

Research Article

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Dynamics investigation on a Kadomtsev–Petviashvili equation with variable coefficients

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Abstract: In this work, we investigate a generalized Kadomtsev–Petviashvili equation with variable coefficients and self-consistent sources in plasma and fluid mechanics. The multiple rogue wave solutions, including 1-, 3-, and 6-order rogue waves, are presented by three different functions under a nonlinear transformation. Based on the Hirota bilinear method and a more complex assumption, new lump solutions are constructed, which have not been seen in other literature. The dynamic properties of the obtained results are illustrated graphically.

Keywords: rogue wave, self-consistent sources, Kadomtsev–Petviashvili equation, dynamic properties

1 Introduction

The rogue wave originated from the ocean is a sudden wave with large amplitude and very short duration, which has great destructive power to ships and structures on the sea [1,2]. This phenomenon has received continuous attention from researchers in oceanography, physics, and other nonlinear science fields. At present, rogue wave has extended from the ocean to nonlinear optical systems, plasma, hydrodynamics, atmosphere, Bose–Einstein condensation, and superfluid [3–7]. At present, the existing method to obtain rogue wave solutions includes the physics-informed neural network (PINN) method, bilinear derivative method, Darboux transformation method, Riemann–Hilbert method, homoclinic wave trial method, and so on [8–11]. Based on the symbolic calculation method established by the bilinear derivative,

Ma [12] has obtained the rogue wave solutions and the interaction solutions with other solitons of Kadomtsev–Petviashvili–Ito equation [13], extended Hirota–Satsuma–Ito equation [14], extended second KP equation [15], and so on, which greatly promoted the development of rogue wave theory.

In this article, we will investigate the following generalized variable-coefficient Kadomtsev–Petviashvili equation (vcKPe) with self-consistent sources [16]:

$$f(t)u_x^2 + f(t)uu_{xx} + g(t)u_{xxx} + l(t)u_x + m(t)u_{yy} + n(t)u_{xy} + q(t)u_{xx} + u_{xt} = 0, \quad (1)$$

where $u = u(x, y, t)$. Eq. (1) is used to describe the evolution of small amplitude ion acoustic waves propagating in plasma under transverse disturbance. It can also be derived as surface and inner water wave models. KP equation is widely regarded as the natural generalization of classical KdV equation in two-dimensional space, and it is applied in almost all physical fields [1]. The Gramian-type and lump solutions of Eq. (1) have been studied in refs [17] and [18]. The breather wave and interaction solutions were obtained in ref. [19]. Next, our work is mainly to find the multiple rogue wave solutions of Eq. (1) and obtain more 1-order rogue wave solutions using a more complex assumption than ref. [19].

Based on the result for ref. [18], Eq. (1) has the following bilinear form:

$$\begin{aligned} &[g(t)D_x^4 + D_t D_x + q(t)D_x^2 + m(t)D_y^2 + n(t)D_y D_x] \xi \cdot \xi \\ &= \xi[g(t)\xi_{xxxx} + m(t)\xi_{yy} + n(t)\xi_{xy} + q(t)\xi_{xx} + \xi_{xt}] + 3g(t)\xi_{xx}^2 \\ &- 4g(t)\xi_x \xi_{xxx} - m(t)\xi_y^2 - n(t)\xi_x \xi_y - q(t)\xi_x^2 - \xi_t \xi_x = 0, \end{aligned} \quad (2)$$

with

$$f(t) = \tau g(t)e^{\int l(t)dt}, \quad u = \frac{12e^{-\int l(t)dt}}{\tau}(\ln \xi)_{xx}, \quad (3)$$

where τ is an arbitrary constant, $\xi = \xi(x, y, t)$.

The organization of this article is as follows. Section 2 obtains the multiple rogue wave solutions, which contain 1-, 3-, and 6-order rogue wave; Section 3 derives the new lump solutions by a more complex assumption with variable coefficients; Section 4 concludes this article.

