

Research Article

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Study on abundant analytical solutions of the new coupled Konno–Oono equation in the magnetic field

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Abstract: In this article, we focus on investigating the new coupled Konno–Oono equation that arises in the magnetic field. An effective technology called the Exp-function method (EFM) is utilized to find abundant analytical solutions. By this method, four families (28 sets) of the exact solutions, such as bright solitary, dark solitary, bright–dark solitary, double-bright solitary, double-dark solitary and kinky bright–dark solitary wave solutions, are constructed. The performances of the real, imaginary and absolute parts of the solutions are presented in the form of 3D contours. The results show that the EFM is a promising method to construct abundant analytical solutions for the partial differential equations arising in physics.

Keywords: traveling wave solutions, exp-function method, coupled Konno–Oono equation, partial differential equations

Abbreviations

EFM	exp-function method
KOE	Konno–Oono equation
PDE	partial differential equation
ODE	ordinary differential equation

1 Introduction

As we all know, nonlinear partial differential equations (PDEs) can be used to describe many complex natural

phenomena involving in hydrodynamics [1,2], thermal science [3–8], plasma physics [9,10], biomedical science [11–14], optics [15–23] and so on [24–31]. In the current study, we aim to investigate the new coupled Konno–Oono equation (KOE), which reads as [32,33]:

$$\begin{cases} p_{xt} - 2pq = 0, \\ q_t + 2pp_x = 0. \end{cases} \quad (1)$$

Eq. (1) is a special case of the nonlinear KOE system, which is first introduced by Konno–Oono as the coupled integrable dispersionless system in ref. [34]. Eq. (1) plays a key role in the magnetic field and its exact solution has always been the focus of the research. Many scientists have made outstanding contributions. Alam *et al.* employed the generalized (G'/G)-expansion method to find its exact solutions that expressed in the form of hyperbolic functions, trigonometric functions and rational functions in the work of Alam and Belgacem [35]. In ref. [36], the sine-Gordon expansion method is used by Yel *et al.* to construct the new soliton solutions. In ref. [37], Mirhosseini-Alizamini *et al.* applied the new extended direct algebraic method to find the exact solutions. In ref. [38], Torvattanabun *et al.* utilized the extended simplest equation method to seek the new exact solutions. In ref. [39], three effective methods that are simplified extended tanh-function method, variational direct method and He's frequency formulation are used to develop the exact solutions of Eq. (1). The Exp-function method (EFM) which was proposed by Chinese mathematician, Dr. Ji-Huan He, is a powerful tool to develop the abundant exact traveling wave solutions of the PDEs. Up to now, the new coupled KOE has not been investigated by the EFM. So this article will give a study on the new coupled KOE by the EFM. We arrange the overall structure of this article as follows. In Section 2, we give a brief introduction of the EFM. In Section 3, the EFM is applied to construct the exact solutions. In Section 4, we plot the behaviors of some solutions by the 3D contour and give the corresponding physical explanations. In Section 5, we come to a conclusion.

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2 The EFM

Step 1 Consider a PDE as:

$$f(\zeta, \zeta_x, \zeta_{xx}, \zeta_{xxx}, \dots, \zeta_t, \dots) = 0. \quad (2)$$

using the following traveling wave transformation:

$$\zeta(x, t) = \zeta(\varphi), \quad \varphi = \omega x + kt, \quad (3)$$

where ω and k are arbitrary nonzero constants.

With help of the transformation, we can convert Eq. (2) into an ordinary differential equation (ODE) as:

$$f(\zeta, \zeta_\varphi, \zeta_{\varphi\varphi}, \dots, \dots) = 0. \quad (4)$$

Step 2 Suppose the solution of Eq. (4) is [40–44]:

$$\zeta(\varphi) = \frac{\sum_{i=-p}^u a_i \exp(i\varphi)}{\sum_{j=-g}^s b_j \exp(j\varphi)}, \quad (5)$$

where u , p , s and g are positive integers that can be determined later, and a_i and b_j are unknown constants.

