Research Article

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Nonlocal magneto-thermoelastic infinite half-space due to a periodically varying heat flow under Caputo–Fabrizio fractional derivative heat equation

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Abstract: This article deals with a new modified heat conduction model with fractional order that includes the Caputo–Fabrizio differential operator (CF) and the thermal relaxation time. This new approach to the CF fractional derivative has attracted many researchers because it includes a nonsingular kernel. The nonlocal theory proposed by Eringen has also been applied to demonstrate the effect of scale-dependent thermoelastic materials. The problem of thermal isotropic semi-infinite space is addressed as an application of the presented model. The medium is exposed to regularly changing heat sources and is initially placed in a continuous external magnetic field. The system of governing equations was expressed in the field of the Laplace transform, and the problem in this field was solved by the state-space operation. The inverse of the transformed expressions of physical quantities is found numerically using Zakian's algorithm. The effects of the nonlocal parameter, the fractal order parameter, and the magnetic field were graphically presented and analyzed in detail. Some of the previous investigations were extracted in some special cases.

Keywords: Caputo–Fabrizio derivative operator, nonlocal theory, magneto-thermoelasticity, periodic heat source, state space approach

1 Introduction

Nowadays, mathematical models derived from the fractional order have been given great importance because they are more accurate and more practical than traditional order models [1]. Motivated by the development of fractional calculus, many researchers have focused on searching for solutions to nonlinear differential equations by creating arithmetic or analytical techniques in order to find approximate solutions [2]. Over recent decades, fractional calculus has been used to model many physical problems. The main reason and motivation for using fractional derivatives is that many different systems represent memory and history, in addition to the fact that nonlocal effects are often difficult to model using integer derivatives. Given that the concept of the fractional derivative is very interesting to explain some of the phenomena represented in different differential equations, this concept has been generalized in many ways to the fractional derivatives. Also, in many areas of research, the use of fractional-order derivatives has appeared in many applications, such as bioengineering, economics, geophysics, and biology [3].

Calculus of nonintegral orders is a fractional calculus that has developed very rapidly in recent decades due to its applications in many different scientific fields [4,5]. To better explain the dynamics of various certain real-world phenomena, researchers aim to find the best definitions of the fractional operators. The reader can refer to [6] for an excellent evaluation of the major achievements of fractional calculus up to 1974, while information for an analysis of progress since then to the present can be found in refs. [7,8].

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Both the Liouville and Caputo definitions were inspired by real-world considerations and the implementation of fractional arithmetic models, which are now naturalistic and known as the Riemann-Liouville or Caputo definitions. Each author created new fractional derivatives, which have never been used earlier, as they can be used to successfully model many real problems. The existence of some nonlocal systems with real-world applications to explain inhomogeneities and fluctuations that cannot be modeled by traditional local arithmetic or fractional calculus using singular kernels has motivated many researchers to define fractional models that are described by nonsingular kernels. However, until now, there has been some complaint about the overly complex mathematical expression of the concept of MF equations and the multiples that follow.

In this context, Caputo and Fabrizio [9,10] proposed a new concept for defining fractional derivatives, assuming that the temporal and spatial components are considered separate representations. The first description works on time variables, where the actual powers in solutions of standard fractional derivatives are converted to integer powers with some simplification of formulas and calculations. In this case, the Laplace transform is suitable for the application. The second clarification is related to spatial variables, and therefore, it is more appropriate to deal with the Fourier transform for a nonlocal fractional derivative.

In the past 2 years, many studies have been presented on the results of the new fractional Caputo–Fabrizio derivative, for example, as the related fractional integral and solutions of many straight fractional differential equations [11]. In refs. [12–14], the extreme theory of finite value issues is discussed as it plays an important role in solving many fractional propagation equations. Also, in many areas of mathematical modeling, the Caputo–Fabrizio fractional derivative has been used to model many real problems, for example, in the modeling of spring mass damper motion [15,16], the nonlinear Fisher diffusion equation [17], the elasticity model [18], the fluid transmission line model, and the Korteweg-de Vries-Burgers functional differential equation [15,19]. In addition to the Caputo–Fabrizio fractional derivative, some researchers have provided other definitions of fractional derivatives with nonsingular kernels [20–29].

The size dependence of mechanical behavior in micro- and nanomaterials is controversial because of the applicability of the traditional continuum theory. Previous research discovered that the independent classical continuity theory cannot capture the small effect, and thus, the mechanical behavior of micro- and nanostructures cannot be correctly predicted. Hence, nonclassical continuum theories have been generalized to include the effects of nanomaterials and nanomaterials. Examples of these theories presented are strain gradient theory, classical couple stress model, nonlocal elasticity theory, and modified couple stress model [30].

The theory of nonlocal elasticity has been applied to study nanomechanical uses such as elastic wave scattering, composite wave propagation, dislocation dynamics, mechanical fractures, and surface potential fluids. Studies have shown that it is more accurate than the classical theory in describing nanostructures. In 1972, in an attempt to deal with small-scale structural problems, Eringen introduced the theory of nonlocal continuity mechanics [31–33]. Nonlocal continuity theories consider the stress at some points to be a function of the stress at any point in the structure, while classical continuity mechanics states that the stress at some points depends only on the stress at that point. In this nonlocal theory, the equilibrium laws include nonlocal field residuals, and these residuals are related to the constitutive equations in which the constraint requirements are balanced by thermodynamics and stabilization. Also, component equations and nonpositional residuals work with deformation gradients and motions of all body points. On the basis of the nonlocal theory of thermoelasticity, Inan and Eringen [34] studied the propagation of thermoelastic waves in the plates. Wang and Dhaliwal [35] also introduce the energy and work equation in nonlocal generalized thermoelasticity and prove that the initial value and boundary problems have a unique solution.

Abouelregal *et al.* [36] introduced a nonlocal Bernoulli–Euler model and analyzed the thermoelastic interactions in nanoscale beams based on couple stress and generalized thermoelasticity theories. Nonlocal nanoscale beams have been discussed by Abouelregal *et al.* [37] according to the theories of Euler and Bernoulli and modified couple stress. It is also assumed that the thermal conductivity of the nanobeam depends on the change in temperature. Abouelregal *et al.* [38] introduced a new differential equation system that describes the theory of nonlocal thermoelasticity with higher derivatives and two-phase lag. To achieve this model, he used Eringen's nonlocal continuum theory and Taylor's expansive methodology for higher-order time derivatives. Koutsoumaris *et al.* [39] expressed the nonlocal continuum theory, either in integral or differential form, which is widely used to explain size effect phenomena in micro- and nanostructures. Liew *et al.* [40] introduced a literature review of recent research studies on the applications of a nonlocal elasticity theory in the modeling and simulation of graphene sheets. Rajneesh *et al.* [41] investigated the transient analysis of a nonlocal thermoelastic micro-stretch thick circular plate with phase lags. Solutions for various problems support nonlocal theory [42–47].

