Research Article

Esra Karatas Akgül, Ali Akgül, Wasim Jamshed*, Zulfiqar Rehman, Kottakkaran Sooppy Nisar, Mohammed S. Alqahtani, and Mohamed Abbas

# Analysis of respiratory mechanics models with different kernels

https://doi.org/10.1515/phys-2022-0027
received December 13, 2021; accepted March 28, 2022

**Abstract:** In this article, we investigate the mechanics of breathing performed by a ventilator with different kernels by an effective integral transform. We mainly obtain the solutions of the fractional respiratory mechanics model. Our goal is to give the underlying model flexibly by making use of the advantages of the non-integer order operators. The big advantage of fractional derivatives is that we can formulate models describing much better the systems with memory effects. Fractional operators with different memories are related to different types of relaxation process of the non-local dynamical systems. Additionally, since we consider the utilisation of different kinds of fractional derivatives, most often having benefit in the implementation, the similarities and differences can be obviously seen between these derivatives.

**Keywords:** Sumudu transforms, fractional derivatives, respiratory mechanics models

*** Corresponding author: Wasim Jamshed,** Department of Mathematics, Capital University of Science and Technology (CUST), Islamabad, 44000, Pakistan, e-mail: wasiktk@hotmail.com
**Esra Karatas Akgül, Ali Akgül:** Department of Mathematics, Siirt University, Art and Science Faculty, TR-56100 Siirt, Turkey
**Zulfiqar Rehman:** Department of Mathematics, Air University, Islamabad, 44000, Pakistan
**Kottakkaran Sooppy Nisar:** Department of Mathematics, College of Arts and Sciences, Prince Sattam Bin Abdulaziz University, Wadi Aldawaser, Wadi 11991, Saudi Arabia
**Mohammed S. Alqahtani:** Radiological Sciences Department, College of Applied Medical Sciences, King Khalid University, Abha 61421, Saudi Arabia; Bioimaging Unit, Space Research Centre, Michael Atiyah Building, University of Leicester, Leicester, LE1 7RH, United Kingdom
**Mohamed Abbas:** Electrical Engineering Department, College of Engineering, King Khalid University, Abha 61421, Saudi Arabia; Computers and Communications Department, College of Engineering, Delta University for Science and Technology, Gamasa 35712, Egypt

## 1 Introduction

The mechanical features of the respiratory system can be schematised in terms of resistance, compliance and inertance. The latter is considered less valuable in describing the overall mechanical behaviour at the breathing frequencies. Many techniques for the assessment of respiratory mechanics are related on static measures following flow interruption or on the frequency/time domain, off-line identification of lumped variable models of the respiratory system [1].

Fractional analysis, which has many implementations in science and engineering, is a valuable mathematical analysis branch in which derivatives and integrals are extended to non-integer orders. The notions of fractional derivative and fractional integral were discussed in more detail in the 18th and 19th centuries. Many works demonstrate the benefits of the non-local fractional derivatives and integrals providing us to see real-world problems associated with the memory effect. The area of fractional calculus, which has been discussed for centuries, has provided many different descriptions of fractional derivatives and integrals to offer, and one of the most valuable descriptions is the Riemann–Liouville (RL) definition. This definition for arbitrary derivative and integral of any function was offered in the late 19th century with a complex analysis approximation. Many fractional derivatives are special cases of this general definition of RL. This important description is acquired by generalizing the Cauchy integral formula under favour of the iterated derivatives of a complex analytical function, and there are many non-local fractional derivative descriptions acquired from iterating some local derivatives in literature. Thus, one of the most wide and comprehensible ways of describing fractional calculus is to investigate the description of RL [2].

Differential equations including fractional derivatives are created in order to be able to make more precise measurements in understanding and modeling events in nature and to produce solutions with less errors. They solve these equations with integral transformations. The Sumudu transform is just one of these integral transformations, it does not lose the uniqueness and form of the function. Converting it to a function increases its usefulness. The exact operations of some linear fractional differentials including Caputo, Caputo–Fabrizio, Atangana–Baleanu and Constant Caputo fractional derivatives will be obtained [3–7]. For more details see refs. [8–15].

We arranged this study as follows: We present the fundamental definitions of fractional calculus to use for the main results of the article in Section 2. Then, we apply the Sumudu transformation to the mechanics models with power-law, exponential-decay, Mittag-Leffler and hybrid kernels in Section 3. We present the conclusion in the last section.

