Research Article

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The exact solutions of the stochastic fractional-space Allen–Cahn equation

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Abstract: The fundamental objective of this article is to find exact solutions to the stochastic fractional-space Allen–Cahn equation, which is derived in the Itô sense by multiplicative noise. The exact solutions to this equation are required since it appears in many discipline areas including plasma physics, quantum mechanics and mathematical biology. The tanh–coth method is used to generate new hyperbolic and trigonometric stochastic and fractional solutions. The originality of this study is that the results produced here expand and improve on previously obtained results. Furthermore, we use Matlab package to display 3D surfaces of analytical solutions derived in this study to demonstrate the effect of stochastic term on the solutions of the stochastic-fractional-space Allen–Cahn equation.

Keywords: stochastic Allen–Cahn equation, fractional-space Allen–Cahn equation, tanh–coth method

1 Introduction

Fractional derivatives have attracted a lot of attention in recent decades due to their possible applications in a variety of fields, such as finance [1–3], biology [4], physics [5–8], hydrology [9,10] and biochemistry and chemistry [11]. Since derivatives of fractional order allow the memory and heredity qualities of various substances to be described, these fractional-order equations are more suited than integer-order equations [12].

On the other hand, random perturbations arise from many natural sources in the practically physical system. They cannot be denied and the presence of noise can lead to some statistical properties and important phenomena. As a result, stochastic differential equations were developed, and they began to play an increasingly significant role in modeling phenomena in chemistry, biology, physics, fluid mechanics, oceanography and atmosphere, *etc*.

Recently, some related research on approximate solutions of fractional differential equations with stochastic term have been explored such as Liu and Yan [13], Mohammed [14], Zou [15,16], Ahmad *et al*. [17,18], Li and Yang [19], Kamrani [20] and Taheri *et al*. [21].

In this article, the fractional-space Allen–Cahn equation induced by multiplicative noise in the Itô sense is taken into account as follows:

$$\frac{\partial u}{\partial t} = D_x^{2\alpha} u + u - u^3 + \rho u \beta_t, \quad \text{for } 0 < \alpha \leq 1, \tag{1}$$

where α is a parameter that defines the order of the fractional space derivative and $\beta(t)$ is the standard Brownian motion and $\beta_t = \frac{d\beta}{dt}$. Throughout this study, we take into account $\beta(t)$, which is a function of t only.

When $\rho = 0$ and $\alpha = 1$, Eq. (1) is known as the classical Cahn–Allen equation. It appears in a variety of scientific applications, including plasma physics, quantum mechanics and mathematical biology.

For $\alpha = 1$, Mohammed *et al*. [22] used three different methods including the tanh–coth and the generalized $\frac{G'}{G}$ -expansion, the Riccati–Bernoulli sub-ordinary differential equation (ODE) methods to get the stochastic exact solutions for Eq. (1). On the other hand, there are several ways to find the exact solutions of deterministic Eq. (1) with integer-order (*i.e.* $\rho = 0$ and $\alpha = 1$) such as the double exp-function method [23], the modified simple equation method [24], the Haar

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wavelet method [25], the tanh–coth method [26] and the first integral method [27].

The purpose of this article is to find the exact solutions of the stochastic fractional-space Allen–Cahn Eq. (1) derived by a one-dimensional multiplicative white noise by using the tanh–coth method. Furthermore, we expand and improve on some earlier results. The obtained solutions would be quite useful in explaining certain exciting physical phenomena. This is the first work to provide exact solutions to the stochastic fractional-space Allen–Cahn Eq. (1). Also, we discuss the effect of stochastic term on the exact solutions of the stochastic fractional-space Allen–Cahn Eq. (1) by utilizing MATLAB program to plot some graphs.

This article is organized as follows. In Section 2, we define the order α of Jumarie's derivative and we state some significant properties of modified Riemann–Liouville derivative. In Section 3, we use appropriate wave transformation to find the wave equation of stochastic Allen–Cahn Eq. (1). In Section 4, the tanh–coth method is applied to obtain the exact fractional stochastic solutions of the Allen–Cahn equation. While in Section 5, we see the effect of noise term on the exact solutions of the Allen–Cahn Eq. (1). Finally, we present the conclusions of this article.

