

Research Article

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Structure of analytical ion-acoustic solitary wave solutions for the dynamical system of nonlinear wave propagation

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Abstract: In the present study, the ion-acoustic solitary wave solutions for Kadomtsev–Petviashvili (KP) equation, potential KP equation, and Gardner KP equation are constructed. The nonlinear KP equations are studying the nonlinear process of waves without collisions plasma and having non-isothermal electrons and cold ions. Two-dimensional ion-acoustic solitary waves (IASWs) in magnetized plasma are consisting of electrons and ions. We obtained the ion-acoustic solitary wave solutions same as dark and bright, kink and anti-kink wave solitons. The physical phenomena of various structures for IASWs are represented graphically with symbolic computations. These results are more helpful in the development of soliton dynamics, quantum plasma, dynamic of adiabatic parameters, fluid dynamics, and industrial phenomena.

Keywords: modified mathematical method, Kadomtsev–Petviashvili equation, potential KP equation, Gardner KP equation, ion-acoustic solitary wave solutions, solitons

1 Introduction

Kadomtsev and Petviashvili in 1970, first time introduced the important nonlinear Kadomtsev–Petviashvili (KP) equation, which is the generalized Korteweg–de Vries (KdV) equation of two space variables [1]. The KP equation explains the phenomena for weakly dispersive waves. The KP equation can be used as a model of long wavelength

water waves having weak nonlinear restoring forces and frequency dispersion. From the last five decades, a much analytical and numerical research has been carried out on various forms of KP equation. In recent years, KP equation has large deal of interest because of its explicit solutions including multi-solitons, periodic, and rational solutions in variables x and y [2]. The solutions and explicit solutions of KP equations are obtained by using the elementary techniques, see ref. [2,3]. The numerical solutions of KP equation by numerical approach are found in ref. [4]. By adomian decomposition method, the numerical solutions including solitons are derived in ref. [5].

The dusty plasma is ionized gas, which contains a little particle of matter in solid form, has size range from tens of nanometers to hundreds of microns. The solitons are solitary waves which represent an important feature for nonlinearity marvels in a system of space. The nonlinear partial differential equations (NLPDEs) have a specified type of contained results and possess different important features [6–16].

The study of dust plasmas containing huge dust particles is very significant to understand various behaviors for industrial physical applications, space, and astrophysical [17]. The dust plasma has potential applications in the research for space medium, e.g., zone of asteroid, cometary tails, planet rings, earth environments, medium of interstellar, and astrophysical [18–22]. The grains in the form of dust have negative charge because the number of various processes of charging, for example, ultra violet radiation, field emission, and current of plasma [23–25]. The effects of nonthermal supply for dust fluid and ions temperature on random amplitude applied in solitary construction of electrostatic and effective models, and exist in hot nonthermal dusty plasmas models containing hot nonthermal distributed ions and dust fluid [26,27].

The Sagdeev pseudo-potential and solitons of rarefactive are finding Boltzmannian electrons and ions for dust-acoustic solitary waves under dust plasma [28].

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Sagdeev reported that ion-acoustic waves under unmagnetized plasma have isothermal electrons, hot and cold ions [29]. He studied oscillation of scientific particles for nonlinear wave in good potential such as equation of integral energy to find the families of basic equations for plasma dynamic. He derived the Sagdeev potential, which is useful to measure the presence of various characteristics of soliton in plasma. In a dynamical system, the number of trapped ions is ignored because of a very small number of ions. The number of trap electrons is more, but the electron distribution is less under an ion-acoustic wave. So the distribution effect of trap electron is not Maxwellian, and these are not counted under the observation.

In the recent years, many researchers constructed the solutions for NLPDEs. Therefore, to construct the solutions for these NLPDEs many scholars and mathematicians developed many techniques. The list of some techniques are the scheme of Backlund transformed, the Hirota bilinear technique, the scheme of Darboux transformed, Exp-function method, the scheme of Jacobian technique, the scheme of trial equation, simple equation technique, the extension of fan-sub equation technique, the extend mapping technique, the technique of sinh-cosh, the direct algebraic technique, the scheme F-expansion technique, the modified tanh technique, the Riccati equation mapping technique, the extension of auxiliary equation technique, and Seadawy techniques [30–53]. We constructed the bright-dark solitons as in form ion-acoustic solitary wave solutions for three nonlinear PDEs by modified technique [54–64].

This article is arranged as follows: Introduction is given in Section 1. In Section 2, the proposed technique is described. Formulation for Sagdeev potential by basic equations is explained in Section 3. In Section 4, we construct solitonic solutions for equations by the described technique. In Section 5, the obtained results are discussed. Conclusion of this study is given in Section 6.

2 Algorithm of proposed method

Consider the NLPDEs in general form as:

$$S(\phi_t, \phi_x, \phi_y, \phi_{tt}, \phi_{xx}, \phi_{yy}, \phi_{xt}, \dots) = 0, \quad (1)$$

where S denotes polynomial function for $\phi(x, y, t)$ and its all derivatives. The main features for proposed technique are explained as follows:

Step 1: Consider the traveling wave transform as:

$$V(\zeta) = \phi(x, y, t), \quad \zeta = \kappa x + \lambda y + \mu t. \quad (2)$$

By using Eq. (2), the ODE (ordinary differential equation) for Eq. (1), we obtain

$$R(V', V'', V''', \dots) = 0, \quad (3)$$

where R denotes polynomial function in $V(\zeta)$ and their derivatives.

Step 2: The trial solution for Eq. (3) is considered as follows:

$$V(\zeta) = \sum_{l=0}^m a_l \Psi(\zeta)^l + \sum_{l=-1}^{-m} b_{-l} \Psi(\zeta)^l + \sum_{l=2}^m c_l \Psi(\zeta)^{l-2} \Psi'(\zeta) + \sum_{l=1}^m d_l \left(\frac{\Psi'(\zeta)}{\Psi(\zeta)} \right)^l, \quad (4)$$

where $a_l, b_l, c_l, d_l, (l = 0, 1, 2, 3, \dots)$ denote constant parameters to be determined later, while $\Psi(\zeta)$ satisfy the following auxiliary equation:

$$\left(\frac{d\Psi}{d\zeta} \right) = \sqrt{\beta_1 \Psi^2(\zeta) + \beta_2 \Psi^3(\zeta) + \beta_3 \Psi^4(\zeta)}. \quad (5)$$

Step 3: Apply the homogeneous rule in Eq. (3), to balance the highest order derivative and nonlinear term to determine m for Eq. (4).

Step 4: Substitute Eq. (4) in Eq. (5) and Eq. (3), and combine each coefficient of $\Psi'^k(\zeta) \Psi^l(\zeta)$ ($k = 0, 1; l = 1, 2, 3, \dots, m$) then each coefficient equates to zero. We obtained a system of algebraic equations and solved them by symbolic computation and obtained parameter values. Substituting parameter values and $\Psi(\zeta)$ in Eq. (4), solutions for Eq. (1) are obtained.

3 Formulation of Sagdeev potential by set of basic equations

Consider the collisionless plasma containing nonisothermal cold ions and electrons. The plasma model considered the free electrons temperature (T_{ef}), with trap electrons temperature (T_{et}), move with good potential, continuously lose energy, and a consequences upper electrons move front and back with good potential and become trap in good potential under plasma. The electron density and effect of trapped electrons [65,66] are defined as:

$$n_e(\phi) = \int f_e(x, r) dx = \exp(\phi)_{er} f_e(\sqrt{\phi}) + \begin{cases} \frac{1}{\sqrt{\sigma}} \{\exp(\sigma\phi)_{er} f(\sqrt{\sigma\phi}), & \text{where } \sigma > 0; \\ \frac{2}{\sqrt{\pi}} \exp\left(-(\sqrt{-\sigma\phi})^2 \int_0^{\sqrt{-\sigma\phi}} \exp(X^2 dX)\right), & \text{where } \sigma < 0. \end{cases} \quad (6)$$

The function of distribution of electron is $f_e(x, r)$ and ratio between temperature of free and trapped electrons is $\sigma = \frac{T_{ef}}{T_{et}}$. By applying Taylor series in Eq. (6), for $\phi < 1$, electron density n_e is obtained as:

$$n_e = 1 + \phi - \frac{4}{3}\beta\phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15}\gamma\phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 + \dots, \quad (7)$$

where n_e denotes electron density and ϕ is the potential normalized by unperturbed electron density n_0 and $\frac{\kappa_B T_{et}}{e}$, respectively. κ_B is the Boltzmann constant, $\beta = \frac{1-\sigma}{\sqrt{\pi}}$, and $\gamma = \frac{1-\sigma^2}{\sqrt{\pi}}$, $\left(\sigma = \frac{T_{ef}}{T_{et}}\right)$. The plasma has flat top and Maxwellian distribution for $\sigma = 0$ and $\sigma = 1$. If $\beta = 0$ and $\gamma = 0$, determine electron densities isothermal plasmas. The conditions are $0 < \beta < \frac{1}{\sqrt{\pi}}$, $0 < \gamma < \frac{1}{\sqrt{\pi}}$, for the non-isothermal plasma. Consider the families of basic equations represent dynamic plasma under describe fluid, followed by Das and Sen (1994) as:

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial x} u = 0, \quad (8)$$

$$-\frac{\partial \phi}{\partial x} = u \frac{\partial n}{\partial x} + \frac{\partial u}{\partial t}. \quad (9)$$

Poisson's equation is accompanied as:

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n, \quad (10)$$

where n is the ion density, normal by the n_0 move by velocity, u normal with ion acoustic speed $c_s = \left(\kappa_B \frac{T_{ef}}{m_i}\right)^{\frac{1}{2}}$. The space x has Debye length $\lambda_D = \left(\frac{\kappa_B T_{ef}}{4\pi n_0 e^2}\right)^{\frac{1}{2}}$ and time t^{-1} has ion-frequency $\left(\frac{4\pi n_0 e^2}{\kappa_B T_{ef}}\right)^{\frac{1}{2}}$. For checking the stable solutions for Eqs. (8)–(10), suppose the physical parameters depend on space and time x, t with relation $(\xi = x - \mu t)$, and the wave Mach number is μ . Consider $\xi \rightarrow \pm\infty$ for bounded solution, the plasma parameters u and ϕ are set to be zero. By applying these conditions the families of basic Eqs. (8)–(10) are changed as:

$$\frac{1}{2} \left(\frac{d\phi}{d\xi} \right)^2 + V(\phi, \mu) = 0. \quad (11)$$

Sagdeev potential [24] is taken as:

$$V(\phi, \mu) = 1 - \exp(\phi) + \frac{8}{15}\beta\phi^{\frac{5}{2}} + \frac{16}{105}\gamma\phi^{\frac{7}{2}} - \mu^2 \left[\left(1 - \frac{2\phi}{\mu^2} \right)^{\frac{1}{2}} - 1 \right]. \quad (12)$$

With parameter selection μ, β , and γ , various types of solutions for Eq. (12) are obtained.