Step 3 Taking Eq. (5) into Eq. (4) and balancing the linear term of the highest and lowest orders, respectively, u , p , s and g can be determined.

Step 4 Substituting the obtained results into Eq. (4) and making the coefficients of $\exp(i\varphi)$ to be zero, then solving the systems, the coefficients a_i and b_j can be determined.

3 The solutions

To construct the exact solutions, the following traveling wave transformation is introduced:

$$p(x, t) = p(\varphi), \quad q(x, t) = q(\varphi), \quad \varphi = x + wt. \quad (6)$$

Applying the aforementioned transformation in Eq. (1), we have:

$$\begin{cases} wp'' - 2pq = 0, \\ wq' + 2pp' = 0, \end{cases} \quad (7)$$

where $p'' = \frac{d^2 p}{d\varphi^2}$, $p' = \frac{dp}{d\varphi}$, $q' = \frac{dq}{d\varphi}$. Integrating the second equation of Eq. (7) with respect to φ once, we have:

$$wq + p^2 + m = 0, \quad (8)$$

where m is the integral constant. Based on Eq. (8), we have:

$$q = -\frac{1}{w}(p^2 + m). \quad (9)$$

Taking the above equation into the first equation of Eq. (7) yields:

$$w^2 p'' + 2p^3 + 2mp = 0. \quad (10)$$

According to the EFM, we suppose the solution of Eq. (10) is:

$$p(\varphi) = \frac{\sum_{i=-p}^u a_i \exp(i\varphi)}{\sum_{j=-g}^s b_j \exp(j\varphi)}, \quad (11)$$

which can be expressed as:

$$p(\varphi) = \frac{a_{-c} \exp(-c\varphi) + \dots + a_u \exp(u\varphi)}{b_{-g} \exp(-g\varphi) + \dots + b_s \exp(s\varphi)}. \quad (12)$$

Putting the above equation into Eq. (10) and balancing the highest order with the highest order nonlinear term as:

$$p''(\varphi) = \frac{\dots + \Lambda_1 \exp[(2s + u)\varphi]}{\dots + \Lambda_2 \exp(3s\varphi)}, \quad (13)$$

$$p^3(\varphi) = \frac{\dots + \Lambda_3 \exp(3u\varphi)}{\dots + \Lambda_4 \exp(3s\varphi)}, \quad (14)$$

we have:

$$2s + u = 3u. \quad (15)$$

It gives:

$$u = s. \quad (16)$$

By the same way, we balance the linear term of lowest order as:

$$p''(\varphi) = \frac{\Delta_1 \exp[-(2g + c)\varphi] + \dots}{\Delta_2 \exp(-3g\varphi) + \dots}, \quad (17)$$

$$p^3(\varphi) = \frac{\Delta_3 \exp(-3c\varphi) + \dots}{\Delta_4 \exp(-3g\varphi) + \dots}. \quad (18)$$

There is:

$$-(2g + c) = -3c, \quad (19)$$

which leads to:

$$g = c. \quad (20)$$

Without losing generality, here, we select $u = s = 1$, $g = c = 1$, $b_1 = 1$, so Eq. (12) can be re-expressed as:

$$p(\varphi) = \frac{a_1 \exp(\varphi) + a_0 + a_{-1} \exp(-\varphi)}{\exp(\varphi) + b_0 + b_{-1} \exp(-\varphi)}. \quad (21)$$

Substituting Eq. (21) into Eq. (10), we have:

$$\frac{[\mathcal{I}_3 \exp(3\varphi) + \mathcal{I}_2 \exp(2\varphi) + \mathcal{I}_1 \exp(\varphi) + \mathcal{I}_0 + \mathcal{I}_{-1} \exp(-\varphi) + \mathcal{I}_{-2} \exp(-2\varphi) + \mathcal{I}_{-3} \exp(-3\varphi)]}{[\exp(\varphi) + b_0 + b_{-1} \exp(-\varphi)]^3} = 0. \quad (22)$$