As far as we know, there is hardly any effort to analyze the nonlocal arrangement and fracture, which is extremely important in material handling applications, particularly in the context of preheated materials because the material adjacent to the surface approaches its melting temperature. In these cases, the theoretical model of nonlocal effect and fractional order is taken into account in the newly amended Fourier law and is necessary to analyze the thermoelastic responses of micro/nanoscale structures. This article presents a thermoelastic investigation of a rotating finite rod that, under Eringen's nonlocal theory, is subject to a moving heat source. The study method demonstrates temporary nonlocal thermal stresses in a rotating rod with a constant angular speed. The changes in temperature, displacement, and distribution of stresses are studied along the axial direction. In all of the fields studied, the effects of the moveable heat source velocity, fractional-order parameter, nonlocal parameters, and magnetic field are considered.

Through studies and experiments, it was found that the classical Fourier model of thermal conductivity is no longer accurate for describing heat diffusion. With its application, only the infinite speed of heat diffusion can be expected. Hence, a variety of generalizations of Fourier's law of thermal conductivity have been proposed to remove inconsistencies and flaws in the classical theory. In this context, as a broad-based theory in the field of thermoelasticity, Lord and Shulman [48] proposed a new model regarding thermal conductivity to replace the classical Fourier's law, and the generalized theory of thermal elasticity was presented at the same time. The heat equation in this theory is a type of wave and thus ensures limited rates of heat and elastic wave propagation. The previous theory was not the only proposal in this context, but many generalized theories of thermoelasticity have been proposed. Many studies were also presented based on these generalized theories to show their effectiveness in solving problems and conflicts of the classical theory [49–53].

This article aims to briefly summarize and clarify the possible future work on nanostructured materials based on the Eringen theory of nonlocal elasticity and with a focus on bending, vibration, and wave propagation within solid materials. A new model of the generalized thermal conductivity equation that includes a fractional derivative without a singular kernel is presented. This proposed model has been applied to explain the combined effect of nonlocal and magnetic field parameters on thermal and mechanical elastic waves in a semi-infinite medium. The medium was subjected to heating by a cyclic heat source and immersed in an external magnetic field of constant intensity. After formulating the system of paired governing equations, it is solved by applying the Laplace transform method. Also, to obtain the numerical results for the temperature, displacement, or nonlocal stress in the medium, a numerical Laplace inversion was performed using the Zakian algorithm [54,55]. Finally, the effect of the fractional derivatives with an exponential core on the studied fields, in addition to the strength of the heat source and magnetic fields, was studied.

2 Modified nonlocal fractional thermoelasticity with exponential kernels

In the light of the Cattaneo–Vernotte model, Lord and Shulman have made a remarkable change [48] to remove the deficiency of classical theory. Cattaneo–Vernotte gave a wider version of Fourier law by integrating the relaxation-time parameter with respect to the heat flow vector as follows:

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right)\vec{q}(\vec{x}, t) = -K\vec{\nabla}\theta(\vec{x}, t). \qquad (1)$$

Here, $\vec{q}$ refers to heat flux vector, K refers to thermal conductivity, $\theta = T - T_0$ refers to the variant of the temperature, T_0 is the reference temperature, and $\vec{x}$ is the position vector. The relaxation time parameter, abbreviated as τ_0, is a nonnegative time delay.

The equation for the energy balance is as follows:

$$\rho C_E \frac{\partial \theta}{\partial t} + T_0 \frac{\partial}{\partial t}(\beta_{ij} e_{ij}) = -q_{i,i} + Q, \qquad (2)$$

where C_E stands for specific heat at constant strain, $\beta_{ij} = c_{ijkl}\alpha_{kl}$ stands for thermal coupling coefficients, α_{kl} stands for linear thermal expansion tensor, c_{ijkl} stands for elastic constants, Q stands for heat source, and ρ represents material density.

Compared with the classical model, mathematical models with derivatives of fractional orders are more accurate and practical. In this context, many academics have sought to investigate solutions to nonlinear differential equations using the fractional operator, motivated by the advancement of fractional calculus, by developing a variety of analytical and computational strategies to obtain approximate solutions. Riemann–Liouville, Caputo, Hilfer, and other fractional operators are used in these differential equations [2,56,57]. Conversely, these operators have been shown to have a power law kernel and are limited in their ability to represent physical situations. To overcome this problem, an alternative kernel fractional differentiation factor with exponential decay was recently introduced by Caputo and Fabrizio [10,11]. The Caputo–Fabrizio factor (CF) of the fractional derivative has attracted many researchers due to the fact that it has a nonsingular kernel. The CF operator is also best suited for modeling a particular class of real-world problems that follow the law of exponential decay. In recent years, the development of mathematical modeling using the CF fractional-order derivative has become an exciting field of study and research [11].

The standard Caputo time fractional derivative of order α is given by

$$D_t^{(\alpha)}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{\dot{f}(\tau)}{(t-\tau)^{\alpha}}\mathrm{d}\tau. \tag{3}$$

Definition (3) holds true for $\in(0,1)$, $f \in H^1(0,b)$ and $b > 0$. When replacing $(t-\tau)^{-\alpha}$ with $\exp\left[-\frac{\alpha}{(1-\alpha)}t\right]$ and $\Gamma(1-\alpha)$ with $(1-\alpha)$, Caputo–Fabrizio [8,58] obtains a new definition of fractional time derivative as follows:

$$D_t^{(\alpha)}f(t) = \frac{1}{1-\alpha}\int_0^t \dot{f}(\tau)\exp\left[-\frac{\alpha}{(1-\alpha)}t\right]\mathrm{d}\tau, \quad t \geq 0,\ 0 < \alpha < 1. \tag{4}$$

The new definition, like the usual Caputo derivative, makes it clear that if $f(t)$ is a constant function, $D_t^{(\alpha)}f(t) = 0$. The main difference between the current and previous definitions is that the new kernel, unlike the previous one, does not have a singularity for $t = \tau$. It can be observed that when $t = \tau$, we get to the traditional first-order derivative, which is expressed as follows:

$$\lim_{\alpha\to 1} D_t^{(\alpha)}f(t) = \dot{f}(t). \tag{5}$$

The Laplace transform is widely known for its use in the study of differential equations. It is also worth noting that this new fractal idea comes with a number of advantages, as the Laplace transform can easily be used as follows:

$$\mathcal{L}[D_t^{(\alpha)}f(t)] = \frac{1}{s+\alpha(1-s)}[s\mathcal{L}[f(t)] - f(0)]. \tag{6}$$

Furthermore, the constitutive equation for heat flow in the generalized LS thermoelastic model (4) is further refined to include a Caputo–Fabrizio (CF) operator of fractional differentiation as follows:

$$(1 + \tau_0\, D_t^{(\alpha)})\vec{q}(\vec{x},t) = -K\ \vec{\nabla}\theta(\vec{x},t). \tag{7}$$

In a finite case, Eq. (7) can be reduced to Cattaneo-Vernotte model at $\alpha \to 1$. Putting Eq. (7) into Eq. (2), the fractional heat transfer equation for generalized thermoelasticity including the Caputo–Fabrizio (CF) operator can be written as follows:

$$(1 + \tau_0 D_t^{(\alpha)})\left[\rho C_E\frac{\partial\theta}{\partial t} + T_0\frac{\partial}{\partial t}(\beta_{ij}e_{ij}) - Q\right] = \nabla.\ (K\ \nabla\theta). \tag{8}$$

