Research Article

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Soliton solutions of Calogero–Degasperis–Fokas dynamical equation *via* modified mathematical methods

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Abstract: New solitary wave solutions of the Calogero–Degasperis–Fokas (CDF) equation *via* two modified methods called improved simple equation and modified F-expansion schemes are investigated. Numerous types of results are obtained in the form of hyperbolic functions, trigonometric functions and elliptic functions. Moreover, some of the derived solutions are illustrated as two-dimensional, three-dimensional and contour graphical images that were plotted with the assistance of computational software Mathematica, which gave useful knowledge to study the physical phenomena of the CDF model. The investigated solutions have fruitful advantages in mathematical physics.

Keywords: Calogero–Degasperis–Fokas equation, modified methods

1 Introduction

Many researchers have used incipient and puissant implements in pristine and applied mathematics to develop incipient research areas such as fractional differential calculus and astronomical applications that have been implemented in genuine-world quandaries [1–3]. The concept of solitary wave solutions goes back to works of John Scott Russell in 1834. The main structure of constructing the soliton solutions is the comeback to inverse scattering transforms [4] which Ablowitz and Clarkson studied for the astronomically immense field of integrable nonlinear evaluation equations (NLEEs). To find more information about solitons, we refer ref. [5] to the intrigued reader. Over the last few decades, nonlinear phenomena have been optically canvassed to have fascinating characteristics in mathematical physics and engineering. The phenomena of NLEEs have magnetized plentiful care and have become one of the most intriguing fields of research. These types of equations are broadly used to explicate intricate physical phenomena arising in fluid mechanics, plasma wave, optical fiber telecommunication, soliton theory and atmospheric science [6–17].

Many efficacious and puissant techniques have been employed to construct wave solutions, inverse scattering transformation [18,19], Backlund transformation scheme [20], auxiliary equation technique [21,22], extended direct algebraic scheme [23,24], Darboux transformations technique [25], homotopy perturbation technique [26,27], extended direct algebraic scheme [28], the modified Kudryashov scheme [29,30], technique of first integral [31,32], sine-Gordon expansion scheme [33,34], projective Riccati equation scheme [35,36], expansion technique of Jacobi elliptic function [37,38], Hirota bilinear scheme [39], Wronskian determinant technique [40] and so on. Due to lot of applications and importance in applied science of the CDF model, many researchers investigated several solutions by employing different schemes. Previous studies [41–43] employed reduction scheme by the spectral transform, exp-function method and improved tanh technique in the CDF equation, respectively. Recently, Jhangeer *et al.* [44] derived solitary wave solution of different types of the CDF equation by applying the (G'/G) expansion schemes. We have employed two modified mathematical methods [45] in the CDF wave model. Our constructed solutions are more powerful and used as fruitful applications in mathematical physics.

This article is organized as follows. In Section 2, the proposed methods are described briefly. In Section 3, the proposed schemes are successfully instigated on the CDF model. Results and discussion and conclusion are explained in Sections 4 and 5, respectively.

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2 Analysis of the proposed methods

Let

$$P_1(U, U_x, U_t, U_{xx}, U_{tt}, \dots) = 0. \tag{1}$$

Consider

$$U = U(\xi), \quad \xi = x - \omega t. \tag{2}$$

Put (2) in (1),

$$P_2(U, U', U'', \dots) = 0. \tag{3}$$

2.1 Improved simple equation method

Let (3) have the solutions:

$$U(\xi) = \sum_{i=-N}^{N} A_i \Psi^i(\xi). \tag{4}$$

Let Ψ satisfies,

$$\Psi' = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3. \tag{5}$$

Putting (4) with (5) in (3), numerous systems of equations are attained, and these systems for the required values of the parameters are solved. Substituting value of determined parameters and value of Ψ in (4), concerned results of (1) are attained.

2.2 Modified F-expansion method

Let formal solution of Eq. (3) be:

$$U = a_0 + \sum_{i=1}^{N} a_i F^i(\xi) + \sum_{i=1}^{N} b_i F^{-i}(\xi). \tag{6}$$

Let F gratify

$$F' = A + BF + CF^2. \tag{7}$$

Put Eq. (6) along with Eq. (7) in Eq. (3). Choosing distinct cases of A, B, C with value of F from Table 1 [46] and subsituting the determined values of a_i, b_i in Eq. (6) completes the destination of Eq. (1).

3 Applications

Let Calogero–Degasperis–Fokas (CDF) equation in ref. [44] be

$$U_t + U_{xxx} - \frac{1}{8}(U_x)^3 - (pe^U + qe^{-U})U_x = 0. \tag{8}$$

Put Eq. (2) in Eq. (8),

$$-\omega U + U''' - \frac{1}{8}(U')^3 - (pe^U + qe^{-U})U' = 0. \tag{9}$$

Consider Painleve transformation $U = \mathrm{Ln}(V)$ in (9),

$$\begin{aligned}&-(pV^2 + q)VV' - 3V''VV' + \frac{15(V')^3}{8}\\&+ V^2(V^{(3)} - \omega V') = 0.\end{aligned} \tag{10}$$

3.1 Application of improved simple equation method

Let solution of (10) be:

$$V = A_2\Psi^2 + A_1\Psi + \frac{A_{-2}}{\Psi^2} + \frac{A_{-1}}{\Psi} + A_0 \tag{11}$$

Put (11) with (5) in (10).

Case 1: $c_3 = 0$,

Family-I (Figures 1 and 2)

$$\begin{aligned}&\omega = \frac{1}{2}(-c_1^2 + 4c_0c_2 - 4\sqrt{pq}), \quad A_2 = \frac{3c_2^2}{2p},\\&A_1 = \frac{\sqrt{3}\sqrt{8c_2^2\sqrt{pq} - 12c_0c_2^3 + 3c_1^2c_2^2} + 3c_1c_2}{2p},\\&A_0 = \frac{c_2(3c_1^2 + 4\sqrt{p}\sqrt{q}) + \sqrt{3}c_1\sqrt{c_2^2(3c_1^2 - 12c_0c_2 + 8\sqrt{p}\sqrt{q})} - 6c_0c_2^2}{4c_2p},\\&A_{-2} = 0, \quad A_{-1} = 0.\end{aligned} \tag{12}$$

Put Eq. (12) in Eq. (11),

$$\begin{aligned}V_1 = &\frac{3\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \varepsilon)\right)\right)^2}{8p}\\&-\left(\frac{\sqrt{3}\sqrt{8c_2^2\sqrt{pq} - 12c_0c_2^3 + 3c_1^2c_2^2} + 3c_1c_2}{4c_2p}\right)\\&\times c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \varepsilon)\right)\\&+ \frac{\sqrt{3}c_1\sqrt{c_2^2(3c_1^2 - 12c_0c_2 + 8\sqrt{qp})}}{4c_2p}\\&+ \frac{c_2(3c_1^2 + 4\sqrt{pq}) - 6c_0c_2^2}{4c_2p}, \quad 4c_0c_2 > c_1^2.\end{aligned} \tag{13}$$

