Research Article

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Travelling waves solutions of the KP equation in weakly dispersive media

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Abstract: The current work focuses on the solutions of the Kadomtsev and Petviashvili (KP) equation, which models nonlinear waves in a dispersive medium. The modified auxiliary equation approach is utilized to find analytical solutions of the KP equation. Consequently, a set of solutions including Jacobi elliptic solutions and solitary and periodic waves solutions is obtained. The geometry of the derived solutions is plotted with an appropriate choice of the parameters. It can be seen that the proposed method is powerful and can be used to solve nonlinear partial differential equations due to its simplicity.

Keywords: the KP equation, exact solution, the modified auxiliary equation method

1 Introduction

In 1970, Kadomtsev and Petviashvili [1] proposed an equatoin as a generalization of the KdV equation. The (2+1)-dimensional Kadomtsev and Petviashvili (KP) equation describes water waves and waves in ferromagnetic media; see [2,3]. The KP equation is given by the following form

$$(u_t + 6uu_x + u_{xxx})_x + \kappa u_{yy} = 0, \qquad (1)$$

where $u(x, y, t)$ is a function in spatial directions x and y, and time t and $\kappa = \pm 1$. This equation has received much attention and many authors have been studied the numerical attention and the exact solutions of the KP equation [4–8]. Rational solutions were obtained by Ablowitz and Satsuma in ref. [9]. Different solutions for the KP equation were founded by Wazwaz using two analytical approaches [2]. By utilizing the Hirota formulation, several Lump solutions were derived in [3]. The exp-expansion method and extended complex method were used to find analytical solutions of the KP equation [10].

Nonlinear partial differential equations (NPDEs) have an important role in describing a great variety of phenomena. For instance, in physics, many problems in fluid mechanics, plasma physics, nonlinear dynamic, and wave motion are described by nonlinear partial differential equations. Moreover, the applications of NPDEs extend to other areas such as engineering, ecology, mechanics, and chemistry; see ref. [11]. Finding the exact solutions, of NPDEs may help us to understand these nonlinear phenomena. Thus, many methods have been proposed earlier to obtain the exact and numerical solutions of NPDEs, for example, Backlund transform, Homotopy perturbation method, *etc* [12–17]. In addition, various powerful methods are introduced recently, for example, F-expansion method, exp-function expansion method, auxiliary equation method, sub-equation method, the extended sine-cosine method, the (G'/G)-expansion method, the direct algebraic method, and other methods; see [18–31]. Also, one of the applications of the Lie symmetry analysis is finding the exact solutions of NPDEs; see [32–35] for more details.

This paper is devoted to find analytical solutions for KP equation in terms of Jacobi elliptic functions and other functions. As a result, more general analytical exact solutions of the KP equation are obtained. These solutions might be useful in the study of fluid physics and nonlinear waves.

This article is organized as follows. In Section 2, the main steps of the proposed method are sketched. In Section 3, the modified auxiliary equation approach is used to find analytical solutions of Eq. (1). Finally, a brief summary of the obtained results is given.

2 The modified auxiliary equation method

The steps of this method are briefly outlined here. Consider the nonlinear partial differential equation shown below

$$Q(u, u_x, u_y, u_t, u_{xy}, u_{xt}, u_{xx}, \dots) = 0, \qquad (2)$$

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where Q is a polynomial in u and its derivatives.

Step 1. To begin, we employ the transformation

$$u(x, y, t) = U(\xi), \quad \xi = x + ry - ct, \tag{3}$$

where ξ converts the variables x, y, t into a single form and r and c are constants to be determined. By using Eq. (3), Eq. (2), can be written in the form

$$G(U, U'', \ldots) = 0, \tag{4}$$

and G is a polynomial in G and its derivatives.

Step 2. It is assumed that Eq. (4) has a solution in the form

$$U(\xi) = \sum_{i=-n}^{n} \lambda_i \, \phi^i(\xi), \tag{5}$$

where n is a positive integer and λ_i are arbitrary constants to be determined. Also, $\phi(\xi)$ satisfies

$$\phi'^2(\xi) = \mu_0 + \mu_1\phi^2(\xi) + \mu_2\phi^4(\xi), \tag{6}$$

where μ_0, μ_1, μ_2 are arbitrary constants. Eq. (6) has the following solutions

Case 1. If $\mu_0 = 1, \mu_1 = -(1 + m^2), \mu_2 = m^2$, then Eq. (6) has a solution $\phi(\xi) = \mathrm{sn}(\xi, m)$, where $\mathrm{sn}(\xi, m)$ defines the Jacobi function and m denotes the elliptic modulus such that $0 < m < 1$.

Case 2. If $\mu_0 = 1 - m^2, \mu_1 = 2m^2 - 1, \mu_2 = -m^2$, then Eq. (6) has a solution $\phi(\xi) = \mathrm{cn}(\xi, m)$, and $\mathrm{cn}(\xi, m)$ defines the Jacobi function and m denotes the modulus where $0 < m < 1$.

Case 3. If $\mu_0 = m^2 - 1, \mu_1 = 2 - m^2, \mu_2 = -1$, then Eq. (6) has a solution $\phi(\xi) = \mathrm{dn}(\xi, m)$, where $\mathrm{dn}(\xi, m)$ defines the Jacobi function and m denotes the modulus such that $0 < m < 1$.

Case 4. If $\mu_0 = m^2, \mu_1 = -(1 + m^2), \mu_2 = 1$, then Eq. (6) has a solution $\phi(\xi) = \mathrm{ns}(\xi, m)$, where $\mathrm{ns}(\xi, m)$ defines the Jacobi function and m denotes the modulus such that $0 < m < 1$.

