

Research Article

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Optimal block replacement policy for two-dimensional products considering imperfect maintenance with improved Salp swarm algorithm

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Abstract: Human society is entering Industry 4.0. Engineering systems are becoming more complex, which increases the difficulties in maintenance support work. Maintenance plays a very important role in the entire life cycle of a system, and in reality, maintenance is not always perfect, and its maintenance degree is between good as new and bad as old, which should be considered in the maintenance strategy. Under the framework of two-dimensional warranty, this work proposes an optimal two-dimensional block replacement strategy based on the minimum expected warranty cost. Two-dimensional block replacement maintenance is imperfect maintenance. The failure rate reduction method is used to describe the maintenance effect of two-dimensional block replacement. In case analysis, the grid search algorithm, genetic algorithm, particle swarm optimization algorithm, and improved salp swarm algorithm (SSA) are used to find the optimal warranty scheme for the laser module. The improved SSA can converge faster and find a warranty scheme that makes the warranty cost lower. Therefore, managers can use these results to reduce costs and get a win-win extended warranty cost. Rationalization suggestions are put forward for managers to make maintenance decisions through comparative analysis and sensitivity analysis.

Keywords: imperfect maintenance, two-dimensional warranty, block replacement, salp swarm algorithm

Nomenclature

C_p	imperfect preventive maintenance cost
C_m	corrective maintenance cost.
C_{Iy}	component life cycle cost borne by users with extended warranty service
C_{In}	component life cycle cost borne by users without extended warranty service
$G(r)$	probability distribution function of utilization rate
M_{Iy}	component life cycle cost borne by manufacturer with extended warranty service
M_{In}	component life cycle cost borne by manufacturer without extended warranty service
r_u	upper limit of utilization rate
r_l	lower limit of utilization rate
(T_0, U_0)	two-dimensional block replacement interval
$[W_B, U_B]$	two-dimensional basic warranty period
$[W_E, U_E]$	two-dimensional extended warranty period
$[W_L, U_L]$	time limit and usage limit of the product life
ω	repair degree
$\lambda(t r)$	failure rate function of component

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1 Introduction

The two-dimensional warranty service strategy generally takes the use time (T) and use degree (U , such as driving mileage, revolutions, etc.) as the constraint boundary for the termination of the warranty period [1]. (Two-dimensional warranty service is adopted for products whose degradation changes with use time and use degree,

which can not only take the use of products into account in the decision-making process of warranty service, but also take into account the interests of both the users and manufacturers. With the progress of industrial production and manufacturing technology, the technical complexity of large products (such as automobiles, ships, and aircraft) is gradually increasing, and the types and quantity of products that degraded with the change in use time and use degree are gradually increasing. The demand for two-dimensional warranty service strategy for products is becoming increasingly prominent [2]. Therefore, in order to meet the modeling requirements of two-dimensional warranty service decision-making and provide corresponding data support for the formulation of warranty service terms, this work mainly studies the modeling of products' two-dimensional warranty service decision-making. Since the two-dimensional warranty strategy is generally used for products whose degradation laws vary with use time and use degree, the scope of the warranty period is also limited by both the use time and use degree [3]. For product warranties, it is in the manufacturer's interest to use a two-dimensional warranty service strategy for products because it limits the high usage of the product, *i.e.*, the higher the usage, the earlier the warranty period ends.

The block replacement policy refers to the replacement of components in batches at a given time kT ($k = 1, 2, \dots$). Even if some components are replaced between two replacement intervals, they must be replaced together when the replacement interval T is reached [4]. The advantage of the block replacement policy is that it is easy for the equipment management department to formulate maintenance plans and implement maintenance activities. This strategy is applicable to electronic components and rubber parts which are relatively low in price and a large number of them are used.

1.1 Motivation

With the increasing technical complexity of products, the application of two-dimensional warranty strategy is becoming more and more popular [5]. For example, the basic warranty period of an engineering project emergency vehicle includes two restrictions: calendar time and motorcycle hours. During the two-dimensional warranty period, if preventive maintenance is carried out for the product, the preventive maintenance interval shall also be two-dimensional [6]. At present, it mainly relies on expert experience to determine the two-dimensional preventive maintenance interval for products, which is lack of scientific decision-making basis [7]. The traditional

one-dimensional maintenance interval determination method cannot meet the needs of two-dimensional warranty service decision-making, so it is necessary to carry out modeling research on the two-dimensional warranty service with two-dimensional preventive maintenance [8].

In the existing literature, the block replacement policy is usually assumed to be a perfect maintenance [9]. This assumption makes the establishment and solution of the model more convenient. However, in fact, the block replacement policy often only replaces individual components in the system. The replacement is difficult to restore the system as new, but is often between "good as new" and "bad as old," that is, imperfect maintenance [10]. Therefore, it is more practical to regard block replacement policy as imperfect maintenance.

At present, with the progress of manufacturing technology, the reliability of products is higher and higher, and the probability of failure within the basic warranty period is lower. Therefore, users are more and more inclined to purchase extended warranty services. The extended warranty service not only meets the user's after-sales service requirements for the products after the basic warranty period, but also provides a new profit source for the manufacturer. This work explores the modeling method of effective integration of basic warranty and extended warranty, and strives to obtain a warranty scheme that satisfies both the manufacturer and the user.

1.2 Contribution

This work studies the two-dimensional non-renewal basic warranty cost model and extended warranty cost model when two-dimensional block replacement maintenance is imperfect maintenance. The failure rate reduction method is used to describe the maintenance effect of two-dimensional block replacement, and the optimal two-dimensional block replacement interval is obtained by optimizing the warranty service cost model. This study proposes a discrete utilization rate based salp swarm algorithm (SSA) to solve the problem. This study also introduces the extended warranty mechanism and calculates the extended warranty cost scheme acceptable to manufacturers and users.

The remainder of this article is organized into six sections. Section 2 provides a literature review of related studies. Section 3 presents the description of the model based on reasonable assumptions. Section 4 introduces the model constructing process. Section 5 introduces the improved SSA for solving the model. Section 6 presents a real case scenario to illustrate the applicability of our model. Section 7 presents the conclusion.

2 Related works

Warranty and maintenance are not the same concept. Maintenance is the specific performance of the implementation of warranty work, and the effect of warranty is finally reflected through the effect of maintenance [11]. The specific relationship between the two is: maintenance is the specific work of the implementation of warranty, warranty decision-making includes the selection of warranty mode and maintenance strategy, and the core work of warranty decision-making is the combination of warranty mode and maintenance strategy, so as to achieve the optimal effect of warranty work. This section mainly combs the existing research from two aspects: warranty mode and maintenance strategy.

2.1 Warranty mode

According to the dimensions contained in the warranty period, warranty can be divided into one-dimensional warranty, two-dimensional warranty, and multi-dimensional warranty. Due to the complexity of multi-dimensional warranty modeling and less application at present, there is less research on it. The research on one-dimensional warranty and two-dimensional warranty is the mainstream at present, and two-dimensional warranty is the forefront of warranty theory research. One-dimensional warranty means that the warranty period is determined based on a single variable, usually calendar time or use degree. The two-dimensional warranty policy is that the warranty period is determined by two variables, usually calendar time and use degree, as shown in Figure 1.

For one-dimensional warranty, Vahdani *et al.* [12] established the renewal warranty service model of multi-stage degraded repairable products, considering the minimum maintenance with non-negligible maintenance time and the replacement maintenance with negligible maintenance time. Xie *et al.* [13] established an overall profit evaluation model considering the units within and outside the warranty period. Aggrawal *et al.* [14] established a warranty service price model considering the product sales cycle, in which the unit failure obeys the exponential distribution. González-Prida *et al.* [15] used generalized renewal process and inhomogeneous Poisson process to study the optimization decision-making problem of warranty service period. Zhu and Xiang [16] adopted the condition based maintenance strategy. For the multi-component system with two stages, they used the multi-stage random integer model to select the components to be maintained within the limited maintenance time, so as to minimize the total maintenance cost and ensure the reliability of the system. Considering the

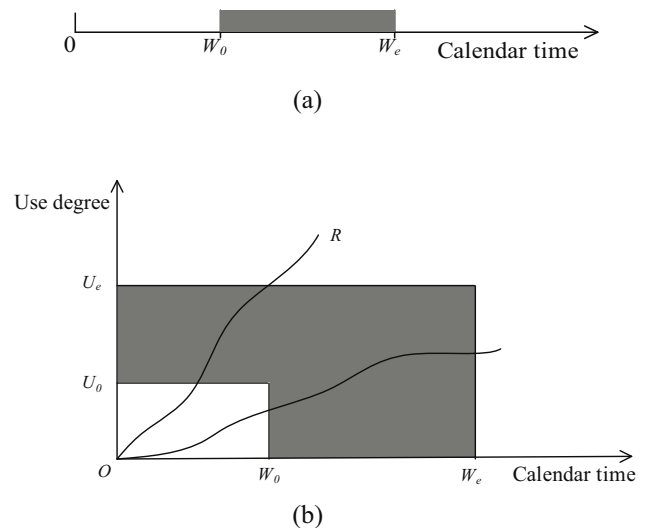


Figure 1: Warranty diagram. (a) One-dimensional warranty and (b) two-dimensional warranty.

dependence between components, Safaei *et al.* [17] proposed the optimal age replacement strategy for parallel systems and series systems.

