar{p}^0) u_{2,\eta}^{(2)} + A_{1122}^0 u_{1,\xi}^{(1)} - \gamma^0 p_t^{(1)} = -\gamma^0 q^*, \ \text{at} \ \eta = 0, \\ &u_i^{(2)} = 0, \ \text{at} \ \eta = 1. \end{aligned} \tag{23}$$

The solution of this system may be obtained as follows:

$$\begin{aligned} u_1^{(2)} = \frac{1}{2}(1-\eta)\Bigg[\left(\frac{2}{\gamma^0}(A_{1221}^0 + \bar{p}^0) - (1+\eta)\delta_1\right) v_{1,\xi\xi}^* \\ - (1+\eta) v_{1,\tau\tau}^* + 2p^*\Bigg], \end{aligned} \tag{24}$$

$$u_2^{(2)} = \frac{1}{2\gamma^0}(1-\eta)^2 (A_{1221}^0 + \bar{p}^0) v_{2,\xi\xi}^* \tag{25}$$

and

$$p_t^{(1)} = \frac{1}{\gamma^0}(\eta\delta_2 + \delta_3) v_{2,\xi\xi}^* - \eta v_{2,\tau\tau}^* + q^*, \tag{26}$$

wherein

$$\begin{aligned} \delta_1 &= \delta^0 + A_{1122}^0 - A_{1111}^0 - A_{2222}^0 - \bar{p}^0, \\ \delta_2 &= \frac{1}{\gamma^0}(A_{1221}^0 + \bar{p}^0)(A_{2222}^0 - \delta^0) + A_{1212}^0, \\ \delta_3 &= \frac{1}{\gamma^0}(A_{1221}^0 + \bar{p}^0)(A_{1122}^0 - A_{2222}^0 - \bar{p}^0). \end{aligned}$$

Hence, the incremental tractions τ_{21} and τ_{22} are represented as

$$\begin{aligned} \tau_{21} &= \varepsilon[(\delta_1 - A_{1221}^0 - \bar{p}^0)\eta v_{1,\xi\xi}^* + \gamma^0(\eta v_{1,\tau\tau}^* - p^*)] + O(\varepsilon^2), \\ \tau_{22} &= \varepsilon\Bigg[\gamma^0 \eta v_{2,\tau\tau}^* + \Bigg(\frac{1}{\gamma^0}(A_{1221}^0 + \bar{p}^0)(A_{1122}^0 - A_{2222}^0 - \bar{p}^0)(1 \\ &\quad - \eta) - (\eta\delta_2 + \delta_3)\Bigg) v_{2,\xi\xi}^* - \gamma^0 q^*\Bigg] + O(\varepsilon^2). \end{aligned} \tag{27}$$

Finally, upon returning to the original dimensional variables, the effective boundary conditions at the interface ($x_2 = 0$) eventually become

$$\begin{aligned} \tau_{21} &= h(\gamma^0 c_0^{-2} v_{1,tt} + \delta_4 v_{1,11}) - P, \\ \tau_{22} &= h(\gamma^0 c_0^{-2} v_{2,tt} + \delta_5 v_{2,11}) - Q, \end{aligned} \tag{28}$$

where

$$\begin{aligned} \delta_4 &= 2A_{1122}^0 - A_{1111}^0 - A_{2222}^0 - 2\bar{p}^0, \quad \text{and} \\ \delta_5 &= \frac{1}{\gamma^0}(A_{1221}^0 + \bar{p}^0)^2 - A_{1212}^0. \end{aligned}$$