Case 5. If $\mu_0 = 1 - m^2, \mu_1 = 2 - m^2, \mu_2 = 1$, then Eq. (6) has a solution $\phi(\xi) = \mathrm{cs}(\xi, m)$, where $\mathrm{cs}(\xi, m)$ defines the Jacobi function *cs* and m denotes the modulus such that $0 < m < 1$.

Case 6. If $\mu_0 = 1, \mu_1 = 2m^2 - 1, \mu_2 = m^2(m^2 - 1)$, then Eq. (6) has a solution $\phi(\xi) = \mathrm{sd}(\xi, m)$, where $\mathrm{sd}(\xi, m)$ defines the Jacobi function sd and m denotes the modulus such that $0 < m < 1$.

Step 3. The positive integer n in Eq. (5) can be found by the use of the balance principle.

Step 4. By substituting (5) and (6) into (4) and putting all terms with the same power of $\phi(\xi)$ to zero, yields a set of overdetermined equations for λ_i. Consequently, a solution of Eq. (2) is obtained.

3 Analytical solutions of the KP equation

The extended auxiliary equation method will be used to solve the KP equation. By using the transformation,

$$u(x, t) = U(\xi), \quad \xi = x + ry - ct, \tag{7}$$

where r and c are nonzero constants, Eq. (1) can be written in the form

$$(\kappa r^2 - c)U'' + 6UU'' + 6U'^2 + U'''' = 0. \tag{8}$$

Using the balance principle in Eq. (8), it is found that $n = 2$. Thus, the solution of Eq. (8) is written as follows:

$$U(\xi) = \lambda_0 + \lambda_1\phi(\xi) + \frac{\lambda_{-1}}{\phi(\xi)} + \lambda_2\phi(\xi)^2 + \frac{\lambda_{-2}}{\phi(\xi)^2}. \tag{9}$$

Substitution Eq. (9) together with Eq. (6) into Eq. (8) and putting the coefficients of $\phi(\xi)$ to zero gives

$$\begin{aligned}
&120\mu_0^2\lambda_{-2} + 60\mu_0\lambda_{-2}^2 = 0,\\
&24\mu_0^2\lambda_{-1} + 72\mu_0\lambda_{-2}\lambda_{-1} = 0,\\
&48\mu_1\lambda_{-2}^2 + 18\mu_0\lambda_{-1}^2 + 120\mu_0\mu_1\lambda_{-2} + 36\mu_0\lambda_{-2}\lambda_0\\
&\quad - 6\mu_0 c\lambda_{-2} + 6\mu_0\kappa\lambda_{-2}r^2 = 0,\\
&20\mu_0\mu_1\lambda_{-1} + 54\mu_1\lambda_{-2}\lambda_{-1} + 12\mu_0\lambda_{-1}\lambda_0\\
&\quad + 12\mu_0\lambda_{-2}\lambda_1 - 2\mu_0 c\lambda_{-1} + 2\mu_0\kappa\lambda_{-1}r^2 = 0,\\
&36\mu_2\lambda_{-2}^2 + 12\mu_1\lambda_{-1}^2 + 16\mu_1^2\lambda_{-2} + 72\mu_0\mu_2\lambda_{-2}\\
&\quad + 24\mu_1\lambda_{-2}\lambda_0 - 4\mu_1 c\lambda_{-2} + 4\mu_1\kappa\lambda_{-2}r^2 = 0,\\
&\mu_1^2\lambda_{-1} + 12\mu_0\mu_2\lambda_{-1} + 36\mu_2\lambda_{-2}\lambda_{-1} + 6\mu_1\lambda_{-1}\lambda_0\\
&\quad + 6\mu_1\lambda_{-2}\lambda_1 - \mu_1 c\lambda_{-1} + \mu_1\kappa\lambda_{-1}r^2 = 0,\\
&6\mu_2\lambda_{-1}^2 + 6\mu_0\lambda_1^2 + 8\mu_1\mu_2\lambda_{-2}\\
&\quad + 12\mu_2\lambda_{-2}\lambda_0 + 8\mu_0\mu_1\lambda_2 + 12\mu_0\lambda_0\lambda_2 - 2\mu_2 c\lambda_{-2}\\
&\quad - 2\mu_0 c\lambda_2 + 2\mu_2\kappa\lambda_{-2}r^2 + 2\mu_0\kappa\lambda_2 r^2 = 0,\\
&\mu_1^2\lambda_1 + 12\mu_0\mu_2\lambda_1 + 6\mu_1\lambda_0\lambda_1 + 6\mu_1\lambda_{-1}\lambda_2\\
&\quad + 36\mu_0\lambda_1\lambda_2 - \mu_1 c\lambda_1 + \mu_1\kappa\lambda_1 r^2 = 0,\\
&12\mu_1\lambda_1^2 + 36\mu_0\lambda_2^2 + 16\mu_1^2\lambda_2 + 72\mu_0\mu_2\lambda_2\\
&\quad + 24\mu_1\lambda_0\lambda_2 - 4\mu_1 c\lambda_2 + 4\mu_1\kappa\lambda_2 r^2 = 0,\\
&20\mu_1\mu_2\lambda_1 + 12\mu_2\lambda_0\lambda_1 + 12\mu_2\lambda_{-1}\lambda_2\\
&\quad + 54\mu_1\lambda_1\lambda_2 - 2\mu_2 c\lambda_1 + 2\mu_2\kappa\lambda_1 r^2 = 0,\\
&18\mu_2\lambda_1^2 + 48\mu_1\lambda_2^2 + 120\mu_1\mu_2\lambda_2\\
&\quad + 36\mu_2\lambda_0\lambda_2 - 6\mu_2 c\lambda_2 + 6\mu_2\kappa\lambda_2 r^2 = 0,\\
&24\mu_2^2\lambda_1 + 72\mu_2\lambda_1\lambda_2 = 0,\\
&120\mu_2^2\lambda_2 + 60\mu_2\lambda_2^2 = 0.
\end{aligned} \tag{10}$$

Solving the resulting system (10) for $\lambda_0, \lambda_1, \lambda_2, \lambda_{-1}, \lambda_{-2}$ yields the following sets

Set 1.

$$\lambda_0 = \frac{1}{6}(-4\mu_1 + c - \kappa r^2), \quad \lambda_1 = 0, \quad \lambda_{-1} = 0, \lambda_2 = 0, \quad \lambda_{-2} = -2\mu_0. \tag{11}$$

On substituting these values into Eq. (9), various exact solutions can be contracted as the following cases.