There are:

$$\begin{aligned}
\mathcal{I}_3 &= 2ma_1 + 2a_1^3, \\
\mathcal{I}_2 &= 2ma_0 + w^2a_0 + 6a_0a_1^2 + 4ma_1b_0 - w^2a_1b_0, \\
\mathcal{I}_1 &= 2ma_{-1} + 4w^2a_{-1} + 6a_0^2a_1 + 6a_{-1}a_1^2 + 4ma_1b_{-1} \\
&\quad - 4w^2a_1b_{-1} + 4ma_0b_0 - w^2a_0b_0 + 2ma_1b_0^2 + w^2a_1b_0^2, \\
\mathcal{I}_0 &= 2a_0^3 + 12a_0a_1b_{-1} + 4ma_0b_{-1} - 6w^2a_0b_{-1} + 4ma_{-1}b_0 \\
&\quad + 3w^2a_{-1}b_0 + 4ma_{-1}b_{-1}b_0 + 3w^2a_{-1}b_{-1}b_0 + 2ma_0b_0^2, \\
\mathcal{I}_{-1} &= 6a_{-1}a_0^2 + 6a_1a_{-1}^2 + 4ma_{-1}b_{-1} - 4w^2a_{-1}b_{-1} \\
&\quad + 2ma_1b_{-1}^2 + 4w^2a_1b_{-1}^2 + 4ma_0b_{-1}b_0 - w^2a_0b_{-1}b_0 + 2ma_{-1}b_0^2 + w^2a_{-1}b_0^2, \\
\mathcal{I}_{-2} &= 6a_{-1}a_0 + 2ma_0b_{-1}^2 + w^2a_0b_{-1}^2 + 4ma_{-1}b_{-1}b_0 - w^2a_{-1}b_{-1}b_0 - w^2a_{-1}b_{-1}b_0, \\
\mathcal{I}_{-3} &= 2a_{-1}^3 + 2ma_{-1}b_{-1}^2.
\end{aligned}$$

In light of Eq. (22), we have:

$$\begin{cases} \mathcal{I}_3 = 0, \mathcal{I}_2 = 0, \mathcal{I}_1 = 0 \\ \mathcal{I}_0 = 0 \\ \mathcal{I}_{-1} = 0, \mathcal{I}_{-2} = 0, \mathcal{I}_{-3} = 0. \end{cases}$$

Solving the above systems, we can obtain the following four families:

Family 1

Case 1

$$\begin{aligned}
a_1 &= -i\sqrt{m}, \quad a_0 = a_0, \quad a_{-1} = \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}}, \\
b_0 &= b_0, \quad b_{-1} = \frac{a_0^2 + mb_0^2}{4m}, \quad w = \pm 2\sqrt{m},
\end{aligned}$$

where a_0 , b_0 and m are free parameters.

From case 1, we can obtain two sets of the solutions as:

$$\begin{cases} p_1(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 + \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x - 2\sqrt{m}t)} \\ q_1(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 + \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (23)$$

or

$$\begin{cases} p_2(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 + \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x + 2\sqrt{m}t)} \\ q_2(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 + \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (24)$$

Case 2

$$a_1 = i\sqrt{m}, \quad a_0 = a_0, \quad a_{-1} = -\frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}}, \quad b_0 = b_0, \quad b_{-1} = \frac{a_0^2 + mb_0^2}{4m}, \quad w = \pm 2\sqrt{m},$$

where a_0 , b_0 and m are free parameters.

