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Rapid Communication

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Radial oscillations of an electron in a Coulomb attracting field

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Abstract: The radial motion of an electron in a Coulomb attracting field based on a new theory of the interaction of a particle with an electromagnetic field is studied. It is shown that the electron can perform radial oscillations due to a repulsive force that exists in the vicinity of the center and prevents the electron from approaching the center. This concept of a previously unknown repulsive force significantly shifts the paradigm of the interaction of a charge with an electromagnetic field. A laboratory experiment is proposed to test the new phenomenon.

Keywords: radial oscillations, Coulomb field, repulsive force, macroatom, qubit

1 Introduction

In ref [1], a new theory was developed to describe the interaction of a charged particle with an electromagnetic field, based on the hypothesis of the Lorentz invariance of the 4-momentum of a particle in an electromagnetic field. This implies the dependence of the potentials acting on a moving charge particle on its velocity according to Lorentz transformations law. The new formalism itself is self-consistent and does not contradict the basic conservation laws; it cannot be refuted or proved theoretically, and therefore its validity or fallacy can only be established empirically.

One of the important problems for which the new formalism leads to results that differ significantly from the traditional ones, allowing experimental verification, is the problem of the motion of a charge in a Coulomb field. The classical consideration of the motion of an electron in a central attracting Coulomb field has been considered in [2] and then in [3–5]. In addition to elliptical and hyperbolic trajectories, the equations also describe a specific spiral trajectory of the electron falling onto the center under certain conditions.

An alternative consideration based on a new theory [1] gives a new understanding of this problem. As mentioned earlier, the fundamental hypothesis of the new formalism postulates the Lorentz invariance of the 4-momentum of an electron in an electromagnetic field. The consistent development of this hypothesis leads to a new theory of the interaction of an electron with an electromagnetic field. It provides an alternative description of some theoretical problems that, not yet experimentally verified, have conflicting interpretations. Here, within the framework of the new theory, we investigate the special problem of the radial motion of an electron in the central Coulomb attracting field. We show that there is a previously unknown repulsive force acting on an electron near the center and preventing the electron from approaching the center. Due to the action of this force, the electron can perform radial oscillations. A laboratory experiment is proposed to test the new phenomenon.

2 Theoretical results

The invariance of the 4-momentum of an electron in an electromagnetic field leads to the following equations relating the total energy E and the generalized momentum $\boldsymbol{P}$ of an electron, moving with velocity $\boldsymbol{v}$ in the presence of an electrostatic scalar potential Φ:

$$E^2 - (c\boldsymbol{P})^2 = (mc^2 + e\Phi)^2 \; \boldsymbol{P} = \boldsymbol{v}E/c^2. \quad (1)$$

Based on Eq. (1), we investigate a one-dimensional radial motion of an electron in a Coulomb attracting field with the potential $\Phi(r) = -q/r$, where $q > 0$ is the positive charge of the center. The following equations for the force $F(r) = dP/dt$ acting on the electron and the radial trajectory $t(r)$ along which the electron moves under the action of this force are obtained from Eq. (1):

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$$F(r) = \frac{1}{\varepsilon}\left(-\frac{eq}{r^2} + \frac{eqr_c}{r^3}\right), \quad (2)$$

$$t(r) = \frac{r_c\varepsilon}{c(1-\varepsilon^2)}\left[\sqrt{\left(\frac{\varepsilon r}{r_c}\right)^2 - \left(1-\frac{r}{r_c}\right)^2} - \frac{1}{\sqrt{1-\varepsilon^2}}\arccos\frac{1-(1-\varepsilon^2)r/r_c}{\varepsilon}\right]. \quad (3)$$

Here, $\varepsilon = E/mc^2$, $r_c = eq/mc^2$ is called the critical radius, and $e > 0$ is the elementary charge.

According to Eq. (2), the force acting on the electron is negative (attractive) at $r > r_c$ and positive (repulsive) at $r < r_c$ (Figure 1). Due to this force, the electron at $\varepsilon < 1$ moves according to Eq. (3) in a finite interval from $r_{min} = r_c/(1+\varepsilon)$ to $r_{max} = r_c/(1-\varepsilon)$. The motion of the electron at $\varepsilon < 1$ is described as follows. When moving toward the center, the electron crosses the critical radius r_c and slows down. Having reached the distance r_{min}, it stops and starts moving backward. Reaching the distance r_{max}, it stops and again starts to move toward the center. Thus, the electron performs radial oscillations in a finite interval (r_{min}, r_{max}). The closer ε to unity, the greater the amplitude of radial oscillations, and the closer the electron approaches the center (blue curve in Figure 1). However, at no energy can the electron reach the center. For low energies ($\varepsilon \ll 1$), small harmonic radial oscillations are established (green curve in Figure 1). For $\varepsilon > 1$, the electron eventually moves to infinity at any initial condition (red curve in Figure 1).

The cause of radial oscillations is the repulsive force acting on the electron at $r < r_c$ described by the second term in Eq. (2). Note the formal analogy (2) with the force of interatomic interaction in diatomic molecules [6,7]. Figure 1 shows the graphs of the forces acting on an electron and radial oscillations for various energies, plotted according to Eqs. (2) and (3). Radial oscillations are not stable: a small transverse momentum applied to an electron causes an orbital motion that is superimposed on the radial oscillations. This results in a complex motion as the electron periodically approaches and recedes from the center along a petal-like trajectory. Thus, within the framework of the new theory, the spiral trajectory of the electron falling on the center is impossible.