Case 1. If $\mu_0 = 1, \mu_1 = -(1 + m^2), \mu_2 = m^2$, then the kp Eq. (1) has a solution in the following form:

$$u_1(x, y, t) = \frac{1}{6}(4 + c + 4m^2 - r^2\kappa) - \frac{2}{\mathrm{sn}(\xi, m)^2}, \tag{12}$$

where $\xi = x + ry - ct$ as defined earlier. Solution (12) leads to

$$u_2(x, y, t) = \frac{1}{6}(4 + c - r^2\kappa) - 2\csc(\xi)^2, \quad u_3(x, y, t) = \frac{1}{6}(8 + c - r^2\kappa) - 2\coth(\xi)^2, \tag{13}$$

when $m \to 0, m \to 1$, respectively. Figure 1 represents solutions u_2, u_3 when $t = 1, \kappa = 1, c = 1$, and $r = 2$. Figure 1 illustrates the periodic soliton solution as in (a) and the dark solitary solution as in (b).

Case 2. If $\mu_0 = 1 - m^2, \mu_1 = 2m^2 - 1, \mu_2 = -m^2$, then Eq. (1) has a solution

$$u_4(x, y, t) = \frac{1}{6}(4 + c - 8m^2 - r^2\kappa) + \frac{2(m^2 - 1)}{\mathrm{cn}(\xi, m)^2}. \tag{14}$$

When $m \to 0$, this solution reduces to

$$u_5(x, y, t) = \frac{1}{6}(4 + c - r^2\kappa) - 2\sec(\xi)^2. \tag{15}$$

The solution (14) is plotted in Figure 2 when $m = 0, m = 0.9$, and $t = 1, \kappa = 1, c = 1$, and $r = 2$.

Case 3. If $\mu_0 = m^2 - 1, \mu_1 = 2 - m^2, \mu_2 = -1$, then Eq. (1) has a solution in the following form:

$$u_6(x, y, t) = \frac{1}{6}(-8 + c + 4m^2 - r^2\kappa) - \frac{2(m^2 - 1)}{\mathrm{dn}(\xi, m)^2}. \tag{16}$$

In Figure 3(a), the solution (16) is shown when $m = 0.3$. The corresponding results for $m = 0.9$ is plotted in Figure 3(b). It can be seen that Figure 3 illustrates the periodic soliton solution of Eq. (16).

Case 4. If $\mu_0 = m^2, \mu_1 = -(1 + m^2), \mu_2 = 1$, then Eq. (1) has a solution:

$$u_7(x, y, t) = \frac{1}{6}(4 + c + 4m^2 - r^2\kappa) - \frac{2m^2}{\mathrm{ns}(\xi, m)^2}. \tag{17}$$

The solution (17) leads to

$$u_8(x, y, t) = \frac{1}{6}(8 + c - r^2\kappa) - 2\tanh(\xi)^2, \tag{18}$$

when $m \to 1$. Figure 4 represents solution (17) when $t = 1, \kappa = 1, c = 1, r = 2$. Figure 4 illustrates the periodic soliton solution as in (a) and the bright solitary solution as in (b).

Case 5. If $\mu_0 = 1 - m^2, \mu_1 = 2 - m^2, \mu_2 = 1$, then Eq. (1) has a solution:

$$u_9(x, y, t) = \frac{1}{6}(-8 + c + 4m^2 - r^2\kappa) + \frac{2(m^2 - 1)}{\mathrm{cs}(\xi, m)^2}. \tag{19}$$

When $m \to 0$, Eq. (19) becomes

$$u_{10}(x, y, t) = \frac{1}{6}(-8 + c - r^2\kappa) - 2\tan(\xi)^2. \tag{20}$$

Solution (19) is plotted in Figure 5 when $m = 0, m = 0.95$, and $t = 1, \kappa = 1, c = 1$, and $r = 2$.

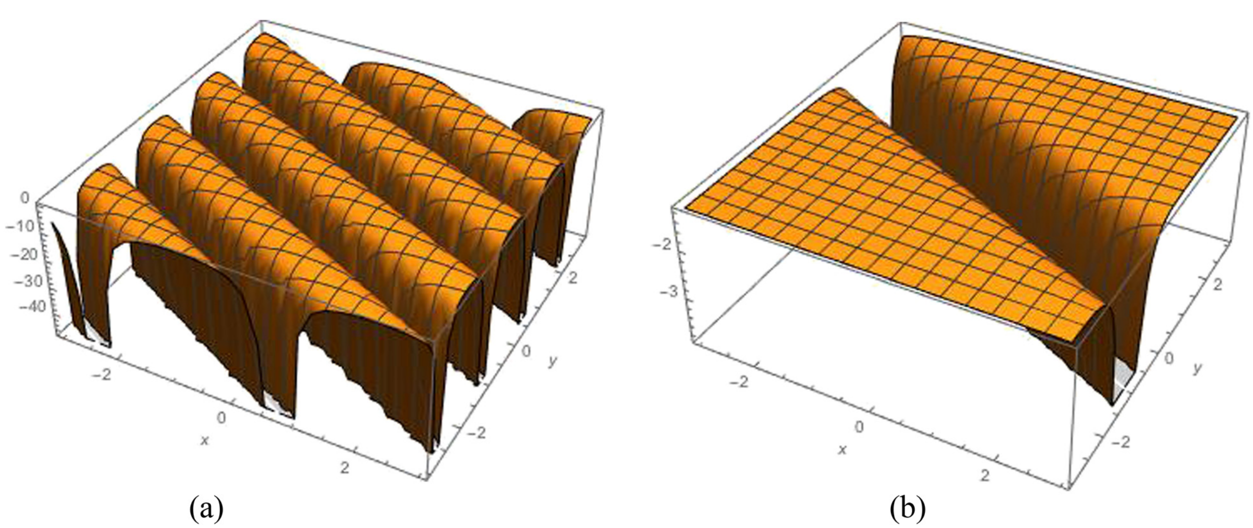


Figure 1: The graphs (a) and (b) the 3D plots of solutions u_2 and u_3, respectively.