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2 Multiple rogue wave solutions

2.1 1-order rogue wave solution

Making the transformation

$$v = x + \omega(t), \quad \xi = \xi(v, y), \quad u = \frac{12e^{-\int l(t)dt}}{\tau} [\ln \xi]_{vv}, \quad (4)$$

where $\omega(t)$ is the unknown function. Compared with the linear transformation in other literature, $\omega(t)$ can be a nonlinear function and adapt to more complex nonlinear integrable systems. Using this transformation, Eq. (2) turns into

$$\begin{aligned} & \xi[g(t)\xi_{vvvv} + m(t)\xi_{yy} + n(t)\xi_{vy} + \xi_{vv}[q(t) - \omega'(t)]] \\ & + 3g(t)\xi_{vv}^2 - 4g(t)\xi_v\xi_{vvv} - m(t)\xi_y^2 - n(t)\xi_v\xi_y \\ & - q(t)\xi_v^2 + \omega'(t)\xi_v^2 = 0. \end{aligned} \quad (5)$$

To investigate the 1-order rogue wave solution, we set

$$\xi = (v - \mu)^2 + \vartheta_1(y - v)^2 + \vartheta_0, \quad (6)$$

where μ , v , ϑ_0 , and ϑ_1 are undetermined constants. Substituting Eq. (6) into Eq. (5), we have

$$\begin{aligned} n(t) &= 0, \quad g(t) = -\frac{1}{3}\vartheta_0\vartheta_1m(t), \\ \omega(t) &= \int [q(t) - \vartheta_1m(t)]dt. \end{aligned} \quad (7)$$

Substituting Eqs. (6) and (7) into Eq. (4), we obtain the following 1-order rogue wave solution:

$$u = \frac{24e^{-\int l(t)dt}[-\mu + \int q(t) - \vartheta_1m(t)dt - x]^2 + \vartheta_1(y - v)^2 + \vartheta_0]}{\tau[\mu + \int q(t) - \vartheta_1m(t)dt - x]^2 + \vartheta_1(y - v)^2 + \vartheta_0]}. \quad (8)$$

The dynamic properties of Eq. (8) are shown in Figures 1 and 2. When all variable coefficients in Eq. (1) are constants, we can observe a rogue wave from Figure 1. However, when the variable coefficient $q(t) = t$ is not constant, two rogue waves are found in Figure 2. Through Figures 1

and 2, we show the influence of variable coefficients on the dynamic properties of the solution (8).

2.2 3-order rogue wave solution

Aim to derive the 3-order rogue wave solution for Eq. (1), suppose that

$$\begin{aligned} \xi &= \mu^2 + v^2 + v^6 + y^6\vartheta_{17} + y^4\vartheta_{16} \\ &+ 2\mu v(y^2\vartheta_{23} + v^2\vartheta_{24} + \vartheta_{22}) \\ &+ 2vy(y^2\vartheta_{20} + v^2\vartheta_{21} + \vartheta_{19}) + v^4y^2\vartheta_{11} + y^2\vartheta_{15} \\ &+ v^2(y^4\vartheta_{14} + y^2\vartheta_{13} + \vartheta_{12}) + v^4\vartheta_{10} + \vartheta_{18}, \end{aligned} \quad (9)$$

where ϑ_i ($i = 10, \dots, 24$) is the unknown constant. Substituting Eq. (9) into Eq. (5), we have

$$\begin{aligned} n(t) &= 0, \quad \vartheta_{14} = \frac{\vartheta_{11}^2}{3}, \quad g(t) = -\frac{1}{90}\vartheta_{13}m(t), \\ \omega(t) &= \frac{1}{3} \int 3q(t) - \vartheta_{11}m(t)dt, \quad \vartheta_{23} = -\vartheta_{11}\vartheta_{24}, \\ \vartheta_{20} &= -\frac{1}{9}\vartheta_{11}\vartheta_{21}, \quad \vartheta_{17} = \frac{\vartheta_{11}^3}{27}, \quad \vartheta_{16} = \frac{17}{225}\vartheta_{10}\vartheta_{11}^2, \\ \vartheta_{15} &= \frac{19}{75}\vartheta_{10}^2\vartheta_{11}, \quad \vartheta_{22} = -\frac{1}{25}\vartheta_{10}\vartheta_{24}, \quad \vartheta_{12} = -\frac{\vartheta_{10}^2}{5}, \\ \vartheta_{19} &= \frac{\vartheta_{10}\vartheta_{21}}{15}, \\ \vartheta_{18} &= -\mu^2 - v^2 + \mu^2\vartheta_{24}^2 + \frac{v^2\vartheta_{21}^2}{3\vartheta_{11}} + \frac{3\vartheta_{10}^3}{25}, \\ \vartheta_{13} &= \frac{6\vartheta_{10}\vartheta_{11}}{5}. \end{aligned} \quad (10)$$

Substituting Eqs (9) and (10) into Eq. (4), we obtain the following 3-order rogue wave solution:

$$\begin{aligned} v &= x + \omega(t), \\ u &= \frac{12e^{-\int l(t)dt}}{\tau} (\ln \xi)_{vv}. \end{aligned} \quad (11)$$

The dynamic properties of Eq. (11) are presented in Figures 3 and 4.