In the view of case 2, we can obtain another two sets of the solutions as:

$$\begin{cases} p_3(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 - \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x - 2\sqrt{m}t)} \\ q_3(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 - \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (25)$$

or

$$\begin{cases} p_4(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 - \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x + 2\sqrt{m}t)} \\ q_4(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 - \frac{i(a_0^2 + mb_0^2)}{4\sqrt{m}} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + \frac{a_0^2 + mb_0^2}{4m} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right]. \end{cases} \quad (26)$$

Family 2**Case 1**

$a_1 = -i\sqrt{m}$, $a_0 = \pm\sqrt{m(4b_{-1} - b_0^2)}$, $a_{-1} = i\sqrt{m}b_{-1}$, $b_0 = b_0$, $b_{-1} = b_{-1}$, $w = \pm 2\sqrt{m}$, where b_0 , b_{-1} and $..$ are free parameters.

In this case, we can obtain four sets of the exact solutions as:

$$\begin{cases} p_5(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_5(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (27)$$

or

$$\begin{cases} p_6(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \\ q_6(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (28)$$

or

$$\begin{cases} p_7(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_7(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (29)$$

or

$$\begin{cases} p_8(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \\ q_8(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} + i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right]. \end{cases} \quad (30)$$

Case 2

$$a_1 = i\sqrt{m}, \quad a_0 = \pm\sqrt{m(4b_{-1} - b_0^2)}, \\ a_{-1} = -i\sqrt{m}b_{-1}, \quad b_0 = b_0, \quad b_{-1} = b_{-1}, \quad w = \pm 2\sqrt{m},$$

where b_0 , b_{-1} and m are free parameters.

In this case, we can obtain another four sets of the exact solutions as:

$$\begin{cases} p_9(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_9(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (31)$$

or

$$\begin{cases} p_{10}(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \\ q_{10}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (32)$$

or

$$\begin{cases} p_{11}(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_{11}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (33)$$

or

$$\begin{cases} p_{12}(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \\ q_{12}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{m(4b_{-1} - b_0^2)} - i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 + b_{-1} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right]. \end{cases} \quad (34)$$

Family 3**Case 1**

$$a_1 = -i\sqrt{m}, \quad a_0 = a_0, \quad a_{-1} = i\sqrt{m}b_{-1}, \quad b_0 = \pm \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}}, \quad b_{-1} = b_{-1}, \quad w = \pm 2\sqrt{m},$$

where a_0 , b_{-1} and m are free parameters.

In this case, we can obtain four sets of the exact solutions as:

$$\begin{cases} p_{13}(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 + i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_{13}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 + i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (35)$$

or

$$\begin{cases} p_{14}(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}) + a_0 + i\sqrt{m} b_{-1} \exp(-x + 2\sqrt{m})}{\exp(x - 2\sqrt{m}) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m})} \\ q_{14}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}) + a_0 + i\sqrt{m} b_{-1} \exp(-x + 2\sqrt{m})}{\exp(x - 2\sqrt{m}) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m})} \right)^2 + m \right] \end{cases} \quad (36)$$

or

$$\begin{cases} p_{15}(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}) + a_0 + i\sqrt{m} b_{-1} \exp(-x - 2\sqrt{m})}{\exp(x + 2\sqrt{m}) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m})} \\ q_{15}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}) + a_0 + i\sqrt{m} b_{-1} \exp(-x - 2\sqrt{m})}{\exp(x + 2\sqrt{m}) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m})} \right)^2 + m \right] \end{cases} \quad (37)$$

or

$$\begin{cases} p_{16}(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}) + a_0 + i\sqrt{m} b_{-1} \exp(-x + 2\sqrt{m})}{\exp(x - 2\sqrt{m}) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m})} \\ q_{16}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}) + a_0 + i\sqrt{m} b_{-1} \exp(-x + 2\sqrt{m})}{\exp(x - 2\sqrt{m}) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m})} \right)^2 + m \right]. \end{cases} \quad (38)$$

Case 2

$$\begin{aligned} a_1 &= i\sqrt{m}, \quad a_0 = a_0, \quad a_{-1} = -i\sqrt{m} b_{-1}, \\ b_0 &= \pm \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}}, \quad b_{-1} = b_{-1}, \quad w = \pm 2\sqrt{m}, \end{aligned}$$

where a_0 , b_{-1} and m are free parameters.