2 Modified Riemann–Liouville derivative and properties

The order α of Jumarie's derivative is defined by ref. [28]:

$$D_x^{\alpha}\phi(x) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)}\dfrac{\mathrm{d}}{\mathrm{d}x}\displaystyle\int_0^x (x-\zeta)^{-\alpha}(\phi(x)-\phi(0))\mathrm{d}\zeta, \\ 0<\alpha<1, \\ [\phi^{(n)}(x)]^{\alpha-n}, \quad n\le\alpha\le n+1,\ n\ge 1, \end{cases}$$

where $\phi:\mathbb{R}\to\mathbb{R}$ is a continuous function but not necessarily first-order differentiable and $\Gamma(.)$ is the Gamma function.

Now, let us state some significant properties of modified Riemann–Liouville derivative as follows:

$$D_x^{\alpha}x^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)}x^{r-\alpha}, \quad r>0,$$
$$D_x^{\alpha}[a\phi(x)] = aD_x^{\alpha}\phi(x),$$
$$D_x^{\alpha}[a\phi(x)+b\psi(x)] = aD_x^{\alpha}\phi(x)+bD_x^{\alpha}\psi(x)$$

and

$$D_x^{\alpha}\phi(u(x)) = \sigma_x\frac{\mathrm{d}\phi}{\mathrm{d}u}D_x^{\alpha}u,$$

where σ_x is called the sigma indexes [29,30].

3 Wave equation of the Allen–Cahn equation

To derive the wave equation of stochastic fractional-space Allen–Cahn Eq. (1), we apply the next wave transformation:

$$u(t,x)=\psi(\xi)e^{[\rho\beta(t)-\rho^2t]}, \quad \xi=c\left(\frac{1}{\Gamma(1+\alpha)}x^{\alpha}-\lambda t\right), \tag{2}$$

where ψ is a deterministic function, ρ is the noise intensity and c,λ are nonzero constants. By differentiating u with regard to t and x we obtain

$$\frac{\mathrm{d}u}{\mathrm{d}t}=\left(-c\lambda\psi'+\rho\psi\frac{\mathrm{d}\beta}{\mathrm{d}t}-\frac{1}{2}\rho^2\psi\right)e^{[\rho\beta(t)-\rho^2t]},$$
$$D_x^{\alpha}u=c\sigma_x\psi'e^{[\rho\beta(t)-\rho^2t]} \quad \text{and} \quad D_x^{2\alpha}u=c^2\sigma_x^2\psi''e^{[\rho\beta(t)-\rho^2t]}. \tag{3}$$

Substituting (3) into Eq. (1), we get the next ODE:

$$c^2\ell^2\psi''+c\lambda\psi'-\psi^3e^{[\rho\beta(t)-\rho^2t]}+(1+\frac{1}{2}\rho^2)\psi=0, \tag{4}$$

where we put $\sigma_x=\ell$. Taking expectation on both sides yields

$$c^2\ell^2\psi''+c\lambda\psi'-\psi^3e^{-2\rho^2t}\mathbb{E}(e^{2\rho\beta(t)})+\left(1+\frac{1}{2}\rho^2\right)\psi=0. \tag{5}$$

Since $\mathbb{E}(e^{\rho Z})=e^{\frac{\rho^2}{2}t}$ for every standard Gaussian random variable Z and for real number ρ, the equality $\mathbb{E}(e^{\rho\beta(t)})=e^{\frac{\rho^2}{2}t}$ as a result of $\rho\beta(t)$ is distributed like $\rho\sqrt{t}Z$. Now Eq. (5) becomes

$$c^2\ell^2\psi''+c\lambda\psi'-\psi^3+\left(1+\frac{1}{2}\rho^2\right)\psi=0. \tag{6}$$

In the following, we apply the tanh–coth method to attain the solutions of the wave Eq. (6). And we, therefore, get the exact solutions of the stochastic fractional-space Allen–Cahn Eq. (1).