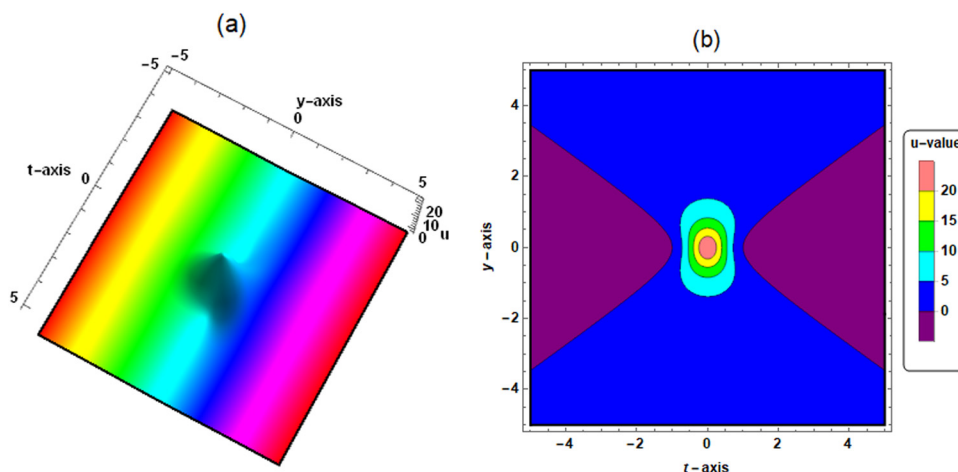


Figure 1: $l(t) = \mu = v = x = 0$, $q(t) = m(t) = \vartheta_0 = 1$, $\vartheta_1 = 2$, $\tau = -1$, (a) 3D plot, (b) contour plot.

2.3 6-order rogue wave solution

In order to present the 6-order rogue wave solution of Eq. (1), we have

$$\begin{aligned} \xi = & v^{12} + y^{12}g_{50} + y^{10}g_{49} + y^4g_{46} \\ & + (\mu^2 + v^2)(v^2 + y^2g_1 + g_0) + y^8g_{48} \\ & + y^6g_{47} + v^{10}(y^2g_{26} + g_{25}) + y^2g_{45} \\ & + v^8(y^4g_{29} + y^2g_{28} + g_{27}) \\ & + 2\mu v[y^6 + y^6g_{64} + y^4g_{63} + v^4(y^2g_{69} + g_{68}) \\ & + y^2g_{62} + v^2(y^4g_{67} + y^2g_{66} + g_{65}) + g_{61}] \\ & + 2vy[y^6 + y^4(v^2g_{57} + g_{56}) + y^2(v^4g_{55} + v^2g_{54} + g_{53}) \\ & + v^6g_{60} + v^4g_{59} + v^2g_{58} + g_{52}] \\ & + v^6(y^6g_{33} + y^4g_{32} + y^2g_{31} + g_{30}) \\ & + v^4(y^8g_{38} + y^6g_{37} + y^4g_{36} + y^2g_{35} + g_{34}) \\ & + v^2(y^{10}g_{44} + y^8g_{43} + y^6g_{42} + y^4g_{41} + y^2g_{40} + g_{39}) + g_{51}, \end{aligned} \quad (12)$$

where g_i ($i = 25, \dots, 69$) is the unknown constant. Substituting Eq. (12) into Eq. (5), we obtain