In this case, we can obtain another four sets of the exact solutions as:

$$\begin{cases} p_{17}(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 - i\sqrt{m} b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_{17}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 - i\sqrt{m} b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (39)$$

or

$$\begin{cases} p_{18}(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 - i\sqrt{m} b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m}t)} \\ q_{18}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 - i\sqrt{m} b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (40)$$

or

$$\begin{cases} p_{19}(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 - i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m}t)} \\ q_{19}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + a_0 - i\sqrt{m}b_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (41)$$

or

$$\begin{cases} p_{20}(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 - i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m}t)} \\ q_{20}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + a_0 - i\sqrt{m}b_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) - \frac{\sqrt{-a_0^2 + 4mb_{-1}}}{\sqrt{m}} + b_{-1} \exp(-x + 2\sqrt{m}t)} \right)^2 + m \right]. \end{cases} \quad (42)$$

Family 4

Case 1

$$\begin{aligned} a_1 &= -i\sqrt{m}, \quad a_0 = \pm\sqrt{-4i\sqrt{m}a_{-1} - mb_0^2}, \\ a_{-1} &= a_{-1}, \quad b_0 = b_0, \quad b_{-1} = -\frac{ia_{-1}}{\sqrt{m}}, \quad w = \pm 2\sqrt{m}, \end{aligned}$$

where a_{-1} , b_0 and m are free parameters.

In this case, we can develop another four sets of the exact solutions as:

$$\begin{cases} p_{21}(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{21}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (43)$$

or

$$\begin{cases} p_{22}(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{22}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x - 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (44)$$

or

$$\begin{cases} p_{23}(x, t) = \frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{23}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right] \end{cases} \quad (45)$$

or

$$\begin{cases} p_{24}(x, t) = \frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{24}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{-i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 - \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right]. \end{cases} \quad (46)$$

Case 2

$$\begin{aligned} a_1 &= i\sqrt{m}, \quad a_0 = \pm\sqrt{4i\sqrt{m}a_{-1} - mb_0^2}, \\ a_{-1} &= a_{-1}, \quad b_0 = b_0, \quad b_{-1} = \frac{ia_{-1}}{\sqrt{m}}, \end{aligned}$$

where a_{-1} , b_0 and m are free parameters.

In this case, we can obtain another four sets of the exact solutions as:

$$\begin{cases} p_{25}(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{25}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (47)$$

or

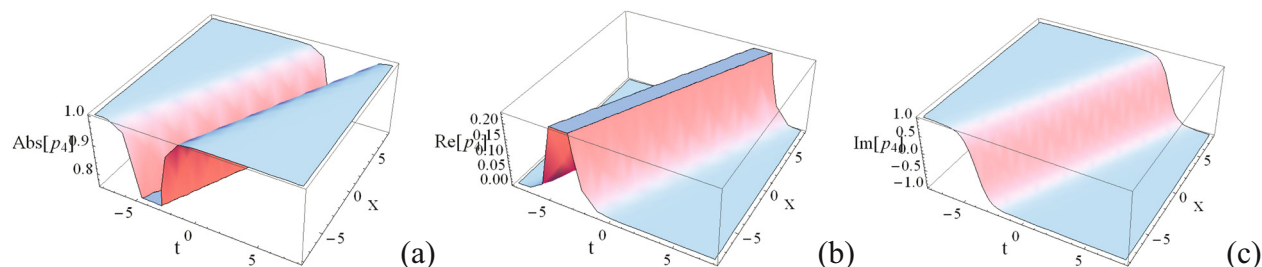
$$\begin{cases} p_{26}(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{26}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) + \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (48)$$