4 The exact solutions of the Allen–Cahn equation

To find the exact solutions of the stochastic fractional-space Allen–Cahn Eq. (1), we are using the tanh–oth method that Malfliet proposed [31]. We define the solution ψ in the following form:

$$\psi(\xi)=\sum_{k=0}^{M}a_k\chi^k, \tag{7}$$

where $\chi = \tanh\xi$ or $\chi = \coth\xi$. First, let us calculate M by equating the order of ψ^3 with the order of ψ'' to obtain

$$M = 1. \tag{8}$$

Hence, Eq. (7) takes the form:

$$u(\xi) = a_0 + a_1\chi. \tag{9}$$

Substituting Eqs. (9) into (6) we obtain

$$-2a_1c^2\ell^2(1-\chi^2)\chi + c\lambda a_1(1-\chi^2) - (a_0 + a_1\chi)^3 + \left(\frac{1}{2}\rho^2 + 1\right)(a_0 + a_1\chi) = 0.$$

Hence,

$$(2a_1c^2\ell^2 - a_1^3)\chi^3 - (c\lambda a_1 + 3a_0a_1^2)\chi^2 + \left(\frac{1}{2}a_1\rho^2 + a_1 - 3a_0^2a_1 - 2a_1c^2\ell^2\right)\chi + c\lambda a_1 - a_0^3 + \frac{1}{2}a_0\rho^2 + a_0 = 0.$$

We have by equating each coefficient of $\chi^k (k = 0, 1, 2, 3)$ to zero:

$$c\lambda a_1 - a_0^3 + a_0\left(\frac{1}{2}\rho^2 + 1\right) = 0,$$

$$a_1\left(\frac{1}{2}\rho^2 + 1\right) - 3a_0^2a_1 - 2a_1c^2\ell^2 = 0,$$

$$c\lambda a_1 + 3a_0a_1^2 = 0$$

and

$$(2a_1c^2\ell^2 - a_1^3) = 0.$$

We solve these equations by using Mathematica to obtain five cases as follows:

First case:

$$a_0 = 0,\ a_1 = \pm\sqrt{\frac{1}{2}\rho^2 + 1},\ c = \pm\frac{1}{2\ell}\sqrt{\rho^2 + 2} \text{ and } \lambda = 0.$$

The solution of wave Eq. (6) in this case is

$$\psi(\xi) = \pm\sqrt{\frac{1}{2}\rho^2 + 1}\tanh\xi \text{ or } \psi(\xi) = \pm\sqrt{\frac{1}{2}\rho^2 + 1}\coth\xi.$$

Therefore, the stochastic fractional-space Allen–Cahn Eq. (1) has the exact solution:

$$u_1(t,x) = \pm\sqrt{\frac{1}{2}\rho^2 + 1}\tanh\left(\frac{1}{\ell}\sqrt{\frac{\rho^2+2}{4}}\frac{1}{\Gamma(1+\alpha)}x^\alpha\right) \times e^{[\rho\beta(t) - \rho^2 t]} \tag{10}$$

or

$$u_2(t,x) = \pm\sqrt{\frac{1}{2}\rho^2 + 1}\coth\left(\frac{1}{\ell}\sqrt{\frac{\rho^2+2}{4}}\frac{1}{\Gamma(1+\alpha)}x^\alpha\right) \times e^{[\rho\beta(t) - \rho^2 t]}. \tag{11}$$

Second case:

$$a_0 = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1},\ a_1 = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1},\ c = \frac{-1}{4\ell}\sqrt{\rho^2 + 2} \text{ and } \lambda = \frac{3\ell}{2}\sqrt{\rho^2 + 2}.$$

In this case Eq. (6) has solution in the following form:

$$\psi(\xi) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}[1 + \tanh\xi] \text{ or}$$

$$\psi(\xi) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}[1 + \coth\xi].$$

Consequently, the stochastic fractional-space Allen–Cahn Eq. (1) has the exact solution:

$$u_3(t,x) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[1 - \tanh\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t) - \rho^2 t]} \tag{12}$$

or

$$u_4(t,x) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[1 - \coth\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t) - \rho^2 t]}. \tag{13}$$