In reality, the two-dimensional warranty strategy is most widely used in automobile products warranty. In the process of two-dimensional warranty service decision modeling, the first step is to determine the two-dimensional warranty period range [18]. Figure 2 lists several two-dimensional warranty service period ranges, of which the rectangular form is a common case. According to the characteristics of product warranty service and the interests of both users and manufacturers, the rectangular warranty period is suitable for most products. The scope of the two-dimensional warranty service period considered in this study is in the rectangular form.

For two-dimensional warranty, by optimizing the two-dimensional warranty service cost model, Banerjee and Bhattacharjee [19] studied the decision-making problem of minimum maintenance or replacement for the first failure in the two-dimensional warranty period. Huang *et al.* [20] established the Bayesian decision-making model of periodic preventive maintenance of units whose degradation process obeys inhomogeneous Poisson process under the proportional cost sharing warranty strategy, and established the optimization model of periodic preventive maintenance in two-dimensional warranty service of repairable units by using binary joint distribution. Taleizadeh and Mokhtarzadeh [21] used the value risk method to formulate pricing scheme and two-dimensional warranty scheme for products sold online and offline. Lin and Chen [22] analyzed the two-dimensional warranty claim data. Breaking the assumption that the utilization

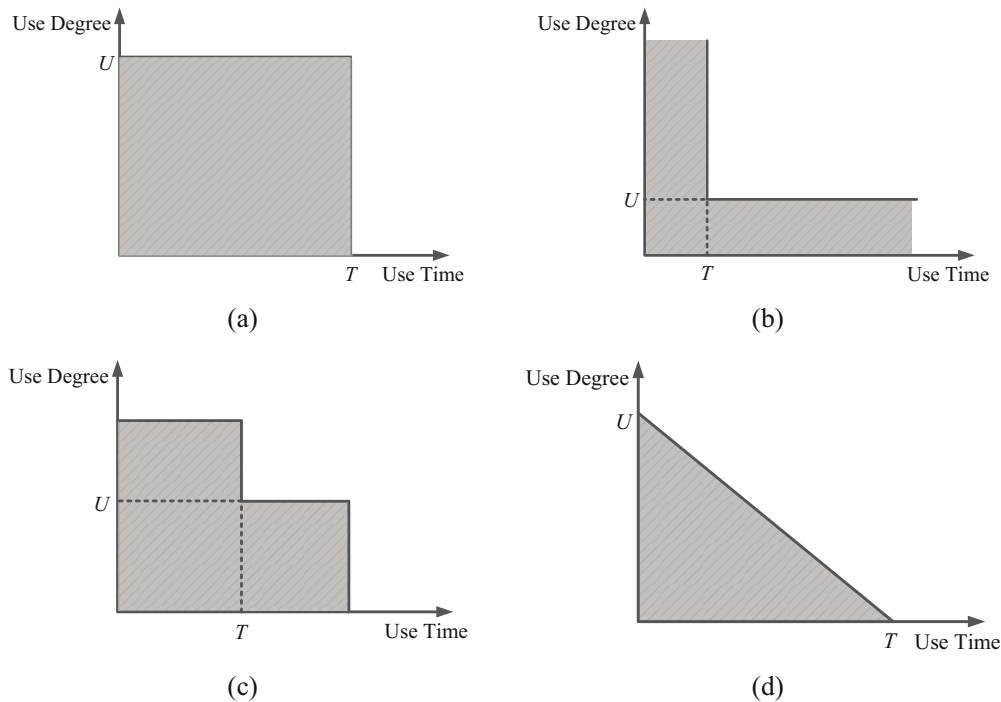


Figure 2: Two-dimensional warranty service period. (a) Rectangular, (b) L-shaped, (c) stepped type and (d) triangular.

rate is a linear function of age, they mined a more accurate failure law by analyzing the time and mileage data at the time of failure. Song [23] proposed a two-dimensional preventive maintenance and replacement strategy. Under this strategy, preventive maintenance actions are arranged according to age or use degree. Each impact before the n -th impact will lead to product failure or increase in product failure rate. If the product has withstood $(n - 1)$ th shock, replace it with a new product at the n th shock. From the perspective of the manufacturer, the average warranty cost in the whole warranty period is obtained by using the renewal theory. According to the literature review, one-dimensional warranty and two-dimensional warranty mostly concentrate in the basic warranty stage, and there is less research on extended warranty.

2.2 Maintenance strategy

The maintenance strategy adopted during the warranty period will have a great impact on the warranty service cost and products' performance during the warranty period. Maintenance strategies can be divided into two categories: corrective maintenance strategy and preventive maintenance strategy. Common preventive maintenance strategies include functional check strategy and replacement strategy.

In engineering practice, the failure mode of many products will show some signs in the process of functional degradation to indicate that the failure is about to occur or is occurring. If this sign is found through functional check, preventive measures can be taken in time to avoid the functional failure of the product [24,25]. Delay time is generally used to describe such degradation process [26], and its basic idea is to divide the formation of products failure into two stages: the formation stage of potential failure and the formation stage of functional failure [27]. The duration of these two stages is called initial defect time u and failure delay time h [28]. When the products undergo functional check at the interval of cycle T , t_i represents the i -th check point ($i = 1, 2, 3, \dots$). There are two situations of product maintenance, as shown in Figure 3. Figure 3(a) shows the failure maintenance after the function failure occurs between the two function check, and Figure 3(b) shows the potential failure maintenance after the potential failure is detected at the function check point.

It is assumed that the probability density function of potential initial defect time u is $g(u)$ and the cumulative distribution function is $G(u)$. The probability density function of failure delay time h is $f(h)$ and the distribution function is $F(h)$. Then, the specific value of failure renewal probability before time t_i is $g(u)duF(t_i - u)$, and the specific value of check renewal probability at time t_i

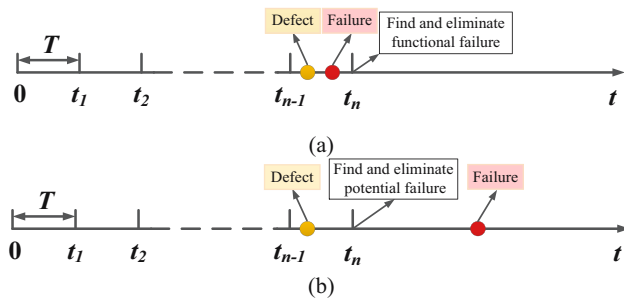


Figure 3: Two situations in function check. (a) Function failure is found and (b) potential failure is found.

is $g(u)du[1 - F(t_i - u)]$. Then, the probability $P_f(t_i - 1, t_i)$ of failure maintenance during $(t_i - 1, t_i)$ is

$$P_f(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} g(u)F(t_i - u)du.$$

The probability $P_m(t_{i-1}, t_i)$ of potential failure maintenance at time t_i is:

$$P_m(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} g(u)[1 - F(t_i - u)]du.$$

Periodic replacement strategy mainly includes block replacement strategy and age replacement strategy. At present, the commonly used periodic replacement strategy is one-dimensional periodic replacement strategy. However, with the increasing technical complexity and advanced performance of products, the failure law of many products is affected by many factors (products running time, service time, driving mileage, etc.). When repairing such products, if only one-dimensional periodic replacement interval about time is given, the effect of other influencing factors on failure law will be ignored, resulting in untimely maintenance.

The two-dimensional age replacement strategy means that if the product does not fail during use, it will be replaced regularly according to the specified two-dimensional age (calendar time and use degree). If a failure occurs within the specified time (two-dimensional age), the failure product shall be repaired, and the age shall be counted again after the repair. Compared with the block replacement strategy, the age replacement strategy has certain flexibility. The two-dimensional age replacement process is shown in Figure 4. For more research on two-dimensional age replacement, please refer to refs [29,30].

The two-dimensional block replacement strategy is to replace the product regularly according to the time and use degree after the product is put into use. The two-dimensional block replacement interval can be expressed

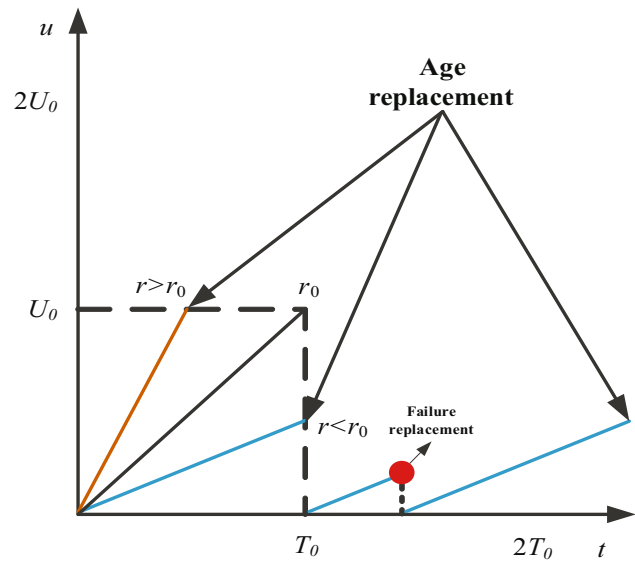


Figure 4: Two-dimensional age replacement strategy.

as (T_0, U_0) , in which the time interval is T_0 and the use degree interval is U_0 . If any one of the time or use degree of the product reaches the threshold of a given interval, the product shall be replaced. Even if the product is replaced due to functional failure between two regular replacements, it needs to be replaced at the scheduled regular replacement time. If the univariate method [31] is selected to describe the two-dimensional failure law of the product, the two-dimensional block replacement process is shown in Figure 5. Ke and Yao [9] considered three different decision criteria and studied the block replacement policy under uncertain environment on the premise that the component life is a random variable. Schouten *et al.* [32] studied the optimal block replacement policy of wind turbine system components under the condition of time-varying cost. Zhang *et al.* [33] considered the duration of the task and gave the optimal plan for block replacement from the perspective of cost and maintainability. Azevedo *et al.* [34] assumed that the product adopts a corrective replacement strategy and an imperfect maintenance strategy when critical failures and non-critical failures occur, respectively, and used a multi-objective genetic algorithm (GA) to obtain a block replacement plan for products. For more research on two-dimensional block replacement, please refer to ref. [35].