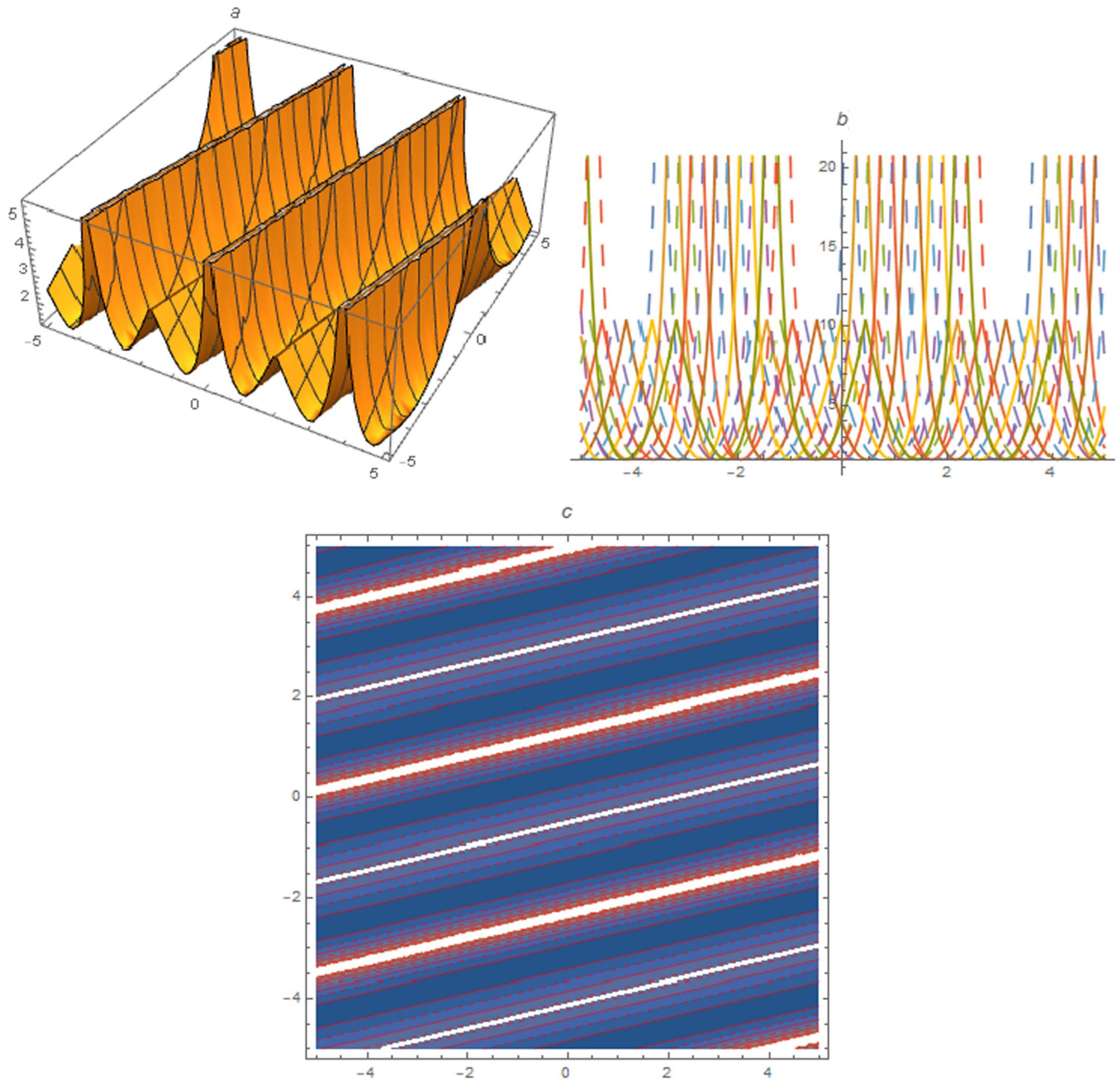

Figure 1: The profile of solution U_1 for $c_1 = 1$, $c_2 = 1$, $c_0 = 1$, $p = 1$, $q = 0.4$, $\varepsilon = 0.5$.

Since $U = \mathrm{Ln}(V)$, then solution of Eq. (10) becomes

$$U_1 = \mathrm{Ln}(V_1). \tag{14}$$

Family-II (Figure 3)

$$\begin{aligned}
A_0 &= \frac{c_0(3c_1^2 + 4\sqrt{p}\sqrt{q}) + \sqrt{3}\,c_1\sqrt{c_0^2(3c_1^2 - 12c_0c_2 + 8\sqrt{p}\sqrt{q})} - 6c_2c_0^2}{4c_0 p},\\
A_{-2} &= \frac{3c_0^2}{2p}, \quad A_2 = 0,\\
A_1 &= 0,\\
A_{-1} &= \frac{\sqrt{3}\sqrt{8c_0^2\sqrt{pq} - 12c_2c_0^3 + 3c_1^2c_0^2} + 3c_0c_1}{2p},\\
\omega &= \frac{1}{2}(-c_1^2 + 4c_0c_2 - 4\sqrt{p}\sqrt{q}).
\end{aligned} \tag{15}$$

Substituting Eq. (15) in Eq. (10),

$$\begin{aligned}
V_2 = &-\frac{(2c_2)(\sqrt{3}\sqrt{8c_0^2\sqrt{pq} - 12c_2c_0^3 + 3c_1^2c_0^2} + 3c_0c_1)}{(2p)\left(c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \varepsilon)\right)\right)}\\
&+ \frac{\sqrt{3}\,c_1\sqrt{c_2^2(3c_1^2 - 12c_0c_2 + 8\sqrt{qp})}}{4c_2 p}\\
&+ \frac{c_2(3c_1^2 + 4\sqrt{pq}) - 6c_0c_2^2}{4c_2 p}\\
&+ \frac{(3c_0^2)\left(\frac{2c_2}{c_1 - \sqrt{4c_2c_0 - c_1^2}\tan\left(\frac{1}{2}\sqrt{4c_2c_0 - c_1^2}(\xi + \varepsilon)\right)}\right)^2}{2p},\\
&4c_0c_2 > c_1^2.
\end{aligned} \tag{16}$$