Due to energy radiation losses, the amplitude of radial oscillations will gradually decrease to a value determined by quantum uncertainties. Finally, near $r = r_c$, a quantum state of an oscillating electron with a minimum energy E_{min} and a minimum amplitude A_{min} is established, which are determined by the following formulas:

$$E_{min} = \sqrt{mc^2(\hbar c/2r_c)} \quad A_{min} = \sqrt{\lambda\!\!\!^{-}_c r_c/2}, \quad (4)$$

where $\hbar$ is Planck constant and $\lambda\!\!\!^{-}_c$ is the Compton wavelength.

The minimum energy is equal to the geometric mean of the rest energy of an electron and the energy of a quantum with a wavelength ($2r_c$). The minimum amplitude is equal to the geometric mean of the Compton wavelength $\lambda\!\!\!^{-}_c$ and half of the critical radius ($r_c/2$). For $r_c = 10$ cm, the corresponding numerical values are $E_{min} \sim 0.71$ eV and $A_{min} \sim 1.4 \times 10^{-5}$ cm.

Thus, according to the new theory, a repulsive force acts on an electron in an attractive Coulomb field near the center. Due to this force, an unusual bound state of an electron can be established near the critical radius at $\varepsilon < 1$. This theoretical result needs experimental verification.

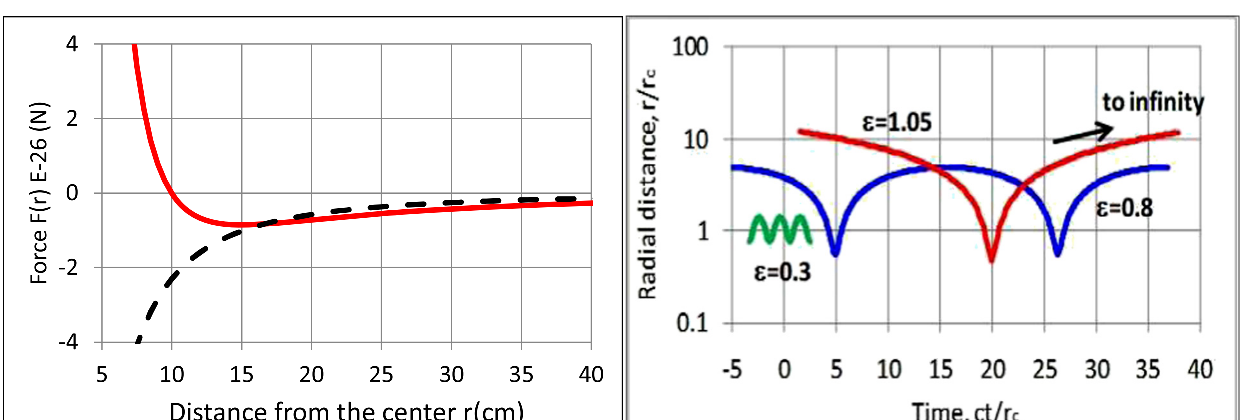

Figure 1: Left panel: the force acting on an electron according to Eq. (2) (solid red curve) and the Coulomb force (dashed black curve) at $\varepsilon = 0.3$ and $r_c = 10$ cm. Right panel: electron radial motion, according to Eq. (3), at $\varepsilon = 0.3$ (green curve), $\varepsilon = 0.8$ (blue curve), and $\varepsilon = 1.05$ (red curve). The time axis is arbitrarily offset for easy viewing.

3 Proposed experiment

The essence of the proposed experiment is to detect the repulsive force on macroscopic scales, where the critical radius r_c can reach large values for sufficiently strongly charged macroscopic bodies. In the experiment the repulsion of the electron beam from a positively charged sphere will be observed. It is necessary to ensure that the critical radius r_c is greater than the radius of the sphere so that a spherical layer exists near the surface of the sphere, where the electron experiences a repulsive rather than an attractive force. This is achieved by charging the sphere to a potential U above 0.51 MV.

Indeed, a conducting sphere with the radius R and potential U has the charge $Q = UR$. This charge is uniformly distributed over the spherical surface, so the potential outside the sphere is equal to the potential of the point charge Q located in the center, and therefore, the critical radius is $r_c = eQ/mc^2 = eUR/mc^2$. Using the megavolt as the potential unit, we have $r_c/R = U/0.51$. Therefore, a conducting sphere with a radius of, say, 10 cm, charged to a potential of 1 MV, has a critical radius $r_c \sim 20$ cm. This is quite enough to detect the repulsive force by observing the repulsion of electrons from the sphere.

Thus, when the potential of the sphere exceeds 0.51 MV, the critical radius is larger than the radius of the sphere, and the electrons experience a repulsive force in the spherical layer $R < r < r_c$. When the potential decreases below 0.51 MV, the critical radius becomes smaller than the radius of the sphere, and the repulsive force disappears. Note, that in the presence of free electrons near a sphere with $U > 0.51$ MV, they will be attracted and collected near the spherical surface $r = r_c$, performing complex oscillations. The number of electrons will increase to $\sim Q/e$, when their total negative charge will compensate for the positive charge of the sphere Q and nullify the attracting field at $r > r_c$. Thus, a cloud of stably oscillating electrons will be formed around a positively charged sphere of any size, if the potential of the sphere exceeds 0.51 MV. Such a macroatom can be considered, as a qubit for a quantum computer.

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