As shown in Figure 5, the shape parameter r_0 of the two-dimensional block replacement area is U_0/T_0 , when $r > r_0$, U_0 is the interval of block replacement, and when $r < r_0$, T_0 is the interval of block replacement. However, periodic replacement strategy mainly concentrates in the basic warranty stage, and lacks extended warranty research.

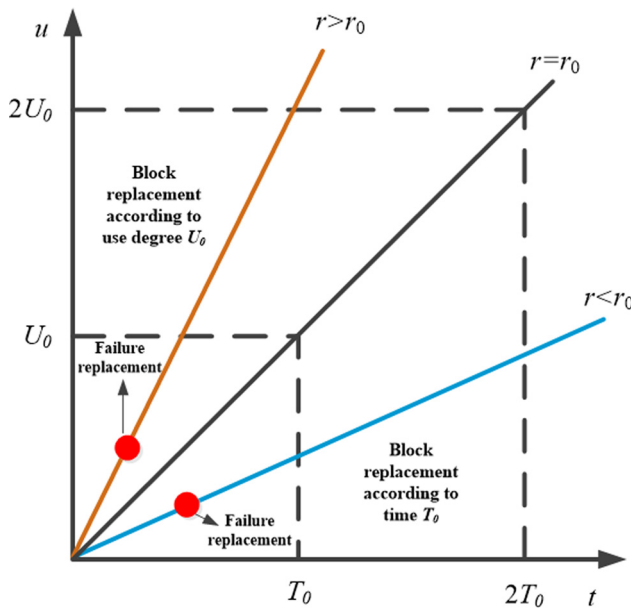


Figure 5: Two-dimensional periodic replacement strategy.

3 Model assumptions

The time limit and usage limit of the two-dimensional basic warranty period are W_B and U_B , and the time limit and usage limit of the two-dimensional extended warranty period are W_E and U_E . Let $\Omega_1 = [0, W_B] \times [0, U_B]$, $\Omega_2 = [0, W_E] \times [0, U_E]$. There is a linear relationship between time and usage. The relationship between products usage u and time t is $u = rt$, r is a random variable and is different for different users, $g(r)$ and $G(r)$ are the probability density function and probability distribution function of r , respectively. The product life is two-dimensional, and W_L and U_L are the time limit and usage limit of the product life, respectively.

Assume that the two-dimensional block replacement maintenance is imperfect maintenance, and the interval

Table 1: Number of preventive maintenances over each interval in product life

Number	Stage	Value
n_1	$[0, W_B)$	$\lfloor W_B/T_0 \rfloor$
n_2	$[0, W_B)$	$\lfloor W_B r/T_0 \rfloor$
n_3	$[0, U_B)$	$\lfloor U_B/U_0 \rfloor$
n_4	$[0, U_B)$	$\lfloor U_B/T_0 r \rfloor$

is (T_0, U_0) . T_0 is the time interval for two-dimensional block replacement, U_0 is the usage interval for two-dimensional block replacement, and imperfect preventive maintenance is performed regardless of which limit comes first. ω is the repair degree of imperfect preventive maintenance, when $\omega = 1$, imperfect preventive maintenance becomes perfect maintenance, and when $\omega = 0$, imperfect preventive maintenance becomes minimum maintenance. Maintenance time can be ignored due as it is small relative to preventive maintenance intervals and products life.

C_p is the cost of imperfect preventive maintenance, and C_m is the cost of corrective maintenance. Let n_k ($k = 1, 2, 3, \dots$) be the number of imperfect preventive maintenance at different stages of the product life cycle, and the specific values are shown in Table 1. The cost of basic warranty shall be borne by the manufacturer, and the cost of extended warranty service shall be borne by the user. Under different usage rates r , there are two implementation situations for two-dimensional imperfect preventive maintenance, as shown in Figure 6.

Case 1. When $r \leq r_0$, the implementation time of imperfect preventive maintenance is $T_j = jT_0$ ($j = 1, 2, 3, \dots$), as shown in Figure 6(a).

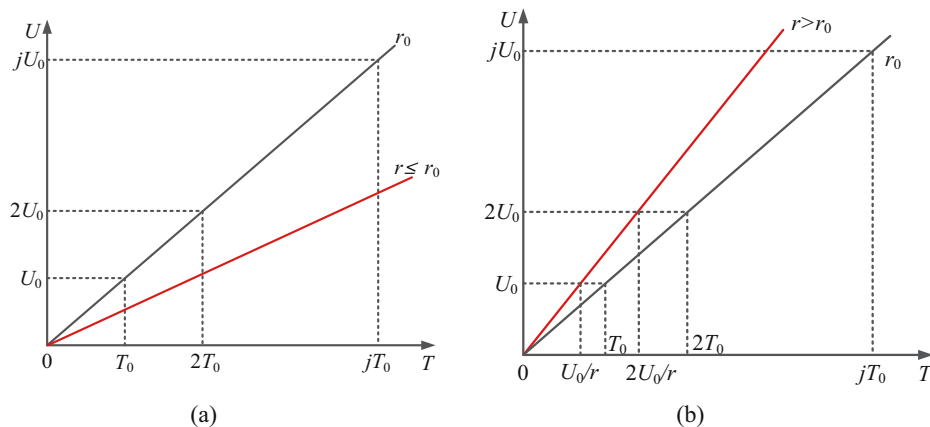


Figure 6: Two-dimensional imperfect preventive maintenance. (a) $r \leq r_0$ and (b) $r > r_0$.

Case 2. When $r > r_0$, the implementation time of imperfect preventive maintenance is $T_j = jU_0/r$ ($j = 1, 2, 3, \dots$), as shown in Figure 6(b).

4 Model construction

4.1 Basic warranty cost model

The failure rate function of the component is:

$$\lambda(t|r) = \theta_0 + \theta_1 r + \theta_2 t^2 + \theta_3 r t^2. \quad (1)$$

Failure rate fallback method is adopted to describe the effect of imperfect preventive maintenance [175]. That is, after an imperfect preventive maintenance, the failure rate of components becomes

$$\lambda_{t^+} = (1 - \omega)\lambda_{t^-}, \quad (2)$$

where λ_{t^-} is the component failure rate before the imperfect preventive maintenance is performed and λ_{t^+} is the component failure rate after the imperfect preventive maintenance is performed. Then, there are two kinds of failure rates of components between the n -th preventive maintenance and the $(n + 1)$ -th preventive maintenance.

Case 1. When $r \leq r_0$, the implementation time of imperfect preventive maintenance is $T_j = jT_0$, and the failure rate of components is

$$\lambda_{(nT_0)^+} = \lambda(t|r) - \omega \sum_{j=0}^{n-1} (1 - \omega)^j \lambda((n - j)T_0|r) \quad (3)$$

$$(j = 1, 2, 3, \dots).$$

Case 2. When $r > r_0$, the implementation time of imperfect preventive maintenance is $U_j = jU_0(T_j = jU_0/r)$, and the failure rate of components is

$$\lambda_{(nT_0)^+} = \lambda(t|r) - \omega \sum_{j=0}^{n-1} (1 - \omega)^j \lambda((n - j)U_0/r|r) \quad (4)$$

$$(j = 1, 2, 3, \dots).$$

Proof. Taking case 1 as an example, the above conclusion is proved by induction. During the first preventive maintenance interval $[0, T_0]$, the failure rate of components is $\lambda(t|r)$, which conforms to the formula of case 1. Assuming that the formula of case 1 is correct when the number of imperfect preventive maintenance is less than or equal to n , that is, after the n -th preventive maintenance, the failure rate of components is

$$\lambda_{(nT_0)^+} = \lambda(t|r) - \omega \sum_{j=0}^{n-1} (1 - \omega)^j \lambda((n - j)T_0|r). \quad (5)$$

Then, the failure rate of components before $(n + 1)$ -th preventive maintenance is

$$\lambda_{[(n+1)T_0]^-} = \lambda((n + 1)T_0|r) - \omega \sum_{j=0}^{n-1} (1 - \omega)^j \lambda((n - j)T_0|r). \quad (6)$$

According to formula 2, the failure rate of components after $(n + 1)$ -th preventive maintenance is

$$\begin{aligned} \lambda_{[(n+1)T_0]^+} &= (1 - \omega)\lambda_{[(n+1)T_0]^-} Q \\ &= \lambda((n + 1)T_0|r) - \left[\omega \lambda((n + 1)T_0|r) + \omega \sum_{j=0}^{n-1} (1 - \omega)^{j+1} \lambda((n - j)T_0|r) \right] \\ &= \lambda((n + 1)T_0|r) - \omega \sum_{j=0}^n (1 - \omega)^j \lambda((n + 1 - j)T_0|r). \end{aligned} \quad (7)$$

The formula proving the method of case 2 is similar to that of case 1.