or

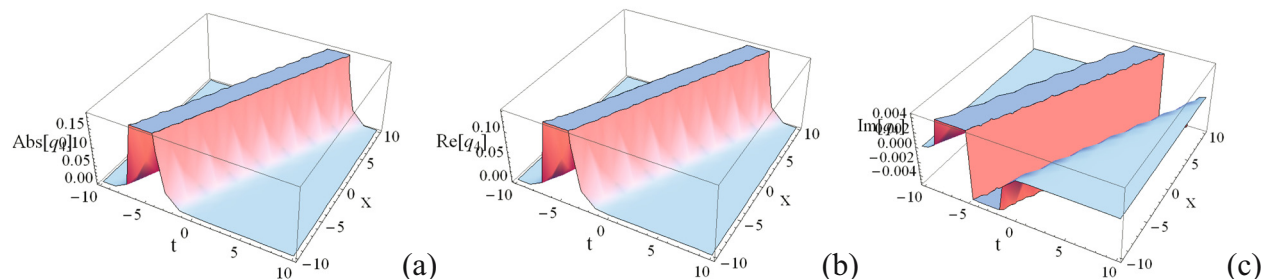
$$\begin{cases} p_{27}(x, t) = \frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{27}(x, t) = -\frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x + 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x - 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right], \end{cases} \quad (49)$$

or

$$\begin{cases} p_{28}(x, t) = \frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \\ q_{28}(x, t) = \frac{1}{2\sqrt{m}} \left[\left(\frac{i\sqrt{m} \exp(x - 2\sqrt{m}t) - \sqrt{-4i\sqrt{m}a_{-1} - mb_0^2} + a_{-1} \exp(-x + 2\sqrt{m}t)}{\exp(x + 2\sqrt{m}t) + b_0 + \frac{ia_{-1}}{\sqrt{m}} \exp(-x - 2\sqrt{m}t)} \right)^2 + m \right]. \end{cases} \quad (50)$$



(1) The behavior of the absolute, real and imaginary parts of $p_4(x, t)$.



(2) The behavior of the absolute, real and imaginary parts of $q_4(x, t)$.

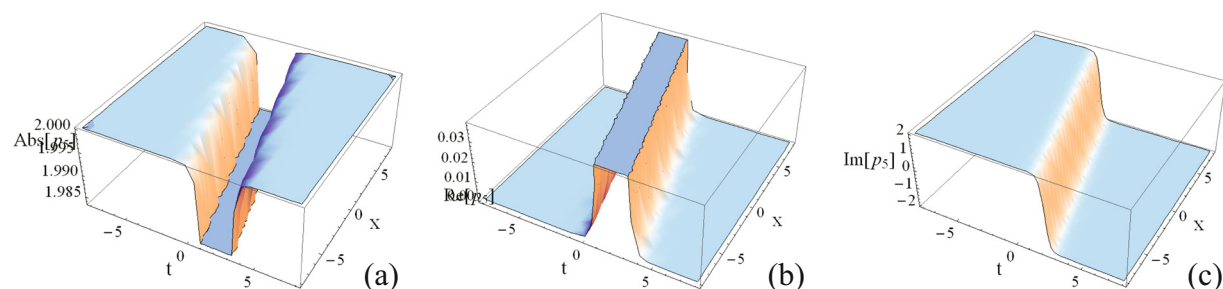
Figure 1: The behaviors of the solution Eq. (26).

4 Results and physical explanation

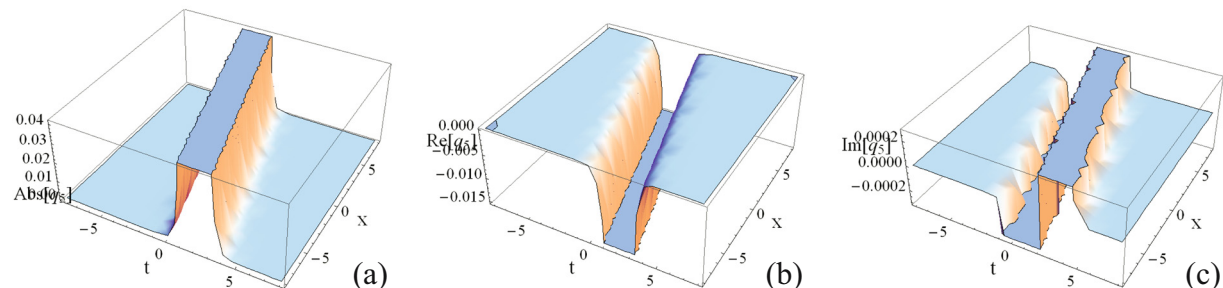
In this section, we present some results obtained in Section 3 in the form of 3D contours and give the corresponding physical explanations. It should be pointed out