Third case:

$$a_0 = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1},\ a_1 = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1},$$

$$c = \frac{-1}{4\ell}\sqrt{\rho^2 + 2} \text{ and } \lambda = \frac{3\ell}{2}\sqrt{\rho^2 + 2}.$$

In this case Eq. (6) has solution in the following form:

$$\psi(\xi) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}[-1 - \tanh\xi] \text{ or}$$

$$\psi(\xi) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}[-1 - \coth\xi].$$

Therefore, the stochastic fractional-space Allen–Cahn Eq. (1) has the exact solution:

$$u_5(t,x) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[-1 + \tanh\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t) - \rho^2 t]} \tag{14}$$

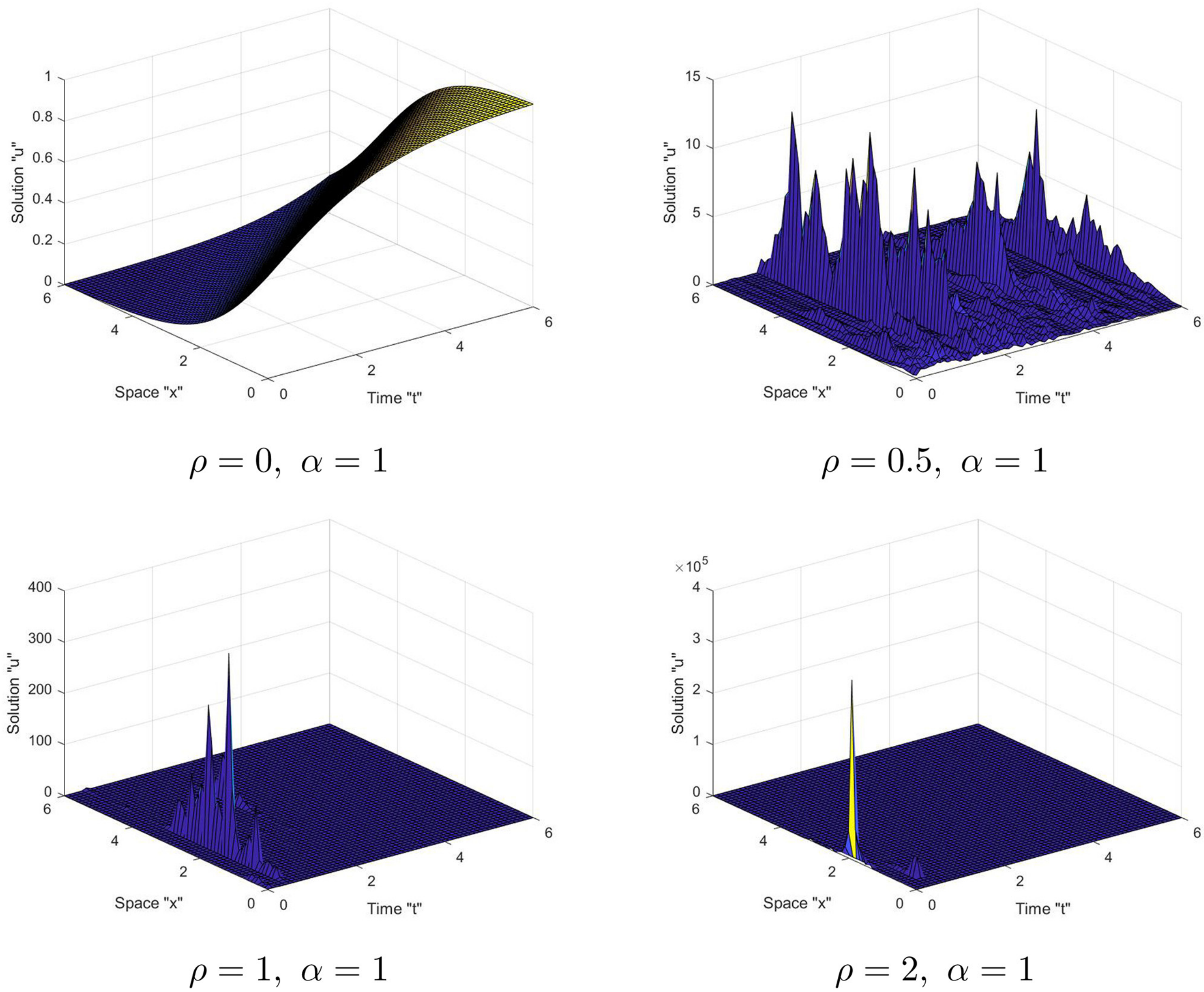

Figure 1: 3D-Graph of solution u_3 in Eq. (12) with $\alpha = 1$.

or

$$u_6(t,x) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[-1 + \coth\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} - \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t)-\rho^2 t]}. \tag{15}$$

Fourth case:

$$a_0 = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1}, \ a_1 = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1},$$
$$c = \frac{1}{4\ell}\sqrt{\rho^2+2} \text{ and } \lambda = \frac{-3\ell}{2}\sqrt{\rho^2+2}.$$

In this case, the solitary wave solution of Eq. (6) is

$$\psi(\xi) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1}[1+\tanh\xi] \text{ or}$$
$$\psi(\xi) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1}[1+\coth\xi].$$

Consequently, the exact solution of the stochastic fractional-space Allen–Cahn Eq. (1) is

$$u_7(t,x) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[1 + \tanh\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t)-\rho^2 t]} \tag{16}$$