First, the basic warranty cost model is established. According to the quantity relationship between r_0 and r_B , there are two cases in the basic warranty: $r_0 \leq r_B$ and $r_0 > r_B$. When $r_0 \leq r_B$, according to different r , there are three cases: $0 < r \leq r_0$, $r_0 < r \leq r_B$, and $r > r_B$. When $0 < r \leq r_0$, the basic warranty cost is

$$E_1(C) = n_1 C_p + C_m \left[\sum_{i=0}^{n_1-1} \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1 - \omega)^j \lambda((i - j)T_0|r) \right] dt \right] + \int_{n_1 T_0}^{W_B} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1 - \omega)^j \lambda((n_1 - j)T_0|r) \right] dt \quad (8)$$

When $r_0 < r \leq r_B$, the basic warranty cost is

$$E_2(C) = n_2 C_p + C_m \left[\sum_{i=0}^{n_2-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1 - \omega)^j \lambda((i - j)U_0/r|r) \right] dt \right] + \int_{n_2 U_0/r}^{W_B} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1 - \omega)^j \lambda((n_2 - j)U_0/r|r) \right] dt \quad (9)$$

When $r > r_B$, the basic warranty cost is

$$E_3(C) = n_3 C_p + C_m \left[\sum_{i=0}^{n_3-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1 - \omega)^j \lambda((i - j)U_0/r|r) \right] dt \right] + \int_{n_3 U_0/r}^{U_B/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1 - \omega)^j \lambda((n_3 - j)U_0/r|r) \right] dt \quad (10)$$

So, when $r_0 \leq r_B$, the basic warranty cost of the components is

$$E(C_{B1}) = \int_0^{r_0} E_1(C)g(r)dr + \int_{r_0}^{r_B} E_2(C)g(r)dr + \int_{r_B}^{\infty} E_3(C)g(r)dr. \quad (11)$$

When $r_0 > r_B$, according to different r , there are three cases: $0 < r \leq r_B$, $r_B < r \leq r_0$, and $r > r_0$. When $0 < r \leq r_B$, the basic warranty cost is

$$E_4(C) = n_1 C_p + C_m \left[\sum_{i=0}^{n_1-1} \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt \right. \\ \left. + \int_{n_1 T_0}^{w_B} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt \right]. \quad (12)$$

When $r_B < r \leq r_0$, the basic warranty cost is

$$E_5(C) = n_4 C_p + C_m \left[\sum_{i=0}^{n_4-1} \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt \right. \\ \left. + \int_{n_4 T_0}^{U_B/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0|r) \right] dt \right]. \quad (13)$$

When $r > r_0$, the basic warranty cost is

$$E_6(C) = n_3 C_p \\ + C_m \left[\sum_{i=0}^{n_3-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)U_0/r|r) \right] dt \right. \\ \left. + \int_{n_3 U_0/r}^{U_B/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt \right]. \quad (14)$$

So, when $r_0 > r_B$, the basic warranty cost of the components is

$$E(C_{B2}) = \int_{r_L}^{r_B} E_4(C)g(r)dr + \int_{r_B}^{r_0} E_5(C)g(r) \\ dr + \int_{r_0}^{r_u} E_6(C)g(r)dr. \quad (15)$$

To sum up, the basic warranty cost under different conditions is

$$E(C_B) = \begin{cases} \int_{r_L}^{r_0} E_1(C)g(r)dr + \int_{r_0}^{r_B} E_2(C)g(r)dr + \int_{r_B}^{r_u} E_3(C)g(r)dr & r_0 \leq r_B \\ \int_{r_L}^{r_B} E_4(C)g(r)dr + \int_{r_B}^{r_0} E_5(C)g(r)dr + \int_{r_0}^{r_u} E_6(C)g(r)dr & r_0 > r_B. \end{cases} \quad (16)$$

4.2 Extended warranty cost scheme

Next from the perspective of the manufacturer and the user, the component life cycle cost borne by all parties is established to determine the win-win extended warranty cost. It is considered to carry out two-dimensional imperfect preventive maintenance only in the basic warranty period, and only corrective maintenance in other life stages. From the perspective of users, assuming that C_{ly} (C_{ln}) is the component life cycle cost borne by users when

they choose extended warranty service (do not choose extended warranty service), there is an upper limit on the cost $E(C_E)$ of extended warranty service to satisfy users, that is

$$C_{ly} + E(C_E) \leq C_{ln}. \quad (17)$$

In calculating C_{ln} , there are two cases: $r_L > r_B$ and $r_L \leq r_B$. For case $r_L > r_B$, when $r_0 \geq r_L > r_B$, if the user will not choose extended warranty service, the full life cycle cost borne by the user is

$$C_{ln} = C_m \left[\int_{r_L}^{r_u} \int_{U_B/r}^{U_L/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt g(r)dr \right. \\ + \int_{r_L}^{r_0} \int_{U_B/r}^{U_L/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0|r) \right] dt g(r)dr \\ + \int_{r_B}^{r_L} \int_{U_B/r}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0|r) \right] dt g(r)dr \\ \left. + \int_{r_L}^{r_B} \int_{w_B}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt g(r)dr \right]. \quad (18)$$

When $r_L > r_0 \geq r_B$, if the user will not choose extended warranty service, the full life cycle cost borne by the user is

$$C_{ln} = C_m \left[\int_{r_L}^{r_u} \int_{U_B/r}^{U_L/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt g(r)dr \right. \\ + \int_{r_0}^{r_L} \int_{U_B/r}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt g(r)dr \\ + \int_{r_B}^{r_0} \int_{U_B/r}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0|r) \right] dt g(r)dr \\ \left. + \int_{r_L}^{r_B} \int_{w_B}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt g(r)dr \right]. \quad (19)$$

When $r_L > r_B > r_0$, if the user will not choose extended warranty service, the full life cycle cost borne by the user is

$$C_{ln} = C_m \left[\int_{r_L}^{r_u} \int_{U_B/r}^{U_L/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt g(r)dr \right. \\ + \int_{r_B}^{r_L} \int_{U_B/r}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt g(r)dr \\ + \int_{r_0}^{r_B} \int_{w_B}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right] dt g(r)dr \\ \left. + \int_{r_L}^{r_0} \int_{w_B}^{T_L} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt g(r)dr \right]. \quad (20)$$

For case $r_L \leq r_B$

$$C_{In} = \begin{cases} C_m \left[\int_{r_0}^{r_u} \int_{U_B/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_B}^{r_0} \int_{U_B/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0/r) dt \right) g(r) dr \\ + \int_{r_L}^{r_B} \int_{W_B}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_L} \int_{W_B}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_L \leq r_B < r \\ \\ C_m \left[\int_{r_B}^{r_u} \int_{U_B/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_0}^{r_B} \int_{W_B}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right) dt \right] g(r) dr \\ + \int_{r_L}^{r_0} \int_{W_B}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_L} \int_{W_B}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_L \leq r_0 \leq r_B \\ \\ C_m \left[\int_{r_B}^{r_u} \int_{U_B/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_L}^{r_B} \int_{W_B}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right) dt \right] g(r) dr \\ + \int_{r_0}^{r_L} \int_{W_B}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_0} \int_{W_B}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_0 < r_L \leq r_B. \end{cases} \quad (21)$$

When calculating C_{Iy} , there are six cases, including $r_L \leq r_E \leq r_B$, $r_E < r_L < r_B$, $r_E < r_B < r_L$, $r_B < r_E < r_L$, $r_B < r_L < r_E$, and $r_L < r_B < r_E$. In order to reduce the number of discussions, let $r_E = r_B$. When $r_L \leq r_E = r_B$,

$$C_{Iy} = \begin{cases} C_m \left[\int_{r_0}^{r_u} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_B}^{r_0} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0/r) dt \right) g(r) dr \\ + \int_{r_L}^{r_B} \int_{W_E}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_L} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_L \leq r_B < r_0 \\ \\ C_m \left[\int_{r_B}^{r_u} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_0}^{r_B} \int_{W_E}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right) dt \right] g(r) dr \\ + \int_{r_L}^{r_0} \int_{W_E}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_L} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_L \leq r_0 \leq r_B \\ \\ C_m \left[\int_{r_B}^{r_u} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_L}^{r_B} \int_{W_E}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right) dt \right] g(r) dr \\ + \int_{r_0}^{r_L} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_0} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_0 < r_L \leq r_B. \end{cases} \quad (22)$$

When $r_B = r_E < r_L$,

$$C_{Iy} = \begin{cases} C_m \left[\int_{r_0}^{r_u} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_L}^{r_0} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0/r) dt \right) g(r) dr \\ + \int_{r_E}^{r_L} \int_{U_E/r}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_B} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_0 \geq r_L > r_B \\ \\ C_m \left[\int_{r_L}^{r_u} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_0}^{r_L} \int_{U_E/r}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr \\ + \int_{r_B}^{r_0} \int_{U_E/r}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0/r) dt \right) g(r) dr + \int_{r_l}^{r_B} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_L > r_0 \geq r_B \\ \\ C_m \left[\int_{r_L}^{r_u} \int_{U_E/r}^{U_L/r} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_B}^{r_L} \int_{U_E/r}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right) dt \right] g(r) dr \\ + \int_{r_0}^{r_B} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right) dt \right] g(r) dr + \int_{r_l}^{r_0} \int_{W_E}^{T_L} \left(\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0/r) dt \right) g(r) dr \right] & r_L > r_B > r_0. \end{cases} \quad (23)$$