4 Applications of described method for nonlinear dynamical equations

Here, we applied the proposed technique for the construction of solitary wave solutions for three nonlinear dynamical equations.

4.1 KP equation

Consider the KP equation as:

$$(\phi_t + 6\phi\phi_x + \phi_{xxx})_x + 3\phi_{yy} = 0, \quad (13)$$

and the transformation of wave as:

$$V(\zeta) = \phi(x, y, t), \quad \zeta = \kappa x + \lambda y + \mu t, \quad (14)$$

putting Eq. (14) in Eq. (13), we obtain:

$$\mu\kappa V'' + 6\kappa^2(V')^2 + 6\kappa^2 V V'' + \kappa^4 V'''' + 3\lambda^2 V'' = 0, \quad (15)$$

and integrate Eq. (15), once with respect to ζ with zero constant of integration, which becomes

$$\mu\kappa V' + 6\kappa^2 V V' + \kappa^4 V''' + 3\lambda^2 V' = 0, \quad (16)$$

and balancing higher order derivative and nonlinear term in Eq. (16), we obtain $m = 2$. The general solution for Eq. (16) is obtained as:

$$V(\zeta) = a_0 + a_1\Psi(\zeta) + a_2\Psi(\zeta)^2 + \frac{b_1}{\Psi(\zeta)} + \frac{b_2}{\Psi(\zeta)^2} + c_2\Psi'(\zeta) + d_1\frac{\Psi'(\zeta)}{\Psi(\zeta)} + d_2\left(\frac{\Psi'(\zeta)}{\Psi(\zeta)}\right)^2. \quad (17)$$

Substitute Eq. (17) in Eq. (16), and combine each coefficient of $\Psi'^k(\zeta)\Psi^l(\zeta)$ ($k = 0, 1; i = 1, 2, 3, 4, 5, \dots, m$), and equate each coefficient to zero. We obtain system of equations. After solving the system of equations by Mathematica, the values of constants are obtained as:

Case-I

$$\begin{aligned} a_0 &= a_0, \quad a_1 = -\beta_2(d_2 + \kappa^2), \quad a_2 = -2\beta_3\kappa^2 - \beta_3d_2, \\ b_1 &= b_2 = c_2 = d_1 = 0, \quad \beta_1 = \frac{\beta_2^2}{4\beta_3}, \quad d_2 = d_2, \\ \mu &= \frac{-24a_0\beta_3\kappa^2 - \beta_2^2\kappa^4 - 12\beta_3\lambda^2 - 6\beta_2^2d_2\kappa^2}{4\beta_3\kappa}. \end{aligned} \quad (18)$$

Substituting Eq. (18) in Eq. (17), then solutions for Eq. (13) are obtained as:

$$\begin{aligned} \phi_1(x, y, t) &= a_0 + \beta_1(d_2 + \kappa^2) \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right) \times \frac{\beta_1 d_2 \varepsilon^2 \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^4}{4 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2} \\ &\quad - \frac{\beta_1^2 \beta_3 (d_2 + 2\kappa^2) \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2}{\beta_2^2}. \end{aligned} \quad (19)$$

$$\begin{aligned} \phi_2(x, y, t) &= a_0 + \beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2 / (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\ &\quad (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 - \frac{1}{2} \beta_2 \sqrt{\frac{\beta_1}{\beta_3}} (-d_2 - \kappa^2) \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right) \\ &\quad - \frac{\beta_1 (d_2 + 2\kappa^2) (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2}{4 (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2}. \end{aligned} \quad (20)$$

$$\begin{aligned} \phi_3(x, y, t) &= a_0 + \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\ &\quad (\eta \sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\ &\quad - \beta_3 (d_2 + 2\kappa^2) \left(\frac{\varepsilon (p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2 \\ &\quad + \beta_2 (-d_2 - \kappa^2) \left(-\frac{\varepsilon (p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right). \end{aligned} \quad (21)$$

(Figure 1).

Case-II

$$\begin{aligned} a_0 &= \frac{-\beta_1 \kappa^4 - 6\beta_1 d_2 \kappa^2 - \kappa \mu - 3\lambda^2}{6\kappa^2}, \\ a_1 &= \pm \sqrt{\beta_1 \beta_3} (2\kappa^2 + d_2), \quad a_2 = -2\beta_3 \kappa^2 - \beta_3 d_2, \\ \beta_2 &= \mp 2\sqrt{\beta_1} \sqrt{\beta_3}, \quad d_2 = d_2, \quad b_1 = b_2 = c_2 = d_1 = 0. \end{aligned} \quad (22)$$

Putting Eq. (22), in Eq. (17), only positive value for a_1 , solutions for Eq. (13) are obtained as:

$$\begin{aligned} \phi_4(x, y, t) &= \frac{\beta_1 d_2 \varepsilon^2 \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^4}{4 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2} \\ &\quad - \frac{\beta_1 (6d_2 \kappa^2 + \kappa^4) + \kappa \mu + 3\lambda^2}{6\kappa^2} \\ &\quad - \frac{2\beta_1^{3/2} \sqrt{\beta_3} (d_2 + \kappa^2) \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)}{\beta_2} \\ &\quad - \frac{\beta_1^2 \beta_3 (d_2 + 2\kappa^2) \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2}{\beta_2^2}. \end{aligned} \quad (23)$$

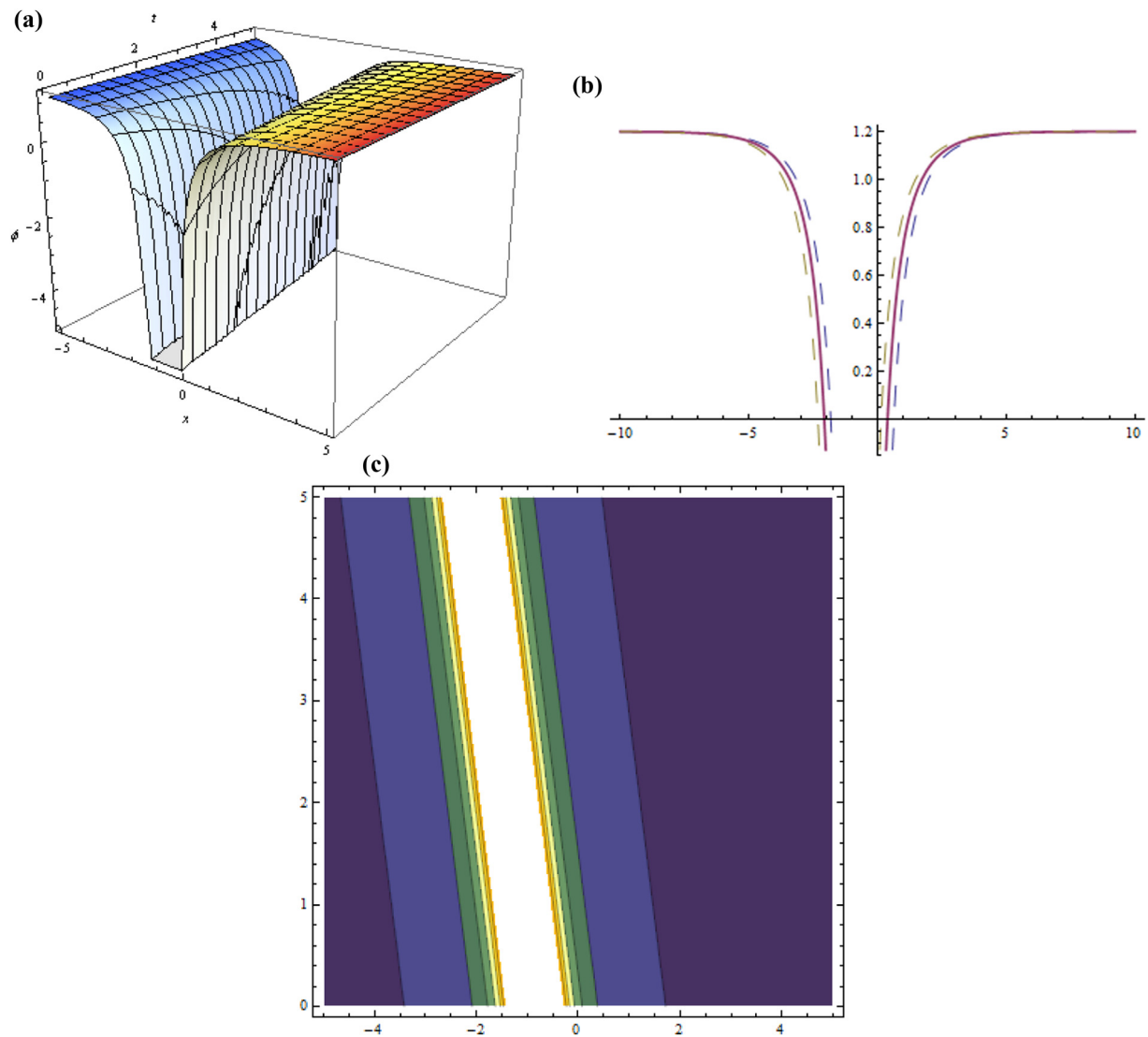


Figure 1: Three dimensional, two dimensional, and contour plots for Eq. (19) represent dark soliton while $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 0.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $a_0 = 1$, $d_2 = 1$, $\gamma = 1$.

$$\begin{aligned}
 \phi_5(x, y, t) = & \beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2 \lambda (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
 & \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
 & - \frac{\beta_1 (d_2 + \kappa^2) (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} \\
 & - \frac{\beta_1 (d_2 + 2\kappa^2) (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2}{4(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2} - \frac{\beta_1 (6d_2 \kappa^2 + \kappa^4) + \kappa \mu + 3\lambda^2}{6\kappa^2}.
 \end{aligned} \tag{24}$$

$$\begin{aligned}
\phi_6(x, y, t) = & \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& (\eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& - \frac{\beta_1(6d_2\kappa^2 + \kappa^4) + \kappa\mu + 3\lambda^2}{6\kappa^2} + 2\sqrt{\beta_1}\sqrt{\beta_3}(d_2 + \kappa^2) \left(-\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right) \\
& - \beta_3(d_2 + 2\kappa^2) \left(\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2.
\end{aligned} \tag{25}$$

(Figure 2).

