

Research Article

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Abundant accurate analytical and semi-analytical solutions of the positive Gardner–Kadomtsev–Petviashvili equation

<https://doi.org/10.1515/phys-2022-0001>

received March 29, 2021; accepted December 15, 2021

Abstract: This research article examines the correctness of two new analytical methods for solving the internal solitary waves of shallow seas. To get the computational solutions to the positive $(2 + 1)$ -dimensional Gardner–Kadomtsev–Petviashvili model, the extended simplest equation and modified Kudryashov methods are used. Numerous new traveling wave solutions in various forms are developed in order to assess the starting conditions required for the variational iteration technique, one of the most accurate semi-analytical methods. The semi-analytical solutions are used to demonstrate the precision of the solutions obtained and the analytical methods employed. The dynamical behavior of internal solitary waves in shallow waters is shown using many three-dimensional drawings. The performance of the used schemes demonstrates their efficacy and power, as well as their capacity to handle a large number of nonlinear evolution equations.

Keywords: positive $(2 + 1)$ -D GKP equation, analytical and semi-analytical solutions

1 Introduction

In recent years, the shallow water wave phenomena have emerged as an attractive option for researching and analyzing the bidirectional propagating water wave surface's dynamical and physical behavior [1]. The flow of a fluid under a pressure surface illustrates the shallow water wave attitude through a set of hyperbolic nonlinear partial differential equations that have been recently constructed [2]. Adhemar Jean Claude Barr'e de Saint–Venant has derived these unidirectional hyperbolic equations as a model of transitory open-channel flow and surface runoff [3–6]. As a result, these equations are referred to as Saint–Venant equations, a contraction of the two-dimensional (2D) shallow water equations [7]. Additionally, the famous Navier–Stokes equations are considered a subset of shallow water wave equations because they are used when the horizontal length scale of the fluid is slightly greater than the vertical length scale, and mass conservation ensures that the fluid's vertical velocity scale is small in comparison to its horizontal speed scale [8,9]. Additionally, the 2D Kadomtsev–Petviashvili–Benjamin–Bona–Mahony equation is included in this hyperbolic collection [10–12]. The shallow water wave has many applications in a variety of disciplines, including electromagnetic theory, astrophysics, electrochemistry, fluid dynamics, plasma physics, acoustics, and cosmology [13–15].

Three kinds of computational schemes (analytical, semi-analytical, and numerical methods) have been derived such as refs [16–20]. Each of these methods has been applied to a large number of nonlinear evolution problems. Numerous computational, semi-analytical, and numerical solutions have been developed, but none of them is universally applicable to all nonlinear evolution equations; therefore, the quest for this unifying approach continues. The tremendous development in computer technology has been the most beneficial instrument in this research; nevertheless, this technology has been utilized only to

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find new methods, and no one has used it to verify the correctness of previously obtained systems [21,22].

This manuscript studies the (2 + 1)-D Gardner–Kadomtsev–Petviashvili equation, which is given by refs [23–25]

$$(\mathcal{G}_t + p\mathcal{G}\mathcal{G}_x \pm \mathcal{G}^2\mathcal{G}_x + \mathcal{G}_{xxx})_x + \mathcal{G}_{yy} = 0, \quad (1)$$

where p, q are arbitrary constants to be evaluated while $\mathcal{G} = \mathcal{G}(x, y, t)$ is a function of space and time which explains the evolution of nonlinear waves where the effect of the surface tension and the viscosity is negligible. Employing the following wave transformation $\mathcal{G}(x, y, t) = \mathcal{K}(\mathfrak{J})$, $\mathfrak{J} = x + y + rt$, where r is the wave velocity, and integrating the result once with zero constants of the integration convert the positive form of equation (1) into the following ordinary differential equation:

$$(r + 1)\mathcal{K} + p\mathcal{K}\mathcal{K}' + q\mathcal{K}^2\mathcal{K}' + \mathcal{K}''' = 0. \quad (2)$$