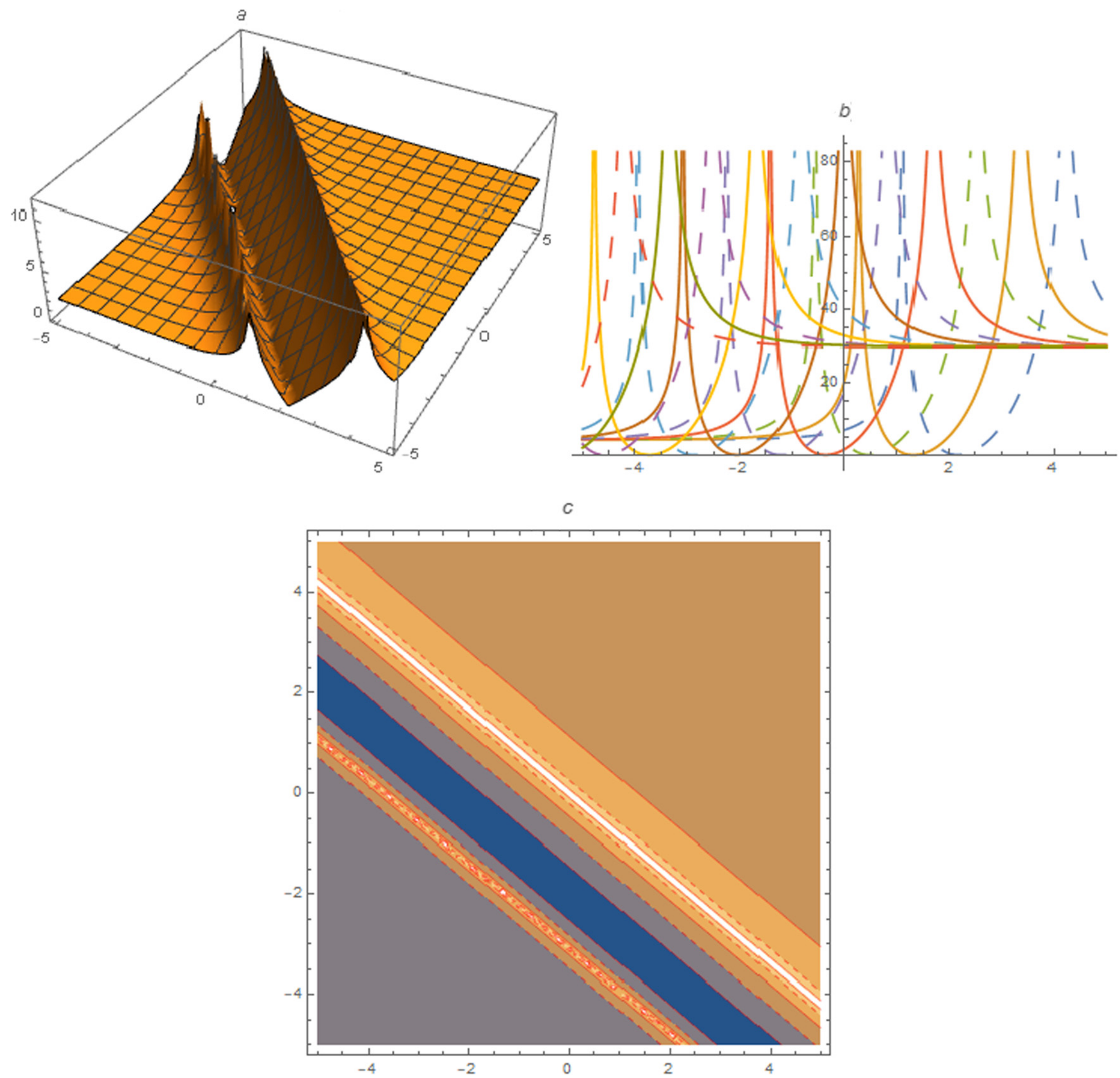

Figure 2: The profile of solution U_4 for $c_2 = -1$, $c_1 = 0.21$, $p = 0.01$, $q = 0.3$, $\varepsilon = 0.001$.

Since $U = \mathrm{Ln}(V)$, then solution of Eq. (10) can be written as:

$$U_2 = \mathrm{Ln}(V_2). \tag{17}$$

Case 2: $c_0 = 0$, $c_3 = 0$,

$$\begin{gathered} A_{-1} = 0, \\ A_0 = \frac{\sqrt{3}c_1\sqrt{c_2^2(3c_1^2 + 8\sqrt{qp})} + c_2(3c_1^2 + 4\sqrt{qp})}{4c_2 p}, \\ A_{-2} = 0, \quad \omega = -\frac{c_1^2}{2} - 2\sqrt{pq}, \quad A_2 = \frac{3c_2^2}{2p}. \end{gathered} \tag{18}$$

Put Eq. (18) in Eq. (10),

$$\begin{aligned} V_3 = {} & \frac{\sqrt{3}c_1\sqrt{c_2^2(3c_1^2 + 8\sqrt{qp})} + c_2(3c_1^2 + 4\sqrt{qp})}{4c_2 p} \\ & + \frac{\sqrt{3}\sqrt{8c_2^2\sqrt{qp} + 3c_1^2c_2^2} + 3c_1c_2}{2p} \\ & \times \left(\frac{c_1\exp(c_1(\xi + \varepsilon))}{1 - c_2\exp(c_1(\xi + \varepsilon))}\right) \\ & + \frac{(3c_2^2)\left(\frac{c_1\exp(c_1(\xi + \varepsilon))}{1 - c_2\exp(c_1(\xi + \varepsilon))}\right)^2}{2p}, \quad c_1 > 0. \end{aligned} \tag{19}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (19) can be written as:

$$U_3 = \mathrm{Ln}(V_3) \tag{20}$$

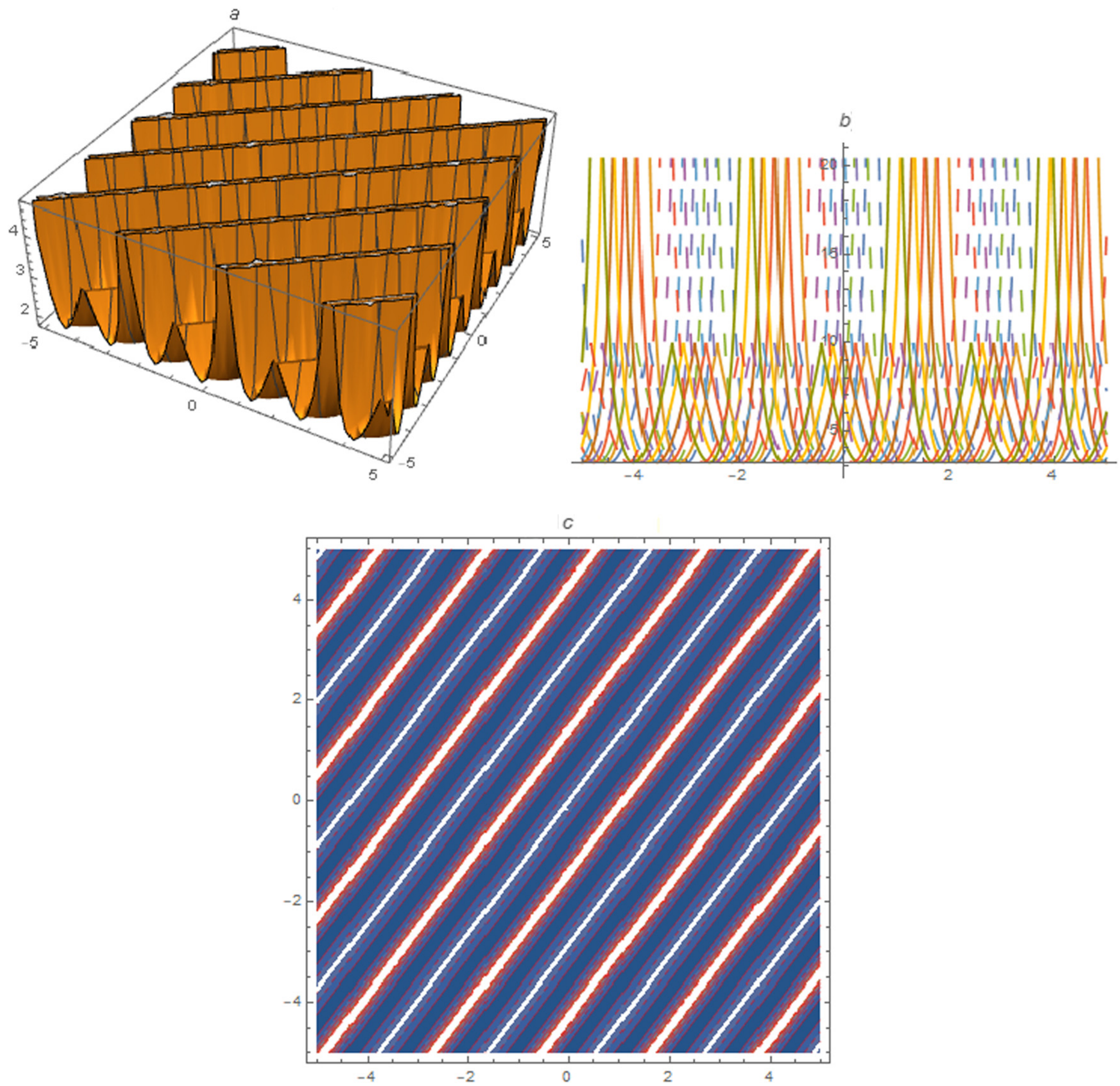

Figure 3: The profile of solution U_5 for $c_2 = 1$, $c_0 = 1.21$, $p = 1.01$, $q = 0.3$, $\varepsilon = 0.001$.