In a specific case ($\alpha \to 1$) from the previous equation, the heat transfer equation given by Lord and Shulman [48] can be derived as follows:

$$\nabla.\ (K\ \nabla\theta) = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left[\rho C_E\frac{\partial\theta}{\partial t} + \gamma T_0\frac{\partial}{\partial t}(\beta_{ij}e_{ij}) - Q\right]. \tag{9}$$

The basic equation for elastic and homogeneous materials based on the theory of nonlocal elasticity can be expressed as follows [31–33]:

$$\tau_{ij} = \int_V \mathbb{K}(|\vec{x}' - \vec{x}|, \xi)\sigma_{ij}\mathrm{d}V, \tag{10}$$

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} - \gamma\theta\delta_{ij}, \tag{11}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{12}$$

where τ_{ij} is the nonlocal stress tensor, σ_{ij} is the local stress tensor, ε_{ij} is the strain tensor, and u_i are the components of the displacement vector. Also, $\gamma = (3\lambda + 2\mu)\alpha_t$, λ and μ are Lamé's constants, α_t is the thermal expansion coefficient, and δ_{ij} is the Kronecker delta.

The influence of the stress at position $\vec{x}'$ on the stress at the location $\vec{x}$ in the elastic body is defined by the nonlocal kernel $\mathbb{K}(|\vec{x}' - \vec{x}|, \xi)$. The parameter $\xi = e_0 a/l$ is determined by the material constants. The internal characteristic length denoted by the parameter a, while the external characteristic length is denoted by l. The constant e_0 is determined by the experiment. Furthermore, when $\xi \to 0$ is used, the kernel function $\mathbb{K}$ becomes the Dirac delta function $\delta(|\vec{x}' - \vec{x}|)$.

The kernel $\mathbb{K}$ is determined based on the material, situation, and the problem studied. The most popular kernel functions are the exponential and Gaussian, with the modified Bessel function sometimes being employed. Eringen [31] explained that given the kernel in the form of a Bessel function, the following equation can be obtained:

$$(1 - \xi^2\nabla^2)\tau_{ij} = \sigma_{ij}. \tag{13}$$

The equation of motion given by the linear momentum equilibrium is as follows:

$$\tau_{ij,j} + F_i = \rho\ddot{u}_i. \tag{14}$$

where F_i are the components of the body force.

The form of the nonlocal kinematic equation can be obtained by combining (13) and (14) as follows:

$$\sigma_{ij,j} + (1 - \xi^2\nabla^2)F_i = \rho(1 - \xi^2\nabla^2)\ddot{u}_i. \tag{15}$$

The equations of motion may be derived in terms of the temperature θ and the displacements u_i as follows:

$$\begin{aligned}(\lambda + \mu)u_{j,ij} + \mu\nabla^2 u_{i,jj} - \gamma\theta_{,i} + (1 - \xi^2\nabla^2)F_i \\ = \rho(1 - \xi^2\nabla^2)\ddot{u}_i.\end{aligned} \tag{16}$$

The induced magnetic field $\vec{h}$ and the induced electric field $\vec{E}$ are constructed using magnetic field $\vec{H}$ and current density $\vec{J}$. Maxwell's equations for the electromagnetic field for an ideally conducting medium can be written as follows [58]:

$$\begin{aligned}&\vec{J} = \nabla \times \vec{h}, \quad \nabla \times \vec{E} = -\mu_0\frac{\partial \vec{h}}{\partial t},\\ &\vec{J} = \sigma_0\left(\vec{E} + \mu_0\left(\frac{\partial \vec{u}}{\partial t} \times \vec{H}\right)\right),\\ &\vec{h} = \nabla \times (\vec{u} \times \vec{H}), \nabla\cdot \vec{h} = 0,\end{aligned} \tag{17}$$

where μ_0 is the magnetic permeability and σ_0 is the electric conductivity.

3 Modeling and formulation of the problem

In this section, the problem to which the proposed model is applied is defined and described, and then, a system of governing equations is constructed according to the constraint conditions imposed on the problem. Now, we will consider that an isotropic body has the shape of an infinite half-space $x \geq 0$ at a reference temperature of T_0 with a periodically changing heat source spread over the surface along the x-direction so that the x-axis lies inside the body in a direction perpendicular to the free surface plane. It will also be assumed that the problem is one dimensional, and therefore, the turbulence of the medium will also be one dimensional. As a result, the components of the displacement vector $\vec{u}$ are as follows:

$$u_x = u(x, t), \quad u_y = u_z = 0. \tag{18}$$

The applied magnetic field $\vec{H}$ is assumed to act orthogonal to the axial direction of the material, *i.e.*, $\vec{H} = (0, H_x, 0)$. The Lorentz force $\vec{F} = \vec{J} \times \vec{H}$ coming from the application of a longitudinal magnetic field $\vec{H}$, as indicated in motion Eq. (16), may be stated as follows using Maxwell's equations:

$$\vec{F} = (f_x, f_y, f_z) = -\sigma_0\mu_0 H_x^2\left(\frac{\partial \boldsymbol{u}}{\partial t}, 0, 0\right). \tag{19}$$

In the case of a one-dimensional problem, the constitutive relationship (13) representing nonlocal thermal stress is as follows:

$$\left(1 - \xi^2\frac{\partial^2}{\partial x^2}\right)\tau_{xx} = (\lambda + 2\mu)\frac{\partial \boldsymbol{u}}{\partial x} - \gamma\theta. \tag{20}$$

The equation of motion (16) can be written using Eqs. (19) and (20) as follows:

$$\begin{aligned}(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial\theta}{\partial x} - \sigma_0\mu_0 H_x^2\left(1 - \xi^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial u}{\partial t}\\ = \rho\left(1 - \xi^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 u}{\partial t^2}.\end{aligned} \tag{21}$$

The generalized thermal conductivity Eq. (8) based on the fractional Caputo–Fabrizio derivatives can be written in the following form:

$$(1 + \tau_0 D_t^{(\alpha)})\left[\rho C_E\frac{\partial\theta}{\partial t} + \gamma T_0\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) - Q\right] = K\frac{\partial^2\theta}{\partial x^2}. \tag{22}$$

The following nondimensional variables will be used:

$$\begin{aligned}&\{x', u'\} = c_0\omega_0\{x, u\}, \quad \{t', \tau_0'\} = c_0^2\omega_0\{t, \tau_0\},\\ &\xi' = c_0^2\omega_0^2\xi, \quad \theta' = \frac{\theta}{T_0}, \quad \sigma_x' = \frac{\sigma_x}{\mu}, \quad Q' = \frac{Q}{KT_0c_0^2\omega_0^2},\\ &c_0^2 = \frac{(\lambda + 2\mu)}{\rho}, \omega_0 = \frac{\rho C_E}{K}.\end{aligned} \tag{23}$$