## 2 Mathematical background

**Definition 1.** We describe the Sumudu transform over

$$A = \{Y(t) | \exists N, \tau_1, \tau_2 > 0, |Y(t)| < N \exp(|t|/\tau_j), \text{ if } t \in (-1)^j \times [0, \infty)\}, \tag{1}$$

by ref. [16]:

$$G(u) = S[Y(t)] = \int_0^\infty Y(ut)\exp(-t)\mathrm{d}t, \ u \in (-\tau_1, \tau_2). \tag{2}$$

**Definition 2.** We describe the Mittag-Leffler kernels $E_\eta(\tau)$ and $E_{\eta,\beta}(\tau)$ as ref. [17]:

$$E_\eta(\tau) = \sum_{j=0}^{\infty} \frac{\tau^j}{(\eta j + 1)} \ (\tau \in \mathbb{C}, \mathrm{Re}(\eta) > 0), \tag{3}$$

and

$$E_{\eta,\beta}(\tau) = \sum_{j=0}^{\infty} \frac{\tau^j}{(\eta j + \beta)} \ (\tau, \beta \in \mathbb{C}, \mathrm{Re}(\eta) > 0). \tag{4}$$

**Definition 3.** We present the fractional derivative with the power-law kernel as ref. [17]:

$${}_a^C D_t^\eta Y(t) = \frac{1}{(n-\eta)} \int_a^t (t-\tau)^{n-\eta-1} Y^{(n)}(\tau)\mathrm{d}\tau, \tag{5}$$

and

$${}_b^C D_t^\eta Y(t) = \frac{(-1)^n}{(n-\eta)} \int_t^b (t-\tau)^{n-\eta-1} Y^{(n)}(\tau)\mathrm{d}\tau. \tag{6}$$

**Lemma 1.** *We have the Sumudu transform of the fractional derivative with the power-law kernel as ref.* [16]:

$$S[{}_a^C D_t^\eta Y(t)] = \frac{G[u] - Y(a)}{u^\eta}, \tag{7}$$

*where* $G[u] = S[Y(t)]$.

**Definition 4.** Suppose that $\theta, \lambda : [0, \infty) \to \mathfrak{R}$. Then, we have the convolution of $\theta, \lambda$ by ref. [16]:

$$(\theta \star \lambda) = \int_0^t \theta(t-u)\lambda(u)\mathrm{d}u, \tag{8}$$

thus, we reach

$$S\{(\theta \star \lambda)(t)\} = uS\{\theta(t)\}S\{\lambda(t)\}. \tag{9}$$

**Definition 5.** We describe the fractional derivative with the exponential-decay kernel as ref. [17]:

$${}_a^{CFC} D_t^\eta Y(t) = \frac{M(\eta)}{1-\eta} \int_a^t Y'(x)\exp(-\lambda(t-x))\mathrm{d}x, \tag{10}$$

and

$${}_b^{CFC} D_t^\eta Y(t) = \frac{-M(\eta)}{1-\eta} \int_t^b Y'(x)\exp(-\lambda(t-x))\mathrm{d}x. \tag{11}$$

**Lemma 2.** *We have the Sumudu transform of fractional derivative with the exponential-decay kernel as ref.* [16]:

$$S[{}_a^{CFC} D_t^\eta Y(t)] = \frac{M(\eta)}{(1-\eta)} \frac{G[u]}{\left(1 + \frac{\eta}{1-\eta}u\right)} - \frac{M(\eta)}{(1-\eta)} \frac{Y(a)}{\left(1 + \frac{\eta}{1-\eta}u\right)}. \tag{12}$$

**Definition 6.** We describe the fractional derivative with the Mitttag-Leffler kernel as ref. [17]:

$${}_a^{ABC} D_t^\eta Y(t) = \frac{B(\eta)}{1-\eta} \int_a^t Y'(x) E_\eta(-\lambda(t-x)^\eta)\mathrm{d}x, \tag{13}$$

and

$${}_a^{ABC} D_t^\eta Y(t) = \frac{-B(\eta)}{1-\eta} \int_t^b Y'(x) E_\eta(-\lambda(t-x)^\eta)\mathrm{d}x. \tag{14}$$

**Lemma 3.** *We have the Sumudu transform of the fractional derivative with the Mitttag-Leffler kernel as ref.* [16]:

$$S[{}_a^{ABC} D_t^\eta Y(t)] = \frac{B(\eta)}{(1-\eta)} \frac{G[u]}{\left(1 + \frac{\eta}{1-\eta}u^\eta\right)} - \frac{B(\eta)}{(1-\eta)} \frac{Y(a)}{\left(1 + \frac{\eta}{1-\eta}u^\eta\right)}. \tag{15}$$