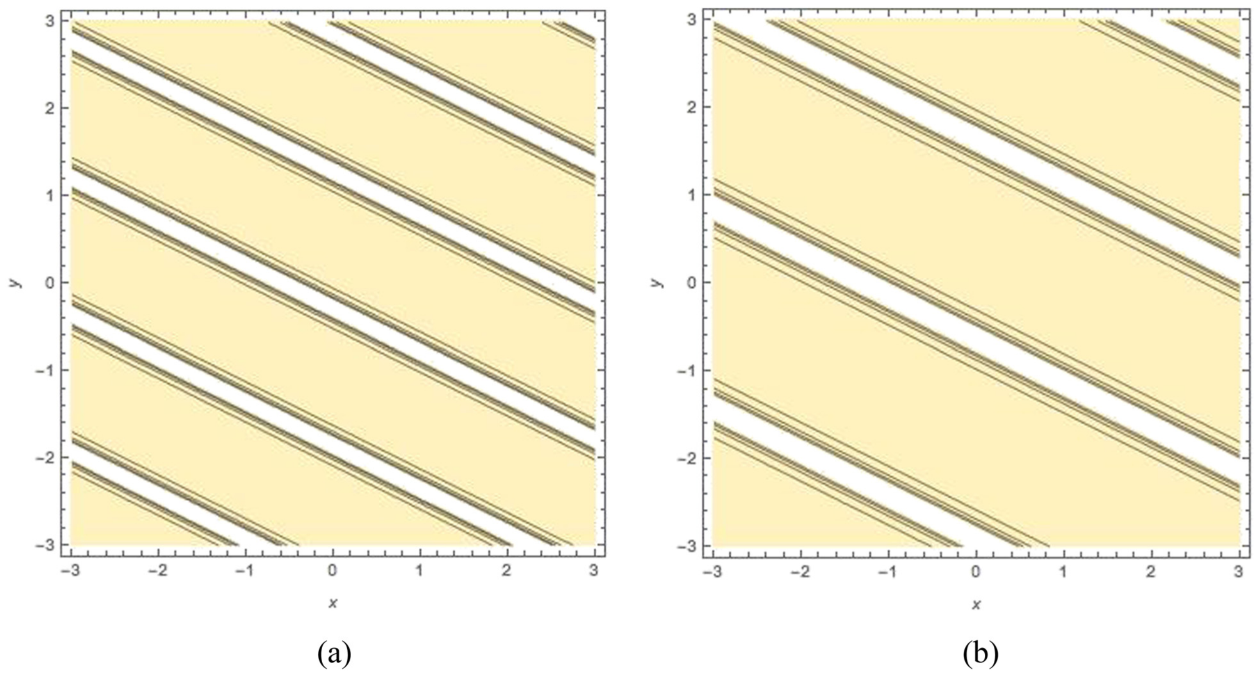


Figure 2: The graphs (a) and (b) contour plots of solution (14) when $m = 0$ and $m = 0.9$, respectively.

Case 6. If $\mu_0 = 1$, $\mu_1 = 2m^2 - 1$, $\mu_2 = m^2(m^2 - 1)$, then Eq. (1) has a solution:

$$u_{11}(x, y, t) = \frac{1}{6}(4 + c - 8m^2 - r^2\kappa) - \frac{2}{\mathrm{sd}(\xi, m)^2}. \quad (21)$$

When $m \to 1$, Eq. (21) becomes

$$u_{12}(x, y, t) = \frac{1}{6}(-4 + c - r^2\kappa) - 2\mathrm{csch}(\xi)^2. \quad (22)$$

Figure 6 represents the solution (21) when $t = 1$, $\kappa = 1$, $c = 1$, $r = 2$. Figure 6 illustrates the periodic soliton solution as in (a) and the dark solitary solution as in (b).

Set 2.

$$\lambda_0 = \frac{1}{6}(-4\mu_1 + c - \kappa r^2), \lambda_1 = 0, \lambda_{-1} = 0,$$
$$\lambda_2 = -2\mu_2, \lambda_{-2} = 0. \quad (23)$$

Putting these values into Eq. (9) leads to the following cases.

Case 1. If $\mu_0 = 1$, $\mu_1 = -(1 + m^2)$, $\mu_2 = m^2$, then Eq. (1) has a solution:

$$u_{13}(x, y, t) = \frac{1}{6}(4 + c + 4m^2 - r^2\kappa) - 2m^2\mathrm{sn}(\xi, m)^2. \quad (24)$$