The velocity of equal elastic wave propagation is indicated by the value c_0. After applying the nondimensional parameters described earlier, the governing PDEs (20)–(22) have the following form (removing primes):

$$\left(1 - \xi^2\frac{\partial^2}{\partial x^2}\right)\tau_{xx} = \beta^2\frac{\partial u}{\partial x} - b\theta, \tag{24}$$

$$\left(1 - \xi^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \frac{b}{\beta^2}\frac{\partial\theta}{\partial x} - \varepsilon\left(1 - \xi^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial u}{\partial t}, \tag{25}$$

$$\frac{\partial^2\theta}{\partial x^2} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left[\frac{\partial\theta}{\partial t} + g\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}\right) - Q\right], \tag{26}$$

where

$$\beta^2 = \frac{(\lambda+2\mu)}{\mu}, \quad b = \frac{\gamma T_0}{\mu}, \quad \varepsilon = \frac{\sigma_0\mu_0 H_x^2}{\rho c_0^2\omega_0}, \quad g = \frac{\gamma}{\rho C_E}. \tag{27}$$

To investigate the issue, PDEs (24)–(26) must be solved, and temperature θ, displacement u, and nonlocal thermal stress τ_{xx} distributions within the medium must be calculated. To solve the issue, the following basic circumstances are taken into account.

$$\theta(x,0)=\frac{\partial\theta(x,0)}{\partial t}=0=u(x,0)=\frac{\partial u(x,0)}{\partial t}, \quad (28)$$

$$u(x,t)\to 0\,,\ \theta(x,t)\to 0 \text{ as } x\to\infty. \quad (29)$$

It will be taken into account that the surface of the surrounding plane of the medium at $x = 0$ is heated by a periodic heat source $Q(x, t)$ radiating its energy with a constant density Q_0 in the direction of the x-axis. This nondimensional heat source can be represented as follows [59]:

$$Q(x,t)=Q_0\delta(x)\sin\left(\frac{\pi t}{t_0}\right), \qquad 0\le t\le t_0, \quad (30)$$

where t_0 is a constant parameter and $\delta(x)$ symbolizes Dirac's delta function.

4 Solution in the transformed domain

In engineering and some other areas of research, the Laplace transform is a useful tool for solving differential equations. But the problem with using the Laplace transform method is the difficulty of inverting complex transformed functions into the real field using traditional methods. As a result, some numerical algorithms are used to calculate the inverses, as we will discuss in this research. The Laplace transform described by the relationship below can be used to transform Eqs. (24)–(26)

$$\bar f(x,t)=\int_0^\infty f(x,t)e^{-st}\mathrm{d}t, \quad (31)$$

to obtain the following:

$$\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar\tau_{xx}=\frac{\mathrm{d}\bar u}{\mathrm{d}x}-\overline{\theta}, \quad (32)$$

$$\frac{\mathrm{d}^2\bar\theta}{\mathrm{d}x^2}=\left(1+\frac{s\tau_0}{s+\alpha(1-s)}\right)\left[s\bar\theta+s\varepsilon\frac{\mathrm{d}\bar u}{\mathrm{d}x}-\frac{\pi t_0 Q_0}{\pi^2+s^2t_0^2}\delta(x)\right], \quad (33)$$

$$s^2\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar u=\frac{\mathrm{d}^2\bar u}{\mathrm{d}x^2}-\frac{\mathrm{d}\bar\theta}{\mathrm{d}x}-sg\left(1-\xi^2\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right)\bar u.$$

Eq. (33) may be expressed as a vector matrix, such as [60–62]:

$$\frac{\mathrm{d}\vec V(x,s)}{\mathrm{d}x}=A(s)\vec V(x,s)+f(x,s), \quad (34)$$

with

$$\vec V(x,s)=\begin{pmatrix}\bar\theta\\ \bar u\\ \frac{\mathrm{d}\bar\theta}{\mathrm{d}x}\\ \frac{\mathrm{d}\bar u}{\mathrm{d}x}\end{pmatrix},\quad A(s)=\begin{pmatrix}0&0&1&0\\0&0&0&1\\ \alpha_4&0&0&\alpha_5\\ 0&\frac{\alpha_2}{\alpha_1}&\frac{\alpha_3}{\alpha_1}&0\end{pmatrix}, \quad (35)$$

$$f(x,s)=\begin{pmatrix}0\\0\\-\alpha_6\delta(x)\\0\end{pmatrix},$$

where

$$\alpha_1=1+s\xi^2(s+g),\quad \alpha_2=s(s+g),$$
$$\alpha_4=s\left(1+\frac{s\tau_0}{s+\alpha(1-s)}\right),$$
$$\alpha_5=s\varepsilon\left(1+\frac{s\tau_0}{s+\alpha(1-s)}\right), \quad (36)$$
$$\alpha_6=\frac{\pi t_0 Q_0\left(1+\frac{s\tau_0}{s+\alpha(1-s)}\right)}{\pi^2+s^2t_0^2}.$$

The eigenvalue technique, which is explored in depth in refs. [49–51], can be used to analyze solutions for the functions $\bar u(x)$ and $\bar\theta(x)$. We obtain the following results when we apply the eigenvalue technique to the system (34) under the regular conditions case (29):

$$\bar u(x,s)=\frac{\alpha_6(\mathrm{e}^{-k_2x}-\mathrm{e}^{-k_1x})}{2(k_1^2-k_2^2)}, \quad (37)$$

$$\bar\theta(x,s)=\frac{\alpha_6(k_1(\alpha_2-\alpha_1k_2^2)\mathrm{e}^{-k_2x}-k_2(\alpha_2-\alpha_1k_1^2)\mathrm{e}^{-k_1x})}{2k_1k_2(k_1^2-k_2^2)}. \quad (38)$$

The variables k_1^2 and k_1^2 denote the roots of the following equation with the positive real parts:

$$k^4-m_1k^2+m_2=0, \quad (39)$$

where m_1 and m_2 are coefficients that fulfill the relations:

$$m_1=\alpha_4+\frac{\alpha_2}{\alpha_1}+\frac{\alpha_5}{\alpha_1},\quad m_2=\frac{\alpha_2\alpha_4}{\alpha_1}. \quad (40)$$

We can derive the nonlocal stress $\bar\tau_{xx}$ by substituting Eqs. (37) and (38) into (32)

$$\begin{aligned}\bar{\tau}_{xx}(x,s) &= \frac{\alpha_6 k_1 e^{-k_1 x}}{2(k_1^2-k_2^2)(1-\xi^2 k_1^2)} - \frac{\alpha_6 k_2 e^{-k_2 x}}{2(k_1^2-k_2^2)(1-\xi^2 k_2^2)} \\ &+ \frac{\alpha_6 k_2(\alpha_2-\alpha_1 k_1^2)e^{-k_1 x}}{2k_1 k_2(k_1^2-k_2^2)(1-\xi^2 k_1^2)} \\ &- \frac{\alpha_6 k_1(\alpha_2-\alpha_1 k_2^2)e^{-k_2 x}}{2k_1 k_2(k_1^2-k_2^2)(1-\xi^2 k_2^2)}.\end{aligned} \tag{41}$$

Also, the strain $\bar{e}(x,s)$ can be determined as follows:

$$\bar{e}(x,s) = \frac{d\bar{u}(x,s)}{dx} = \frac{\alpha_6(k_1 e^{-k_1 x} - k_2 e^{-k_2 x})}{2(k_1^2-k_2^2)}. \tag{42}$$

An approximate numerical technique is often used to invert the fields studied in the Laplace field because the analytic inversion of these functions is sometimes difficult to achieve. In the literature, several computing strategies can be used to do the inversion of Laplace transforms. Each of these methods often has a distinct application as it can be used for a specific purpose. In this presented work, the numerical results of evaluating the investigated fields will be obtained in the real-time domain, and the approximation approach of Zakian [54,55,63] will be applied.