$$\begin{aligned} n(t) = 0, \quad g(t) = -\frac{1}{3}g_0g_1m(t), \quad \omega(t) = \int q(t) - g_1m(t)dt, \\ g_{26} = 6g_1, \\ g_{29} = 15g_1^2, \quad g_{28} = 230g_0g_1, \quad g_{33} = 20g_1^3, \\ g_{32} = \frac{1540}{3}g_0g_1^2, \quad g_{31} = \frac{18620}{9}g_0^2g_1, \\ g_{69} = -9g_1, \quad g_{37} = \frac{1460}{3}g_0g_1^3, \quad g_{36} = \frac{37450}{9}g_0^2g_1^2, \\ g_{55} = -\frac{5}{g_1^2}, \quad g_{38} = 15g_1^4, \quad g_{35} = \frac{24500}{3}g_0^3g_1, \\ g_{67} = -5g_1^2, \quad g_{66} = -\frac{230}{3}g_0g_1, \quad g_{42} = \frac{35420}{9}g_0^2g_1^3, \\ g_{41} = -\frac{4900}{9}g_0^3g_1^2, \quad g_{54} = -\frac{190g_0}{3g_1^2}, \quad g_{44} = 6g_1^5, \\ g_{57} = -\frac{9}{g_1}, \quad g_{43} = 190g_0g_1^4, \quad g_{40} = \frac{188650}{27}g_0^4g_1, \\ g_{64} = 5g_1^3, \quad g_{63} = 15g_0g_1^2, \quad g_{62} = \frac{535}{9}g_0^2g_1, \quad g_{50} = g_1^6, \\ g_{49} = \frac{58}{3}g_0g_1^5, \quad g_{48} = \frac{1445}{3}g_0^2g_1^4, \\ g_{45} = \frac{v^2}{g_1^6} + g_1\left(\frac{300896750g_0^5}{729} - v^2\right), \\ g_{60} = \frac{5}{g_1^3}, \quad g_{47} = \frac{798980}{81}g_0^3g_1^3, \quad g_{25} = \frac{98g_0}{3}, \\ g_{46} = \frac{16391725}{243}g_0^4g_1^2, \quad g_{58} = -\frac{665g_0^2}{9g_1^3}, \\ g_{68} = \frac{13g_0}{3}, \quad g_{56} = -\frac{7g_0}{3g_1}, \quad g_{34} = -\frac{5187875g_0^4}{243}, \\ g_{59} = \frac{35g_0}{g_1^3}, \quad g_{53} = -\frac{245g_0^2}{9g_1^2}, \quad g_{65} = -\frac{245g_0^2}{9}, \\ g_{30} = \frac{75460g_0^3}{81}, \quad g_{52} = \frac{18865g_0^3}{81g_1^3}, \\ g_{51} = v^2\left(\frac{1}{g_1^7} - 1\right)g_0 + \frac{878826025g_0^6}{6561}, \quad g_{27} = \frac{245g_0^2}{3}, \\ g_{61} = \frac{12005g_0^3}{81}, \quad g_{39} = v^2\left(\frac{1}{g_1^7} - 1\right) + \frac{159786550g_0^5}{729}. \end{aligned} \quad (13)$$

Substituting Eqs (12) and (13) into Eq. (4), we have the following 6-order rogue wave solution:

$$v = x + \omega(t), \quad u = \frac{12e^{-\int l(t)dt}}{\tau}(\ln \xi)_{vv}. \quad (14)$$

The dynamic properties of Eq. (14) are discussed in Figures 5 and 6.

3 New lump solutions

Yuan et al. [19] have studied the lump solution and interaction solutions of Eq. (1). Here, we want to consider the following more complex assumption [20] to find more new lump solutions for Eq. (1)

$$\xi = \alpha_7 + (\alpha_3 + x\alpha_1 + y\alpha_2)^2 + (\alpha_6 + x\alpha_4 + y\alpha_5)^2, \quad (15)$$

where $\alpha_i = \alpha_i(t)$ ($1 \leq i \leq 7$) is unknown functions. Substituting Eq. (15) into Eq. (2), we obtain