$$V_4 = \frac{\sqrt{3}c_1\sqrt{c_2^2(3c_1^2 + 8\sqrt{qp})} + c_2(3c_1^2 + 4\sqrt{qp})}{4c_2 p} + \frac{\sqrt{3}\sqrt{8c_2^2\sqrt{qp} + 3c_1^2c_2^2} + 3c_1c_2}{2p} \times \left(-\frac{c_1\exp(c_1(\xi + \varepsilon))}{c_2\exp(c_1(\xi + \varepsilon)) + 1}\right) + \frac{3c_2^2}{2p}\left(-\frac{c_1\exp(c_1(\xi + \varepsilon))}{c_2\exp(c_1(\xi + \varepsilon)) + 1}\right)^2, \quad c_1 < 0. \tag{21}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (21) can be written as:

$$U_4 = \mathrm{Ln}(V_4). \tag{22}$$

Case 2: $c_1 = c_3 = 0$,
Family-I

$$A_0 = \frac{2\sqrt{p}\sqrt{q} - 3c_0c_2}{2p}, \quad A_{-2} = A_{-1} = 0,$$
$$A_2 = \frac{3c_2^2}{2p}, \quad A_1 = \sqrt{3}\sqrt{\frac{2c_2^2\sqrt{q}}{p^{3/2}} - \frac{3c_0c_2^3}{p^2}}, \tag{23}$$
$$\omega = -2(\sqrt{p}\sqrt{q} - c_0c_2).$$

Put Eq. (23) in Eq. (10),

$$V_5 = \frac{2\sqrt{p}\sqrt{q} - 3c_0c_2}{2p} + \sqrt{3}\sqrt{\frac{2c_2^2\sqrt{q}}{p^{3/2}} - \frac{3c_0c_2^3}{p^2}} \times \left(\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi + \varepsilon))\right) + \frac{(3c_2^2)\left(\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi + \varepsilon))\right)^2}{2p}, \quad c_0c_2 > 0. \tag{24}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (24) can be written as:

$$U_5 = \mathrm{Ln}(V_5) \tag{25}$$

$$V_6 = \frac{2\sqrt{p}\sqrt{q} - 3c_0c_2}{2p} + \sqrt{3}\sqrt{\frac{2c_2^2\sqrt{q}}{p^{3/2}} - \frac{3c_0c_2^3}{p^2}} \times \left(\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))\right) + \frac{(3c_2^2)\left(\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))\right)^2}{2p}, \quad c_0c_2 < 0. \tag{26}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (26) can be written as:

$$U_6 = \mathrm{Ln}(V_6). \tag{27}$$

Family-II

$$A_0 = \frac{2\sqrt{p}\sqrt{q} - 3c_0c_2}{2p}, \quad A_{-2} = \frac{3c_0^2}{2p}, \quad A_{-1} = -\sqrt{3}\sqrt{\frac{2c_0^2\sqrt{q}}{p^{3/2}} - \frac{3c_0^3c_2}{p^2}}, \quad A_2 = 0, \quad A_1 = 0, \quad \omega = -2(\sqrt{p}\sqrt{q} - c_0c_2). \tag{28}$$

Put (28) in (10),

$$V_7 = \frac{2\sqrt{p}\sqrt{q} - 3c_0c_2}{2p} - \frac{\sqrt{3}\sqrt{\frac{2c_0^2\sqrt{q}}{p^{3/2}} - \frac{3c_0^3c_2}{p^2}}}{\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi+\varepsilon))} + \frac{3c_0^2}{(2p)\left(\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi+\varepsilon))\right)^2} \quad c_0c_2 > 0. \tag{29}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (29) can be written as

$$U_7 = \mathrm{Ln}(V_7). \tag{30}$$

$$V_8 = \frac{2\sqrt{p}\sqrt{q} - 3c_0c_2}{2p} - \frac{\sqrt{3}\sqrt{\frac{2c_0^2\sqrt{q}}{p^{3/2}} - \frac{3c_0^3c_2}{p^2}}}{\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))} + \frac{3c_0^2}{(2p)\left(\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))\right)^2} \quad c_0c_2 < 0. \tag{31}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (31) can be written as

$$U_8 = \mathrm{Ln}(V_8). \tag{32}$$

Family-III (Figure 4)

$$\begin{aligned} A_0 &= \frac{-9c_0c_2pq + 16c_0^2c_2^2\sqrt{p}\sqrt{q} + 144c_0^3c_2^3 - p^{3/2}q^{3/2}}{p^2q - 16c_0^2c_2^2p}, \\ A_{-2} &= \frac{3c_0^2}{2p}, \quad A_{-1} = \sqrt{6}\sqrt{-\frac{c_0^2\sqrt{q}}{p^{3/2}} - \frac{6c_2c_0^3}{p^2}} \\ A_2 &= \frac{3c_2^2}{2p}, \quad A_1 = -\frac{\sqrt{6}\,c_2\sqrt{-\frac{c_0^2(6c_0c_2 + \sqrt{p}\sqrt{q})}{p^2}}}{c_0}, \\ \omega &= \frac{2(4c_0c_2pq - 16c_0^2c_2^2\sqrt{qp} - 64c_0^3c_2^3 + p^{3/2}q^{3/2})}{pq - 16c_0^2c_2^2}. \end{aligned} \tag{33}$$

Put (33) in (10),

$$\begin{aligned} V_9 = {} & \frac{-9c_0c_2pq + 16c_0^2c_2^2\sqrt{p}\sqrt{q} + 144c_0^3c_2^3 - p^{3/2}q^{3/2}}{p^2q - 16c_0^2c_2^2p} \\ & - \left(\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi+\varepsilon))\right) \times \left(\frac{\sqrt{6}\,c_2\sqrt{-\frac{c_0^2(6c_0c_2 + \sqrt{p}\sqrt{q})}{p^2}}}{c_0}\right) \\ & + \frac{3c_2^2}{2p}\left(\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi+\varepsilon))\right)^2 \\ & + \frac{\sqrt{6}\sqrt{-\frac{c_0^2\sqrt{q}}{p^{3/2}} - \frac{6c_2c_0^3}{p^2}}}{\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi+\varepsilon))} \\ & + \frac{3c_0^2}{(2p)\left(\sqrt{\frac{c_0c_2}{c_2}}\tan(\sqrt{c_0c_2}(\xi+\varepsilon))\right)^2} \quad c_0c_2 > 0. \end{aligned} \tag{34}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (34) can be written as:

$$U_9 = \mathrm{Ln}(V_9). \tag{35}$$

$$V_{10} = \frac{-9c_0c_2pq + 16c_0^2c_2^2\sqrt{p}\sqrt{q} + 144c_0^3c_2^3 - p^{3/2}q^{3/2}}{p^2q - 16c_0^2c_2^2p} - \left(\frac{\sqrt{6}\,c_2\sqrt{-\frac{c_0^2(6c_0c_2+\sqrt{p}\sqrt{q})}{p^2}}}{c_0}\right) \times \frac{(3c_2^2)\left(\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))\right)^2}{2p} + \sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon)) \times \frac{+\sqrt{6}\sqrt{-\frac{c_0^2\sqrt{q}}{p^{3/2}} - \frac{6c_2c_0^3}{p^2}}}{\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))} \times \frac{+\frac{3c_0^2}{2p}}{\left(\sqrt{-\frac{c_0c_2}{c_2}}\tanh(\sqrt{-c_0c_2}(\xi+\varepsilon))\right)^2},\quad c_0c_2 < 0. \tag{36}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (36) can be written as:

$$U_{10} = \mathrm{Ln}(V_{10}). \tag{37}$$

3.2 Applications of modified F-expansion method

Let (10) have solution:

$$V = a_2F^2 + a_1F + a_0 + \frac{b_2}{F^2} + \frac{b_1}{F}. \tag{38}$$

Substitute (38) with(7) in (10).

Case 1

For $A = 0, B = 1, C = -1$,

$$a_0 = \frac{1}{4}\left(-\frac{4\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}}}{p} + \frac{3}{p}\right),\quad a_2 = \frac{3}{2p},\quad a_1 = \frac{-\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}} - 3}{2p},\quad b_1 = 0,\quad b_2 = 0,\quad \omega = \frac{1}{2}(4\sqrt{p}\sqrt{q} - 1). \tag{39}$$

Put (39) in (38),

$$V_{11} = \frac{1}{4}\left(-\frac{4\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}}}{p} + \frac{3}{p}\right) + \frac{-\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}} - 3}{4p}\tanh\left(\frac{\xi}{2}\right) + 1 + \frac{3}{8p}\left(\tanh\left(\frac{\xi}{2}\right) + 1\right)^2. \tag{40}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (40) can be written as:

$$U_{11} = \mathrm{Ln}(V_{11}). \tag{41}$$

Case 2

$A = 0, B = -1, C = 1$

$$a_0 = \frac{1}{4}\left(-\frac{4\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}}}{p} + \frac{3}{p}\right),\quad a_2 = \frac{3}{2p},\quad a_1 = \frac{-\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}} - 3}{2p},\quad b_1 = 0,\quad b_2 = 0,\quad \omega = \frac{1}{2}(4\sqrt{qp} - 1). \tag{42}$$

Put (42) in (38),

$$V_{12} = \frac{1}{4}\left(-\frac{4\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}}}{p} + \frac{3}{p}\right) + \frac{-\sqrt{3}\sqrt{3-8\sqrt{p}\sqrt{q}} - 3}{4p}\left(1 - \coth\left(\frac{\xi}{2}\right)\right) + \frac{6}{16p}\left(1 - \coth\left(\frac{\xi}{2}\right)\right)^2. \tag{43}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (43) can be written as:

$$U_{12} = \mathrm{Ln}(V_{12}). \tag{44}$$

Case 3

For $A = \frac{1}{2},\ B = 0,\ C = -\frac{1}{2}$ (Figure 5),

$$a_0 = \frac{8\sqrt{p}\sqrt{q} + 3}{8p},\quad a_2 = \frac{3}{8p},\quad a_1 = \sqrt{\frac{3\sqrt{q}}{2p^{3/2}} + \frac{9}{16p^2}},\quad b_1 = 0,\quad b_2 = 0,\quad \omega = \frac{1}{2}(-4\sqrt{qp} - 1). \tag{45}$$

Put (45) in (38),

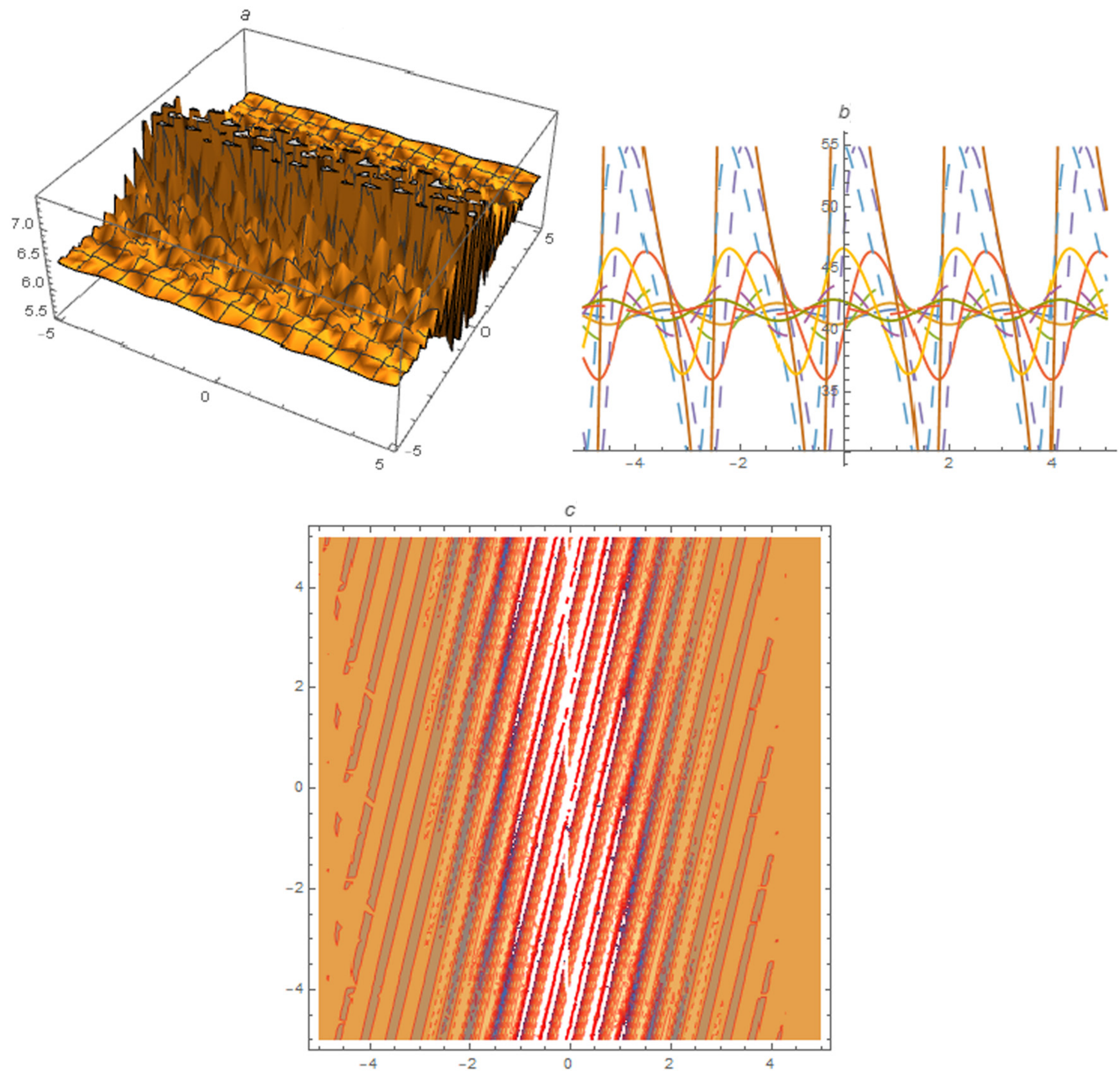

Figure 4: The profile of solution U_7 for $c_2 = -2.1$, $c_0 = -1$, $p = 0.01$, $q = -2.3$, $\varepsilon = 0.1$.