that in the following content, the labelled (a), (b) and (c) represent the contours of absolute part, real part and imaginary part respectively.

By using $a_0 = 1$, $b_0 = 1$ and $m = 1$, we plot the behaviors of the solution Eq. (26) in Figure 1. For $p_4(x, t)$, the

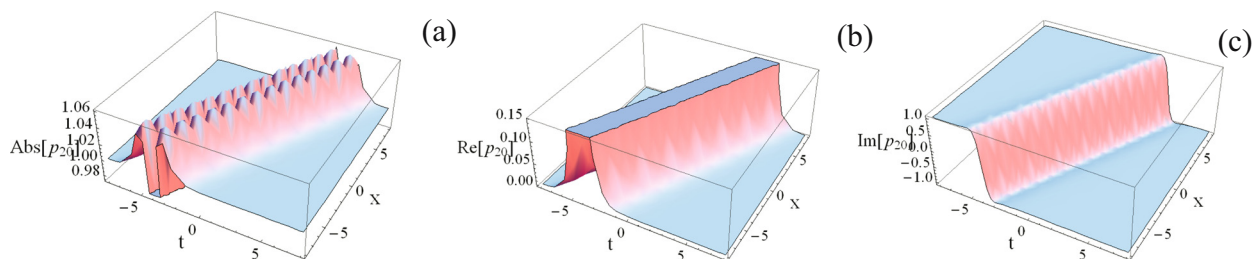


(1) The behavior of the absolute, real and imaginary parts of $p_5(x, t)$.

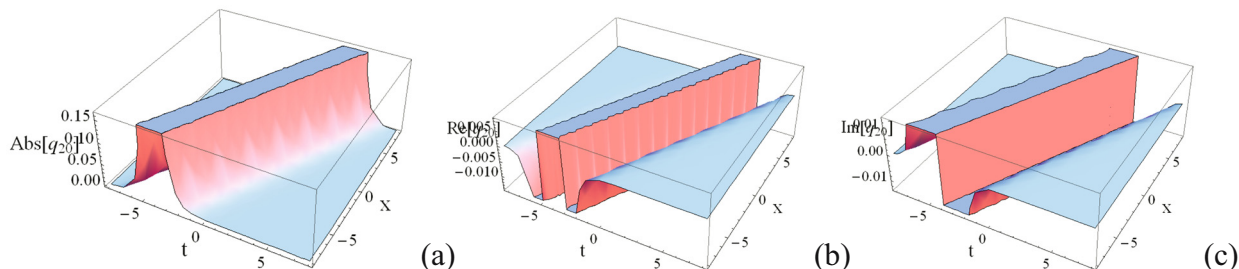


(2) The behavior of the absolute, real and imaginary parts of $q_5(x, t)$.

Figure 2: The behaviors of the solution of Eq. (27).



(1) The behavior of the absolute real and imaginary parts of $p_{20}(x, t)$.