4 Asymptotic formulation for surface waves

Here, having successfully determined the effective boundary conditions in Eq. (28), we now proceed further to derive a pseudo-differential equation for the vertical and longitudinal displacements of the substrate, thereby generalizing the forgoing results to case of the pre-stressed incompressible elastic half-plane, and extending the hyperbolic-elliptic equation in the study of Khajiyeva *et al.* [42]. To begin with, we follow the consideration in the study of Dowaikh and Ogden [51] by considering the displacement components – *via* the Cauchy–Riemann identities, to be of the following forms:

$$u_1 = \psi_{,2} \quad \text{and} \quad u_2 = -\psi_{,1}, \tag{29}$$

where ψ is the potential function, which satisfies the incompressibility condition. Thus, the coupled equations of motion expressed in Eq. (5) would now be rewritten in terms of the potential function ψ as follows:

$$\begin{aligned} A_{1111}\psi_{,112} + A_{2222}\psi_{,222} - \delta\psi_{,112} - p_{t,1} = \rho_e\psi_{,2tt}, \\ A_{1212}\psi_{,111} + A_{2222}\psi_{,122} - \delta\psi_{,122} + p_{t,2} = \rho_e\psi_{,1tt}. \end{aligned} \quad (30)$$

Then, upon eliminating p_t from the above equation, we obtain

$$\alpha\psi_{,1111} + 2\beta\psi_{,1122} + \gamma\psi_{,2222} = \rho_e(\psi_{,11tt} + \psi_{,22tt}), \quad (31)$$

where

$$\alpha = A_{1212} \quad \gamma = A_{2121}, \quad 2\beta = A_{1111} + A_{2222} - 2\delta,$$

and satisfying

$$\alpha > 0 \quad \text{and} \quad \beta + \sqrt{\alpha\gamma} > 0,$$

such that the obtained traction boundary conditions in Eq. (28) are also expressed at the interface ($x_2 = 0$) as follows:

$$\begin{aligned} &\gamma\psi_{,22} - (\gamma - \sigma_2)\psi_{,11} = h(\gamma^0 c_0^{-2}\psi_{,2tt} + \delta_4\psi_{,112}) - P, \\ &(2\beta + \gamma - \sigma_2)\psi_{,112} + \gamma\psi_{,222} - \rho_e\psi_{,2tt} \\ &\quad = h(\gamma^0 c_0^{-2}\psi_{,11tt} + \delta_5\psi_{,1111}) + Q_{,1}. \end{aligned} \quad (32)$$

Now, let us introduce the slow-time perturbation scheme, by introducing the following dimensionless variables:

$$\hat{\xi} = \frac{x_1 - c_R t}{\lambda}, \quad \hat{\eta} = \frac{x_2}{\lambda}, \quad \hat{\tau} = \frac{c_R\varepsilon}{\lambda}t, \quad (33)$$

where $\hat{\tau}$ is the slow time, λ is the typical wave length, and c_R denotes the Rayleigh wave speed.

Then Eq. (31) is re-cast in terms of a single fourth order partial differential equation (PDE) as follows:

$$\begin{aligned} &(\alpha - \rho_e c_R^2)\psi_{,\hat{\xi}\hat{\xi}\hat{\xi}\hat{\xi}} + (2\beta - \rho_e c_R^2)\psi_{,\hat{\xi}\hat{\xi}\hat{\eta}\hat{\eta}} + \gamma\psi_{,\hat{\eta}\hat{\eta}\hat{\eta}\hat{\eta}} \\ &\quad + 2\varepsilon\rho_e c_R^2(\psi_{,\hat{\xi}\hat{\xi}\hat{\xi}\hat{\tau}} + \psi_{,\hat{\xi}\hat{\eta}\hat{\eta}\hat{\tau}}) - \varepsilon^2\rho_e c_R^2(\psi_{,\hat{\xi}\hat{\xi}\hat{\tau}\hat{\tau}} + \psi_{,\hat{\eta}\hat{\eta}\hat{\tau}\hat{\tau}}) = 0, \end{aligned} \quad (34)$$