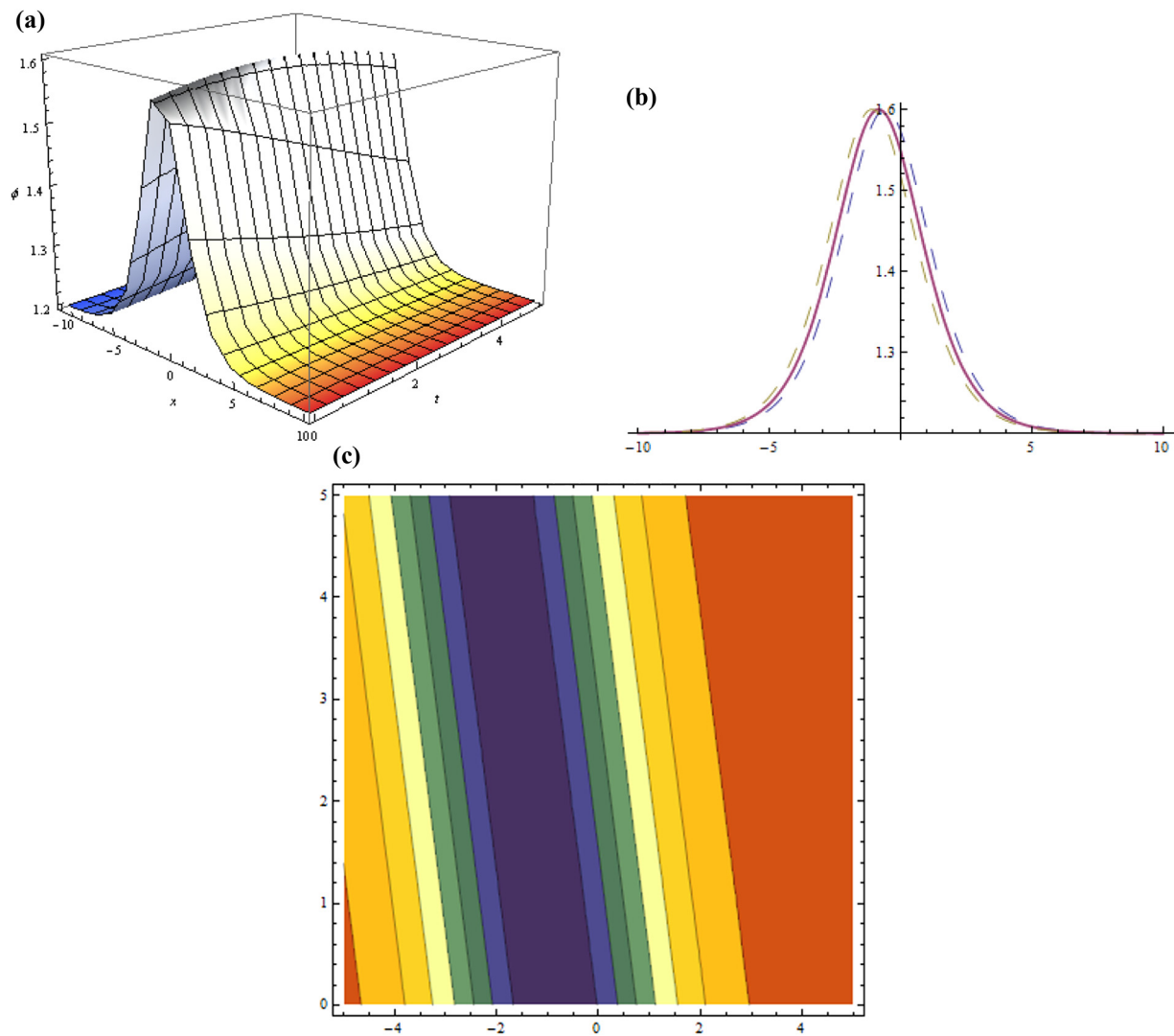


Figure 2: Three dimensional, two dimensional, and contour plots for Eq. (20) representing bright soliton while $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 0.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $a_0 = 1$, $d_2 = 1$, $y = 1$.

Case-III

$$\begin{aligned}
 a_0 &= \frac{-\beta_1 \kappa^4 - 6\beta_1 d_2 \kappa^2 - \kappa \mu - 3\lambda^2}{6\kappa^2}, \\
 a_1 &= \pm 2d_2 \sqrt{\beta_1} \sqrt{\beta_3}, \\
 a_2 &= -\beta_3 d_2, \quad \beta_2 = \mp 2\sqrt{\beta_1} \sqrt{\beta_3}, \\
 b_1 &= b_2 = c_2 = d_1 = 0, \quad d_2 = d_2.
 \end{aligned} \quad (26)$$

Substituting Eq. (26), in Eq. (17), only positive value for a_1 , solutions for Eq. (13) are obtained as:

4.2 Potential KP equation

Consider the potential KP equation as:

$$\phi_{xt} + \frac{3}{2}\phi_x \phi_{xx} + \frac{1}{4}\phi_{xxxx} + \frac{3}{4}\phi_{yy} = 0. \quad (30)$$

We apply the wave transformation:

$$V(\zeta) = \phi(x, y, t), \quad \zeta = \kappa x + \lambda y + \mu t. \quad (31)$$

Putting Eq. (31) in Eq. (30), we obtain:

$$\begin{aligned}
 \phi_7(x, y, t) &= -\frac{\kappa \mu + 3\lambda^2}{6\kappa^2} + \beta_1 \left(d_2 \left(\frac{\varepsilon^2 \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]}{4 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2} - 1 \right) - \frac{\kappa^2}{6} \right) \\
 &\quad - \frac{2\beta_1^{3/2} \sqrt{\beta_3} d_2 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)}{\beta_2} - \frac{\beta_1^2 \beta_3 d_2 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\kappa x + \lambda y + \mu t + \zeta_0) \right] + 1 \right)^2}{\beta_2^2},
 \end{aligned} \quad (27)$$

$$\begin{aligned}
 \phi_8(x, y, t) &= \beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon)^2 \mathcal{A}(\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 \\
 &\quad \times (\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 - \beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} + 1 \right) \\
 &\quad - \frac{1}{4} \beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} + 1 \right)^2 - \frac{\beta_1 (6d_2 \kappa^2 + \kappa^4) + \kappa \mu + 3\lambda^2}{6\kappa^2}.
 \end{aligned} \quad (28)$$

(Figure 3).

$$\begin{aligned}
 \phi_9(x, y, t) &= \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 \\
 &\quad \times (\eta \sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2) \\
 &\quad - \frac{\beta_1 (6d_2 \kappa^2 + \kappa^4) + \kappa \mu + 3\lambda^2}{6\kappa^2} + 2\sqrt{\beta_1} \sqrt{\beta_3} d_2 \left(-\frac{\varepsilon (p + \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} - 1 \right) \\
 &\quad - \beta_3 d_2 \left(\frac{\varepsilon (p + \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} + 1 \right)^2,
 \end{aligned} \quad (29)$$

where $\zeta = \kappa x + \lambda y + \mu t$ (Figure 4).

$$\mu \kappa V'' + \frac{3}{2} \kappa^3 V' V'' + \frac{1}{4} \kappa^4 V'''' + \frac{3}{4} \lambda^2 V'' = 0. \quad (32)$$