From the perspective of the manufacturer, when the manufacturer provides extended warranty service, the life cycle cost of components borne by the manufacturer is M_{Iy} . when the manufacturer does not provide extended warranty service, the life cycle cost of the components borne by the manufacturer is M_{In} . There should be a lower limit for the cost of extended warranty service to the satisfaction of the manufacturer, that is,

$$M_{Iy} - E(C_E) \leq M_{In}, \quad (24)$$

where M_{In} is equal to the basic warranty service cost. For M_{Iy} , assume $r_E = r_B$ as well. When $r_0 \leq r_B$

$$\begin{aligned} M_{Iy} = & \int_{r_l}^{r_0} \left[n_1 C_p + C_m \sum_{i=0}^{n_1-1} \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt \right. \\ & \left. + C_m \int_{n_1 T_0}^{W_E} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt \right] g(r) dr \\ & + \int_{r_0}^{r_B} \left[n_2 C_p + C_m \sum_{i=0}^{n_2-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)U_0/r|r) \right] dt \right. \\ & \left. + C_m \int_{n_2 U_0/r}^{W_E} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right] dt \right] g(r) dr \\ & + \int_{r_B}^{r_u} \left[n_3 C_p + C_m \sum_{i=0}^{n_3-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)U_0/r|r) \right] dt \right. \\ & \left. + C_m \int_{n_3 U_0/r}^{U_E/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt \right] g(r) dr. \end{aligned} \quad (25)$$

When $r_0 > r_B$

$$\begin{aligned} M_{Iy} = & \int_{r_l}^{r_B} \left[n_1 C_p + \sum_{i=0}^{n_1-1} C_m \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt + C_m \int_{n_1 T_0}^{W_E} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt \right] g(r) dr \\ & + \int_{r_B}^{r_0} \left[n_4 C_p + \sum_{i=0}^{n_4-1} C_m \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt + C_m \int_{n_4 T_0}^{U_E/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0|r) \right] dt \right] g(r) dr \\ & + \int_{r_0}^{r_u} \left[n_3 C_p + \sum_{i=0}^{n_3-1} C_m \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)U_0/r|r) \right] dt + C_m \int_{n_3 U_0/r}^{U_B/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt \right] g(r) dr. \end{aligned} \quad (26)$$

Let $A_C = C_{In} - C_{Iy}$ and $A_M = M_{Iy} - M_{In}$. A_C indicates that the user is willing to pay the maximum value of the extended warranty service cost, while A_M indicates that the manufacturer is willing to provide the minimum value of the extended warranty service cost. Obviously, only when $A_C \geq A_M$, the warranty service cost can reach the value satisfactory to both the user and the manufacturer. When $A_C < A_M$, there is no warranty service cost to the satisfaction of both the user and the manufacturer. According to the maintenance method in this model, $A_C = A_M$ can be obtained. At this time, the value of extended warranty service cost satisfactory to both the user and the

manufacturer is unique. When different maintenance methods are selected, there may be a value range of extended warranty service cost that is satisfactory to both the user and the manufacturer.

5 Algorithm

The purpose of this study is to minimize the basic warranty cost, get the optimal preventive maintenance plan, and make the manufacturers and users win-win extended warranty cost. The solution steps of the model are:

Step 1: By minimizing $E(C_B)$, the optimal two-dimensional block replacement interval is obtained.

Step 2: Based on $E(C_B)$, the extended warranty cost that makes the manufacturer and the user win-win is obtained by making $A_C = A_M$.

Step 3: The optimal preventive maintenance plan and the win-win extended warranty cost are got.

Therefore, solving the optimal preventive maintenance interval by minimizing $E(C_B)$ is the key to solving this model, which essentially belongs to an optimization problem with extremely complex objective function. Intelligent optimization algorithms are more and more widely used in solving various complex engineering problems. SSA is a new optimization algorithm proposed by Seyedali Mirjalili of Australia in 2017, which simulates the swarming behavior of salps when navigating and foraging in the ocean [36]. Ref. [36] shows that SSA is a new swarm intelligence

optimization algorithm superior to particle swarm optimization (PSO) algorithm and GA in optimization performance, so it has been widely used in various practical engineering optimization problems. Therefore, this study intends to solve the preventive maintenance scheme and the lowest cost within the initial warranty period by using SSA. Combined with the characteristics of the model, this study proposes a discrete utilization rate based SSA to solve this kind of warranty decision-making problem. Finally, the grid search algorithm (GSA), PSO algorithm, and GA are used to solve the model. Compared with the results obtained by the improved SSA, the advantages of

the improved SSA in solving such problems are verified. The steps of the discrete utilization rate based SSA are as follows:

Step 1: SSA program starts running. A group of (T_0, U_0) is given, and $r_0 = U_0/T_0$.

Step 2: When $r \leq \min(r_0, r_B)$, the expected warranty cost can be calculated by the method in Section 4.1. It can be simply expressed as

$$C_{\text{exp}}(1) = \int_{r_l}^{\min(r_0, r_u)} E(C|r) dG(r). \quad (27)$$

Step 3: $[\min(r_0, r_B), r_u]$ is divided into k intervals on average, and each interval can be expressed as: $[r_{a-1}, r_a]$, $a = 1, 2, \dots, k$. r_a can be expressed as

$$r_a = r_1 + a \times \frac{r_u - r_1}{k}, \quad a = 1, 2, \dots, k. \quad (28)$$

Step 4: The utilization rate of each interval is replaced by the average utilization rate of the interval. The average utilization rate of the interval can be expressed as

$$\bar{r}_a = \int_{r_{a-1}}^{r_a} \frac{sg(s)}{G(r_a) - G(r_{a-1})} ds. \quad (29)$$

Step 5: The probability that the utilization rate falls in each interval is

$$P_a = G(r_a) - G(r_{a-1}). \quad (30)$$

Step 6: The expected warranty cost of each interval is calculated. The calculation formula is

$$C_{\text{exp}a} = E(C|\bar{r}_a)P_a \quad a = 1, 2, \dots, k. \quad (31)$$

Step 7: When $r > \min(r_0, r_B)$, the expected warranty cost rate is

$$C_{\text{exp}}(2) = \sum_{a=1}^k C_{\text{exp}a} \quad a = 1, 2, \dots, k. \quad (32)$$

Step 8: Then, the total expected warranty cost of the system is

$$E(C_B) = C_{\text{exp}}(1) + C_{\text{exp}}(2). \quad (33)$$

Step 9: Step 1 is returned and the next iteration is started.

Same as other population-based optimization techniques, the locations of the salps are defined in an n -dimensional search space, where n is the number of variables for a given problem. Therefore, the positions of all salps are stored in an n -dimensional matrix. It is assumed that there is a food source in the search space as the target of the population.

The leader's position is updated by the following formula

$$x_j^1 = \begin{cases} F_j + c_1((ub_j - lb_j)c_2 + lb_j)c_3 \geq 0 \\ F_j - c_1((ub_j - lb_j)c_2 + lb_j)c_3 < 0, \end{cases} \quad (34)$$

where x_j^1 represents the position of the first salp (the leader) in the j -th dimension, F_j is the location of the food source on the j -th dimension, ub_j and lb_j are the upper and lower bounds on the j -th dimension, respectively, and c_2 and c_3 are random numbers.

Eq. (34) shows that the leader only updates the relative position with the food source. c_1 is the most important parameter in SSA because it is used to balance exploration and utilization:

$$c_1 = 2 \exp \left[- \left(\frac{4l}{L} \right)^2 \right], \quad (35)$$

where l is the current number of iterations and L is the maximum number of iterations. The following formula is used to update the location of the followers

$$x_j^i = \frac{1}{2} at^2 + v_0 t, \quad (36)$$

where $i \geq 2$, x_j^i indicates the position of the i -th follower in the j -th dimension, t is the time, v_0 is the initial velocity, $a = v_{\text{final}}/v_0$, where $v_{\text{final}} = (x - x_0)/t$.

Since time is the number of iterations in optimization, the difference of iteration time is 1. Considering $v_0 = 0$, the formula (36) can be expressed as

$$x_j^i = \frac{1}{2}(x_j^i + x_j^{i-1}). \quad (37)$$

The pseudo code of SSA is as follows

SSA algorithm

Initialize the salp population (T_0, U_0) considering ub and lb

while (end condition is not satisfied)

Calculate the $E(C_B)$ (fitness) of each search agent (salp)

F = the best search agent

Update c_1 by Eq. (28)

for each salp (T_0, U_0)

if the salp is the leader

Update the position of the leading salp by Eq. (27)

Else

Update the position of the follower salp by Eq. (30)

end

end

Amend the salps based on the upper and lower bounds of variables

end

return F

Table 2: Parameters setting

Parameters	Value
j	2
ub_j	$[5, 1 \times 10^5]$
lb_j	$[0, 0]$
L	400

6 Case analysis

A laser module that was made in China is analyzed as an example. The working principle of this type of laser module is to charge the energy storage capacitor through the current generated by the power supply, and generate instantaneous discharge for the discharge of xenon lamp, so as to promote the laser crystal to generate laser after stimulated radiation. Its internal structure is precise and difficult to maintain. The manufacturer is usually responsible for the basic warranty and extended warranty of the product.