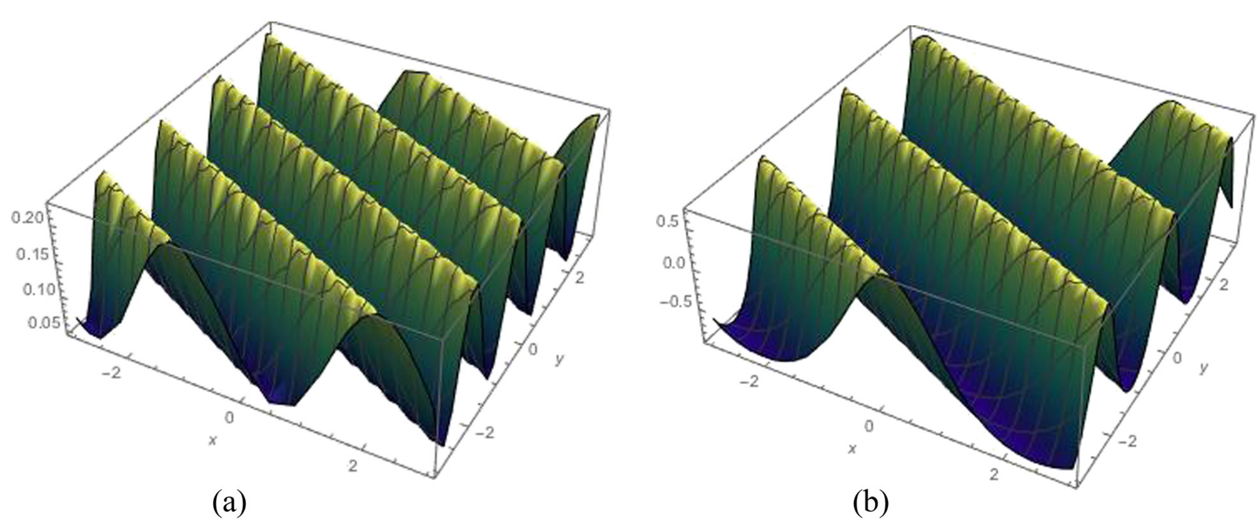


Figure 3: The graphs (a) and (b) the 3D plots of solution (16) when $m = 0.3$ and $m = 0.9$, respectively.

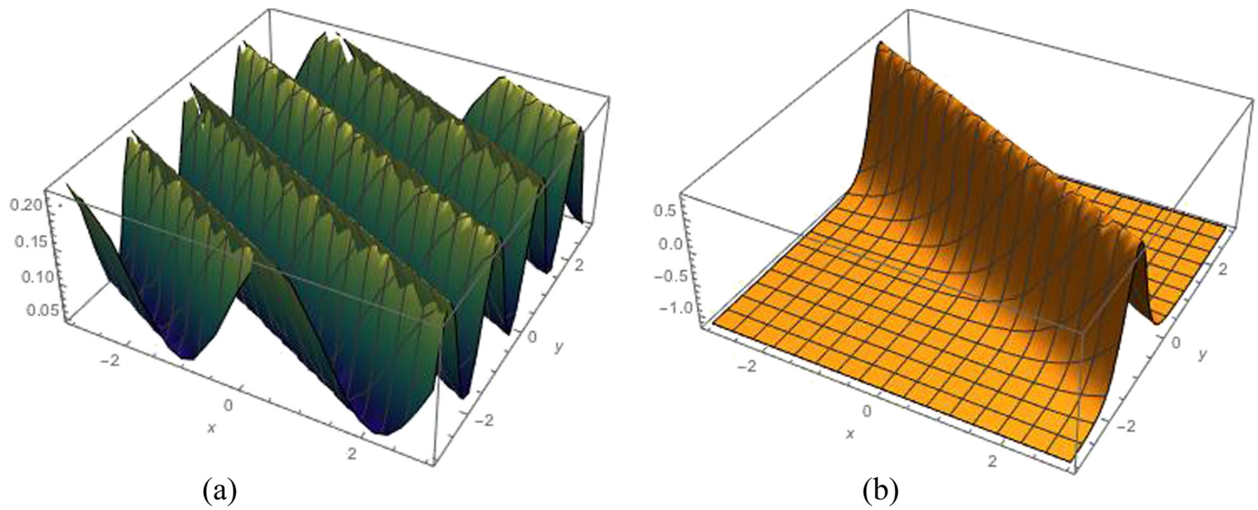


Figure 4: The graphs (a) and (b) the 3D plots of solution (17) when $m = 0.3$ and $m = 1$, respectively.

Case 2. If $\mu_0 = 1 - m^2$, $\mu_1 = 2m^2 - 1$, $\mu_2 = -m^2$, then Eq. (1) has a solution:

$$u_{14}(x, y, t) = \frac{1}{6}(4 + c - 8m^2 - r^2\kappa) + 2m^2\mathrm{cn}(\xi, m)^2. \quad (25)$$

When $m \to 1$, Eq. (25) becomes

$$u_{15}(x, y, t) = \frac{1}{6}(-4 + c - r^2\kappa) + 2\mathrm{sech}(\xi)^2. \quad (26)$$

The solution (25) is plotted in Figure 7 when $t = 1$, $\kappa = 1$, $c = 1$, $r = 2$, and $m = 1$, $m = 0.3$ for (a) and (b), respectively. Figure 7 illustrates the bright solitary solution as in (a) and the periodic soliton solution as in (b).

Case 3. If $\mu_0 = m^2 - 1$, $\mu_1 = 2 - m^2$, $\mu_2 = -1$, then Eq. (1) has a solution in the following form:

$$u_{16}(x, y, t) = \frac{1}{6}(-8 + c + 4m^2 - r^2\kappa) + 2\mathrm{dn}(\xi, m)^2. \quad (27)$$

Case 4. If $\mu_0 = m^2$, $\mu_1 = -(1 + m^2)$, $\mu_2 = 1$, then Eq. (1) has a solution:

$$u_{17}(x, y, t) = \frac{1}{6}(4 + c + 4m^2 - r^2\kappa) - 2\ \mathrm{ns}(\xi, m)^2. \quad (28)$$