$$\begin{aligned} m(t) = \frac{3g(t)(\alpha_1^2 + \alpha_4^2)^3}{\tau(\alpha_1\alpha_5 - \alpha_2\alpha_4)^3}, \quad \alpha_7' = \frac{2\alpha_7(\alpha_4\alpha_2' - \alpha_5\alpha_1')}{\alpha_2\alpha_4 - \alpha_1\alpha_5}, \\ \alpha_5' = \frac{(-\alpha_2^2 - \alpha_5^2)\alpha_1' + (\alpha_1\alpha_2 + \alpha_4\alpha_5)\alpha_2'}{\alpha_2\alpha_4 - \alpha_1\alpha_5}, \\ \alpha_4' = \frac{(-\alpha_1\alpha_2 - \alpha_4\alpha_5)\alpha_1' + (\alpha_1^2 + \alpha_4^2)\alpha_2'}{\alpha_2\alpha_4 - \alpha_1\alpha_5(t)}, \\ \alpha_6' = [3g(t)[\alpha_1^2 + \alpha_4^2]^2[-\alpha_4\alpha_2^2 + 2\alpha_1\alpha_5\alpha_2 + \alpha_4\alpha_5^2] \\ - \tau[\alpha_2\alpha_4 - \alpha_1\alpha_5]^2[n(t)\alpha_5[\alpha_2\alpha_4 - \alpha_1\alpha_5] \\ + q(t)\alpha_4[\alpha_2\alpha_4 - \alpha_1\alpha_5] + [\alpha_2\alpha_3 + \alpha_5\alpha_6]\alpha_1' \\ - [\alpha_1\alpha_3 + \alpha_4\alpha_6]\alpha_2']]/[\tau[\alpha_2\alpha_4 - \alpha_1\alpha_5]^3], \\ \alpha_3' = [3g(t)[2\alpha_2\alpha_4\alpha_5 + \alpha_1(\alpha_2^2 - \alpha_5^2)](\alpha_1^2 + \alpha_4^2)^2 \\ + \tau(\alpha_2\alpha_4 - \alpha_1\alpha_5)^2[n(t)\alpha_2(\alpha_1\alpha_5 - \alpha_2\alpha_4) \\ + q(t)\alpha_1(\alpha_1\alpha_5 - \alpha_2\alpha_4) + (\alpha_2\alpha_6 - \alpha_3\alpha_5)\alpha_1' \\ + (\alpha_3\alpha_4 - \alpha_1\alpha_6)\alpha_2']]/[\tau(\alpha_2\alpha_4 - \alpha_1\alpha_5)^3]. \end{aligned} \quad (16)$$

Substituting Eqs (15) and (16) into Eq. (3), the new lump solutions of Eq. (1) can be derived as follows:

$$\begin{aligned} u = \frac{12}{\tau}e^{-\int l(t)dt}[[2\alpha_1^2 + 2\alpha_4(t)^2]/[\tau(\alpha_2\alpha_4 - \alpha_1\alpha_5) \\ + (\alpha_3 + x\alpha_1 + y\alpha_2)^2 + (\alpha_6 + x\alpha_4 + y\alpha_5)^2] \\ - [[2\alpha_1(\alpha_3 + x\alpha_1 + y\alpha_2) + 2\alpha_4(\alpha_6 + x\alpha_4 + y\alpha_5)]^2] \\ /[[\tau(\alpha_2\alpha_4 - \alpha_1\alpha_5) + (\alpha_3 + x\alpha_1 + y\alpha_2)^2] \\ + (\alpha_6 + x\alpha_4 + y\alpha_5)^2]^2]. \end{aligned} \quad (17)$$

The dynamic properties of Eq. (17) are shown in Figures 7 and 8. Compared with the results in ref. [19], solution (17) contains more arbitrary parameters.