The preventive maintenance cost of laser transmitter $C_p = 400$ CNY, the corrective maintenance cost $C_m = 200$ CNY, and the improvement factor ω of the manufacturer's imperfect preventive maintenance is 0.42. The basic warranty period of the product is (5 years, 10×10^4 KM), and the extended warranty period is (10 years, 20×10^4 KM). The life cycle of the product is (30 years, 60×10^4 KM). The parameters of the product failure rate function are $\theta_0 = 0.1$, $\theta_1 = 0.15$, $\theta_2 = 0.08$, and $\theta_3 = 0.14$. The product utilization rate r follows the Weibull distribution, and its probability density function is:

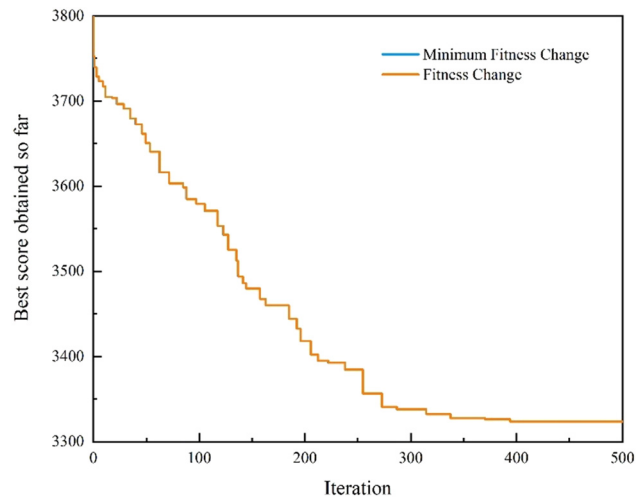


Figure 8: The schematic diagram of PSO algorithm iteration.

$$g(r) = \frac{\alpha}{\beta} \left(\frac{r}{\beta} \right)^{\alpha-1} e^{-\left(\frac{r}{\beta} \right)^{\alpha}},$$

where sale parameter $\alpha = 1.8$, shape parameter $\beta = 1.2$. It is necessary to determine the optimal two-dimensional block replacement interval and the extended warranty cost acceptable to users and manufacturers.

6.1 Problem solving

Discrete utilization rate based SSA is first used to solve the optimal preventive maintenance plan and the minimum basic maintenance cost of the laser module. Referring to the provisions of ref. [36] and the actual situation of the

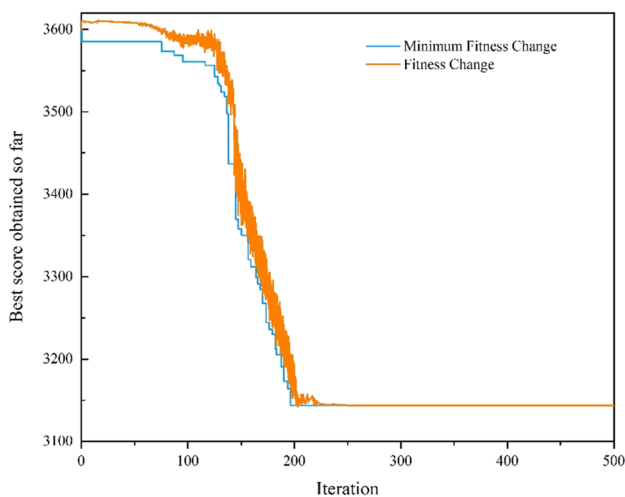


Figure 7: The schematic diagram of improved SSA iteration.

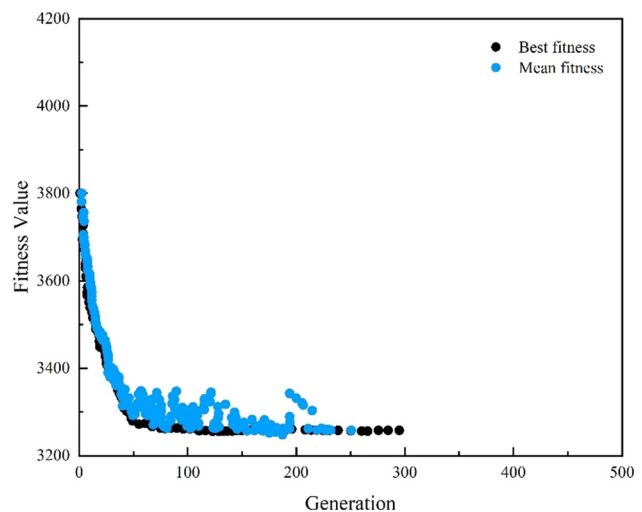
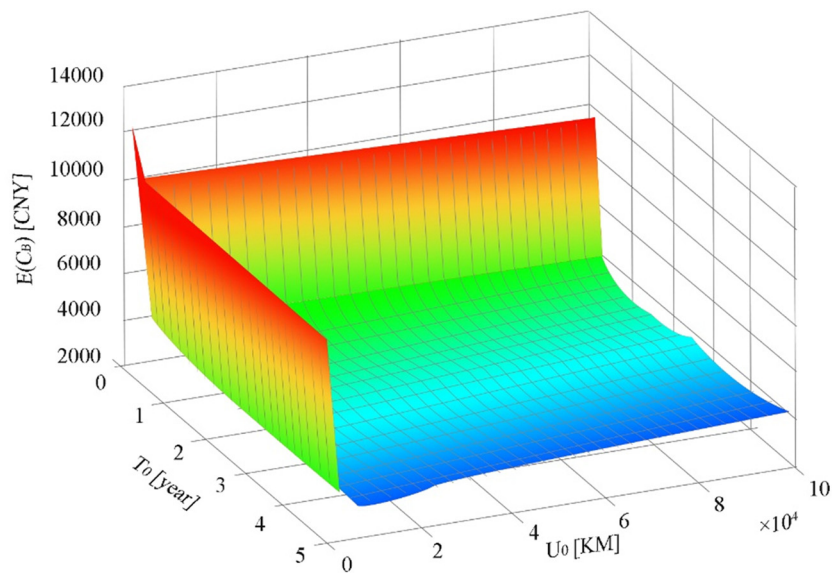


Figure 9: The schematic diagram of GA iteration.

Table 3: The results and operation time of each algorithm

Algorithm	The minimum warranty cost (CNY)	The optimal preventive maintenance interval	Operation time
GSA	5527.6	(1.2 years, 9,500 KM)	1,257 s
PSO	3323.7	(1.33 years, 9,873 KM)	89 s
GA	3257.6	(1.31 years, 10,004 KM)	156 s
SAA	3143.9	(1.32 years, 10,065 KM)	84 s

**Figure 10:** Warranty cost of different preventive maintenance periods.

laser module, the parameters of the algorithm are specified as follows (Table 2).

After the algorithm runs, the optimal preventive maintenance interval $(T_0^*, U_0^*) = (1.32 \text{ years}, 10,065 \text{ KM})$ and the minimum warranty cost is 3143.9 CNY. The schematic diagram of algorithm iteration is shown in Figure 7.

In order to show the advantages of the discrete utilization rate based SSA, it is compared with GSA, PSO algorithm, and GA. Next the GSA, PSO algorithm, and GA are used for optimization, respectively. The step length of the GSA is [0.2 years, 5,000 KM]. The minimum warranty cost calculated by the algorithm is 3527.6 CNY,

Table 4: Warranty cost of different preventive maintenance periods

U_0 (KM)	T_0 (years)							
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
10,000	3164.0	3164.1	3154.6	3192.5	3161.497	3164.704	3165.697	3174.3
20,000	3197.8	3275.4	3285.3	3411.5	3319.4	3319.7	3331.4	3353.8
30,000	3216.4	3343.4	3374.9	3617.7	3458.4	3457.5	3484.0	3529.9
40,000	3212.9	3363.3	3404.3	3733.5	3524.8	3523.9	3567.9	3646.4
50,000	3208.6	3362.8	3438.0	3813.7	3566.8	3568.9	3631.3	3744.5
60,000	3204.4	3359.4	3435.1	3830.4	3561.5	3556.3	3636.3	3783.6
70,000	3200.7	3356.6	3433.1	3830.8	3585.9	3556.6	3651.6	3829.3
80,000	3196.2	3353.3	3430.9	3829.8	3588.9	3598.1	3682.0	3867.6
90,000	3191.7	3350.3	3428.5	3828.0	3589.8	3605.2	3716.5	3919.6

and the corresponding optimal preventive maintenance interval is (1.2 years, 9,500 KM).

PSO algorithm and GA are used to solve the problem. The iteration number is 500 and the population number is 100. The replication strategy of GA is elite selection method, with a proportion of 10, a crossover probability of 0.8, and a mutation probability of 0.01. The inertia weight of PSO algorithm is 0.9, the self-adjustment weight is 1.49, and the social-adjustment weight is 1.49. The initial value of (T_0, U_0) is set as (1 years, 1×10^4 KM). The minimum warranty cost calculated by PSO algorithm is 3323.7 CNY, and the corresponding optimal preventive maintenance interval is (1.33 years, 9,873 KM). The minimum warranty cost calculated by GA is 3257.6 CNY, and the corresponding optimal

Table 5: The win-win two-dimensional extended warranty cost

Δ	0.2	0.4	0.6	0.8	1
C_p (CNY)	80	180	200	225	240
$E(C_E)$ (CNY)	3143.2	4554.1	3349.4	2511.7	1752.0

preventive maintenance interval is (1.31 years, 10,004 KM). Schematic diagrams of algorithm iteration of PSO algorithm and GA are shown in Figures 8 and 9.

The results and operation time of each algorithm are shown in Table 3.