Using the homogeneous balance principles and the following auxiliary equations for extended simplest equation (ESE) and modified Kudryashov (MKud) methods [26–29] for equation (2), respectively, $\mathfrak{F}'(\mathfrak{J}) = l_1 + l_2\mathfrak{F}(\mathfrak{J}) + l_3\mathfrak{F}(\mathfrak{J})^2$ and $\mathfrak{E}'(\mathfrak{J}) = \ln(k)(\mathfrak{E}(\mathfrak{J})^2 - \mathfrak{E}(\mathfrak{J}))$, where l_1, l_2, l_3, k are arbitrary constants to be constructed later, given $n = 1$. Thus, the general solutions of equation (2) based on the suggested analytical schemes (ESE and MKud) [30–33] are formulated in the following forms:

$$\mathcal{K}(\mathfrak{J}) = \begin{cases} \sum_{i=-n}^n a_i \mathfrak{F}(\mathfrak{J})^i = a_1 \mathfrak{F}(\mathfrak{J}) + \frac{a_{-1}}{\mathfrak{F}(\mathfrak{J})} + a_0, \\ \sum_{i=0}^n a_i \mathfrak{E}(\mathfrak{J})^i = a_1 \mathfrak{E}(\mathfrak{J}) + a_0, \end{cases} \quad (3)$$

where a_0, a_1, a_{-1} are arbitrary constants.

The rest of the article is organized as follows. Section 2 investigates the analytical and semi-analytical solutions of the considered model. Section 3 explains the paper's contributions and novelty. Section 4 gives the conclusion of the article.

2 Accuracy of computational solutions

Investigating the computational solutions of the investigated model with the ESE and MKud methods then checking the solutions' accuracy by employing the VI semi-analytical

schemes, shows the applied methods' performance and accuracy as following:

2.1 ESE method's solutions

Calculating the abovementioned parameters through the ESE method's framework gives the following families:

Family I

$$a_1 \rightarrow 0, \quad r \rightarrow \frac{a_{-1}^2(-l_2^2) - 2a_{-1}^2 l_1 l_3 + 6a_0 a_{-1} l_1 l_2 - 6a_0^2 l_1^2 - a_{-1}^2}{a_{-1}^2},$$

$$p \rightarrow -\frac{6(a_{-1} l_1 l_2 - 2a_0 l_1^2)}{a_{-1}^2}, \quad q \rightarrow -\frac{6l_1^2}{a_{-1}^2}.$$

Family II

$$a_{-1} \rightarrow 0, \quad r \rightarrow \frac{a_1^2(-l_2^2) - 2a_1^2 l_1 l_3 + 6a_0 a_1 l_1 l_2 - 6a_0^2 l_1^2 - a_1^2}{a_1^2},$$

$$p \rightarrow -\frac{6(a_1 l_1 l_2 - 2a_0 l_1^2)}{a_1^2}, \quad q \rightarrow -\frac{6l_1^2}{a_1^2}.$$

Family III

$$a_{-1} \rightarrow -\frac{i\sqrt{6}l_1}{\sqrt{q}}, \quad a_1 \rightarrow -\frac{i\sqrt{6}l_3}{\sqrt{q}},$$

$$r \rightarrow i\sqrt{6}a_0 l_2 \sqrt{q} + a_0^2 q - l_2^2 + 4l_1 l_3 - 1,$$

$$p \rightarrow -2a_0 q - i\sqrt{6}l_2 \sqrt{q}, \quad \text{where } (q < 0).$$

Thus, the computational solutions of the positive nonlinear (2 + 1)-D GKP equation are constructed as follows: For $l_2 = 0$, $l_1 l_3 > 0$, we obtain

$$\mathcal{G}_{I,1}(x, y, t)$$

$$= a_0 - \frac{a_{-1}\sqrt{l_1 l_3}}{l_1}$$

$$\times \cot\left(\frac{\sqrt{l_1 l_3}(a_{-1}^2(-\eta + 2l_1 l_3 t + t - x - y) + 6a_0^2 l_1^2 t)}{a_{-1}^2}\right), \quad (4)$$

$$\mathcal{G}_{I,2}(x, y, t)$$

$$= a_0 - \frac{a_{-1}\sqrt{l_1 l_3}}{l_1} \tan$$

$$\times \left(\frac{\sqrt{l_1 l_3}(a_{-1}^2(-\eta + 2l_1 l_3 t + t - x - y) + 6a_0^2 l_1^2 t)}{a_{-1}^2}\right), \quad (5)$$

$$\mathcal{G}_{II,1}(x, y, t)$$

$$= a_0 - \frac{a_1\sqrt{l_1 l_3}}{l_3} \tan$$

$$\times \left(\frac{\sqrt{l_1 l_3}(a_1^2(-\eta + 2l_1 l_3 t + t - x - y) + 6a_0^2 l_3^2 t)}{a_1^2}\right), \quad (6)$$