subject to

$$\begin{aligned} &\gamma\psi_{,\hat{\eta}\hat{\eta}} - (\gamma - \sigma_2)\psi_{,\hat{\xi}\hat{\xi}} - \varepsilon(\gamma^0 k_0 + \delta_4)\psi_{,\hat{\xi}\hat{\xi}\hat{\eta}} \\ &\quad + \frac{\gamma^0 c_R^2}{c_0^2}(2\varepsilon^2\psi_{,\hat{\xi}\hat{\eta}\hat{\tau}} - \varepsilon^3\psi_{,\hat{\eta}\hat{\tau}\hat{\tau}}) = -\lambda^2 P, \\ &(2\beta + \gamma - \sigma_2 - \rho_e c_R^2)\psi_{,\hat{\xi}\hat{\xi}\hat{\eta}} + \gamma\psi_{,\hat{\eta}\hat{\eta}\hat{\eta}} \\ &\quad + \varepsilon[2\varepsilon\rho_e c_R^2\psi_{,\hat{\xi}\hat{\eta}\hat{\tau}} - (\gamma^0 k_0 + \delta_5)\psi_{,\hat{\xi}\hat{\xi}\hat{\xi}\hat{\xi}}] \\ &\quad + \varepsilon^2[2\gamma^0 c_0^{-2}c_R^2\psi_{,\hat{\xi}\hat{\xi}\hat{\xi}\hat{\tau}} - \rho_e c_R^2\psi_{,\hat{\eta}\hat{\tau}\hat{\tau}}] \\ &\quad - \varepsilon^3\gamma^0 c_0^{-2}\psi_{,\hat{\xi}\hat{\xi}\hat{\tau}\hat{\tau}} = \lambda^2 Q_{,\hat{\xi}}, \end{aligned} \quad (35)$$

where

$$k_0^2 = \frac{c_R^2}{c_0^2}.$$

Let us now expand the displacements ψ *via* asymptotic series as follows:

$$\psi = \frac{\lambda^2}{\varepsilon}(\psi^{(0)}(\hat{\xi}, \hat{\eta}, \hat{\tau}) + \varepsilon\psi^{(1)}(\hat{\xi}, \hat{\eta}, \hat{\tau}) + \ldots),$$

then, at the leading order, Eq. (34) becomes

$$(\alpha - \rho_e c_R^2)\psi^{(0)}_{,\hat{\xi}\hat{\xi}\hat{\xi}\hat{\xi}} + (2\beta - \rho_e c_R^2)\psi^{(0)}_{,\hat{\xi}\hat{\xi}\hat{\eta}\hat{\eta}} + \gamma\psi^{(0)}_{,\hat{\eta}\hat{\eta}\hat{\eta}\hat{\eta}} = 0, \quad (36)$$

this PDE is elliptic, hence, it can be represented in operator form as

$$\Delta_1\Delta_2\psi^{(0)} = 0, \quad (37)$$

where

$$\hat{\kappa}_1^2\hat{\kappa}_2^2 = \frac{1}{\gamma}(\alpha - \rho_e c_R^2) \quad \hat{\kappa}_1^2 + \hat{\kappa}_1^2 = \frac{1}{\gamma}(2\beta - \rho_e c_R^2). \quad (38)$$

For each $\hat{\kappa}_i^2 > 0$, the solution of Eq. (37) can be expressed in terms of a pair of plane harmonic functions as

$$\psi^{(0)} = \sum_{i=1}^{2}\psi_i^{(0)}(\hat{\xi}, \hat{\kappa}_i^2\hat{\eta}, \hat{\tau}), \quad (39)$$

where $\psi_i^{(0)}(\hat{\xi}, \hat{\kappa}_i^2\hat{\eta}, \hat{\tau})$ are the arbitrary harmonic functions in the first two arguments. Substituting these solutions given in Eq. (39) into the boundary conditions expressed in Eq. (35) at leading order, and implying the Cauchy–Riemann identities expressed earlier, and further integrating the second boundary condition with respect to ξ, we arrive at

$$\sum_{i=1}^{2} c_{1i}\psi^{(0)}_{i,\hat{\xi}\hat{\xi}} = 0, \quad \text{and} \quad \sum_{i=1}^{2} c_{2i}\psi^{(0)}_{i,\hat{\xi}\hat{\xi}} = 0, \quad (40)$$

where

$$c_{1i} = \gamma(\hat{\kappa}_i^2 + 1) - \sigma_2, \quad c_{2i} = \hat{\kappa}_i(2\beta - \rho_e c_R^2 - \sigma_2 + \gamma(1 - \hat{\kappa}_i^2)),$$