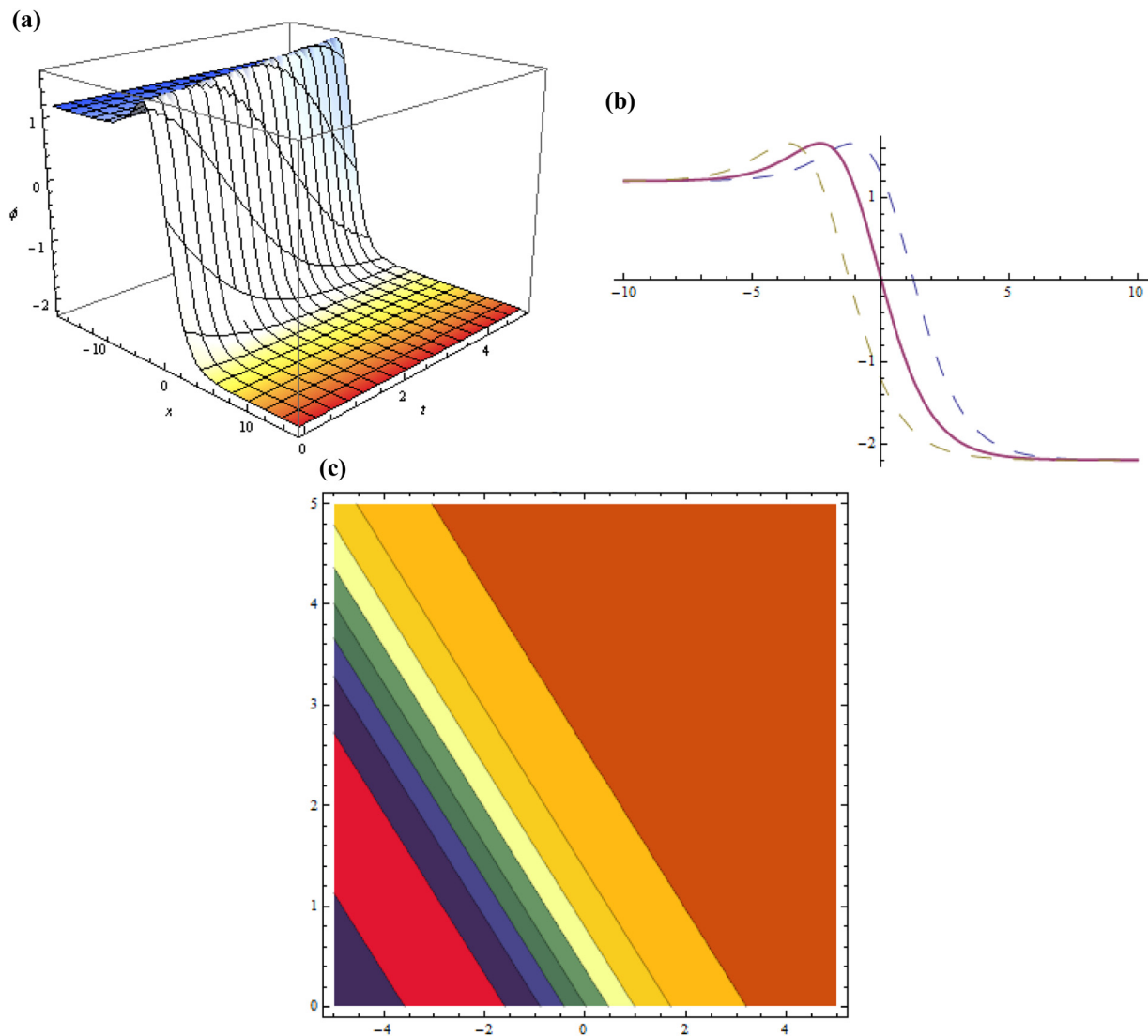


Figure 3: Three dimensional, two dimensional, and contour plots for Eq. (21) representing anti-kink wave soliton when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 2.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $p = 0.3$, $a_0 = 1$, $d_2 = 1$, $\gamma = 1$.

Integrating Eq. (32), once with respect to ζ and integration constant equate to zero, we obtain:

$$\left(\frac{3}{4}\lambda^2 + \mu\kappa\right)V' + \frac{3}{4}\kappa^3(V')^2 + \frac{1}{4}\kappa^4V''' = 0. \quad (33)$$

Balancing highest order derivative and nonlinear term in Eq. (33), we obtain $m = 2$. The general solution for Eq. (33) is given as:

$$V(\zeta) = a_0 + a_1\Psi(\zeta) + a_2\Psi(\zeta)^2 + \frac{b_1}{\Psi(\zeta)} + \frac{b_2}{\Psi(\zeta)^2} + c_2\Psi'(\zeta) + d_1\frac{\Psi'(\zeta)}{\Psi(\zeta)} + d_2\left(\frac{\Psi'(\zeta)}{\Psi(\zeta)}\right)^2. \quad (34)$$

Putting Eq. (34) in Eq. (33), and after solving, we obtain:

Case-I

$$a_1 = \pm 2d_2\sqrt{\beta_1\beta_3}, \quad a_2 = -\beta_3d_2, \quad d_1 = -2\kappa, \quad d_2 = d_2, \quad \mu = \frac{-\beta_1\kappa^4 - 3\lambda^2}{4\kappa}, \quad \beta_2 = \mp 2\sqrt{\beta_1\beta_3}, \quad b_1 = b_2 = c_2 = 0. \quad (35)$$

Substitute Eq. (35) in Eq. (34), only positive value for a_1 , then solutions for Eq. (30) are obtained as:

$$\begin{aligned}
 \phi_1(x, y, t) = & a_0 + (\sqrt{\beta_1} \left(\varepsilon \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^2 \beta_2^2 (\sqrt{\beta_1} d_2 \varepsilon + 2\kappa (\cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)] - 1)) \right. \\
 & - 8\beta_1 \beta_2 \sqrt{\beta_3} d_2 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^3 \\
 & \left. - 4\beta_1^{3/2} \beta_3 d_2 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^4 \right) \Bigg) / 4\beta_2^2 \left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2,
 \end{aligned} \quad (36)$$

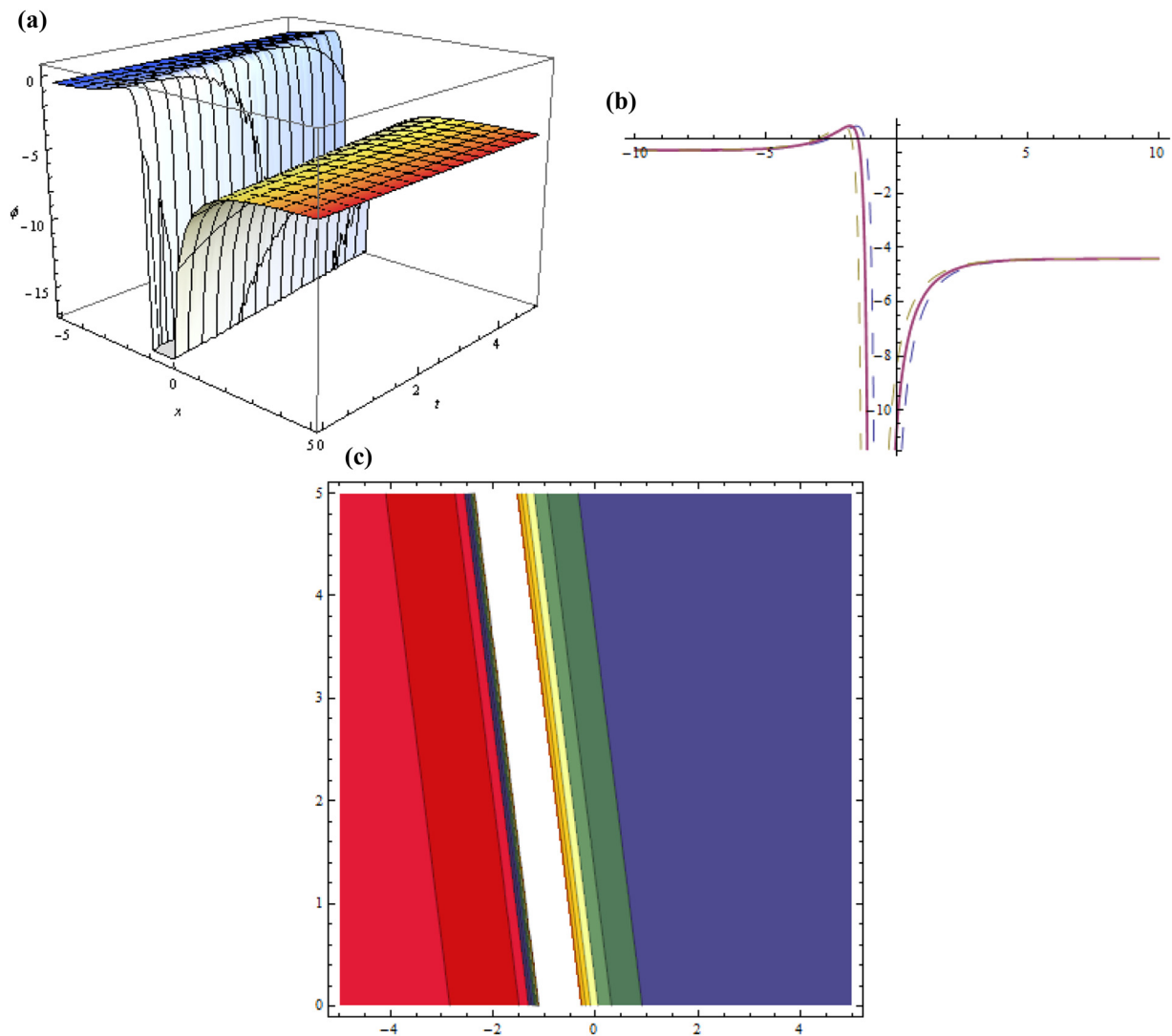


Figure 4: Three dimensional, two dimensional, and contour plots for Eq. (23) representing dark soliton when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 0.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $a_0 = 1$, $d_2 = 1$, $y = 1$.

$$\begin{aligned}
\phi_2(x, y, t) = & a_0 - 2\sqrt{\beta_1}\kappa\varepsilon(\eta \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)/(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] \\
& \eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]) \\
& + \beta_1 d_2(\eta\varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2\lambda(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& - \beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right) - \frac{1}{4}\beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2,
\end{aligned} \tag{37}$$

$$\begin{aligned}
\phi_3(x, y, t) = & a_0 - 2\sqrt{\beta_1}\kappa\varepsilon(\eta\sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)/(\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] \\
& \eta\sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]) \\
& + \beta_1 d_2 \varepsilon^2 (\eta\sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& (\eta\sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& + 2\sqrt{\beta_1}\sqrt{\beta_3} d_2 \left(-\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right) - \beta_3 d_2 \left(\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2.
\end{aligned} \tag{38}$$

(Figure 5).

Case-II

$$a_1 = \pm\sqrt{\beta_3}(\kappa + 2\sqrt{\beta_1}d_2), \quad a_2 = -\beta_3 d_2, \quad d_1 = -\kappa, \quad \mu = \frac{-\beta_1 \kappa^4 - 3\lambda^2}{4\kappa}, \quad d_2 = d_2, \quad \beta_2 = -2\sqrt{\beta_1\beta_3}, \quad b_1 = b_2 = c_2 = 0. \tag{39}$$

Substituting Eq. (39) in Eq. (34), only positive value for a_1 , then solutions for Eq. (30) are obtained as:

$$\begin{aligned}
\phi_4(x, y, t) = & a_0 + \frac{\sqrt{\beta_1}\kappa\varepsilon \operatorname{csch}\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right]^2}{2\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 2} + \frac{\beta_1 d_2 \varepsilon^2 \operatorname{csch}\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right]^4}{4\left(\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 1\right)^2} \\
& - \frac{\beta_1 \sqrt{\beta_3}(2\sqrt{\beta_1}d_2 + \kappa)\left(\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 1\right)}{\beta_2} - \frac{\beta_1^2 \beta_3 d_2 \left(\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 1\right)^2}{\beta_2^2},
\end{aligned} \tag{40}$$

$$\begin{aligned}
\phi_5(x, y, t) = & a_0 - \sqrt{\beta_1}\kappa\varepsilon(\eta \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)/(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] \\
& \eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] - \frac{1}{2}\sqrt{\beta_1}(2\sqrt{\beta_1}d_2 + \kappa)\left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1\right) \\
& + \beta_1 d_2(\eta\varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2\lambda(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) - \frac{1}{4}\beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2,
\end{aligned} \tag{41}$$