**Definition 7.** We describe the fractional derivative with the hybrid kernel as ref. [18]:

$$
\begin{aligned}
&{}^{CPC}_{\;\;a}D_t^{\eta}Y(t)\\
&= \frac{1}{(1-\eta)}\int_a^t (k_1(\eta)Y(\tau)+k_0(\eta)Y'(\tau))(t-\tau)^{-\eta}\mathrm{d}\tau.
\end{aligned} \tag{16}
$$

**Lemma 4.** *We have the Sumudu transform of the fractional derivative with the hybrid kernel as:*

$$
\begin{aligned}
&S\{{}^{CPC}_{\;\;a}D_t^{\eta}Y(t)\}\\
&\quad = k_1(\eta)S\{Y(t)\}u^{1-\eta} + k_0(\eta)[S\{Y(t)\} - Y(a)]u^{-\eta}.
\end{aligned} \tag{17}
$$

**Lemma 5.** *We have the following relations ref.* [16]:

$$S[E_\eta(-\lambda t^\eta)] = \frac{1}{1+\lambda u^\eta}, \tag{18}$$

$$S[1 - E_\eta(-\lambda t^\eta)] = \frac{\lambda u^\eta}{1+\lambda u^\eta}. \tag{19}$$

# 3 Implementations

We construct the analytical solutions of fractional respiratory mechanics models by Sumudu transform in this section.

## 3.1 Respiratory mechanics models with Caputo fractional derivative

First, we consider the respiratory mechanics model equations with the power-law kernel as:

$$R[{}^{C}_{0}D_t^{\eta}Y_i(t)] + \left(\frac{1}{C}\right)Y_i(t) + P_{\mathrm{m}} = P_{\mathrm{d}}, \quad 0 \le t \le t_j, \tag{20}$$

$$R[{}^{C}_{0}D_t^{\eta}Y_e(t)] + \left(\frac{1}{C}\right)Y_e(t) + P_{\mathrm{m}} = 0, \quad t_j \le t \le t_b, \tag{21}$$

$$Y_i(0) = Y_e(t_b) = 0, \tag{22}$$

$$Y_i(t_j) = Y_e(t_j) = Y_\tau. \tag{23}$$

We implement the Sumudu transformation to both sides of the Eq. (22) under the condition $Y_i(0) = 0$, then we have

$$S\{R[{}^{C}_{0}D_t^{\eta}Y_i(t)]\} + S\left\{\left(\frac{1}{C}\right)Y_i(t)\right\} = S\{P_{\mathrm{d}} - P_{\mathrm{m}}\}, \tag{24}$$

$$\frac{S\{Y_i(t)\} - Y_i(0)}{u^\eta} + \left(\frac{1}{CR}\right)S\{Y_i(t)\} = \frac{P_{\mathrm{d}} - P_{\mathrm{m}}}{R}, \tag{25}$$

$$S\{Y_i(t)\} = \frac{P_{\mathrm{d}} - P_{\mathrm{m}}}{R}\frac{1}{u^{-\eta} + \frac{1}{CR}}, \tag{26}$$

and

$$S\{Y_i(t)\} = C(P_{\mathrm{d}} - P_{\mathrm{m}})\frac{\frac{1}{CR}u^\eta}{1 + \frac{1}{CR}u^\eta}. \tag{27}$$

Applying the inverse Sumudu transform of above equation yields

$$Y_i(t) = C(P_{\mathrm{d}} - P_{\mathrm{m}})\left[1 - E_\eta\left(\frac{-1}{CR}t^\eta\right)\right]. \tag{28}$$

Similar operations can be done for the Eq. (23). Then, we reach

$$S\{R[{}^{C}_{0}D_t^{\eta}Y_e(t)]\} + S\left\{\left(\frac{1}{C}\right)Y_e(t)\right\} = S\{-P_{\mathrm{m}}\}, \tag{29}$$

$$\frac{S\{Y_e(t)\} - Y_e(t_{\mathrm{b}})}{u^\eta} + \left(\frac{1}{CR}\right)S\{Y_e(t)\} = \frac{-P_{\mathrm{m}}}{R}, \tag{30}$$

$$S\{Y_e(t)\} = \frac{-P_{\mathrm{m}}}{R}\frac{1}{u^{-\eta} + \frac{1}{CR}}, \tag{31}$$

and

$$S\{Y_e(t)\} = -CP_{\mathrm{m}}\frac{\frac{1}{CR}u^\eta}{1 + \frac{1}{CR}u^\eta}. \tag{32}$$

Applying the inverse Sumudu transform of above equation yields

$$Y_e(t) = -CP_{\mathrm{m}}\left[1 - E_\eta\left(\frac{-1}{CR}t^\eta\right)\right]. \tag{33}$$