then, the classical Rayleigh wave equation follows as a solvability condition

$$c_{22}c_{11} - c_{12}c_{21} = 0, \quad (41)$$

with the relation between the potentials, given as

$$\psi_1^{(0)} = \vartheta\psi_2^{(0)}, \quad \text{at } \hat{\eta} = 0, \quad (42)$$

where

$$\vartheta = -\frac{c_{11}}{c_{12}}.$$

At the next order, the general solution of the potential function ψ is obtained by Khajiyeva *et al.* [42] as follows:

$$\psi^{(1)}(\hat{\xi}, \hat{\eta}, \hat{\tau}) = \sum_{i=1}^{2}(\psi_i^{(1,0)} + \hat{\eta}e_i\bar{\psi}^{(0)}_{i,\hat{\tau}}), \quad (43)$$

with

$$e_i = \frac{\rho_e c_R^2(\hat{\kappa}_i^2 - 1)}{\gamma\hat{\kappa}_i - (\hat{\kappa}_j - \hat{\kappa}_i)}, \quad i \neq j = 1, 2,$$

where $\psi_i^{(1,0)} = \psi_i^{(1,0)}(\hat{\xi}, \hat{\kappa}_i^2\hat{\eta}, \hat{\tau})$ are once again arbitrary plane harmonic functions in the first two arguments and bar denote a harmonic conjugate.

More so, at the next order, the boundary conditions given in Eq. (35) at $\hat{\eta} = 0$ yield

$$\begin{aligned} &\gamma\psi^{(1)}_{,\hat{\eta}\hat{\eta}} - (\gamma - \sigma_2)\psi^{(1)}_{,\hat{\xi}\hat{\xi}} - (\gamma^0 k_0^2 + \delta_4)\psi^{(0)}_{,\hat{\xi}\hat{\xi}\hat{\eta}} = -P,\\ &(2\beta + \gamma - \sigma_2 - \rho_e c_R^2)\psi^{(1)}_{,\hat{\xi}\hat{\xi}\hat{\eta}} + \gamma\psi^{(1)}_{,\hat{\eta}\hat{\eta}\hat{\eta}} + 2\rho_e c_R\psi^{(0)}_{,\hat{\xi}\hat{\gamma}\hat{\tau}}\\ &\quad - (\gamma^0 k_0^2 + \delta_5)\psi^{(0)}_{,\hat{\xi}\hat{\xi}\hat{\xi}\hat{\xi}} = Q_{,\hat{\xi}}. \end{aligned} \tag{44}$$

Therefore, upon inserting Eqs (39) and (43) into conditions (44), and thereafter making use of the Cauchy–Riemann identities, we arrive at $\hat{\eta} = 0$ the following:

$$\begin{aligned} &\sum_{i=1}^{2}\{-c_{1i}\psi^{(1,0)}_{i,\hat{\xi}\hat{\xi}} + 2\gamma e_i\hat{\kappa}_i\psi^{(0)}_{i,\hat{\xi}\hat{\tau}} + \hat{\kappa}_i(\gamma^0 k_0^2 + \delta_4)\bar{\psi}^{(0)}_{i,\hat{\xi}\hat{\xi}\hat{\xi}}\} = -P,\\ &\sum_{i=1}^{2}\{c_{2i}\psi^{(1,0)}_{i,\hat{\xi}\hat{\xi}} - g_i\psi^{(0)}_{i,\hat{\xi}\hat{\tau}} - (\gamma^0 k_0^2 + \delta_5)\bar{\psi}^{(0)}_{i,\hat{\xi}\hat{\xi}\hat{\xi}}\} = \bar{Q}_{,\hat{\xi}}, \end{aligned} \tag{45}$$

where

$$g_i = e_i(2\beta - \rho_e c_R^2 - \sigma_2 + \gamma(1 - 3\hat{\kappa}_i^2)) - 2\hat{\kappa}_i\rho_e c_R.$$