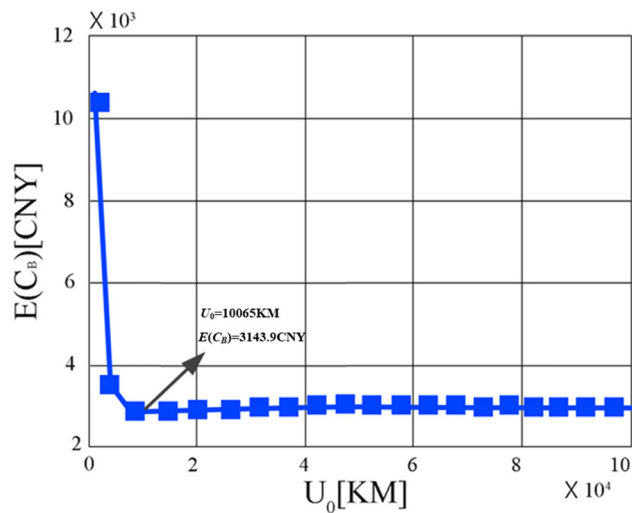


Figure 11: Warranty cost curve when $T_0 = 1.3$ years.

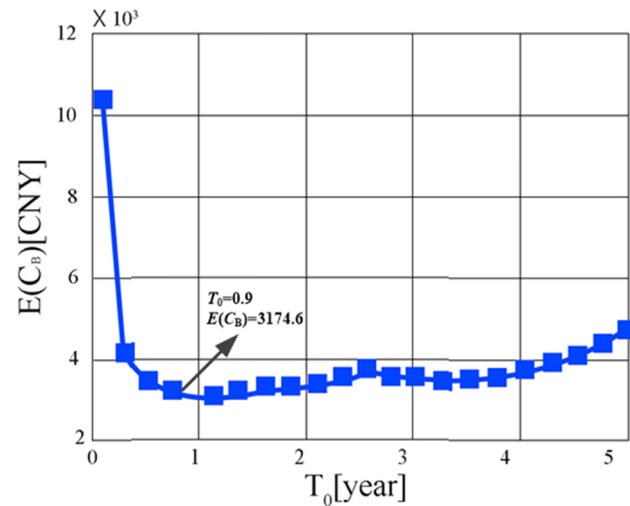


Figure 13: Warranty cost curve when PM is implemented only by age.

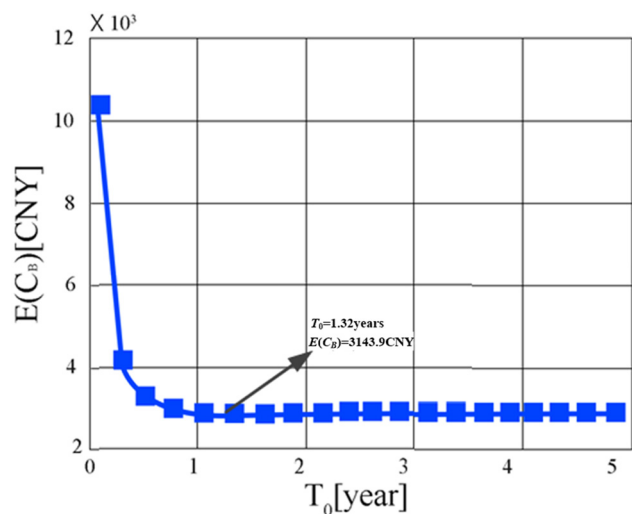


Figure 12: Warranty cost curve when $U_0 = 10,065$ KM.

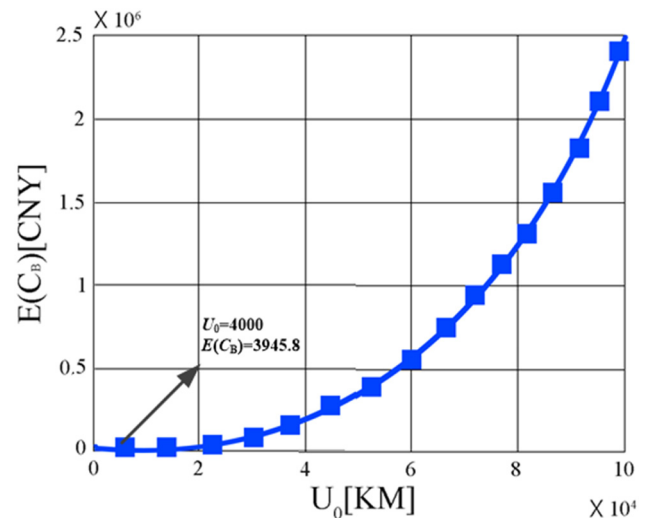


Figure 14: Warranty cost curve when PM is implemented only by usage.

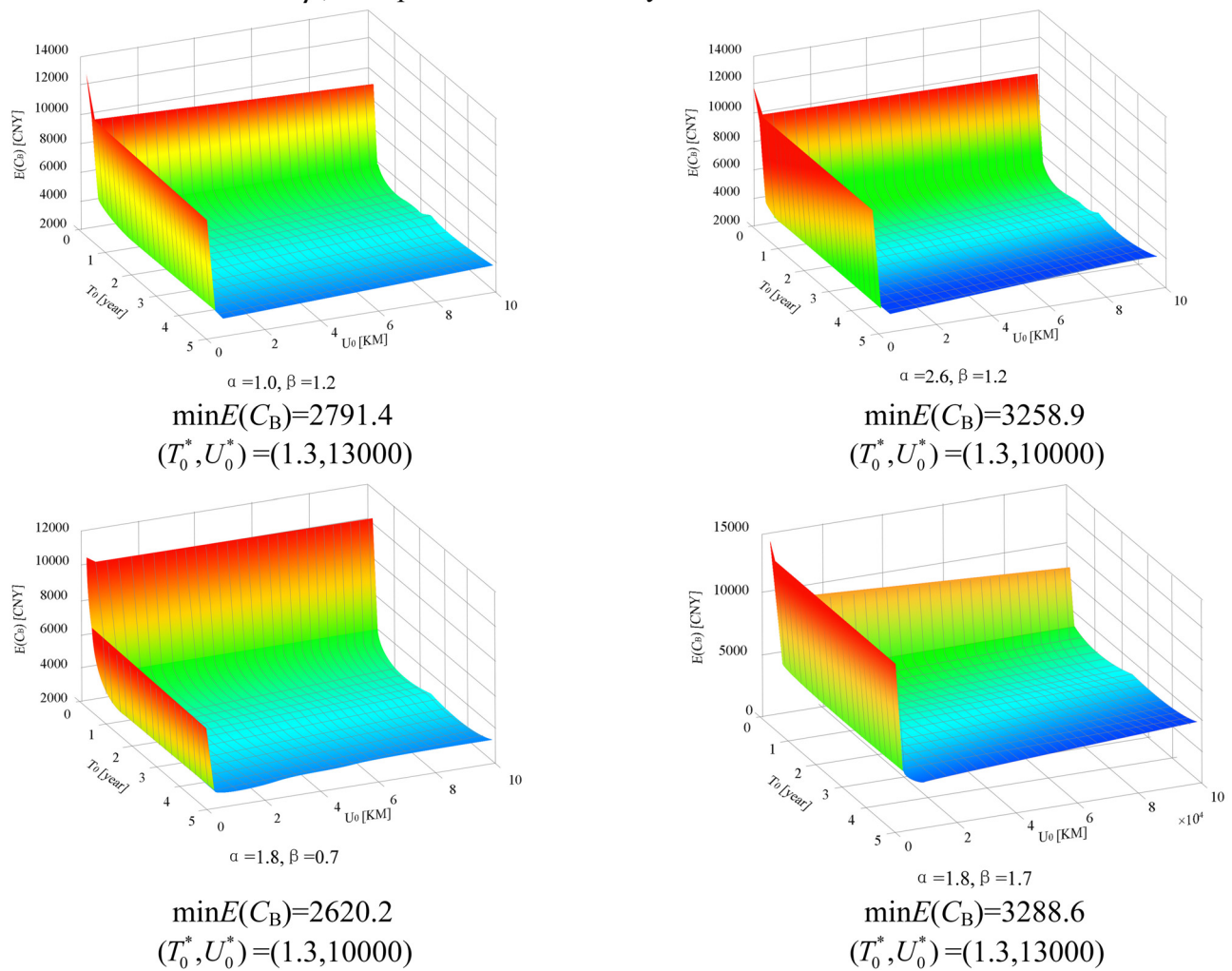


Figure 15: Sensitivity analysis of basic warranty cost model with different α and β . $\min E(C_B) = 2791.4$ and $= (1.3, 13,000)$. $\min E(C_B) = 3258.9$ and $= (1.3, 10,000)$. $\min E(C_B) = 2620.2$ and $= (1.3, 10,000)$. $\min E(C_B) = 3288.6$ and $= (1.3, 13,000)$.

Through comparison, it can be found that the discrete utilization rate based SSA can get lower warranty cost, and can converge earlier with higher operational efficiency, so it has more advantages in solving this model.

Through GSA, the basic warranty service fees under different preventive maintenance intervals are shown in Figure 10, and some results are shown in Table 4.