## 3.2 Respiratory mechanics models with Caputo–Fabrizio fractional derivative

We discuss the respiratory mechanics model equations with Caputo–Fabrizio fractional derivative:

$$R[{}^{CFC}_{\;\;0}D_t^{\eta}Y_i(t)] + \left(\frac{1}{C}\right)Y_i(t) + P_{\mathrm{m}} = P_{\mathrm{d}}, \quad 0 \le t \le t_j, \tag{34}$$

$$R[{}^{CFC}_{\;\;0}D_t^{\eta}Y_e(t)] + \left(\frac{1}{C}\right)Y_e(t) + P_{\mathrm{m}} = 0, \quad t_j \le t \le t_b, \tag{35}$$

$$Y_i(0) = Y_e(t_b) = 0, \tag{36}$$

$$Y_i(t_j) = Y_e(t_j) = Y_\tau. \tag{37}$$

Begin with solving the Eq. (34) under the initial condition $Y_i(0) = 0$, by virtue of the Sumudu transformation, we have

$$S\left\{R[{}^{CFC}_{\;\;0}D_t^{\eta}Y_i(t)]\right\} + S\left(\frac{1}{C}\right)Y_i(t)\Big\} = S\{P_{\mathrm{d}} - P_{\mathrm{m}}\}, \tag{38}$$

Then, we have

$$\frac{M(\eta)}{1-\eta}\frac{S\{Y_i(t)\}}{1+\frac{\eta}{1-\eta}u} - \frac{M(\eta)}{(1-\eta)}\frac{Y_i(0)}{\left(1+\frac{\eta}{1-\eta}u\right)} + \frac{1}{CR}S\{Y_i(t)\} = \frac{P_d - P_m}{R}, \tag{39}$$

and

$$S\{Y_i(t)\} = \frac{\frac{P_d-P_m}{R}}{\frac{1}{CR} + \frac{M(\eta)}{(1-\eta)\left(1+\frac{\eta u}{1-\eta}\right)}}, \tag{40}$$

and

$$S\{Y_i(t)\} = C(P_d - P_m) - C(P_d - P_m) \times \frac{M(\eta)}{M(\eta) + \frac{1}{CR}(1-\eta)}\frac{1}{1 - \frac{-\frac{1}{CR}\eta u}{M(\eta)+\frac{1}{CR}(1-\eta)}}. \tag{41}$$

Implementing the inverse Sumudu transform yields:

$$Y_i(t) = C(P_d - P_m) - C(P_d - P_m) \times \frac{M(\eta)}{M(\eta) + \frac{1}{CR}(1-\eta)}\exp\left(\frac{-\frac{1}{CR}\eta t}{M(\eta) + \frac{1}{CR}(1-\eta)}\right). \tag{42}$$

The similar operations can be done for the Eq. (35). Thus, we reach

$$S\{R[{}_0^{CFC}D_t^{\eta}Y_e(t)]\} + S\left\{\left(\frac{1}{C}\right)Y_e(t)\right\} = S\{-P_m\}, \tag{43}$$

Then, we obtain

$$\frac{M(\eta)}{1-\eta}\frac{S\{Y_e(t)\}}{1+\frac{\eta}{1-\eta}u} - \frac{M(\eta)}{(1-\eta)}\frac{Y_e(t_b)}{\left(1+\frac{\eta}{1-\eta}u\right)} + \frac{1}{CR}S\{Y_e(t)\} = \frac{-P_m}{R}, \tag{44}$$

and

$$S\{Y_e(t)\} = \frac{\frac{-P_m}{R}}{\frac{1}{CR} + \frac{M(\eta)}{(1-\eta)\left(1+\frac{\eta u}{1-\eta}\right)}}, \tag{45}$$

and

$$S\{Y_e(t)\} = -CP_m + CP_m\frac{M(\eta)}{M(\eta) + \frac{1}{CR}(1-\eta)}\frac{1}{1 - \frac{-\frac{1}{CR}\eta u}{M(\eta)+\frac{1}{CR}(1-\eta)}}. \tag{46}$$

Applying the inverse Sumudu transform yields:

$$Y_e(t) = -CP_m + CP_m\frac{M(\eta)}{M(\eta) + \frac{1}{CR}(1-\eta)} \times \exp\left(\frac{-\frac{1}{CR}\eta t}{M(\eta) + \frac{1}{CR}(1-\eta)}\right). \tag{47}$$