Next, plugging the relation determined in Eq. (42) into (45), we obtain at $\hat{\eta} = 0$ as follows:

$$\begin{aligned} &-c_{11}\psi^{(1,0)}_{1,\hat{\xi}\hat{\xi}} - c_{12}\psi^{(1,0)}_{2,\hat{\xi}\hat{\xi}} + b_1\psi^{(0)}_{1,\hat{\xi}\hat{\tau}} + (\hat{\kappa}_1 + \vartheta\hat{\kappa}_2)\\ &\quad\times(\gamma^0 k_0^2 + \delta_4)\bar{\psi}^{(0)}_{1,\hat{\xi}\hat{\xi}\hat{\xi}} = -P,\\ &c_{21}\psi^{(1,0)}_{1,\hat{\xi}\hat{\xi}} + c_{22}\psi^{(1,0)}_{2,\hat{\xi}\hat{\xi}} - b_2\psi^{(0)}_{1,\hat{\xi}\hat{\tau}} - (1 + \vartheta)\\ &\quad\times(\gamma^0 k_0^2 + \delta_5)\bar{\psi}^{(0)}_{1,\hat{\xi}\hat{\xi}\hat{\xi}} = \bar{Q}, \end{aligned} \tag{46}$$

where

$$b_1 = 2\gamma(e_1\hat{\kappa}_1 + \vartheta e_2\hat{\kappa}_2), \quad \text{and} \quad b_2 = g_1 + \vartheta g_2.$$

Furthermore, upon combining the previous equations, we have at $\hat{\eta} = 0$

$$\begin{aligned} &(c_{22}c_{11} - c_{12}c_{21})\psi^{(1,0)}_{2,\hat{\xi}\hat{\xi}} + (c_{21}b_1 - c_{11}b_2)\psi^{(0)}_{1,\hat{\xi}\hat{\tau}}\\ &\quad - [c_{11}(1 + \vartheta)(\gamma^0 k_0^2 + \delta_5) - c_{21}(\hat{\kappa}_1 + \vartheta\hat{\kappa}_2)(\gamma^0 k_0^2\\ &\quad + \delta_4)]\bar{\psi}^{(0)}_{1,\hat{\xi}\hat{\xi}\hat{\xi}} = -(c_{21}P - c_{11}\bar{Q}). \end{aligned} \tag{47}$$

Moreover, from the solvability condition given in Eq. (41), the first term in Eq. (47) vanishes, therefore, we deduce

$$\psi_{1,\hat{\xi}\hat{\tau}}^{(0)} + \frac{c_R b_0}{2\hat{\kappa}_1}\psi_{1,\hat{\xi}\hat{\xi}\hat{\eta}}^{(0)} = -\frac{c_R}{2B}(c_{21}P - c_{11}\bar{Q}), \quad \text{at } \hat{\eta} = 0, \tag{48}$$

where

$$B = \frac{c_R}{2}(c_{21}b_1 - c_{11}b_2)$$

and

$$b_0 = \frac{1}{B}[c_{11}(1 + \vartheta)(\gamma^0 k_0^2 + \delta_5) - c_{21}(\hat{\kappa}_1 + \vartheta\hat{\kappa}_2)(\gamma^0 k_0^2 + \delta_4)]. \tag{49}$$

In addition, Eq. (48) is rearranged in terms of the original variables (x_1, x_2, t) as follows:

$$\begin{aligned} &\psi_{1,11} - \frac{1}{c_R^2}\psi_{1,tt} + \frac{hb_0}{\hat{\kappa}_1}\psi_{1,112}\\ &\quad = -\frac{1}{B}(c_{21}P - c_{11}\bar{Q}), \quad \text{at } x_2 = 0, \end{aligned} \tag{50}$$

where the elliptic equation for the potential ψ_1 is obtained as

$$[\partial_{22}^2 + \hat{\kappa}_1^2\partial_{11}^2]\psi_1 = 0, \tag{51}$$

over the interior, along with the boundary condition on the surface of the substrate ($x_2 = 0$) given in (50). Then, Eq. (50) can be expressed in terms of the surface displacements $u_i(x_1, 0, t)$ as follows:

$$u_{1,11} - \frac{1}{c_R^2}u_{1,tt} + \frac{hb_0}{\hat{\kappa}_1}u_{1,112} = \frac{(\hat{\kappa}_1 + \vartheta\hat{\kappa}_2)}{B}(c_{21}\bar{P}_{,1} + c_{11}Q_{,1}) \tag{52}$$

and

$$u_{2,11} - \frac{1}{c_R^2}u_{1,tt} + \frac{hb_0}{\hat{\kappa}_1}u_{2,112} = \frac{(1 + \vartheta)}{B}(c_{21}P_{,1} - c_{11}\bar{Q}_{,1}). \tag{53}$$