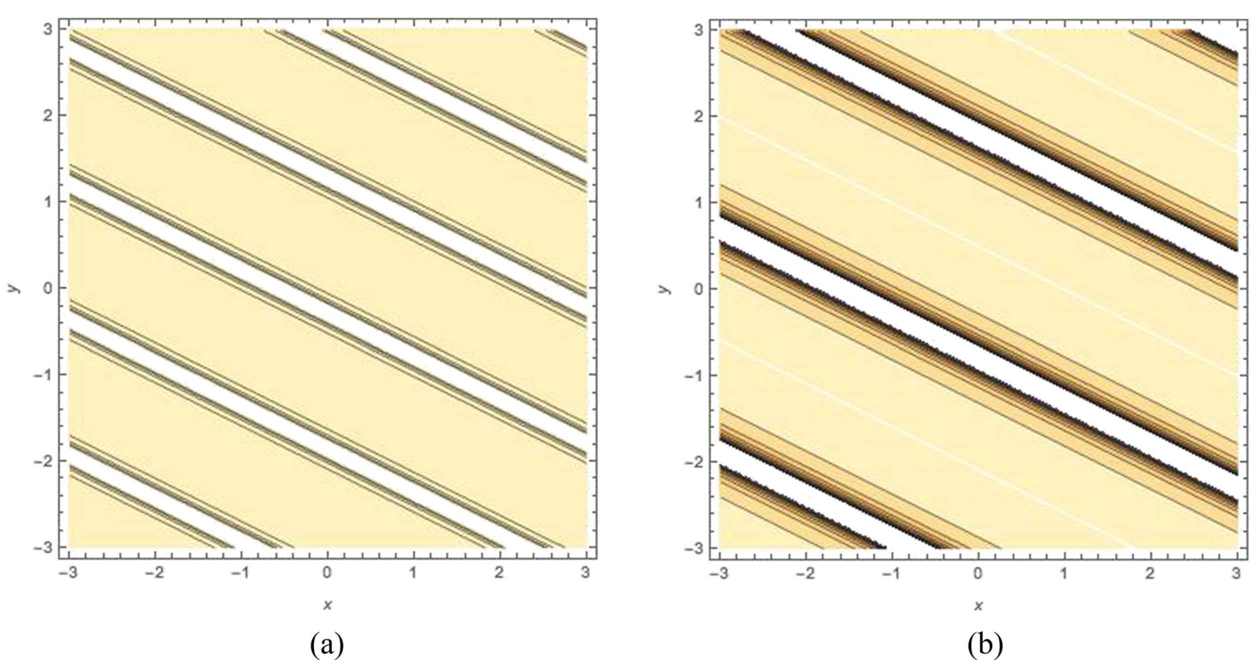


Figure 5: The graphs (a) and (b) the contour plots of solution (19) when $m = 0$ and $m = 0.95$, respectively.

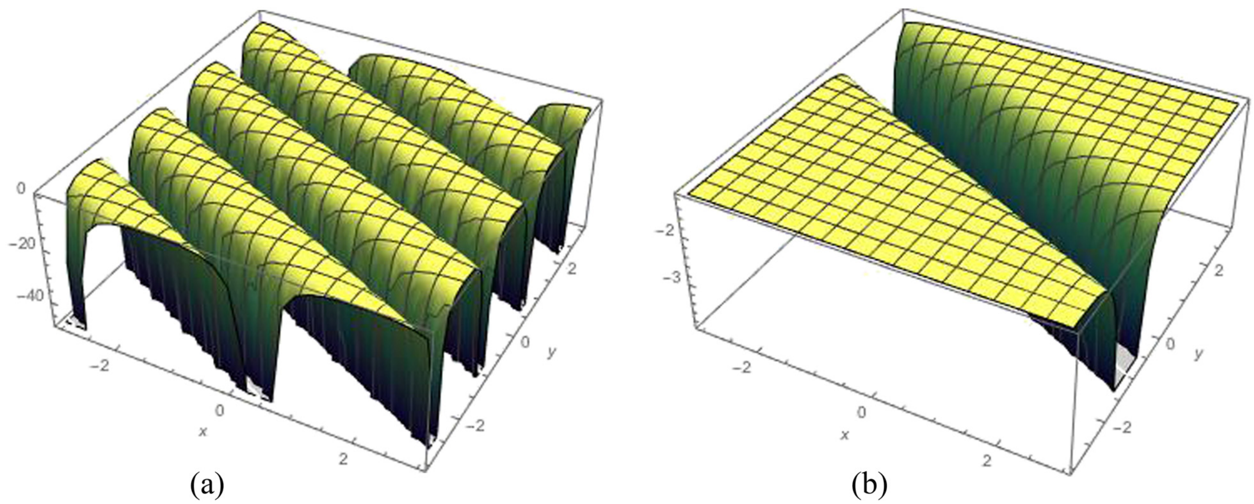


Figure 6: The graphs (a) and (b) 3D plots of solution (21) when $m = 0.4$ and $m = 1$, respectively.

Case 5. If $\mu_0 = 1 - m^2, \mu_1 = 2 - m^2, \mu_2 = 1$, then Eq. (1) has a solution:

$$u_{19}(x, y, t) = \frac{1}{6}(-8 + c + 4m^2 - r^2\kappa) - 2\,\mathrm{cs}(\xi, m)^2. \quad (29)$$

When $m \to 0$, Eq. (29) becomes

$$u_{20}(x, y, t) = \frac{1}{6}(-8 + c - r^2\kappa) - 2\cot(\xi)^2. \quad (30)$$

Case 6. If $\mu_0 = 1, \mu_1 = 2m^2 - 1$, and $\mu_2 = m^2(m^2 - 1)$, then Eq. (1) has a solution:

$$\begin{aligned} u_{21}(x, y, t) = {} & \frac{1}{6}(4 + c - 8m^2 - r^2\kappa) \\ & - 2m^2(m^2 - 1)\mathrm{sd}(\xi, m)^2. \end{aligned} \quad (31)$$

Set 3.

$$\begin{aligned} & \lambda_0 = \frac{1}{6}(-4\mu_1 + c - \kappa r^2), \lambda_1 = 0, \lambda_{-1} = 0, \\ & \lambda_2 = -2\mu_2, \lambda_{-2} = -2\mu_0. \end{aligned} \quad (32)$$

Again by substituting these values into Eq. (9), the following solutions can be obtained.