## 3.3 Respiratory mechanics models with Atangana–Baleanu fractional derivative

We discuss the respiratory mechanics model equations with Atangana–Baleanu fractional derivative,

$$R[{}_0^{ABC}D_t^{\eta}Y_i(t)] + \left(\frac{1}{C}\right)Y_i(t) + P_m = P_d, \quad 0 \le t \le t_j, \tag{48}$$

$$R[{}_0^{ABC}D_t^{\eta}Y_e(t)] + \left(\frac{1}{C}\right)Y_e(t) + P_m = 0, \quad t_j \le t \le t_b, \tag{49}$$

$$Y_i(0) = Y_e(t_b) = 0, \tag{50}$$

$$Y_i(t_j) = Y_e(t_j) = Y_\tau. \tag{51}$$

Begin with solving the Eq. (48) under the initial condition $Y_i(0) = 0$, by virtue of the Sumudu transformation, we have

$$S\{R[{}_0^{ABC}D_t^{\eta}Y_i(t)]\} + S\left\{\left(\frac{1}{C}\right)Y_i(t)\right\} = S\{P_d - P_m\}, \tag{52}$$

Then, we get

$$\frac{B(\eta)}{(1-\eta)}\frac{S\{Y_i(t)\}}{\left(1+\frac{\eta}{1-\eta}u^{\eta}\right)} - \frac{B(\eta)}{(1-\eta)}\frac{Y_i(0)}{\left(1+\frac{\eta}{1-\eta}u^{\eta}\right)} + \frac{1}{CR}S\{Y_i(t)\} = \frac{P_d - P_m}{R}, \tag{53}$$

and

$$S\{Y_i(t)\} = \frac{P_d - P_m}{R}\left(\frac{1-\eta+\eta u^{\eta}}{B(\eta) + \frac{1}{CR}(1-\eta+\eta u^{\eta})}\right), \tag{54}$$

$$\begin{aligned} S\{Y_i(t)\} &= \frac{P_d - P_m}{R}\left(\frac{1-\eta}{B(\eta) + \frac{1}{CR}(1-\eta+\eta u^{\eta})}\right) \\ &+ \frac{P_d - P_m}{R}\left(\frac{\eta u^{\eta}}{B(\eta) + \frac{1}{CR}(1-\eta+\eta u^{\eta})}\right), \end{aligned} \tag{55}$$

$$\begin{aligned} S\{Y_i(t)\} &= \frac{P_d - P_m}{R}\left(\frac{1-\eta}{B(\eta) + \frac{1}{CR}(1-\eta)}\right)\frac{1}{1 + \frac{\frac{1}{CR}\eta u^{\eta}}{B(\eta)+\frac{1}{CR}(1-\eta)}}, \\ &+ \frac{P_d - P_m}{R}\left(\frac{1}{B(\eta) + \frac{1}{CR}(1-\eta)}\right)\frac{\eta u^{\eta}}{1 + \frac{\frac{1}{CR}\eta u^{\eta}}{B(\eta)+\frac{1}{CR}(1-\eta)}}. \end{aligned}$$

Implementing the inverse Sumudu transform yields:

$$Y_i(t) = \frac{P_d - P_m}{R}\left(\frac{1-\eta}{B(\eta) + \frac{1}{CR}(1-\eta)}\right)E_{\eta}\left(\frac{-\frac{1}{CR}\eta t^{\eta}}{B(\eta) + \frac{1}{CR}(1-\eta)}\right),$$

$$+C(P_\mathrm{d}-P_\mathrm{m})\left[1-E_\eta\left(\frac{-\frac{1}{CR}\eta t^\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}\right)\right].$$

Similar operations can be done for Eq. (49). Then, we obtain

$$\frac{B(\eta)}{(1-\eta)}\frac{S\{Y_e(t)\}}{\left(1+\frac{\eta}{1-\eta}u^\eta\right)}-\frac{B(\eta)}{(1-\eta)}\frac{Y_e(t_b)}{\left(1+\frac{\eta}{1-\eta}u^\eta\right)}+\frac{1}{CR}S\{Y_e(t)\}=\frac{-P_\mathrm{m}}{R}, \quad (56)$$

and

$$S\{Y_e(t)\}=-\frac{P_\mathrm{m}}{R}\left(\frac{1-\eta+\eta u^\eta}{B(\eta)+\frac{1}{CR}(1-\eta+\eta u^\eta)}\right), \quad (57)$$

$$S\{Y_e(t)\}=-\frac{P_\mathrm{m}}{R}\left(\frac{1-\eta}{B(\eta)+\frac{1}{CR}(1-\eta+\eta u^\eta)}\right)-\frac{P_\mathrm{m}}{R}\left(\frac{\eta u^\eta}{B(\eta)+\frac{1}{CR}(1-\eta+\eta u^\eta)}\right), \quad (58)$$

$$S\{Y_e(t)\}=\frac{-P_\mathrm{m}}{R}\left(\frac{1-\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}\right)\frac{1}{1+\frac{\frac{1}{CR}\eta u^\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}},$$

$$-\frac{P_\mathrm{m}}{R}\left(\frac{1}{B(\eta)+\frac{1}{CR}(1-\eta)}\right)\frac{\eta u^\eta}{1+\frac{\frac{1}{CR}\eta u^\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}}.$$