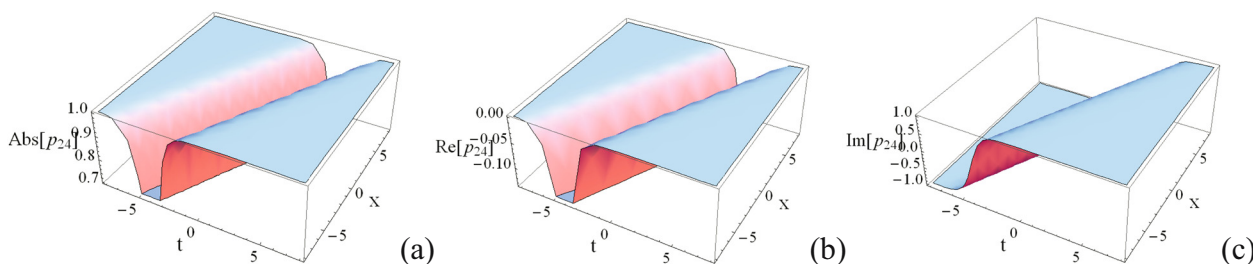
(2) The behavior of the absolute real and imaginary parts of $q_{20}(x, t)$.

Figure 3: The behaviors of the solution Eq. (42).

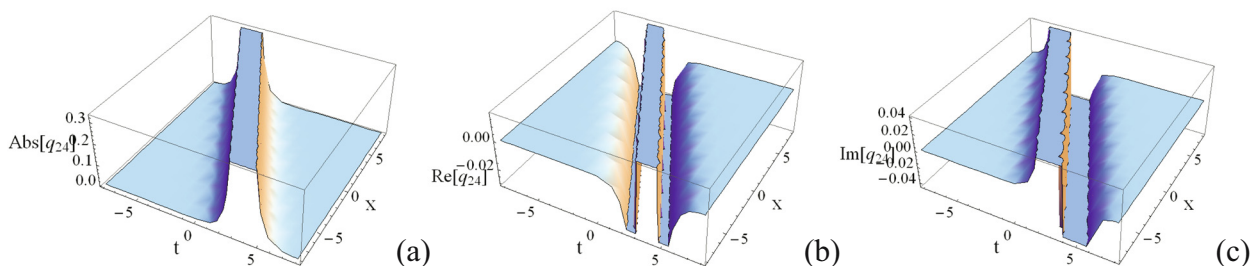
contours of the absolute and real parts are dark solitary wave and bright solitary wave, respectively, and the contour of the imaginary part is kinky bright-dark solitary wave. As for $q_4(x, t)$, the contours of the absolute and real parts are both bright solitary waves, but the imaginary part is the bright-dark solitary wave.

By selecting $b_0 = 1$, $b_1 = 1$ and $m = 1$, the behaviors of the solution Eq. (27) are described in Figure 2. By comparing with Figure 1, we can come to the same conclusion.

If we select $a_0 = 1$, $b_1 = 1$ and $m = 1$, we describe the performances of Eq. (32) in Figure 3. Obviously, the 3D contours of the absolute, real and imaginary parts of



(1) The behavior of the absolute, real and imaginary parts of $p_{24}(x, t)$.



(2) The behavior of the absolute, real and imaginary parts of $q_{24}(x, t)$.

Figure 4: The behaviors of the solution Eq. (46).

$p_{20}(x, t)$ are double bright solitary, bright wave and kinky bright dark solitary wave. As for $q_{20}(x, t)$, they are bright solitary, double dark solitary and bright–dark solitary.

When selecting $a_{-1} = 1$, $b_0 = 1$ and $m = 1$, we plot the solution of Eq. (46) in Figure 4. It can be seen that the 3D contours of absolute and real parts of $p_{24}(x, t)$ are both dark solitary waves, and the contour of the imaginary part is kinky bright–dark solitary. For the $q_{24}(x, t)$, the performances of the absolute, real and imaginary parts are bright solitary, double-dark solitary and bright–dark solitary, respectively.

5 Conclusion

In this article, the EFM is employed to study the new coupled KOE. With the help of this method, four families (28 sets) of the exact solutions, such as bright solitary, dark solitary, bright–dark solitary, double-bright solitary, double-dark solitary and kinky bright–dark solitary wave solutions, are constructed. The performances of some solutions are presented through the 3D contours. The obtained results reveal that the EFM is an effective method to construct abundant exact solutions of the nonlinear equations in physics.

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