5 Approximate Zakian method

By using the numerical inversion methods, the numerical solutions of the physical fields (37), (38), (41), and (42) are obtained in the time domain. Consideration will be given to the application of the Zakian technique [54,55,63] to approximate the functions in the time domain by applying the following relationship:

$$f(x,t) = \frac{2}{t}\sum_{i=1}^{\mathcal{N}} \mathrm{Re}\left\{x, \mathcal{R}_i \bar{F}\left(\frac{\hbar_i}{t}\right)\right\}. \tag{43}$$

As shown in the aforementioned formula, this strategy is easy to implement and easy to use. In the forms $\mathcal{R}_i$ and $\hbar_i$, the conjugated pairs may be real or complex. The number of terms that will be summed or shortened from the infinite sequence is the parameter $\mathcal{N}$, which can be optimized and set based on the applied model. In Table 1, different values for $\mathcal{R}_i$ and $\hbar_i$ will be listed at $\mathcal{N} = 5$.

6 Special cases

We can obtain the following special cases from the introduced equation system and proposed model.

6.1 Models of thermoelasticity of differential operators with integer order

Calculations for classical and generalized models of thermoelasticity of differential operators with integer orders are performed when $\alpha \to 1$.

- The nonlocal coupled thermoelasticity theory (NCTE) can be obtained by setting $\tau_0 = 0$ and $\xi > 0$.
- The classical coupled theory of thermoelasticity (CTE) can be obtained by taking $\tau_0 = \xi = 0$.
- The generalized nonlocal theory of thermoelasticity with relaxation time (NLS) can be obtained when $\tau_0 > 0$ and taking $\xi > 0$.
- The generalized theory of thermoelasticity with relaxation time (LS) can be obtained by assuming $\tau_0 > 0$ and putting $\xi = 0$.

6.2 Models of thermoelasticity of Caputo–Fabrizio fractional differential operators

The computations for classical and generalized models of thermoelasticity of Caputo–Fabrizio fractional-order differential operators are performed when $0 < \alpha < 1$.

- The generalized nonlocal theory of thermoelasticity with fractional order and relaxation time (CFNLS) can be obtained by letting $\tau_0 \geq 0$ and taking $\xi > 0$.
- The generalized theory of thermoelasticity with fractional order and relaxation time (CFLS) can be obtained by assuming $\tau_0 \geq 0$ and putting $\xi = 0$.

Table 1: The values of the parameters $\hbar_i$ and $\mathcal{R}_i$ for $\mathcal{N} = 5$ [64]

j	$\hbar_i$	$\mathcal{R}_i$
1	12.83767675 + i666063445	−36902.08210 + i196990.4257
2	2 12.22613209 + i5.012718792	61277.02524 − i95408.62551
3	3 10.93430308 + i8.409673116	−28916.56288 + i18169.18531
4	4 8.776434715 + i11.92185389	4655.361138 − i1.901528642
5	5 5.225453361 + i15.72952905	−118.7414011 − i141.3036911

7 Numerical results

In this section, some numerical results will be presented to verify the proposed model and study the behavior of the temperature θ, the displacement u, the strain e, and the nonlocal thermal stress τ_{xx}. For this purpose, the inverse approximation method (43) proposed by Zakian [43–45] is applied to Eqs. (37), (38), (41), and (42). The Mathematica programming language is used for all numerical arithmetic operations. Copper was taken into account for the purpose of numerical calculations. The corresponding physical parameters of this material are as follows [65]:

$K = 386\ \mathrm{W\,m^{-1}\,K^{-1}},\ C_E = 384.56\ \mathrm{J/kg\,K},$
$\alpha_t = 1.78 \times 10^{-5}\ \mathrm{K^{-1}},$
$E = 128\ \mathrm{GPa},\ T_0 = 293\ \mathrm{K},\ \rho = 8954\ \mathrm{kg\,m^{-3}},$
$\sigma_0 = 10^{-9}/36\,\pi\ \mathrm{F\,m^{-1}},$
$t = 0.2\ \mathrm{s},\ \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}},\ H_x = 10^{-7}/4\pi\ \mathrm{A\,m^{-1}},$
$\nu = 0.36.$

The numerical calculations of field variables for different orders of the nonlocal Caputo–Fabrizio (CF) differential operator α, the nonlocal parameter ξ, the heat source density, and the magnetic field H_x were studied. The results were analyzed graphically at different x positions within the medium in Figures 1–12. The study and discussion of numerical calculations as well as graphs were divided into three groups.

7.1 Verification of the results

A numerical comparison with the results of Abouelregal *et al.* [66] was done to validate the numerical results and

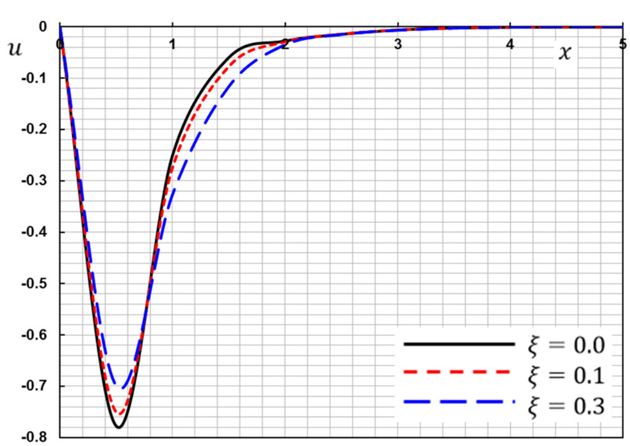

Figure 1: The varaiation of the displacement u with nonlocal parameter ξ.

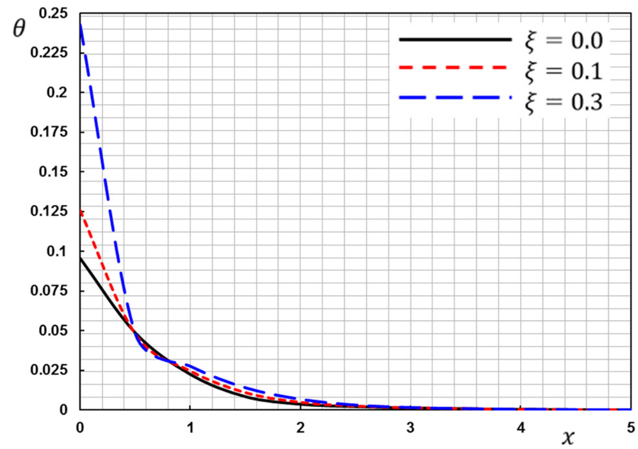

Figure 2: The varaiation of the temperature θ with nonlocal parameter ξ.