Implementing the inverse Sumudu transform yields:

$$Y_e(t)=\frac{-P_\mathrm{m}}{R}\left(\frac{1-\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}\right)E_\eta\left(\frac{-\frac{1}{CR}\eta t^\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}\right),$$

$$-CP_\mathrm{m}\left[1-E_\eta\left(\frac{-\frac{1}{CR}\eta t^\eta}{B(\eta)+\frac{1}{CR}(1-\eta)}\right)\right].$$

## 3.4 Respiratory mechanics models with constant proportional Caputo fractional derivative

We consider the respiratory mechanics model equations with constant proportional Caputo fractional derivative as:

$$R[{}_0^{CPC}D_t^\eta Y_i(t)]+\left(\frac{1}{C}\right)Y_i(t)+P_\mathrm{m}=P_\mathrm{d},\quad 0\le t\le t_j, \quad (59)$$

$$R[{}_0^{CPC}D_t^\eta Y_e(t)]+\left(\frac{1}{C}\right)Y_e(t)+P_\mathrm{m}=0,\quad t_j\le t\le t_b, \quad (60)$$

$$Y_i(0)=Y_e(t_b)=0, \quad (61)$$

$$Y_i(t_j)=Y_e(t_j)=Y_\tau. \quad (62)$$

Begin with solving the Eq. (59) under the initial condition $Y_i(0)=0$, by virtue of the Sumudu transformation, we have

$$S\{R[{}_0^{CPC}D_t^\eta Y_i(t)]\}+S\left\{\left(\frac{1}{C}\right)Y_i(t)\right\}=S\{P_\mathrm{d}-P_\mathrm{m}\}, \quad (63)$$

Then, we obtain

$$k_1(\eta)S\{Y_i(t)\}u^{1-\eta}+k_0(\eta)[S\{Y_i(t)\}-Y_i(0)]u^{-\eta}+\left(\frac{1}{CR}\right)S\{Y_i(t)\}=\frac{P_\mathrm{d}-P_\mathrm{m}}{R}, \quad (64)$$

and

$$\begin{aligned}S\{Y_i(t)\}&=\frac{C(P_\mathrm{d}-P_\mathrm{m})}{1+\frac{k_1(\eta)}{CR}u^{1-\eta}+\frac{k_0(\eta)}{CR}u^{-\eta}}\\&=C(P_\mathrm{d}-P_\mathrm{m})\left[1-\frac{-k_1(\eta)u^{1-\eta}-k_0(\eta)u^{-\eta}}{CR}\right]^{-1}\\&=C(P_\mathrm{d}-P_\mathrm{m})\sum_{j=0}^{\infty}\left[\frac{-k_1(\eta)u^{1-\eta}-k_0(\eta)u^{-\eta}}{CR}\right]^j\\&=C(P_\mathrm{d}-P_\mathrm{m})\sum_{j=0}^{\infty}\left(\frac{1}{CR}\right)^j\sum_{d=0}^{j}\binom{j}{d}[-k_1(\eta)u^{1-\eta}]^{j-d}[-k_0(\eta)u^{-\eta}]^d\\&=C(P_\mathrm{d}-P_\mathrm{m})\sum_{j=0}^{\infty}\sum_{d=0}^{j}(-1)^j\frac{k_1(\eta)^{j-d}k_0(\eta)^d}{(CR)^j}\binom{j}{d}u^{(1-\eta)(j-d)-\eta d}.\end{aligned}$$

Implementing the inverse Sumudu transform yields:

$$Y_i(t)=C(P_\mathrm{d}-P_\mathrm{m})\times\sum_{j=0}^{\infty}\sum_{d=0}^{j}(-1)^j\frac{k_1(\eta)^{j-d}k_0(\eta)^d}{(CR)^j}\binom{j}{d}\times\frac{t^{(1-\eta)(j-d)-\eta d}}{((1-\eta)(j-d)-\eta d+1)}. \quad (65)$$