$$V_{13} = \frac{8\sqrt{p}\sqrt{q} + 3}{8p} + \sqrt{\frac{3\sqrt{q}}{2p^{3/2}} + \frac{9}{16p^2}} \times (\coth[\xi] + \operatorname{csch}[\xi]) + \frac{3}{8p}(\coth(\xi) + \operatorname{csch}(\xi))^2. \tag{46}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (46) can be written as:

$$U_{13} = \mathrm{Ln}(V_{13}). \tag{47}$$

Case 4

$A = 1,\ B = 0,\ C = -1,$

$$a_0 = \frac{2\sqrt{p}\sqrt{q} + 3}{2p}, \quad a_2 = \frac{3}{2p}, \quad a_1 = \sqrt{3}\sqrt{\frac{2\sqrt{q}}{p^{3/2}} + \frac{3}{p^2}}, \quad b_1 = 0, \quad b_2 = 0, \quad \omega = -2(\sqrt{p}\sqrt{q} + 1). \tag{48}$$

Put (48) in (38),

$$V_{14} = \frac{2\sqrt{p}\sqrt{q} + 3}{2p} + \sqrt{3}\sqrt{\frac{2\sqrt{q}}{p^{3/2}} + \frac{3}{p^2}} \times (\tanh[\xi]) + \frac{3}{2p}\tanh^2[\xi]. \tag{49}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (49) can be written as:

$$U_{14} = \mathrm{Ln}(V_{14}). \tag{50}$$

Case 5

$A = C = 1/2,\ B = 0,$

$$a_0 = \frac{8\sqrt{p}\sqrt{q} - 3}{8p}, \quad a_2 = \frac{3}{8p},$$
$$a_1 = \frac{1}{4}\sqrt{3}\sqrt{\frac{8\sqrt{q}}{p^{3/2}} - \frac{3}{p^2}}, \tag{51}$$
$$b_1 = 0, \quad b_2 = 0, \quad \omega = \frac{1}{2}(1 - 4\sqrt{p}\sqrt{q}).$$

Put (51) in (38),

$$V_{15} = \frac{8\sqrt{p}\sqrt{q} - 3}{8p} + \frac{1}{4}\sqrt{3}\sqrt{\frac{8\sqrt{q}}{p^{3/2}} - \frac{3}{p^2}} \times (\sec[\xi] + \tan[\xi]) + \frac{3}{8p}(\tan(\xi) + \sec(\xi))^2. \tag{52}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (49) can be written as:

$$U_{15} = \mathrm{Ln}(V_{15}). \tag{53}$$

Case 6

$A = C = -1/2,\ B = 0,$

$$a_0 = \frac{8\sqrt{p}\sqrt{q} - 3}{8p}, \quad a_2 = \frac{3}{8p},$$
$$a_1 = \frac{1}{4}\sqrt{3}\sqrt{\frac{8\sqrt{q}}{p^{3/2}} - \frac{3}{p^2}}, \tag{54}$$
$$b_1 = 0, \quad b_2 = 0, \quad \omega = \frac{1}{2}(1 - 4\sqrt{p}\sqrt{q}).$$

Put (54) in (38),

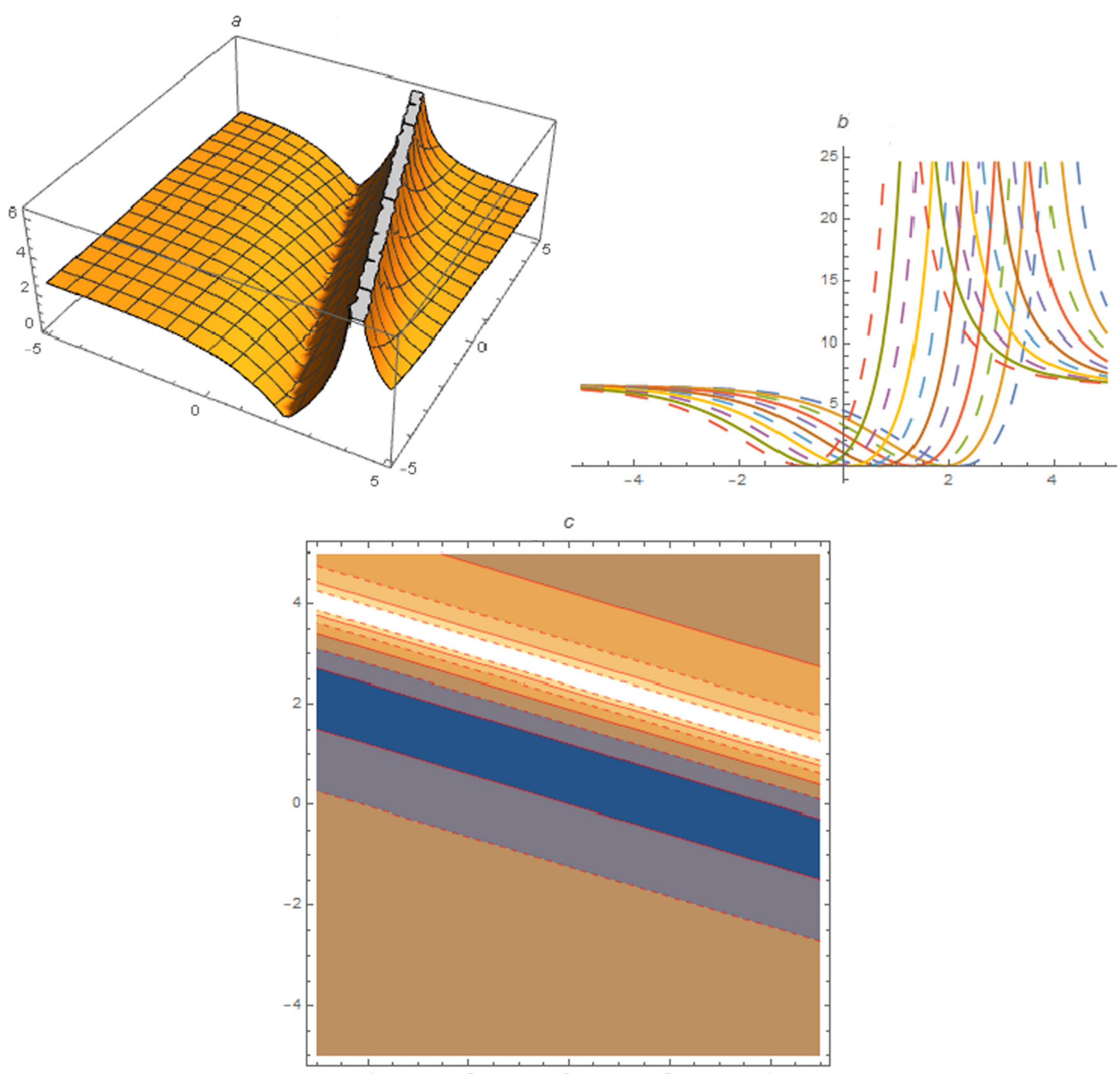

Figure 5: The profile of solution U_{11} for $p = 0.1$, $q = 0.1$.