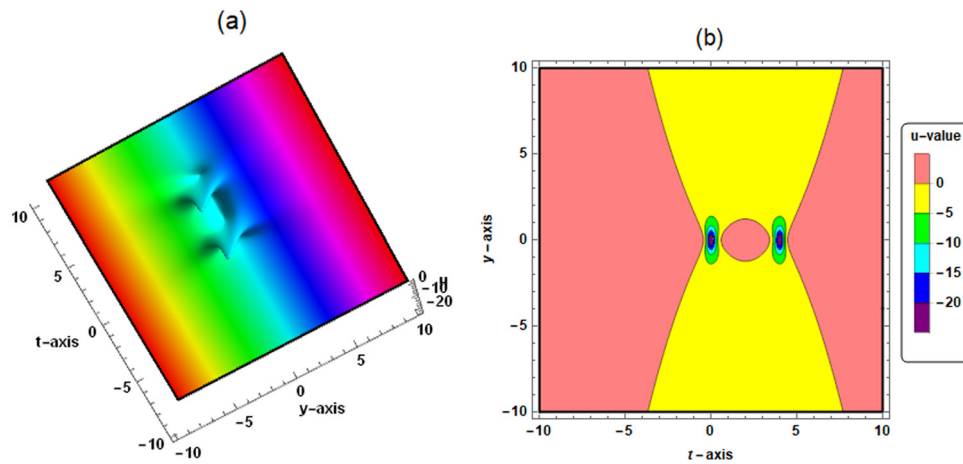


Figure 2: $l(t) = \mu = v = x = 0$, $q(t) = t$, $m(t) = \vartheta_0 = 1$, $\vartheta_1 = 2$, $\tau = -1$, (a) 3D plot, (b) contour plot.

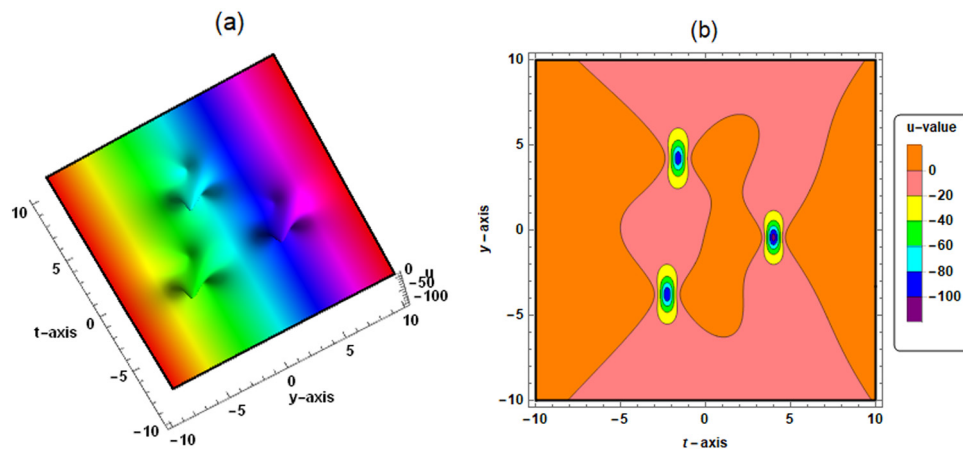


Figure 3: $l(t) = x = 0$, $\mu = v = 10$, $q(t) = m(t) = \vartheta_{11} = \vartheta_{21} = 1$, $\tau = -1$, $\vartheta_{24} = \vartheta_{10} = 2$, (a) 3D plot, (b) contour plot.

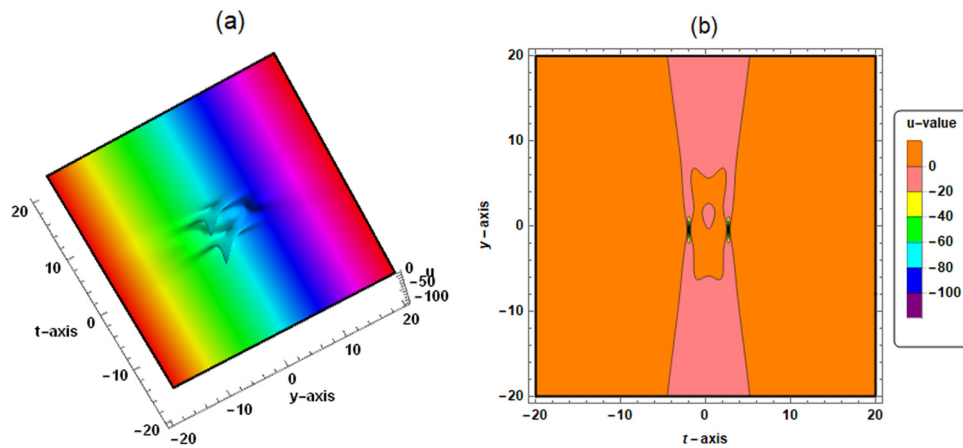


Figure 4: $l(t) = x = 0$, $\mu = v = 10$, $q(t) = t$, $m(t) = \vartheta_{11} = \vartheta_{21} = 1$, $\tau = -1$, $\vartheta_{24} = \vartheta_{10} = 2$, (a) 3D plot, (b) contour plot.