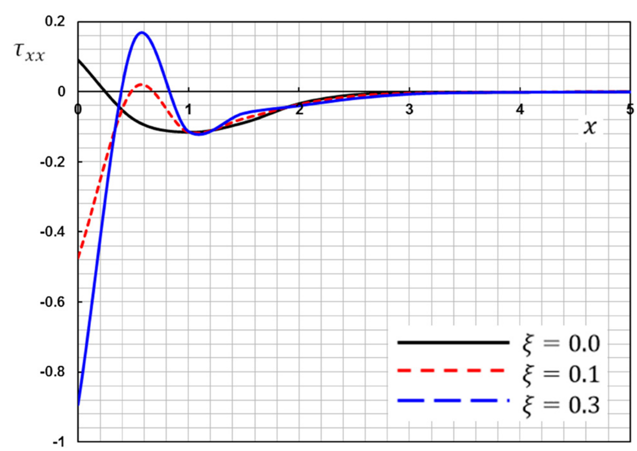

Figure 3: The varaiation of the nonlocal stress τ_{xx} with nonlocal parameter ξ.

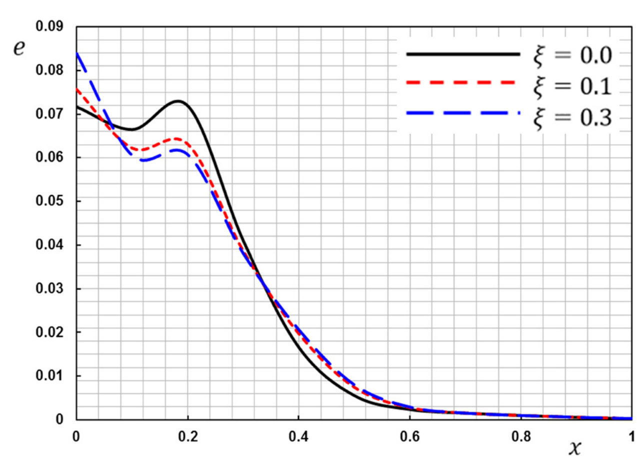

Figure 4: The varaiation of the strain e with nonlocal parameter ξ.

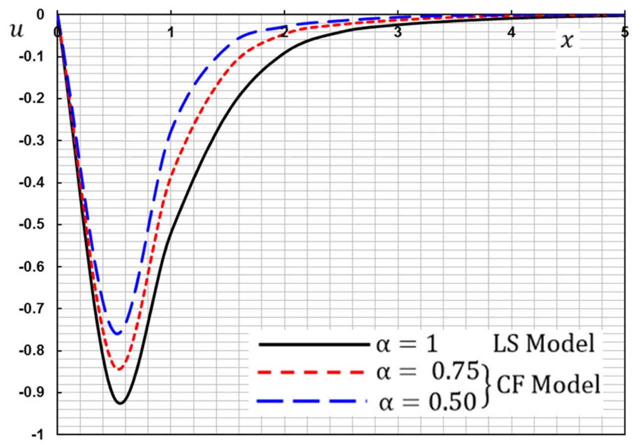

Figure 5: The displacement u for various fractional-order parameter α.

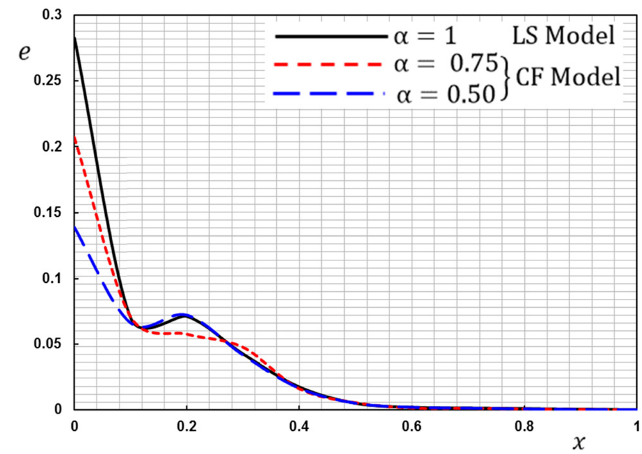

Figure 8: The distributions of the strain e for various fractional-order parameter α.

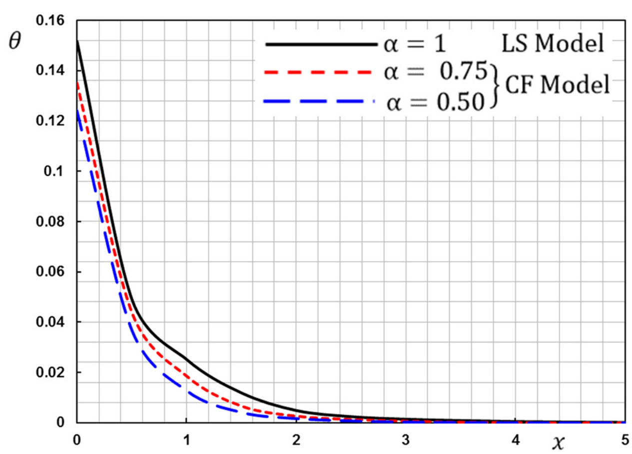

Figure 6: The temperature θ for various fractional-order parameter α.

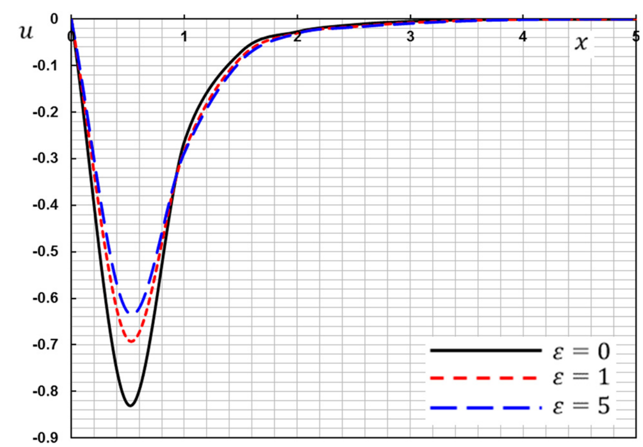

Figure 9: The displacement u under the effect of the applied magnetic field.

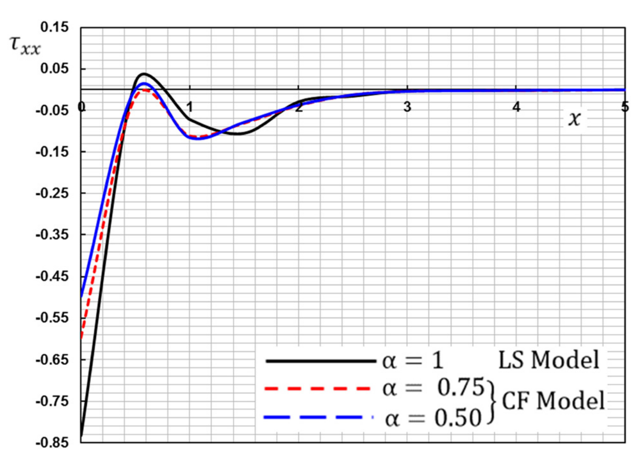

Figure 7: The nonlocal thermal stress τ_{xx} for various fractional-order parameter α.