$$V_{16} = \frac{8\sqrt{p}\sqrt{q} - 3}{8p} + \frac{4}{16}\sqrt{3}\sqrt{\frac{8\sqrt{q}}{p^{3/2}} - \frac{3}{p^2}}\sec(\xi) - \tan(\xi) + \frac{24}{64p}(\sec(\xi) - \tan(\xi))^2. \quad (55)$$

Since $U = \mathrm{Ln}(V)$, then Eq. (55) can be written as:,

$$U_{16} = \mathrm{Ln}(V_{16}). \quad (56)$$

Case 7

$A = C = -1,\ B = 0,$

$$a_0 = \frac{2\sqrt{p}\sqrt{q} - 3}{2p}, \quad a_2 = \frac{3}{2p}, \quad a_1 = \sqrt{3}\sqrt{\frac{2\sqrt{q}}{p^{3/2}} - \frac{3}{p^2}}, \quad b_1 = 0, \quad b_2 = 0, \quad \omega = -2(\sqrt{p}\sqrt{q} - 1). \quad (57)$$

Put (57) in (38),

$$V_{17} = \frac{2\sqrt{p}\sqrt{q} - 3}{2p} + \sqrt{3}\sqrt{\frac{2\sqrt{q}}{p^{3/2}} - \frac{3}{p^2}}(\coth[\xi]) + \frac{3}{2p}\coth^2[\xi]. \quad (58)$$

Since $U = \mathrm{Ln}(V)$, then Eq. (58) can be written as:

$$U_{17} = \mathrm{Ln}(V_{17}). \quad (59)$$

Case 8

$A = 0\ B = 0$ (Figure 6),

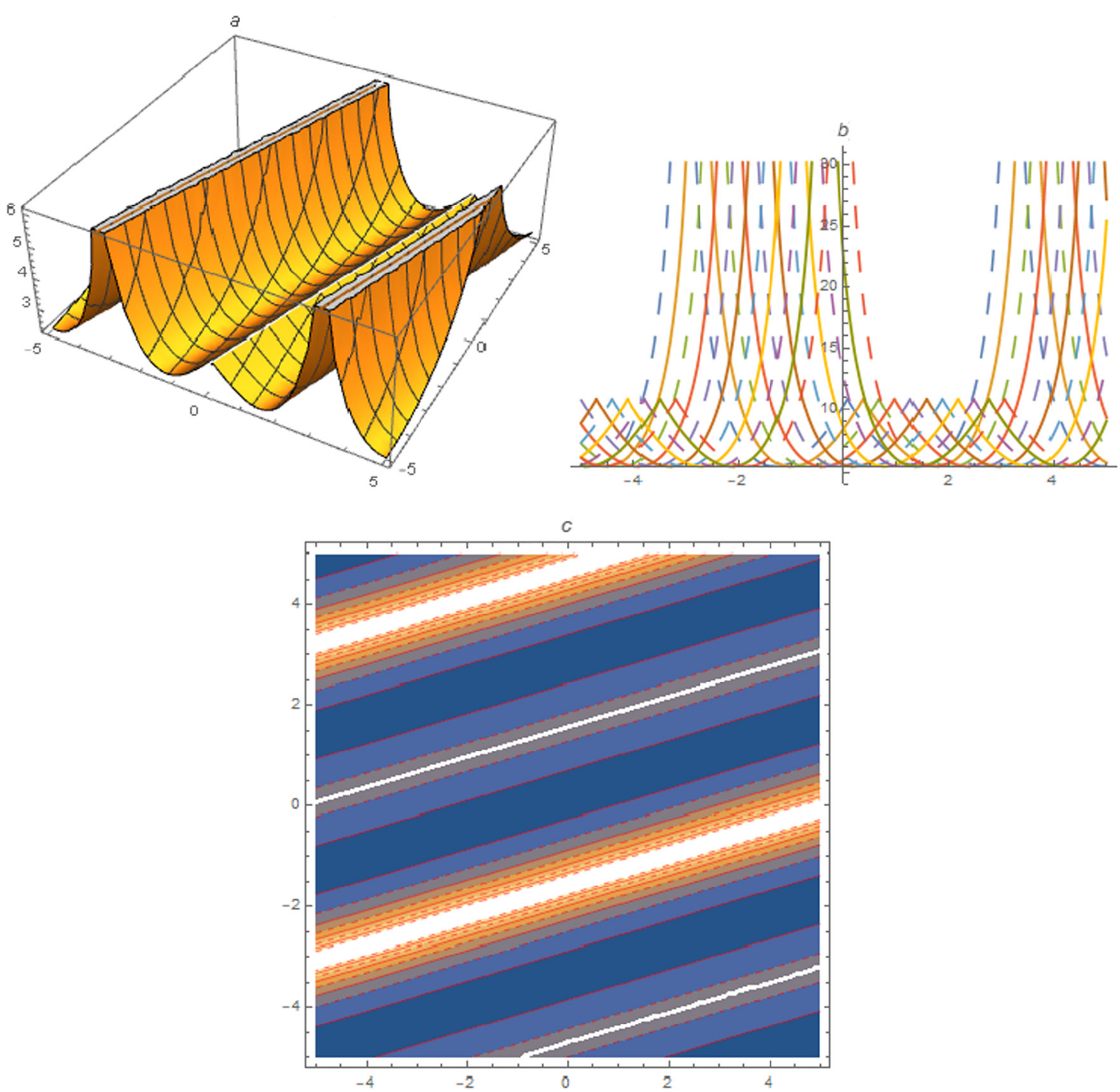

Figure 6: The profile of solution U_{16} for $p = 0.1$, $q = 0.1$.

$$a_0 = \frac{\sqrt{q}}{\sqrt{p}}, \quad a_2 = \frac{3C^2}{2p},$$
$$a_1 = \frac{\sqrt{6}\,C\sqrt[4]{q}}{p^{3/4}}, \tag{60}$$
$$b_1 = 0, \quad b_2 = 0, \quad \omega = -2\sqrt{p}\sqrt{q}.$$

Put (60) in (38),

$$V_{18}\frac{\sqrt{q}}{\sqrt{p}} + \frac{(-1)(\sqrt{6}\,C\sqrt[4]{q})}{p^{3/4}(\mathrm{C}\xi + \eta)} + \frac{3C^2}{2p}\left(-\frac{1}{\mathrm{C}\xi} + \eta\right)^2. \tag{61}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (61) can be written as:

$$U_{18} = \mathrm{Ln}(V_{18}). \tag{62}$$

Case 9

$B = 0$ $C = 0$ (Figure 7),

$$a_0 = \frac{\sqrt{q}}{\sqrt{p}}, \quad a_2 = 0, \quad a_1 = 0, \quad b_1 = \frac{\sqrt{6}\,A\sqrt[4]{q}}{p^{3/4}},$$
$$b_2 = \frac{3A^2}{2p}, \quad \omega = -2\sqrt{p}\sqrt{q}. \tag{63}$$

Put (63) in (38),

$$V_{19} = \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{6}\,A\sqrt[4]{q}}{p^{3/4}}\frac{1}{\mathrm{A}\xi} + \frac{3A^2}{2p}\frac{1}{\mathrm{A}\xi^2}. \tag{64}$$