Taking $r=j-d$ gives:

$$\begin{aligned}Y_i(t)&=C(P_\mathrm{d}-P_\mathrm{m})\sum_{d=0}^{\infty}\sum_{r=0}^{\infty}\frac{(d+r)!}{d!r!}(-1)r+d\frac{k_1(\eta)^r k_0(\eta)^d}{(CR)^{d+r}}\frac{t^{(1-\eta)r-\eta d}}{((1-\eta)r-\eta d+1)}\\&=C(P_\mathrm{d}-P_\mathrm{m})\sum_{d=0}^{\infty}\sum_{r=0}^{\infty}\frac{(d+r)!}{d!r!}\left[\frac{-k_1(\eta)}{CR}t^{1-\eta}\right]^r\left[\frac{-k_0(\eta)}{CR}t^{-\eta}\right]^d\frac{1}{((1-\eta)r-\eta d+1)}.\end{aligned}$$

Then, we obtain [19]:

$$Y_i(t) = C(P_{\rm d} - P_{\rm m})E^{1}_{(1-\eta),-\eta,1}\left(\frac{k_1(\eta)}{CR}t^{1-\eta}, \frac{-k_0(\eta)}{CR}t^{\eta}\right). \quad (66)$$

Similar operations are done for (60). Then, we have

$$S\{R[{}_0^{CPC}D_t^{\eta}Y_e(t)]\} + S\left\{\left(\frac{1}{C}\right)Y_e(t)\right\} = S\{-P_{\rm m}\}, \quad (67)$$

Then, we obtain

$$\begin{aligned} &k_1(\eta)S\{Y_e(t)\}u^{1-\eta} + k_0(\eta)[S\{Y_e(t)\} - Y_e(t_b)]u^{-\eta} \\ &+ \left(\frac{1}{CR}\right)S\{Y_e(t)\} = \frac{-P_{\rm m}}{R}, \end{aligned} \quad (68)$$

and

$$\begin{aligned} S\{Y_e(t)\} &= \frac{-CP_{\rm m}}{1 + \frac{k_1(\eta)}{CR}u^{1-\eta} + \frac{k_0(\eta)}{CR}u^{-\eta}} \\ &= -CP_{\rm m}\left[1 - \frac{-k_1(\eta)u^{1-\eta} - k_0(\eta)u^{-\eta}}{CR}\right]^{-1} \\ &= -CP_{\rm m}\sum_{j=0}^{\infty}\left[\frac{-k_1(\eta)u^{1-\eta} - k_0(\eta)u^{-\eta}}{CR}\right]^{j} \\ &= -CP_{\rm m}\sum_{j=0}^{\infty}\left(\frac{1}{CR}\right)^{j}\sum_{d=0}^{j}\binom{j}{d}[-k_1(\eta)u^{1-\eta}]^{j-d}[-k_0(\eta)u^{-\eta}]^{d} \\ &= -CP_{\rm m}\sum_{j=0}^{\infty}\sum_{d=0}^{j}(-1)^{j}\frac{k_1(\eta)^{j-d}k_0(\eta)^{d}}{(CR)^{j}}\binom{j}{d}u^{(1-\eta)(j-d)-\eta d}. \end{aligned}$$

Applying the inverse Sumudu transform yields:

$$\begin{aligned} Y_i(t) = &-CP_{\rm m}\sum_{j=0}^{\infty}\sum_{d=0}^{j}(-1)^{j}\frac{k_1(\eta)^{j-d}k_0(\eta)^{d}}{(CR)^{j}}\binom{j}{d} \\ &\times\frac{t^{(1-\eta)(j-d)-\eta d}}{((1-\eta)(j-d)-\eta d+1)}. \end{aligned} \quad (69)$$

Choosing $r = j - d$ yields

$$\begin{aligned} Y_i(t) = &-CP_{\rm m}\sum_{d=0}^{\infty}\sum_{r=0}^{\infty}\frac{(d+r)!}{d!r!}(-1)r \\ &+ d\frac{k_1(\eta)^{r}k_0(\eta)^{d}}{(CR)^{d+r}}\frac{t^{(1-\eta)r-\eta d}}{((1-\eta)r-\eta d+1)} \\ = &-CP_{\rm m}\sum_{d=0}^{\infty}\sum_{r=0}^{\infty}\frac{(d+r)!}{d!r!}\left[\frac{-k_1(\eta)}{CR}t^{1-\eta}\right]^{r}\left[\frac{-k_0(\eta)}{CR}t^{-\eta}\right]^{d} \\ &\times\frac{1}{((1-\eta)r-\eta d+1)}. \end{aligned}$$