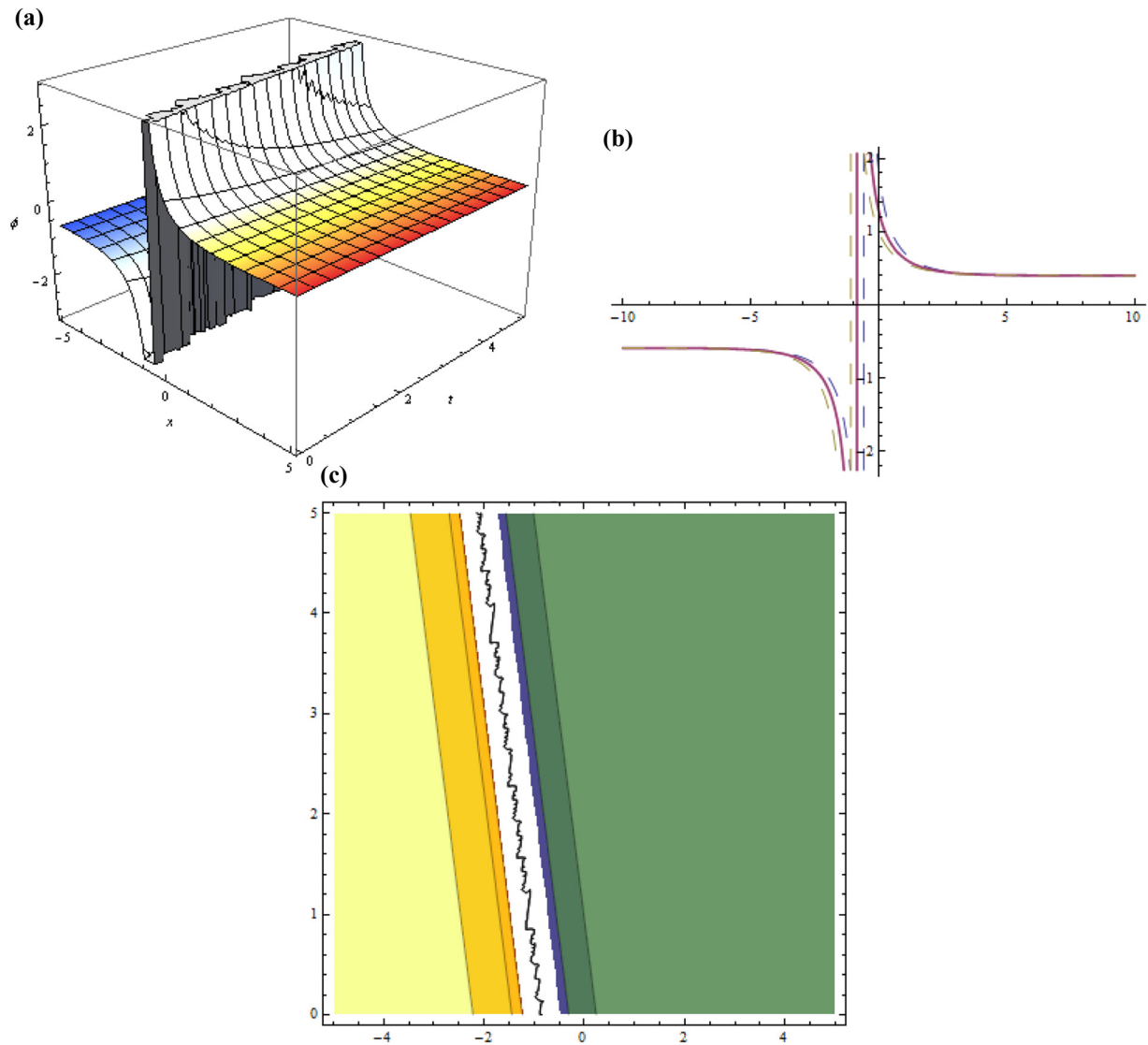


Figure 5: Three dimensional, two dimensional, and contour plots for Eq. (36) representing solitary wave when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 0.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $a_0 = 1$, $d_2 = 1$, $\gamma = 1$.

$$\begin{aligned}
 \phi_6(x, y, t) = & a_0 + \sqrt{\beta_1} \kappa \varepsilon (\eta(-\sqrt{p^2 + 1}) \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] - 1) \\
 & / (\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]) \\
 & \times \eta\sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \\
 & \beta_1 d_2 \varepsilon^2 (\eta\sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
 & \times (\eta\sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]^2) \\
 & + \sqrt{\beta_3} (2\sqrt{\beta_1} d_2 + \kappa) \left(-\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right) \\
 & - \beta_3 d_2 \left(\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta\sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2.
 \end{aligned} \tag{42}$$

(Figure 6).

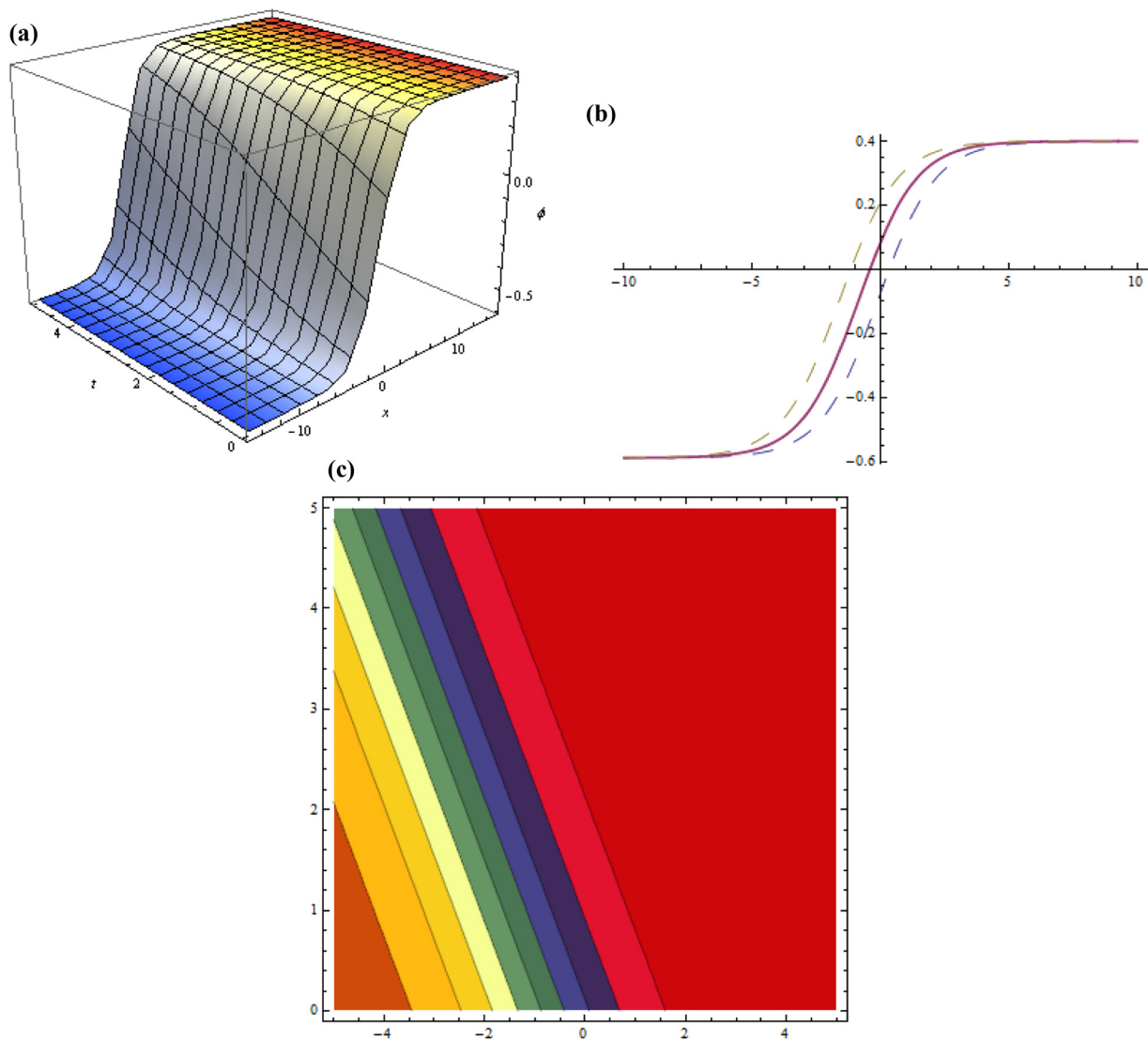


Figure 6: Three dimensional, two dimensional, and contour plots for Eq. (37) representing kink wave soliton when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 1.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $a_0 = 1$, $d_2 = 1$, $y = 1$.

Case-III

$$\begin{aligned} a_1 &= -\beta_2 d_2, \quad a_2 = -\frac{\beta_2^2 d_2}{4\beta_1}, \quad d_1 = -2\kappa, \\ \mu &= \frac{-\beta_1 \kappa^4 - 3\lambda^2}{4\kappa}, \quad d_2 = d_2, \quad \beta_3 = \frac{\beta_2^2}{4\beta_1}, \\ b_1 &= b_2 = c_2 = 0. \end{aligned} \quad (43)$$

Substituting Eq. (43) in Eq. (34), then solutions for Eq. (30), we obtain:

$$\begin{aligned} \phi_7(x, y, t) &= a_0 + \frac{\sqrt{\beta_1} \kappa \varepsilon \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^2}{\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1} \\ &+ \frac{1}{4} \left(3 + 2\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] \right. \\ &- \varepsilon^2 \coth^2 \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] \\ &\left. + \frac{\varepsilon^2 \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^4}{\left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2} \right) d_2 \beta_1. \end{aligned} \quad (44)$$

$$\begin{aligned}
\phi_8(x, y, t) = & a_0 - 2\sqrt{\beta_1} \kappa \varepsilon (\eta \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] \\
& \times \eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]) + \beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2 \\
& \mathcal{A}(\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& + \frac{1}{2} \beta_2 \sqrt{\frac{\beta_1}{\beta_3}} d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right) - \frac{\beta_2^2 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2}{16\beta_3}.
\end{aligned} \quad (45)$$

$$\begin{aligned}
\phi_9(x, y, t) = & a_0 - 2\sqrt{\beta_1} \kappa \varepsilon (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1) / \\
& \times (\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] \eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]) \\
& + \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]^2) \\
& - \beta_2 d_2 \left(-\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right) - \frac{\beta_2^2 d_2 \left(\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2}{4\beta_1}.
\end{aligned} \quad (46)$$

(Figure 7).