Since $U = \mathrm{Ln}(V)$, then Eq. (64) can be written as:

$$U_{19} = \mathrm{Ln}(V_{19}). \tag{65}$$

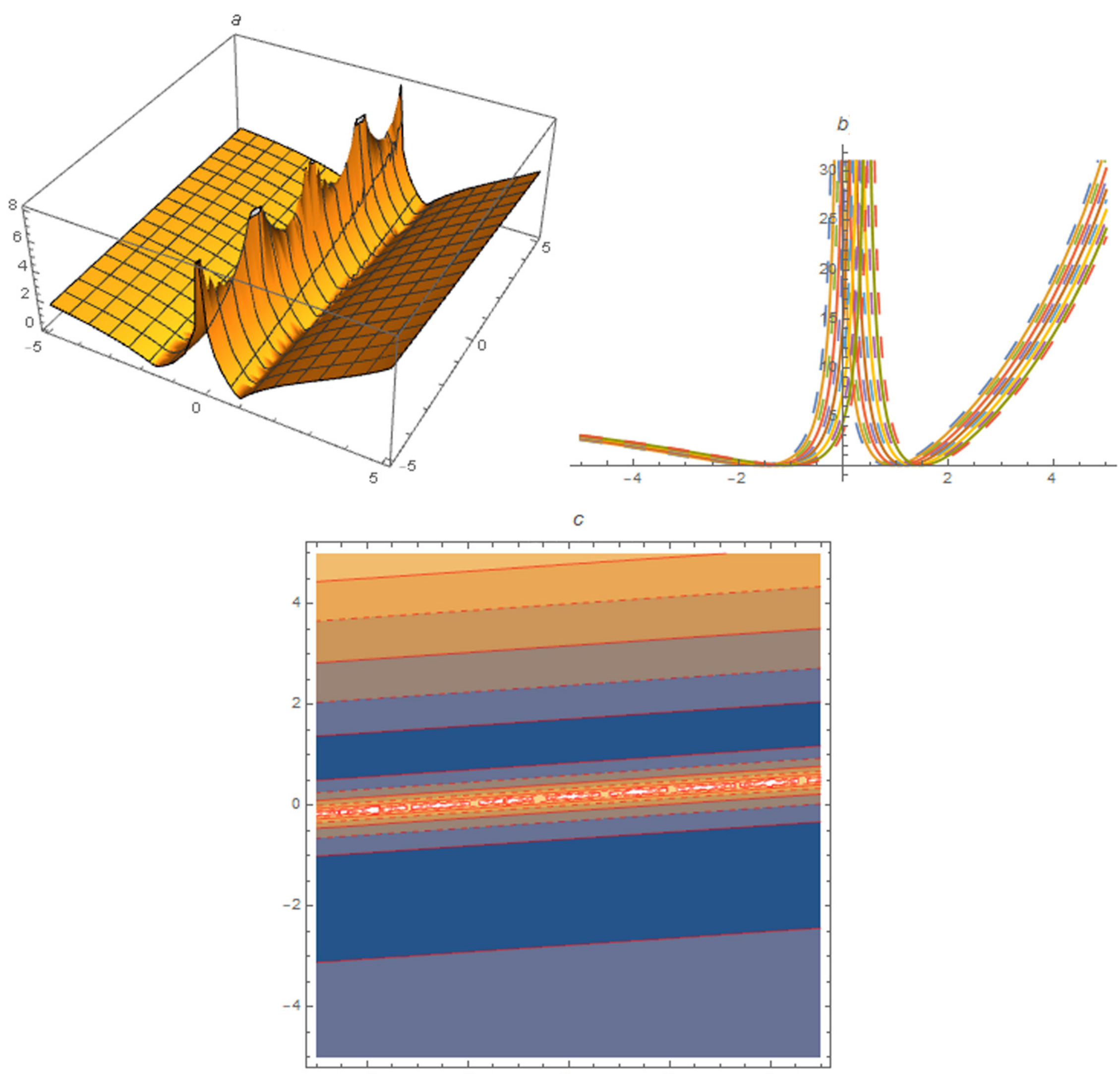

Figure 7: The profile of solution U_{20} for $A = 0.5$, $B = -0.1$, $p = 0.001$, $q = 1$.

Case 10

$C = 0,$

$$a_1 = 0, \quad b_1 = \frac{\sqrt{3}\sqrt{3A^2B^2 + 8A^2\sqrt{p}\sqrt{q}} + 3AB}{2p}, \qquad (66)$$
$$b_2 = \frac{3A^2}{2p}, \quad \omega = \frac{1}{2}(-B^2 - 4\sqrt{p}\sqrt{q}).$$

Put (66) in (38),

$$V_{20} = \frac{\frac{\sqrt{3}AB\sqrt{A^2(3B^2+8\sqrt{p}\sqrt{q})}}{p} + \frac{3A^2B^2}{p} + \frac{4A^2\sqrt{q}}{\sqrt{p}}}{4A^2} + \frac{\sqrt{3}\sqrt{3A^2B^2 + 8A^2\sqrt{p}\sqrt{q}} + 3AB}{2p} \times \left(\frac{1}{\frac{\exp(B\xi) - A}{B}}\right) + \frac{3A^2}{2p}\left(\frac{1}{\frac{\exp(B\xi) - A}{B}}\right)^2. \qquad (67)$$

Since $U = \text{Ln}(V)$, then Eq. (67) can be written as:

$$U_{20} = \text{Ln}(V_{20}). \qquad (68)$$

4 Results and discussion

In this section, we give some comparison of our novel construed obtained results with previous results in previous research literature. The spectral transform scheme was employed to construct solution on the CDF model by authors [41]. Similarly, exp-function technique was used to construct solution of CDF equation by authors in ref. [42]. Moreover, improved tanh method and (G'/G)-expansion method were employed to achieve different types of solutions on CDF in refs [43,44], respectively. Since we have investigated solution of the CDF model by employing improved simple equation and modified F-expansion schemes to construct a variety of powerful and better solutions and compared with solutions in refs [41–44]. However, our some solutions have few similarities with other solutions due to the followings. Our solutions $U_{11} = \text{Ln}(V_{11})$ in Eq. (40) and $U_{17} = \text{Ln}(V_{17})$ in Eq. (39) have some terms similar to the solution of $U_{2,4}(x, t)$ in Eq. (24) and $U_{2,5}(x, t)$ in Eq. (39), respectively, in ref. [47]. Our solution $U_{12} = \text{Ln}(V_{20})$ in Eq. (68) has approximately few terms likely similar to the solution mentioned of $\Psi(\xi)$ in Eq. (54) in ref. [48]. Our solution $U_6 = \text{Ln}(V_6)$ in Eq. (26) is approximately similar to the solution mentined $u_4(\eta)$ in Eq. (16) in ref. [49].

The remaining results are new and have no similarities to those in research literature. The obtained results play important role in different fields of mathematical physics. Hence from the results and discussion segment and graphical description of some solutions it is concluded that our proposed methods are powerful and best tools to solve nonlinear partial differential equations.

5 Conclusion

Two analytical mathematical methods are employed to obtain novel wave solutions of the CDF model. The results obtained *via* improved SE and modified F-expansion schemes are in the form of hyperbolic, trigonometric and exponential functions. By choosing the particular values of the parameters under the constrain conditions, some solutions are plotted in the form of two-dimensional, three-dimensional and contour types with the assistance of Mathematica software. The investigated results are clear to us that our presented techniques are effective and good tools for other fractional nonlinear partial differential equations (FNPDEs).

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