Then, we acquire [19]:

$$Y_i(t) = -CP_{\rm m}E^{1}_{(1-\eta),-\eta,1}\left(\frac{k_1(\eta)}{CR}t^{1-\eta}, \frac{-k_0(\eta)}{CR}t^{\eta}\right). \quad (70)$$

## 4 Conclusion

In this work, the mechanical process model applied by the ventilator has been discussed with some fractional derivatives. We investigated the effect of the four different kernels by Sumudu transform in this article. We obtained very original results in this work. We can see the effect of the power-law, exponential-decay law, Mittag-Leffler kernel and hybrid kernel impacts on the solutions. We proved the efficiencey of the Sumudu transform for the mechanical process for different fractional derivatives.

**Acknowledgments:** The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University (KKU) for funding this research project Number (RGP.1/213/42).

**Funding information:** Deanship of Scientific Research at King Khalid University (KKU) for funding this research project Number (RGP.1/213/42).

**Author contributions:** All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

**Conflict of interest:** The authors state no conflict of interest.

## References

[1] Nucci G, Cobelli C. Mathematical models of respiratory mechanics. Model Methodol Physiol Med. 2001;2001:279–304.
[2] Acay B, Inc M. Respiratory mechanics models in the frame of non-local fractional operators. J Frac Calc Nonlinear Sys. 2021;1(1):21–45.
[3] Watugala GK. Sumudu transform: a new integral transform to solve differential equations and control engineering problems. J Math Educ Sci Technol. 1993;24(1):35–43.
[4] Asiru MA. Further properties of the Sumudu transform and its applications. Int J Math Educ Sci Technol. 2002;33(2):441–9.
[5] Belgacem FBM. Introducing and analysing deeper Sumudu properties. Nonlinear Stud. 2006;13(1):23–41.
[6] Tchuenche JM, Mbare NS. An application of the double Sumudu transform. Appl Math Sci. 2007;1:31–9.
[7] Kılıc A, Eltayeb H. On the applications of Laplace and Sumudu transforms. J Frankl Inst. 2010;347(5):848–62.
[8] Özköse F, Yılmaz S, Yavuz M, Öztürk İ, Şenel MT, Bağcı BŞ, et al. A fractional modeling of tumor–immune system interaction related to lung cancer with real data. Eur Phys J Plus. 2022;137(1):1–28.
[9] Hammouch Z, Yavuz M, Özdemir N. Numerical solutions and synchronization of a variable-order fractional chaotic system. Math Model Numer Simul Appl. 2021;1(1):11–23.

[10] Özköse F, Yavuz M. Investigation of interactions between COVID-19 and diabetes with hereditary traits using real data: A case study in Turkey. Computers Biol Med. 2021;141:105044.
[11] Veeresha P, Yavuz M, Baishya C. A computational approach for shallow water forced Korteweg–De Vries equation on critical flow over a hole with three fractional operators. An Int J Optim Control Theories Appl (IJOCTA). 2021;11(3):52–67.
[12] Naik PA, Yavuz M, Qureshi S, Zu J, Townley S. Modeling and analysis of COVID-19 epidemics with treatment in fractional derivatives using real data from Pakistan. Eur Phys J Plus. 2020;135(10):1–42.
[13] Yusuf A, Qureshi S, Mustapha UT, Musa SS, Sulaiman TA. Fractional modeling for improving scholastic performance of students with optimal control international. J Appl Comput Math. 2022;8(1):1–20.
[14] Qureshi S, Chang MM, Shaikh AA. Analysis of series RL and RC circuits with time-invariant source using truncated M, Atangana beta and conformable derivatives. J Ocean Eng Sci. 2021;6(3):217–27.
[15] Qureshi S, Yusuf A, Aziz S. Fractional numerical dynamics for the logistic population growth model under Conformable Caputo: a case study with real observations. Phys Scr. 2021;96(11):114002.
[16] Atangana A, Akgül A. Can transfer function and Bode diagram be obtained from Sumudu transform. Alex Eng J. 2020;59(4):1971–84.
[17] Acay B, Bas E, Abdeljawad T. Fractional economic models based on market equilibrium in the frame of different type kernels. Chaos Solitons Fractals. 2019;130:109438.
[18] Baleanu D, Fernandez A, Akgül A. On a fractional operator combining proportional and classical differintegrals. Mathematics. 2020;8(3):360.
[19] Fernandez A, Kürt C, Özarslan MA. A naturally emerging bivariate Mittag-Leffler function and associated fractional-calculus operators. arXiv 2020, arXiv:2002.12171.