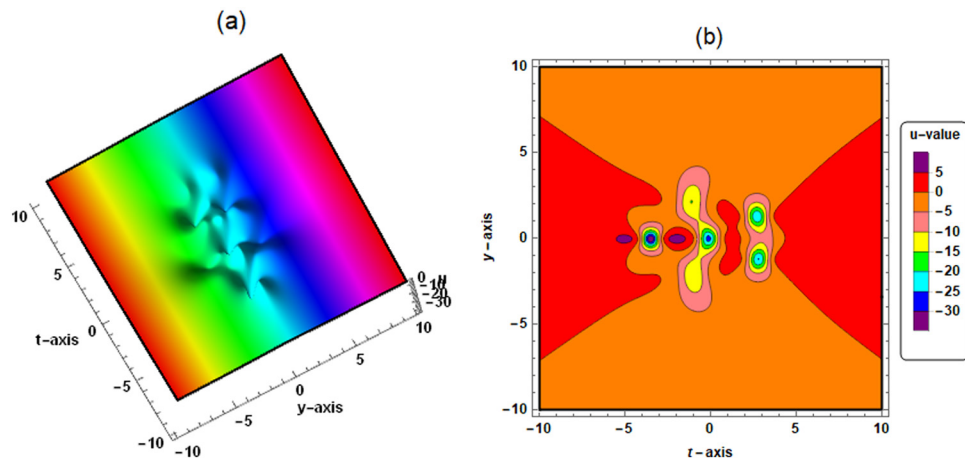


Figure 5: $l(t) = x = 0$, $\mu = \nu = 1,000$, $q(t) = m(t) = \vartheta_0 = 1$, $\tau = -1$, $\vartheta_1 = 2$, (a) 3D plot, (b) contour plot.

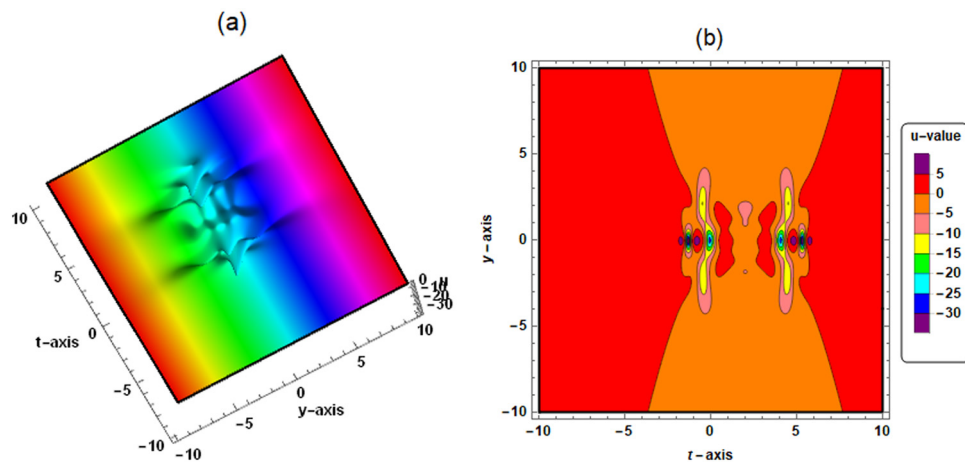


Figure 6: $q(t) = t$, $l(t) = x = 0$, $\mu = \nu = 1,000$, $q(t) = m(t) = \vartheta_0 = 1$, $\tau = -1$, $\vartheta_1 = 2$, (a) 3D plot, (b) contour plot.