Case 1. If $\mu_0 = 1, \mu_1 = -(1 + m^2), \mu_2 = m^2$, then Eq. (1) has a solution,

$$\begin{aligned} u_{22}(x, y, t) = {} & \frac{1}{6}(4 + c + 4m^2 - r^2\kappa) - \frac{2}{\mathrm{sn}(\xi, m)^2} \\ & - 2m^2\mathrm{sn}(\xi, m)^2. \end{aligned} \quad (33)$$

This solution becomes

$$\begin{aligned} u_{23}(x, y, t) = {} & \frac{1}{6}(8 + c - r^2\kappa) - 2\coth(\xi)^2 \\ & - 2\tanh(\xi)^2, \end{aligned} \quad (34)$$

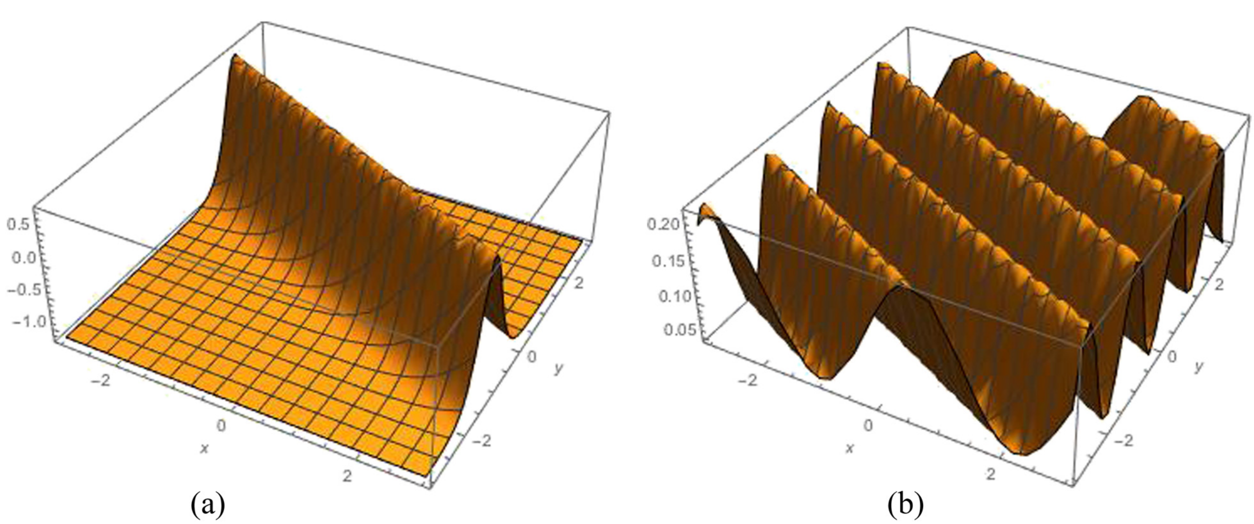
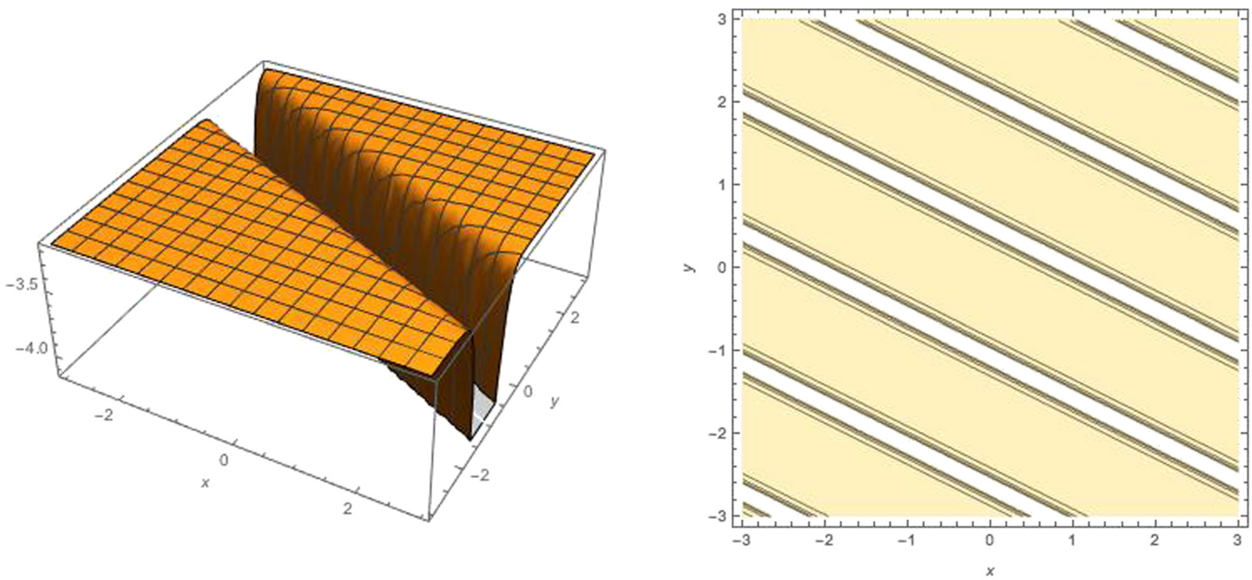


Figure 7: The graphs (a) and (b) 3D plots of solution (25) when $m = 1$ and $m = 0.3$, respectively.