or

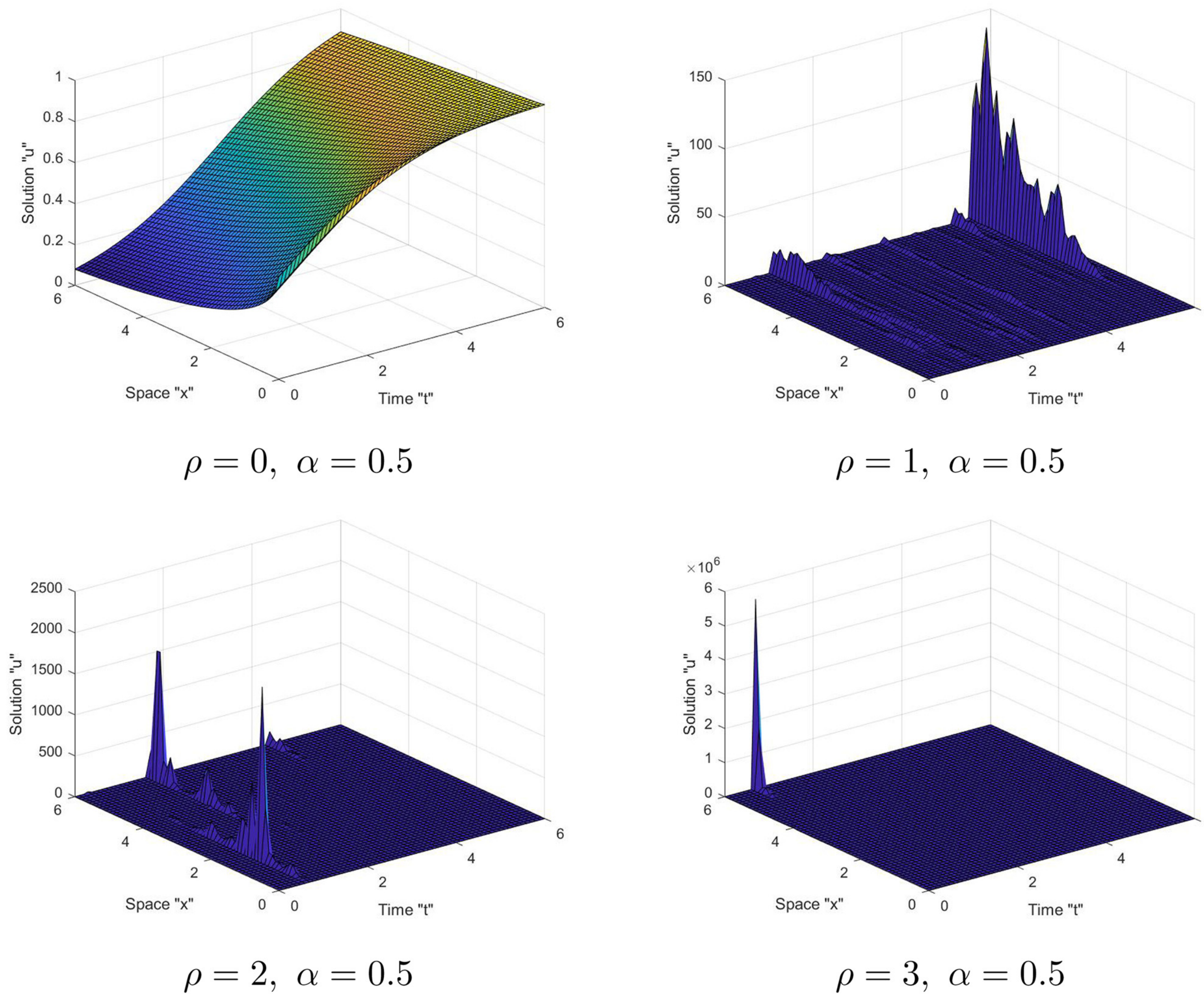

Figure 2: 3D-Graph of solution u_3 in Eq. (12) with $\alpha = 0.5$.

$$u_8(t,x) = \frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[1 + \coth\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t)-\rho^2 t]}. \tag{17}$$

Fifth case:

$$a_0 = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1},\ a_1 = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1},$$
$$c = \frac{1}{4\ell}\sqrt{\rho^2+2} \text{ and } \lambda = \frac{-3\ell}{2}\sqrt{\rho^2+2}.$$

The solution of Eq. (6) in this case is

$$\psi(\xi) = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1}\,[1+\tanh\xi] \text{ or}$$
$$\psi(\xi) = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2+1}\,[1+\coth\xi].$$

Therefore, the exact solution of the stochastic fractional-space Allen–Cahn Eq. (1) is

$$u_9(t,x) = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\left[1 + \tanh\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\right]e^{[\rho\beta(t)-\rho^2 t]} \tag{18}$$