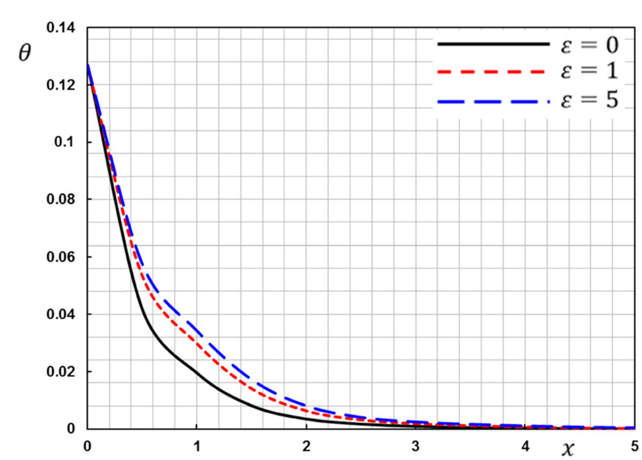

Figure 10: The temperature θ under the effect of the applied magnetic field.

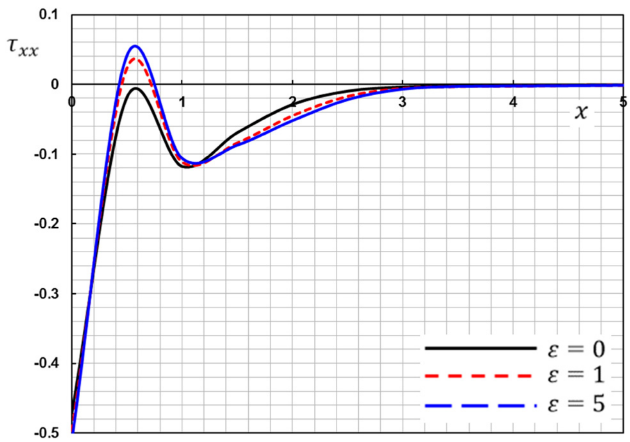

Figure 11: The nonlocal stress τ_{xx} under the effect of the applied magnetic field.

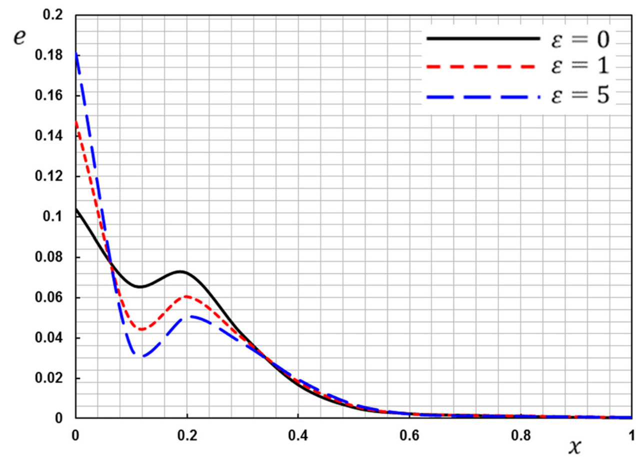

Figure 12: The stain e under the effect of the applied magnetic field.

the validity of the proposed thermoelastic model containing Caputo–Fabrizio fractional-order derivatives. The governing equations in ref. [66] have been applied using the nonlocal Moore–Gibson–Thompson thermoelastic model, while in the current model, the nonlocal thermoelastic fractional model with one relaxation time [48] has been introduced and applied. Table 1 compares the current dimensionless temperature and thermal stress with the similar temperature θ and nonlocal thermal stress τ_{xx} from reference [66] when the Caputo–Fabrizio (CF) differential operator is absent ($\alpha = 0$) (Table 2).

From Table 2, it was discovered that the fractional-order parameter α of the Caputo–Fabrizio derivative operators has a significant effect on the studied fields despite the agreement in behavior and the convergence in numerical values. In addition, according to the data presented in the table, the values of the temperature θ and thermal stress in the case of the nonlocal thermoelastic model are larger than in the fractional nonlocal thermoelastic model version. This means that the presence of fractional-order parameter α in the proposed model reduces and relaxes the mechanical and thermal wave behavior.

7.2 The effect of the nonlocal parameter

Based on new technologies in nanotechnology, the demand for small devices is increasing. To design the tools of such devices, it is important to understand their dynamic behavior. In the past few years, great emphasis has been placed on the static and dynamic behavior of such nanostructures. This section aims to focus on the investigation of flexural, vibration, and diffusion properties of nanostructures based on the theory of nonlocal elasticity proposed by Eringen.

The results obtained to investigate the effect of the nonlocal parameter on temperature, displacement, strain,

Table 2: Comparison of the temperature θ and thermal stress τ_{xx} with ref. [66]

x	Temperature θ		Thermal stress τ_{xx}	
	Present	**Ref. [66]**	**Present**	**Ref. [66]**
0.0	0.49124	0.546272	−0.155985	−0.214435
0.1	0.318432	0.339959	−0.0988131	−0.0578964
0.2	0.0423645	0.0449211	−0.143225	−0.13072
0.3	0.016466	0.0190259	−0.0641543	−0.0890927
0.4	0.0088852	0.00956508	−0.022701	−0.0300153
0.5	0.0017586	0.00189095	−0.00748039	−0.00763243
0.6	0.000629663	0.000708719	−0.00339493	−0.00395257
0.7	0.000274229	0.000296909	−0.00189069	−0.00314844
0.8	0.00006661	7.22195×10^{-5}	−0.00099898	−0.002033
0.9	2.29003×10^{-5}	2.54472×10^{-5}	−0.00047606	−0.00105175
1.0	8.84×10^{-6}	9.61×10^{-6}	−0.00021383	−0.000473434

and nonlocal heat pressure are presented in Figures 1–4. It can be seen that the nonlocal parameter $\xi = 0$ denotes the classical local model of thermoelasticity, whereas other values ($\xi = 0.1,\ 0.3$) denote nonlocal models of elasticity and thermoelasticity. In this case, the values $\alpha = 0.75$, $\tau_0 = 0.02$, $Q_0 = 1$, and $\varepsilon = 1$ were taken into account. Figures (1–4) indicate the prominent effect of the nonlocal coefficient ξ on the behavior of all studied domains. It is further shown from the figures that, depending on the value of the nonlocal parameter, the mechanical and thermal waves reach a steady state. The goal of the figures is to show how the size of the half-space affects its vibrations, and we accomplish a fairly noticeable impact of the medium size here. As shown in the figures, the thermoelastic vibrations of the solid alter dramatically as the material's length increased in relation to its height.

Figure 1 shows the difference in displacement u *versus* distance x for different values of the nonlocal parameter ξ. From the figure, it is observed that the absolute value of the displacement first increases with distance very quickly until it reaches its maximum value and then decreases to zero when $x \geq 3$. As the heat source changes periodically over time (a sinusoidal pulse active for a short period), the displacement is large near the turbulence region. It is also observed from the figure that the minimum values of the displacement decrease with the value of the nonlocal parameter ξ. In the field of $0 \leq x \leq 3$, the effect of the parameter appears for a while and then disappears away from the thermal turbulence region.