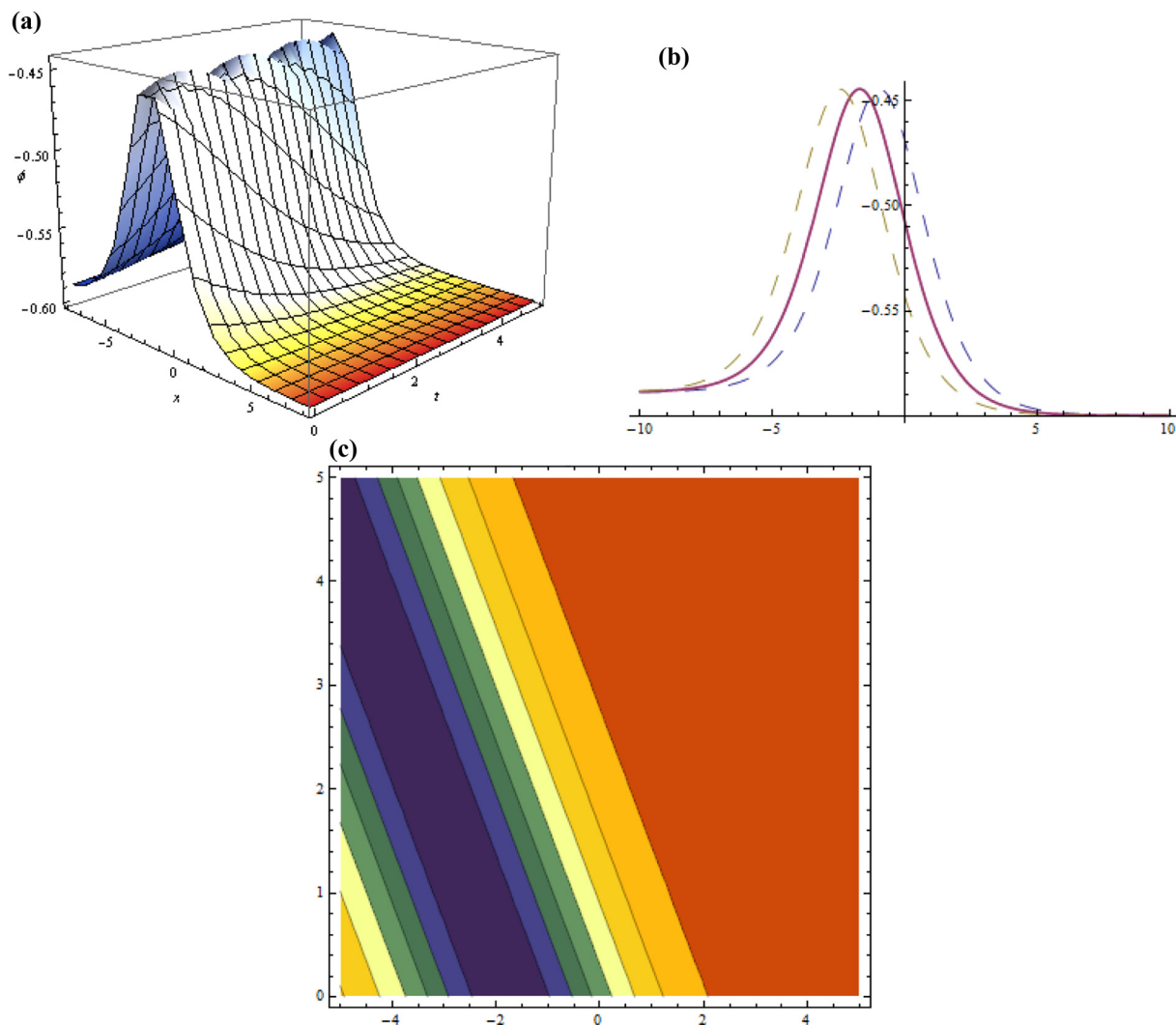


Figure 7: Three dimensional, two dimensional, and contour plots for Eq. (38) representing bright soliton when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 1.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $p = 0.8$, $a_0 = 1$, $d_2 = 1$, $y = 1$.

Case-IV

$$V(\zeta) = \phi(x, y, t), \quad \zeta = \kappa x + \lambda y + \mu t. \quad (52)$$

$$a_1 = \frac{\beta_2 \kappa - 2\sqrt{\beta_1} \beta_2 d_2}{2\sqrt{\beta_1}}, \quad a_2 = -\frac{\beta_2^2 d_2}{4\beta_1},$$

$$b_1 = b_2 = c_2 = 0, \quad d_1 = -\kappa, \quad (47)$$

$$d_2 = d_2, \quad \mu = \frac{-\beta_1 \kappa^4 - 3\lambda^2}{4\kappa}, \quad \beta_3 = \frac{\beta_2^2}{4\beta_1}.$$

Substituting Eq. (52) in Eq. (51), ODE of Eq. (51), we obtain:

$$(\mu \kappa V' + 6\kappa^2 V V' + 6\alpha \kappa^2 V^2 V' + \kappa^4 V''')' + \lambda^2 V'' = 0. \quad (53)$$

Integrating Eq. (53), once with respect to ζ and integration constant equal to zero, Eq. (53) becomes

$$\mu \kappa V' + 6\kappa^2 V V' + 6\alpha \kappa^2 V^2 V' + \kappa^4 V'' + \lambda^2 V' = 0. \quad (54)$$

Substituting Eq. (47) in Eq. (34), then solutions for Eq. (30), we obtain:

$$\begin{aligned} \phi_{10}(x, y, t) = & \frac{1}{4} \left(4a_0 + 2\sqrt{\beta_1} \kappa \left(-\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + \frac{\varepsilon \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^2}{\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1} - 1 \right) \right. \\ & \left. + \left(3 + 2\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] - \varepsilon^2 \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^2 + \frac{\varepsilon^2 \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^4}{\left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2} \right) d_2 \beta_1 \right). \end{aligned} \quad (48)$$

$$\begin{aligned} \phi_{11}(x, y, t) = & a_0 - \sqrt{\beta_1} \kappa \varepsilon (\eta \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + 1) (\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] \eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] \\ & + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)]) + \beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon)^2 (\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 \\ & \times (\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 \\ & - \frac{\beta_2 \sqrt{\frac{\beta_1}{\beta_3}} (\kappa - 2\sqrt{\beta_1} d_2) \left(\frac{\varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} + 1 \right)}{4\sqrt{\beta_1}} - \frac{\beta_2^2 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} + 1 \right)^2}{16\beta_3}. \end{aligned} \quad (49)$$

$$\begin{aligned} \phi_{12}(x, y, t) = & a_0 + \sqrt{\beta_1} \kappa \varepsilon (\eta (-\sqrt{p^2 + 1}) \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + p \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)] - 1) / (\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]) \\ & \times \eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)] \\ & + \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 \\ & \times (\eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])^2 \\ & + \frac{\beta_2 (\kappa - 2\sqrt{\beta_1} d_2) \left(-\frac{\varepsilon (p + \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} - 1 \right)}{2\sqrt{\beta_1}} - \frac{\beta_2^2 d_2 \left(\frac{\varepsilon (p + \sinh[\sqrt{\beta_1} (\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1} (\zeta + \zeta_0)]} + 1 \right)^2}{4\beta_1}, \end{aligned} \quad (50)$$

where $\zeta = \kappa x + \lambda y + \mu t$ (Figure 8).

4.3 Gardner KP equation

Here we take the Gardner KP equation as:

$$(\phi_t + 6\phi\phi_x + 6\alpha\phi^2\phi_x + \phi_{xxx})_x + \phi_{yy} = 0. \quad (51)$$

We apply the wave transformation as:

Balancing the highest order derivative and nonlinear term in Eq. (54), we obtain $m = 2$. The general solution for Eq. (54) is as follows:

$$\begin{aligned} V(\zeta) = & a_0 + a_1 \Psi(\zeta) + a_2 \Psi(\zeta)^2 + \frac{b_1}{\Psi(\zeta)} + \frac{b_2}{\Psi(\zeta)^2} \\ & + c_2 \Psi'(\zeta) + d_1 \frac{\Psi'(\zeta)}{\Psi(\zeta)} + d_2 \left(\frac{\Psi'(\zeta)}{\Psi(\zeta)} \right)^2. \end{aligned} \quad (55)$$

Substituting Eq. (55) in Eq. (54), and after solving, we obtain:

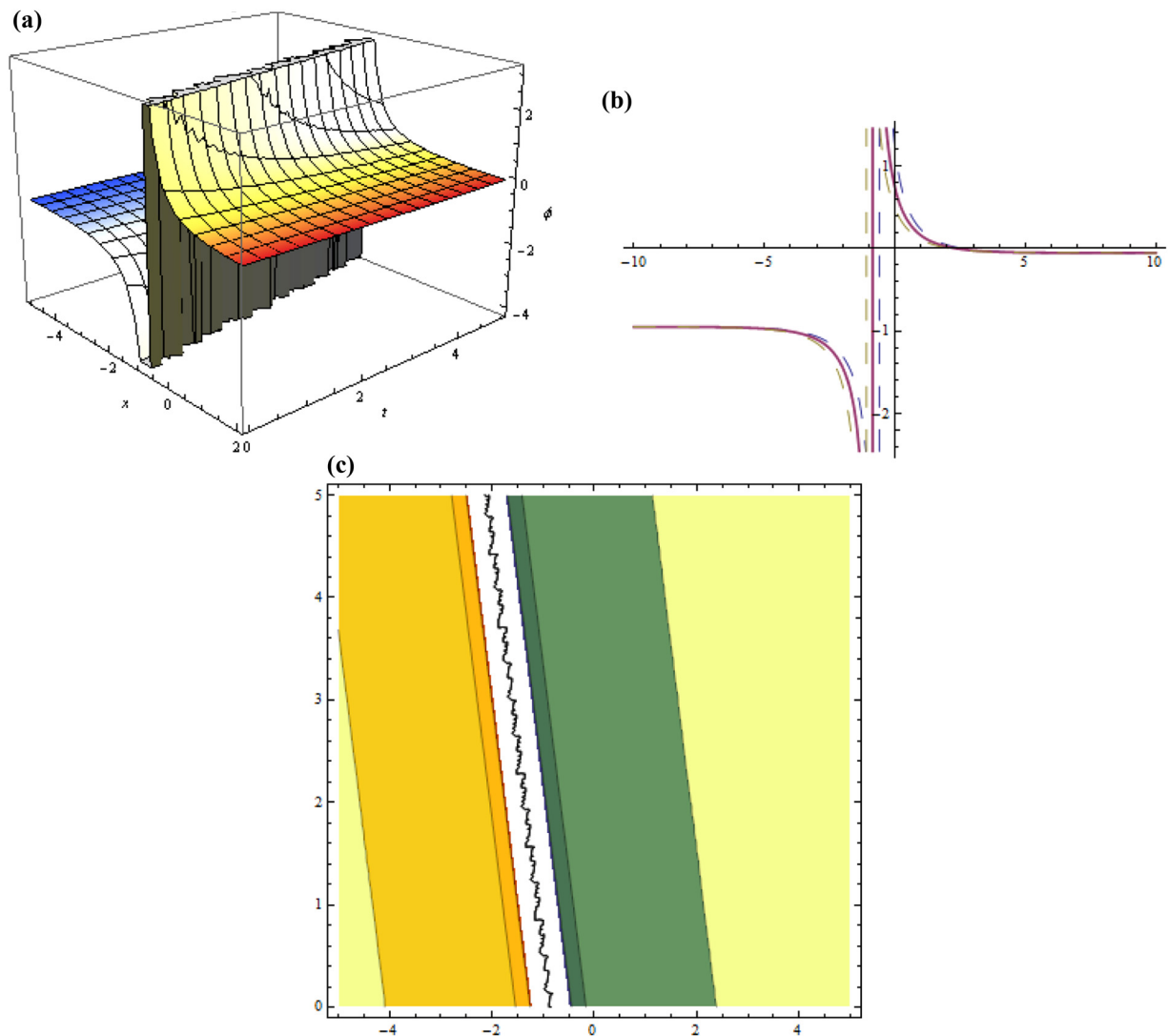


Figure 8: Three dimensional, two dimensional, and contour plots for Eq. (57) representing solitary wave when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 0.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = -1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $\alpha = 1$, $a_0 = 1$, $d_2 = 1$, $\gamma = 1$.

Case-I

$$a_0 = \frac{-\sqrt{\alpha}\beta_2\kappa - 2\sqrt{\beta_3} - 4\alpha\sqrt{\beta_3}\beta_1d_2}{4\alpha\sqrt{\beta_3}},$$

$$a_1 = \frac{-\sqrt{\beta_3}\kappa - \sqrt{\alpha}\beta_2d_2}{\sqrt{\alpha}}, \quad a_2 = \beta_3(-d_2), \quad d_2 = d_2, \quad (56)$$

$$\mu = \frac{3\alpha\beta_2^2\kappa^4 - 8\alpha\beta_1\beta_3\kappa^4 - 8\alpha\beta_3\lambda^2 + 12\beta_3\kappa^2}{8\alpha\beta_3\kappa},$$

$$b_1 = b_2 = c_2 = d_1 = 0,$$

where $\alpha < 0$, and substitute Eq. (56) in Eq. (55), then we obtain solutions for Eq. (51) as:

$$\phi_1(x, y, t) = \frac{1}{4} \left(\frac{\beta_1 d_2 \varepsilon^2 \operatorname{csch} \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right]^4}{\left(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1 \right)^2} - \frac{\frac{\sqrt{\alpha}\beta_2\kappa}{\sqrt{\beta_3}} + 4\alpha\beta_1d_2 + 2}{\alpha} \right. \\ \left. + \frac{4\beta_1(\sqrt{\beta_3}\kappa + \sqrt{\alpha}\beta_2d_2)(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1)}{\sqrt{\alpha}\beta_2} - \frac{4\beta_1^2\beta_3d_2(\varepsilon \coth \left[\frac{1}{2} \sqrt{\beta_1} (\zeta + \zeta_0) \right] + 1)^2}{\beta_2^2} \right). \quad (57)$$

(Figure 9).

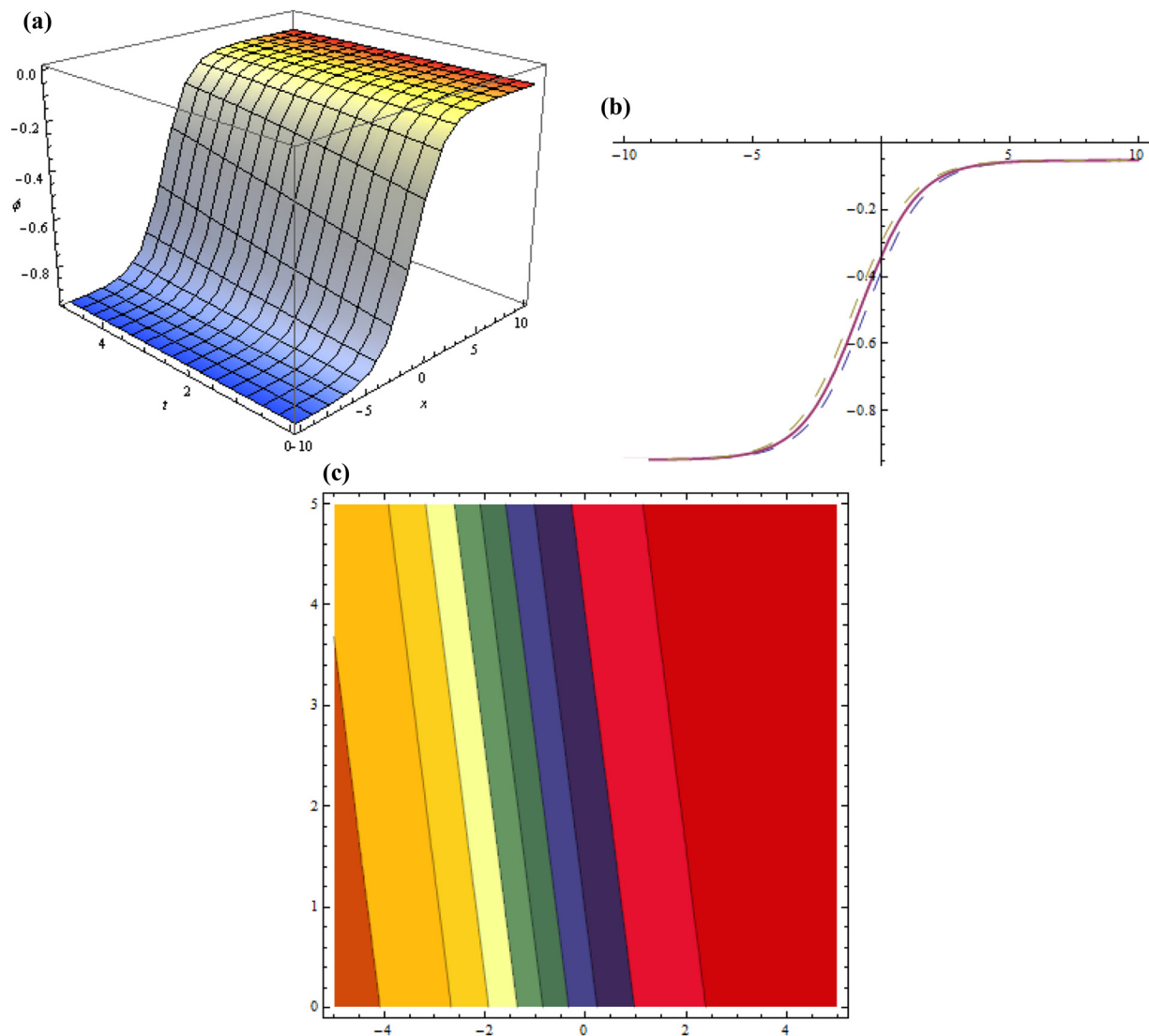


Figure 9: Three dimensional, two dimensional, and contour plots for Eq. (58) representing kink wave soliton when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 1.5$, $\lambda = 1.4$, $\varepsilon = 1$, $\eta = 1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $\alpha = 1$, $a_0 = 1$, $d_2 = 1$, $\gamma = 1$.

$$\begin{aligned}
 \phi_2(x, y, t) = & \frac{1}{4} \left(4\beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2 \lambda (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \right. \\
 & \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 - \beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2 \\
 & \left. - \frac{\frac{\sqrt{\alpha} \beta_2 \kappa}{\sqrt{\beta_3}} + 4\alpha \beta_1 d_2 + 2}{\alpha} - \frac{2\sqrt{\frac{\beta_1}{\beta_3}} (-\sqrt{\beta_3} \kappa - \sqrt{\alpha} \beta_2 d_2) \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)}{\sqrt{\alpha}} \right).
 \end{aligned} \tag{58}$$

$$\begin{aligned}
\phi_3(x, y, t) = & \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta \sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& - \frac{\frac{\sqrt{\alpha}\beta_2\kappa}{\sqrt{\beta_3}} + 4\alpha\beta_1 d_2 + 2}{4\alpha} + \frac{(-\sqrt{\beta_3}\kappa - \sqrt{\alpha}\beta_2 d_2) \left(-\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right)}{\sqrt{\alpha}} \\
& - \beta_3 d_2 \left(\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2.
\end{aligned} \quad (59)$$