or

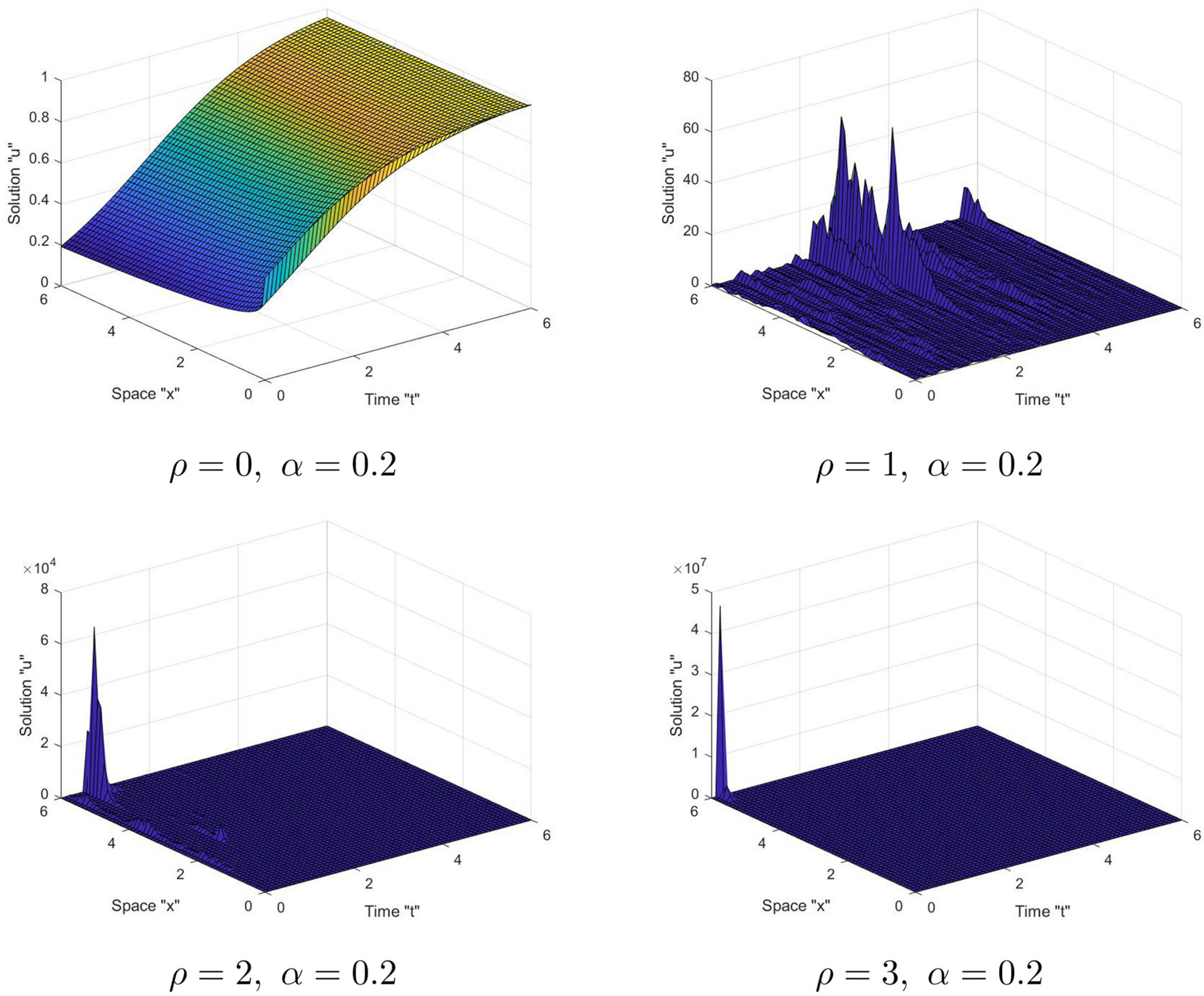

Figure 3: 3D-Graph of solution u_3 in equation (12) with $\alpha = 0.2$.

$$u_{10}(t,x) = -\frac{1}{2}\sqrt{\frac{1}{2}\rho^2 + 1}\Bigg[1 + \coth\left(\frac{1}{4\ell}\sqrt{\rho^2+2}\left(\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{3\ell}{2}\sqrt{\rho^2+2}\,t\right)\right)\Bigg]e^{[\rho\beta(t)-\rho^2 t]}. \tag{19}$$

Remark 1. If we put $\alpha = 1$ in Eqs. (10)–(19), then we get the same results as mentioned in ref. [22].

Remark 2. If we put $\rho = 0$, $\alpha = 1$ in Eqs. (10)–(19), then we obtain the same results as reported in ref. [26].

5 The effect of noise on the solutions of Eq. (1)

Here, we investigate the effect of the noise on the exact solutions of the stochastic fractional-space Allen–Cahn Eq. (1). To describe the behavior of these solutions, we give various graphical representations. We utilize the MATLAB program to plot some figures for different values of ρ (noise intensity). We simulate the solution $u_3(t,x)$ defined in Eq. (12) for $t \in [0,5]$ and $x \in [0,6]$ as follows.

In Figures 1–3, when the intensity of the noise is equal to zero, the surface is less flat, as indicated in the first graph in the table. However, when noise appears and the strength of the noise grows ($\rho = 1, 2, 3$), the surface becomes more planar after minor transit behaviors. This shows that the solutions are stable as a result of the noise effects.

6 Conclusion

By using the tanh–coth method, we derived the exact solutions of the stochastic fractional-space Allen–Cahn equation driven in the Itô sense by multiplicative noise. Furthermore, we expanded and enhanced several results, such as those mentioned in refs [22,26]. These solutions play a key role in understanding some fascinating complicated physical phenomena. Finally, we showed the effect of stochastic term on the exact solutions of the stochastic fractional-space Allen–Cahn equation by using MATLAB package to plot some graphs.

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