Next, Eqs (52) and (53) may be expressed in the form of a pseudo-differential equations on the surface as follows:

$$\begin{aligned} &u_{1,11} - \frac{1}{c_R^2}u_{1,tt} - hb_0\sqrt{-\partial_{11}}\,u_{1,11}\\ &\quad = \frac{(\hat{\kappa}_1 + \vartheta\hat{\kappa}_2)}{\hat{B}}(c_{21}\bar{P}_{,1} + c_{11}Q_{,1}), \quad \text{at } x_2 = 0, \end{aligned} \tag{54}$$

and

$$\begin{aligned} &u_{2,11} - \frac{1}{c_R^2}u_{2,tt} - hb_0\sqrt{-\partial_{11}}\,u_{2,11}\\ &\quad = \frac{(1 + \vartheta)}{B}(c_{21}P_{,1} - c_{11}\bar{Q}_{,1}), \quad \text{at } x_2 = 0. \end{aligned} \tag{55}$$

Note that, for instance, the case of tangential loading of Eq. (52) may be reduced to linear isotropic elasticity as

$$u_{1,11} - \frac{1}{c_R^2}u_{1,tt} + \frac{hb_0}{k_2}u_{1,112} = \frac{(1 - k_2^4)}{4\mu B}Q_{,1}, \tag{56}$$

where in the absence of deformation ($\lambda_1 = \lambda_2 = \lambda_3 = 1$), we find $k_1 = 1$ and $k_2 = \hat{\kappa}_1$, and then the constants b_0 and B become

$$B = k_2^4 - k_2 - 1 + k_2^{-1} \quad \text{and}$$

$$b_0 = \frac{(1 - k_2^4)}{4B}\left[\frac{\rho_c c_R^2}{\mu_c}(1 - 2k_2 - k_2^2) + 8k_2\right],$$

where

$$k_1 = \sqrt{1 - \frac{\rho_s c_R^2}{\lambda_s + 2\mu_s}}, \quad \text{and} \quad k_2 = \sqrt{1 - \frac{\rho_s c_R^2}{\mu_s}},$$

with λ_q and μ_q the elastic constants, and ρ_q are the densities for $q = c, s$ where c indicates the coating layer and s the half-space.

Finally, in the case of no coating, Eqs. (54) and (55) coincide with the recent results reported by Khajiyeva *et al.* [42] for the dynamics of surface waves in a prestressed incompressible half-space. Thus, as the present investigation extended the results of the above-mentioned reference, various directions could be taken to extend the study of Khajiyeva *et al.* [42]; at the same time, it also serves as a benchmark reference for comparative examination.

5 Conclusion

As a concluding note, the present study analyzed a plane inhomogeneous dynamic problem with regard to a coated pre-stressed incompressible half-space. Sufficient perfect continuity conditions are prescribed between the coating layer and the half-space, while the other end surface of the coating is being excited by both the vertical and tangential loads. Moreover, we acquired the related effective boundary conditions within the long-wave assumption *via* the application of the asymptotic approximation method. In addition, the uncoupled pseudo-differential equations are approximately revealed on the surface; these approximate equations serve as the overall postulate for the propagation of surface waves in the governing coated structure. More conceivably, all the results presented recently by Khajiyeva *et al.* [42] could be obtained easily from the presented results as limiting cases of concern upon ignoring the presence and effect of the coating layer. Finally, the present examination can be extended to multiply-coated structures amidst the presence of external forces; in addition, the case of a generalized anisotropic medium can equally be considered as an interesting case of universal concern. Besides, this study is useful in the construction of medical biomaterials – including implants/implantation, which improves the quality of lives, as a particular case of great concern, in addition to its usefulness in the modeling and analysis of coated media.

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