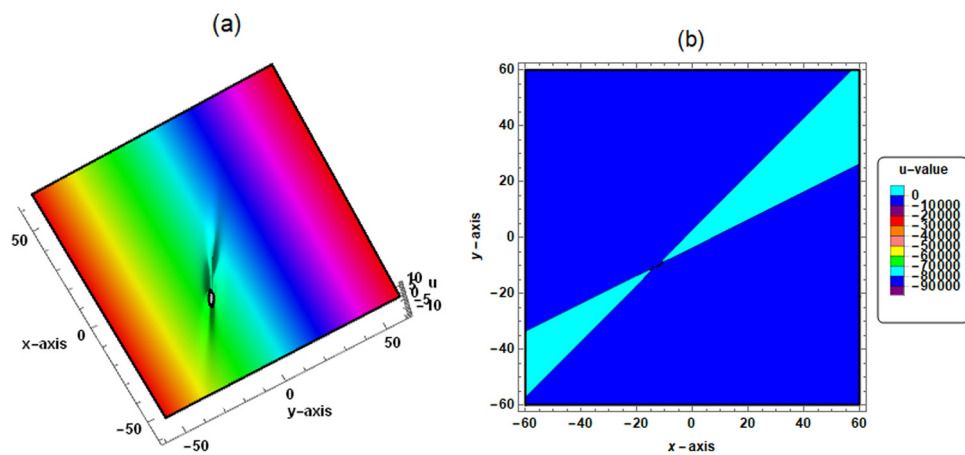


Figure 7: $l(t) = 0$, $\alpha_1(t) = t$, $\alpha_2(t) = -t$, $\tau = 1$, $t = 0.1$, $\alpha_4(t) = -t$, $\alpha_5(t) = 2t$, $q(t) = n(t) = q(t) = 1$, (a) 3D plot, (b) contour plot.

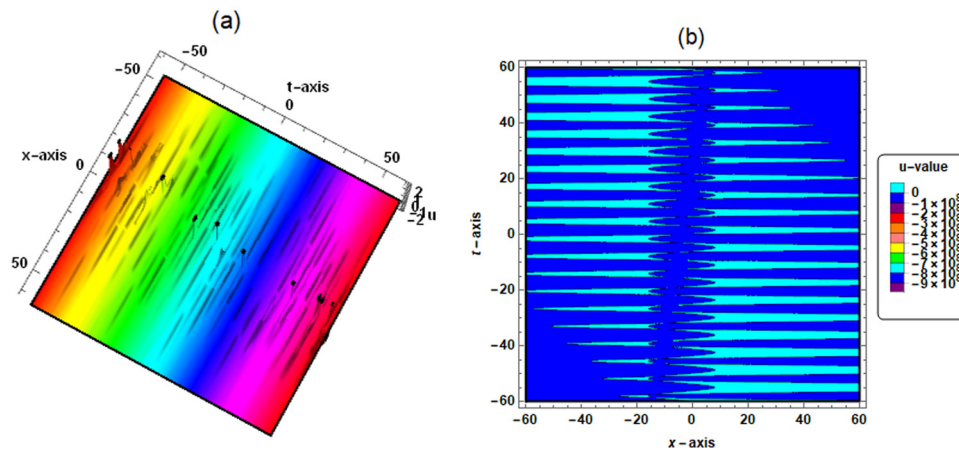


Figure 8: $l(t) = 0$, $\alpha_1(t) = t$, $\alpha_2(t) = -t$, $\alpha_4(t) = -t$, $\alpha_5(t) = 2t$, $q(t) = \cos t$, $q(t) = n(t) = 1$, $\tau = 1$, $y = 0.1$, (a) 3D plot, (b) contour plot.

4 Conclusion

In this article, we investigate a generalized vcKPe with self-consistent sources, which describes the evolution of small amplitude ion acoustic waves propagating in plasma under transverse disturbance. The multiple rogue wave solutions are presented based on symbolic computation [21–43] and Hirota bilinear form, which contain 1-, 3-, and 6-order rogue waves. The dynamic properties of 1-order rogue wave are shown in Figures 1 and 2. We can see that the amplitudes and velocities of the 1-order rogue wave are influenced by some variable coefficients. The 3-order rogue wave is shown in Figures 3 and 4. Three rogue waves can be seen in Figure 3. The influence of the variable coefficients on the 3-order order rogue wave is shown in Figure 4. The 6-order rogue wave is shown in Figures 5 and 6. Six rogue waves can be seen in Figure 5. The influence of the variable coefficients on the 6-order order rogue wave is shown in Figure 6. Using a more complex assumption, we obtain the new lump solutions of Eq. (1), which contain more arbitrary parameters than the results in ref. [19]. Lump solutions (17) under constraints (16) are demonstrated in Figures 7 and 8. Figure 7 illustrates the propagation of a single lump wave. Figure 7 demonstrates the propagation of a periodic lump wave. From the obtained results, the method used in this article can be effectively used to solve the high-order rogue wave solutions of nonlinear integrable systems with variable coefficients.

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Author contributions: The author has accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The author states no conflict of interest.

Data availability statement: Data sharing is not applicable to this article as no datasets were generated or analyzed during this study.

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