In order to more intuitively show the change trend of basic warranty cost with preventive maintenance interval,

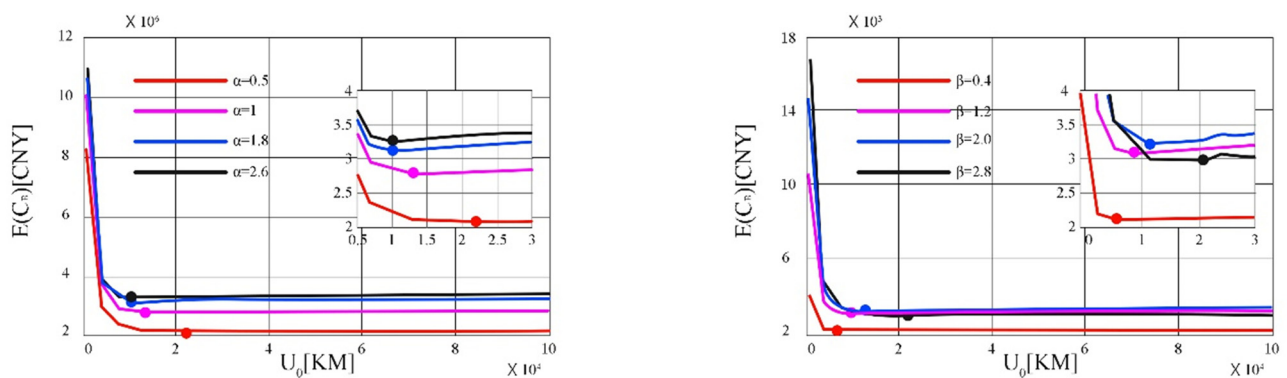


Figure 16: Basic warranty cost considering different α and β when $T_0 = 1.3$ years.

dimension reduction analysis is carried out, Figures 11 and 12. When $T_0 = 1.32$ years, the basic warranty cost at different U_0 is shown in Figure 11. When $U_0 = 10,065$ km, the basic warranty cost at different T_0 is shown in Figure 12.

As can be seen from Figures 11 and 12, with the increase in T_0 or U_0 , the basic warranty service cost first decreases and then increases.

When the preventive maintenance interval is $(T_0^*, U_0^*) = (1.32 \text{ years}, 10,065 \text{ KM})$, the cost of win-win extended warranty service cost under different degrees of preventive maintenance is shown in Table 5. It can be seen that as the degree of preventive maintenance increases, although the cost of imperfect preventive maintenance increases, the cost of extended warranty service does not increase monotonously. The calculation results of the model can provide data and information support for the decision-making of equipment two-dimensional warranty and the formulation of preventive maintenance strategy during the warranty period.

6.2 Comparative analysis

In order to compare and analyze the advantages of two-dimensional imperfect preventive maintenance, the cost model without imperfect preventive maintenance and the cost model of one-dimensional imperfect preventive maintenance are established below. The expected warranty cost without imperfect preventive maintenance is

$$E(C) = C_m \left[\int_0^{r_B} \int_0^{W_B} \lambda(t|r) g(r) dt dr + \int_{r_B}^{\infty} \int_0^{U_B/r} \lambda(t|r) g(r) dt dr \right].$$

When carrying out one-dimensional imperfect preventive maintenance, if the imperfect preventive

maintenance is carried out according to T_0 , the expected cost of basic warranty service is

$$E(C) = \int_0^{r_B} \left[n_1 C_p + C_m \left[\sum_{i=0}^{n_1-1} \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt \right] + \int_{n_1 T_0}^{W_B} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_1-1} (1-\omega)^j \lambda((n_1-j)T_0|r) \right] dt \right] g(r) dr \right] \\ + \int_{r_B}^{\infty} \left[n_4 C_p + C_m \left[\sum_{i=0}^{n_4-1} \int_{iT_0}^{(i+1)T_0} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)T_0|r) \right] dt \right] + \int_{n_4 T_0}^{U_B/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_4-1} (1-\omega)^j \lambda((n_4-j)T_0|r) \right] dt \right] g(r) dr \right].$$

When carrying out one-dimensional imperfect preventive maintenance, if the imperfect preventive maintenance is carried out according to U_0 , the expected cost of basic warranty service is

$$E(C) = \int_0^{r_B} \left[n_2 C_p + C_m \left[\sum_{i=0}^{n_2-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)U_0/r|r) \right] dt \right] + \int_{n_2 U_0/r}^{W_B} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_2-1} (1-\omega)^j \lambda((n_2-j)U_0/r|r) \right] dt \right] g(r) dr \right] \\ + \int_{r_B}^{\infty} \left[n_3 C_p + C_m \left[\sum_{i=0}^{n_3-1} \int_{iU_0/r}^{(i+1)U_0/r} \left[\lambda(t|r) - \omega \sum_{j=0}^{i-1} (1-\omega)^j \lambda((i-j)U_0/r|r) \right] dt \right] + \int_{n_3 U_0/r}^{U_B} \left[\lambda(t|r) - \omega \sum_{j=0}^{n_3-1} (1-\omega)^j \lambda((n_3-j)U_0/r|r) \right] dt \right] g(r) dr \right].$$

Through the calculation, when there is no preventive maintenance, the basic warranty cost is 4120.1 CNY. When carrying the one-dimensional imperfect preventive maintenance, the basic warranty service cost is shown in Figures 13 and 14. When performing imperfect preventive maintenance only according to the time interval, the optimal basic warranty service cost is 3174.6 CNY, and the optimal interval $T_0^* = 0.9$ years. When performing imperfect preventive

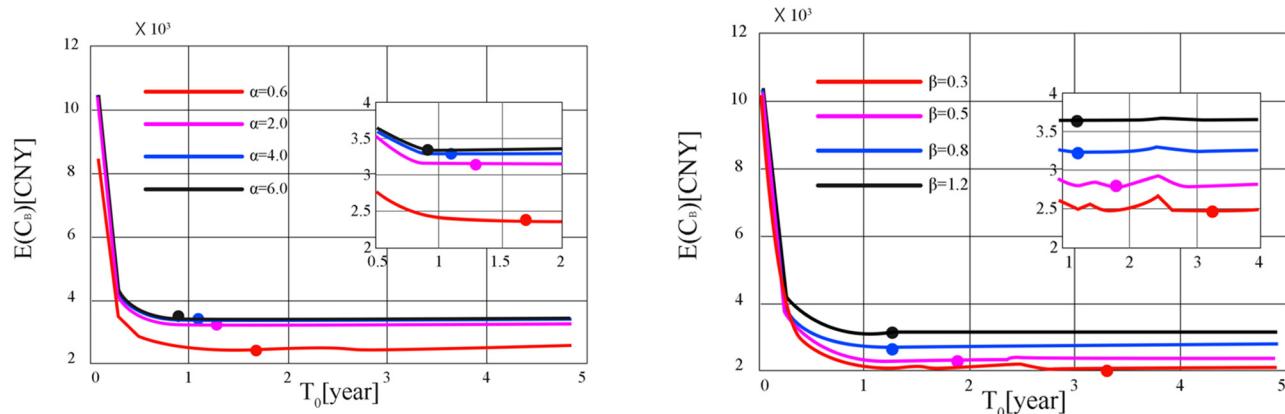


Figure 17: Basic warranty cost considering different α and β when $U_0 = 10,000 \text{ KM}$.

maintenance only according to the usage interval, the optimal basic warranty service cost is 3945.8 CNY, and the optimal interval $U_0^* = 4,000$ KM. It can be seen that two-dimensional imperfect preventive maintenance can effectively reduce the basic warranty service cost.

6.3 Sensitivity analysis

Under the optimal two-dimensional preventive maintenance interval, $r_0 < r_B$. In order to analyze the robustness of the model [37], the change in basic warranty service cost with parameters α and β is analyzed, as shown in Figure 15. As can be seen from Figure 15, with the increase in α and β , the optimal basic warranty service cost increases.

In order to further analyze the influence of α and β on T_0^* and U_0^* . T_0 is set as 1.3 years and the influence of α and β on U_0^* is analyzed, as shown in Figure 16. U_0 is set as 10,000 KM and the influence of α and β on T_0^* is analyzed, as shown in Figure 17. As can be seen from Figure 16, when α increases, the optimal U_0 decreases, and when β increases, the optimal U_0 increases. The effects of α and β on the optimal U_0 are opposite. As can be seen from Figure 17, when α increases, the optimal T_0 increases, and when β increases, the optimal T_0 also increases. It can be seen that the influence of α and β on the optimal T_0 is the same.

7 Conclusion

This study is mainly based on the two-dimensional block replacement strategy, and assuming that the two-dimensional group replacement is imperfect, aiming at minimizing the basic warranty cost, the optimal two-dimensional block replacement interval of the laser module is obtained through the improved SSA. Furthermore, the extended warranty cost scheme that makes the manufacturer and users win-win is obtained. Through result analysis, comparative analysis, and sensitivity analysis, the following conclusions can be obtained:

- 1) The optimal two-dimensional block replacement interval of the laser module (T_0^*, U_0^*) = (1.32 years, 10,065 KM), and the model proposed in this article can provide a scientific basis for the determination of the optimal scheme and the extended warranty cost.
- 2) Compared to one-dimensional warranty and corrective maintenance, two-dimensional imperfect preventive maintenance can effectively reduce the basic warranty service cost.
- 3) The system utilization rate has a great impact on the optimal scheme. The manufacturer should first determine

the utilization rate of potential consumers before formulating the warranty scheme.

- 4) The SSA is effective and can find the optimal solution quickly and accurately.

Some extensions of the proposed model in this article can be considered for future study:

- 1) This study assumes that the warranty object is a single part (single system), without considering the correlation between multiple components. Future research can focus on the correlation between multiple components.
- 2) After determining the optimal two-dimensional block replacement scheme, this work obtains the extended warranty cost acceptable to manufacturers and users. Research on joint optimization of preventive maintenance scheme and extended warranty cost can also be carried out.
- 3) The maintenance strategy mainly adopted in this work is block replacement. In addition to block replacement, age replacement, failure detection, and function inspection are also common preventive maintenance methods. Future research can focus on maintenance decision under different preventive maintenance methods.

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