To study the effect of the nonlocal parameter ξ on temperature distribution θ in the context of generalized models of thermoelasticity involving the Caputo–Fabrizio fractional-order differential operator, the variation in temperature θ *versus* distance x is presented in Figure 2. In line with theoretical and physical observations, low-temperature changes are observed to heat and finally exceed the thermal wavefront located at $x = 3$. The figure shows that the value of temperature θ decreases with respect to the constant x as the parameter ξ increases. This is because, for a short period of time, the heat source varies periodically and the turbulence region decreases. The same behavior has been observed in the literature [64,65].

Figure 3 shows the variance of the nonlocal stress increase τ_{xx} with respect to the distance x in the case of three different values $\xi = 0.0,\ 0.1,\ 0.3$. It is noticed from the figure that with the decrease of the nonlocal parameter ξ, the magnitude of the nonlocal stress increases. It is also noted that the effect of periodically changing heat sources distributed throughout the flat region $x = 0$ will imply these types of activity in all studied field variables. It is first observed that stress and displacement are compressions, then they begin to decrease in size, and finally, they tend to zero in all cases. The maximum stress is close to the half-space boundary near the turbulence region.

Figure 4 represents the strain variance e for different values of nonlocal modulus ξ *versus* distance x. This figure shows that the strain e decreases with increasing distance and eventually reaches zero. We can also see from the figure that the amount of strain e increases with the increase of the parameter ξ in some periods and decreases with it in others. This may be because the heat source periodically changes over time.

From the previous results and discussions, it was found that the micro-scale effect is not obvious for structures with dimensions on the order of micrometers, while it can be observed in nanostructures, and this is consistent with the results obtained by Wang and Liew observed in ref. [67]. Also, when determining the stress at the source of the nano-heating problem, the nonlocal behavior is a critical factor that cannot be ignored [39–41].

7.3 The effect of the fractional-order parameter

In Section 7.3, an investigation was performed into the influence of the fractional order α in the presence of an initial magnetic field on the thermoelastic conducing material. Calculation for $\xi = 0.1$, $\tau_0 = 0.02$, $Q_0 = 1$, and $\varepsilon = 1$ have been carried out. The findings are graphically depicted at various positions of x and are shown in Figure 10. For normal case $\alpha = 1$, the numerical results are consistent with all previous results of applications taken in the sense of generalized nonlocal theory of thermoelasticity with one relaxation time (LS). In the case of $0 < \alpha < 1$, the numerical results based on the Caputo–Fabrizio model of fractional differential heat conduction have been studied.

Figures 5–8 display the spatial variability of the nondimensional thermodynamic temperature fields, stress, displacement, and strain for different values of the fractional differentiation modulus of the Caputo–Fabrizio fractional operator. Note that case $\alpha = 1$ indicates the old mode (nonfractional LS thermoelastic model) and case $\alpha = 0.75,\ 0.5$ indicates the Caputo–Fabrizio model (fractional CF thermoelastic model).

The graphs clearly show that the Caputo–Fabrizio fractional differential operator (CF) (fractional-order factor α) significantly affects all the studied fields. In addition, thermal and mechanical waves reach a steady state according to the values of the fractal order α. It was also found that the increase in the value of the fractal parameter α increases the propagation velocity of the studied waves toward the surface of the solid body affected by the periodic heat source, which disappears faster with increasing distance inside the elastic body.

The numerical results have shown that an increase in the value of the fractional parameter α increases the thermodynamic temperature θ and stress e, while the increase in the value of the fractional parameter α decreases the displacement u and the nonlocal stress τ_{xx}.

Moreover, we are sure that this new CF derivative will play a major role in studying the macroscopic behavior of some materials related to nonlocal exchanges, which predominate in the characterization of materials.

7.4 The effect of the applied magnetic field

In this third case, Figures 9–12 show the diversity of the fields studied when $\alpha = 0.75$ and $\xi = 0.1$ with respect to three different values of the applied magnetic field. The parameter $\varepsilon = \sigma_0\mu_0 H_x^2/(\rho c_0^2\omega_0)$ denotes the amount of magnetic field propagation on the surface perpendicular to the x-axis. We will consider three values of the coefficient, the first case in the absence of a magnetic field ($\varepsilon = 0$), and the second in the presence of a magnetic field ($\varepsilon = 1, 5$). The figures show a significant influence of displacement u, strain e, temperature θ, and nonlocal heat pressure τ_{xx} due to the presence of the magnetic field.

In Figure 9, an increase in the magnetic field value leads to a decrease in the displacement amounts, which is very noticeable at the peaks of the curves. Thus, the magnetic field is able to dampen the thermal expansion of the medium. These results are similar to those in the literature [55]. It is shown in Figure 10 that the applied magnetic field has little effect on changes in the temperature distribution. The temperature changes and increases slightly with the increase of the magnetic field affecting the medium. The same behavior observed in these forms is similar to the behavior of many materials and minerals, and this has been proven in many studies as mentioned in refs. [68,69].

It is shown in Figure 11 that the increase in the magnetic field parameter ε increases the value of the nonlocal pressure field τ_{xx}, which is very noticeable at the starting and peak points as seen from the curves. The nonlocal stress starts with negative values, then increases to its peak and gradually decreases to zero values with increasing distance x. As a result of the fluctuation of the strain behavior, as in the previous cases, it can be seen from Figure 12 that the strain e increases in some periods with the increase of the magnetic field and decreases in others.

8 Conclusion

In this study, a new thermal conductivity model for generalized thermoelasticity is proposed based on the concept of CF's fractional calculus. Fourier's law has been improved to include both the heat flow vector and its fractional time derivatives. Also, the fractional heat conduction model is based on the use of a nonsingular exponentially decreasing kernel, in contrast to the conventional fractional operators. Also, Eringen's nonlocal theory has been used as it accurately formulates nanostructures and covers the gap between experimental and classical results. The governing equations were solved using the Laplace transform and the numerical results for the nondimensional temperature, displacement, and nonlocal pressure were obtained using the Zakian algorithm approximation. The results showed that different field quantities depend not only on instantaneous space and time coordinates but also on effective parameters such as nonlocal modulus and fractional differentiation parameter, in addition to the applied magnetic field. This article provides an explanation of the use of the CF partial derivative as a model for real-world problems involving history and memory as well as nonlocal effects.

It can be inferred from the discussions and results obtained and discussed that the nonlocal parameters have an important influence on the distributions of the studied fields. It was also found that the behavior of different domains reaches a degree of stability based on the nonlocal parameter values. The results also showed the prominent effect of the fractional differential order of the new fractional calculus operator, as they showed its role in reducing the propagation of thermal and mechanical waves within the medium. The results also show the important effect of the applied magnetic field on the behavior of the studied variables, and therefore, its effect on the design process cannot be neglected. The new model was physically validated, and it was observed that the values of the different variables are large near the turbulence region where the effect of the cyclic heat source is fading away from the surface. In other words, the new model predicts infinite speeds of heat wave propagation, contrary to the traditional theories. The results of this work are useful and appreciated as a theoretical basis for nanostructural design, especially for those based on different heat sources.

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