Case-II

$$a_0 = \frac{-2\alpha\beta_1 d_2 - 1}{2\alpha}, \quad a_1 = -\frac{\sqrt{\beta_3}\kappa}{\sqrt{\alpha}}, \quad a_2 = -\beta_3 d_2, \quad d_2 = d_2, \quad \mu = \frac{-2\alpha\beta_1\kappa^4 - 2\alpha\lambda^2 + 3\kappa^2}{2\alpha\kappa}, \quad b_1 = b_2 = c_2 = d_1 = 0, \quad (60)$$

where $\alpha < 0$, substituting Eq. (60) in Eq. (55), then solutions for Eq. (51) become:

$$\begin{aligned}
\phi_4(x, y, t) = & \frac{1}{4} \left(\frac{4\sqrt{\alpha}\beta_1\sqrt{\beta_3}\kappa \left(\varepsilon \coth \left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0) \right] + 1 \right) - 2\beta_2}{\alpha\beta_2} + \right. \\
& \times d_2\beta_1 \left(-4 + \frac{\varepsilon^2 \operatorname{csch} \left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0) \right]^4}{\left(\varepsilon \coth \left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0) \right] + 1 \right)^2} - \frac{4\beta_1\beta_3 \left(\varepsilon \coth \left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0) \right] + 1 \right)^2}{\beta_2^2} \right) \Bigg).
\end{aligned} \quad (61)$$

$$\begin{aligned}
\phi_5(x, y, t) = & \frac{1}{4} \left(-\frac{2}{\alpha} - 4\beta_1 d_2 + 4\beta_1 d_2 4\beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2 / ((\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \right. \\
& \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& - \beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2 + \frac{2\sqrt{\frac{\beta_1}{\beta_3}} \sqrt{\beta_3} \kappa \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)}{\sqrt{\alpha}} \Bigg).
\end{aligned} \quad (62)$$

$$\begin{aligned}
\phi_6(x, y, t) = & -\frac{1}{2\alpha} - \beta_1 d_2 - \frac{\sqrt{\beta_3}\kappa \left(-\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} - 1 \right)}{\sqrt{\alpha}} \\
& + \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta \sqrt{p^2 + 1} + p\varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& - \beta_3 d_2 \left(\frac{\varepsilon(p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2.
\end{aligned} \quad (63)$$

(Figure 10).

Case-III

$$a_0 = \frac{-2\alpha\beta_1 d_2 - 1}{2\alpha}, \quad a_2 = -\beta_3 d_2, \quad d_1 = \pm \frac{\kappa}{\sqrt{\alpha}}, \quad d_2 = d_2, \quad \mu = \frac{4\alpha\beta_1\kappa^4 - 2\alpha\lambda^2 + 3\kappa^2}{2\alpha\kappa}, \quad a_1 = b_1 = b_2 = c_2 = 0, \quad (64)$$

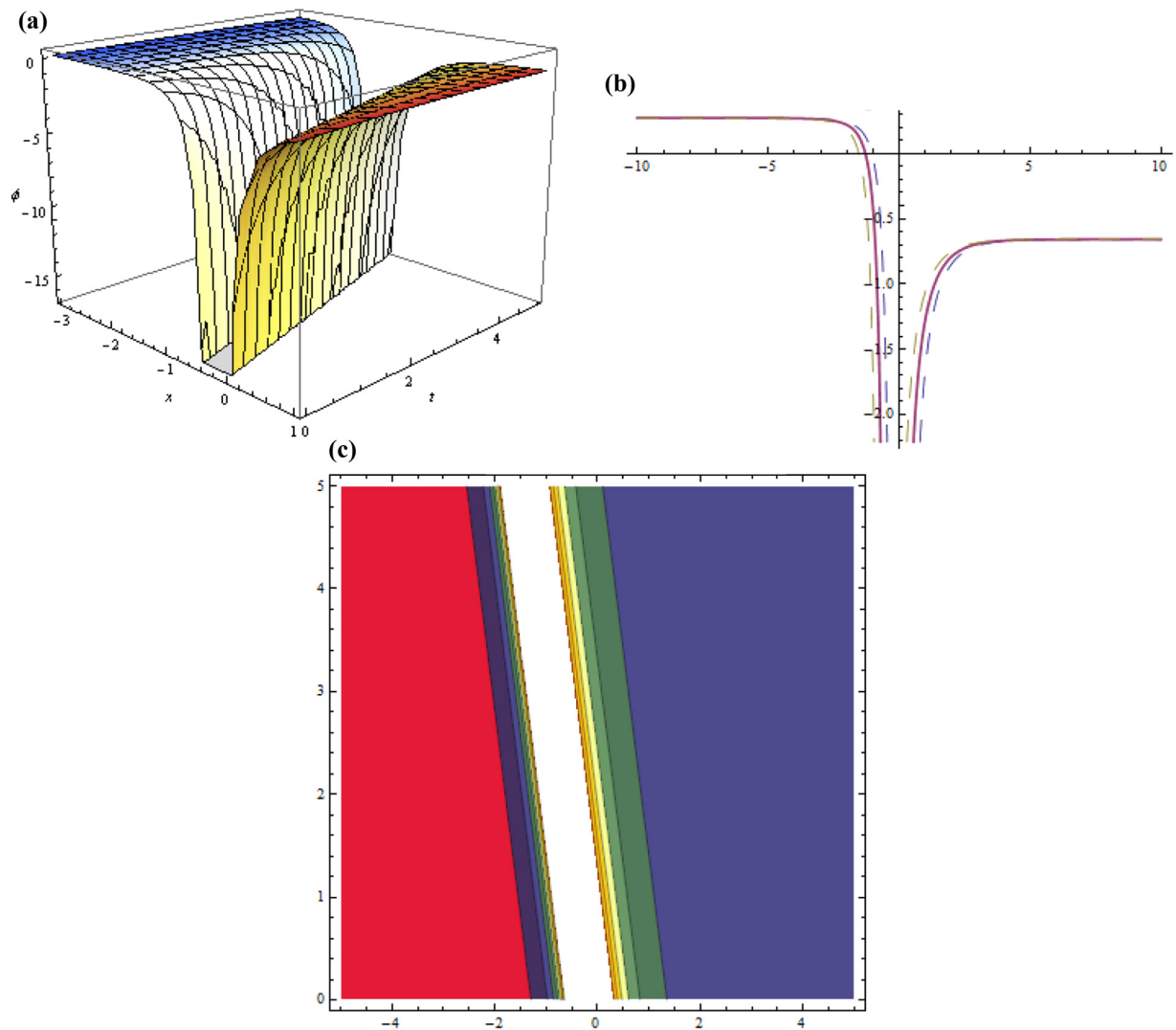


Figure 10: Three dimensional, two dimensional, and contour plots for Eq. (59) representing dark soliton when $\zeta_0 = 0.3$, $\kappa = 2$, $\mu = 0.5$, $\lambda = 1.4$, $\varepsilon = -1$, $\eta = -1$, $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.2$, $p = 0.8$, $\alpha = 1$, $a_0 = 1$, $d_2 = 1$, $y = 1$.

where $\alpha < 0$, substituting Eq. (64) only positive value for d_1 in Eq. (55), then solutions for Eq. (51) become:

$$\begin{aligned} \phi_7(x, y, t) = & \frac{1}{4} \left(-\frac{2}{\alpha} - \frac{2\sqrt{\beta_1}\kappa\varepsilon \operatorname{csch}\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right]^2}{\sqrt{\alpha}\left(\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 1\right)} + \beta_1 d_2 \left(\frac{\varepsilon^2 \operatorname{csch}\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right]^4}{\left(\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 1\right)^2} - 4 \right) \right. \\ & \left. - \frac{4\beta_1^2\beta_3 d_2 \left(\varepsilon \coth\left[\frac{1}{2}\sqrt{\beta_1}(\zeta + \zeta_0)\right] + 1\right)^2}{\beta_2^2} \right). \end{aligned} \quad (65)$$

where $\zeta = \kappa x + \lambda y + \mu t$.

$$\begin{aligned}
\phi_8(x, y, t) = & -\frac{1}{2\alpha} + \sqrt{\beta_1} \kappa \varepsilon (\eta \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1) (\sqrt{\alpha} (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]) \\
& \times \eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]) - \beta_1 d_2 \\
& + \beta_1 d_2 (\eta \varepsilon \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon)^2 (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) - \frac{1}{4} \beta_1 d_2 \left(\frac{\varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]}{\eta + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2.
\end{aligned} \quad (66)$$

$$\begin{aligned}
\phi_9(x, y, t) = & -\frac{1}{2\alpha} + \sqrt{\beta_1} \kappa \varepsilon (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1) / \\
& (\sqrt{\alpha} (\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]) \\
& \eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)]) - \beta_1 d_2 \\
& + \beta_1 d_2 \varepsilon^2 (\eta \sqrt{p^2 + 1} \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] - p \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)] + 1)^2 / ((\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2 \\
& \times (\eta \sqrt{p^2 + 1} + p \varepsilon + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)] + \varepsilon \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])^2) \\
& - \beta_3 d_2 \left(\frac{\varepsilon (p + \sinh[\sqrt{\beta_1}(\zeta + \zeta_0)])}{\eta \sqrt{p^2 + 1} + \cosh[\sqrt{\beta_1}(\zeta + \zeta_0)]} + 1 \right)^2,
\end{aligned} \quad (67)$$

5 Results and discussion

We successfully constructed the new solitary wave solutions including bright and dark, kink and anti-kink wave solitons for three nonlinear PDEs by the proposed new modified technique. Here we discuss the similarity and difference between our new determined results with previous literature which have been found by other methods. The scholar found a numerical solutions including solitons by using the adomian decomposition method which is mentioned in ref. [5]. Our constructed solutions are new, more general, and different from that obtained in the existing literature. These new solutions are obtained in rational, elliptic, trigonometric, and hyperbolic form including singular bright and dark solitons, kink and anti-kink wave solutions.

From the above complete discussion we conclude that our constructed solutions are different and new, which have not been determined in the existing literature. The all calculation shows that the proposed technique is more powerful, effective, and reliable for analytically investigating other various kinds of nonlinear PDEs.

6 Conclusion

In the present work, we successfully proposed modified technique on three NLPDEs. We have determined new ion acoustic solitary wave solutions for KP equation,

potential KP equation, and Gardner KP equation by the modified mathematical method. We obtained the ion-acoustic solitary wave solutions including singular dark solitons, singular bright solitons, kink and anti-kink wave solitons. We also represent physical interpretation of some obtained results by two-dimensional, three-dimensional, and contour graphs by Mathematica to understand the physical structure for ion-acoustic solitary wave. The obtained solutions may have potential applications in quantum plasma, optical fibers, soliton dynamics, laser optics, dynamic of adiabatic parameters, fluid dynamics, and various other fields. The complete calculations show that the proposed technique is more efficient, powerful, straightforward, and fruitful to investigate analytically other various kinds of NLPDEs.

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