(a) (a)

Figure 8: The graphs (a) and (b) 3D plots of solution (33) when $m = 1$ and $m = 0$, respectively.

when $m \to 1$. Solution (33) is plotted in Figure 8 when $t = 1$, $\kappa = 1$, $c = 1$, $r = 2$, and $m = 1$, $m = 0$ for (a) and (b), respectively.

Case 2. If $\mu_0 = 1 - m^2$, $\mu_1 = 2m^2 - 1$, $\mu_2 = -m^2$, then Eq. (1) has a solution:

$$u_{24}(x, y, t) = \frac{1}{6}(4 + c - 8m^2 - r^2\kappa) - \frac{2(1 - m^2)}{\text{cn}(\xi, m)^2} + 2m^2\text{cn}(\xi, m)^2. \quad (35)$$

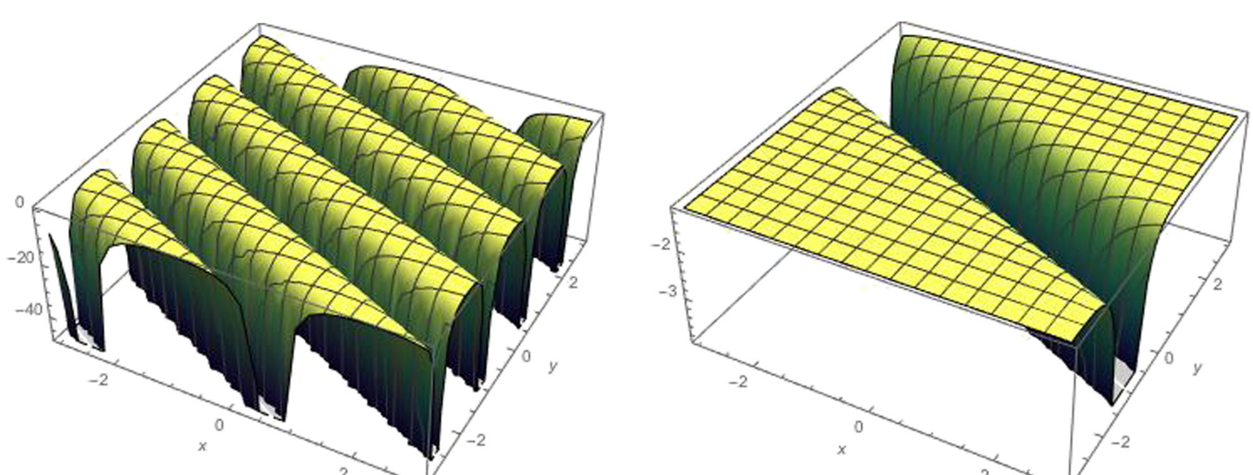
Case 3. If $\mu_0 = m^2 - 1$, $\mu_1 = 2 - m^2$, $\mu_2 = -1$, then Eq. (1) has a solution in the following form:

$$u_{25}(x, y, t) = \frac{1}{6}(-8 + c + 4m^2 - r^2\kappa) - \frac{2(m^2 - 1)}{\text{dn}(\xi, m)^2} + 2\text{dn}(\xi, m)^2. \quad (36)$$

Case 4. If $\mu_0 = m^2$, $\mu_1 = -(1 + m^2)$, $\mu_2 = 1$, then Eq. (1) has a solution:

$$u_{26}(x, y, t) = \frac{1}{6}(4 + c + 4m^2 - r^2\kappa) - \frac{2m^2}{\text{ns}(\xi, m)^2} - 2\,\text{ns}(\xi, m)^2. \quad (37)$$

Case 5. If $\mu_0 = 1 - m^2$, $\mu_1 = 2 - m^2$, $\mu_2 = 1$, then the kp Eq. (1) has a solution in the form:



(a) (b)

Figure 9: The graphs (a) and (b) The 3D plots of the solution (40) when $m = 0.2$ and $m = 1$, respectively.

$$u_{27}(x,y,t)=\frac{1}{6}(-8+c+4m^2-r^2\kappa)+\frac{2(m^2-1)}{\mathrm{cs}(\xi,m)^2} - 2\,\mathrm{cs}(\xi,m)^2. \quad (38)$$

When $m\to 0$, Eq. (38) becomes

$$u_{28}(x,y,t)=\frac{1}{6}(-8+c-r^2\kappa)-2\cot(\xi)^2 - 2\tan(\xi)^2. \quad (39)$$

Case 6. If $\mu_0=1$, $\mu_1=2m^2-1$, $\mu_2=m^2\mu(m^2-1)$, then Eq. (1) has a solution:

$$u_{29}(x,y,t)=\frac{1}{6}(4+c-8m^2-r^2\kappa)-\frac{2}{\mathrm{sd}(\xi,m)^2} - 2m^2(m^2-1)\mathrm{sd}(\xi,m)^2. \quad (40)$$

Solution (40) is plotted in Figure 9 when $t=1$, $\kappa=1$, $c=1$, $r=2$, and $m=0.2$, $m=1$ for (a) and (b), respectively. Figure 9 illustrates the periodic soliton solution as in (a) and the dark solitary solution as in (b).

4 Conclusion

In this article, the modified auxiliary equation method has been employed effectively to derived analytical solutions to the Kadomtsev–Petviashvili equation. These solutions are given in terms of Jacobi elliptic functions. When $m=0$ and $m=1$, solitary and periodic waves solutions are obtained as special cases. Also, the physical explanations of the obtained solutions have been demonstrated in some distinct figures. All of our solutions are verified by inserting them back into Eq. (1). This study shows that the proposed method is powerful, simple, and effective. This technique can be used to solve many problems in fluid mechanics, plasma physics, optical fibers, biology, solid mechanics, *etc*. Finally, the obtained results can be important for computational and experimental studies in water waves. All computations in this article were carried out with the help of Mathematica.

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