$$\mathcal{G}_{\text{II},2}(x, y, t) = a_0 - \frac{a_1 \sqrt{l_1 l_3}}{l_3} \cot \left(\frac{\sqrt{l_1 l_3} (a_1^2 (-\eta + 2l_1 l_3 t + t - x - y) + 6a_0^2 l_3^2 t)}{a_1^2} \right), \quad (7)$$

$$\mathcal{G}_{\text{III},1}(x, y, t) = a_0 - \frac{2i\sqrt{6}\sqrt{l_1 l_3}}{\sqrt{q}} \csc(2\sqrt{l_1 l_3} (t(a_0^2 q + 4l_1 l_3 - 1) + \eta + x + y)). \quad (8)$$

For $l_2 = 0$, $l_1 l_3 < 0$, we obtain

$$\mathcal{G}_{\text{I},3}(x, y, t) = a_0 - \frac{a_{-1} \sqrt{-l_1 l_3}}{l_1} \coth \left(\frac{\sqrt{-l_1 l_3} (a_{-1}^2 (-2l_1 l_3 t - t + x + y) - 6a_0^2 l_1^2 t)}{a_{-1}^2} \mp \frac{\log(\eta)}{2} \right), \quad (9)$$

$$\mathcal{G}_{\text{I},4}(x, y, t) = a_0 - \frac{a_{-1} \sqrt{-l_1 l_3}}{l_1} \tanh \left(\frac{\sqrt{-l_1 l_3} (a_{-1}^2 (-2l_1 l_3 t - t + x + y) - 6a_0^2 l_1^2 t)}{a_{-1}^2} \mp \frac{\log(\eta)}{2} \right), \quad (10)$$

$$\mathcal{G}_{\text{II},3}(x, y, t) = \frac{a_1 \sqrt{-l_1 l_3}}{l_3} \tanh \left(\frac{\sqrt{-l_1 l_3} (a_1^2 (-2l_1 l_3 t - t + x + y) - 6a_0^2 l_3^2 t)}{a_1^2} \mp \frac{\log(\eta)}{2} \right) + a_0, \quad (11)$$

$$\mathcal{G}_{\text{II},4}(x, y, t) = \frac{a_1 \sqrt{-l_1 l_3}}{l_3} \coth \left(\frac{\sqrt{-l_1 l_3} (a_1^2 (-2l_1 l_3 t - t + x + y) - 6a_0^2 l_3^2 t)}{a_1^2} \mp \frac{\log(\eta)}{2} \right) + a_0, \quad (12)$$

$$\mathcal{G}_{\text{III},2}(x, y, t) = a_0 + \frac{2i\sqrt{6}\sqrt{-l_1 l_3}}{\sqrt{q}} \operatorname{csch} \left(2 \left(\sqrt{-l_1 l_3} (t(a_0^2 q + 4l_1 l_3 - 1) + x + y) \mp \frac{\log(\eta)}{2} \right) \right). \quad (13)$$

For $l_1 = 0$, $l_2 > 0$, we obtain

$$\mathcal{G}_{\text{I},5}(x, y, t) = \frac{a_{-1} e^{l_2(-\eta + l_2^2 t + t - x - y)}}{l_2} - \frac{a_{-1} l_3}{l_2} + a_0, \quad (14)$$

$$\mathcal{G}_{\text{II},5}(x, y, t) = \frac{a_1 l_2}{\exp \left(l_2 \left(t \left(-\frac{6a_0 l_3 l_2}{a_1} + \frac{6a_0^2 l_3^2}{a_1^2} + l_2^2 \right) - \eta + t - x - y \right) \right) - l_3} + a_0, \quad (15)$$

$$\mathcal{G}_{\text{III},3}(x, y, t) = \frac{i\sqrt{6} l_2}{\sqrt{q} (-1 + l_3 \exp(l_2(t(i\sqrt{6} a_0 l_2 \sqrt{q} + a_0^2 q - l_2^2 - 1) + \eta + x + y)))} + a_0 + \frac{i\sqrt{6} l_2}{\sqrt{q}}. \quad (16)$$

For $l_1 = 0$, $l_2 < 0$, we obtain

$$\mathcal{G}_{\text{I},6}(x, y, t) = -\frac{a_{-1} e^{l_2(-\eta + l_2^2 t + t - x - y)}}{l_3} - a_{-1} + a_0, \quad (17)$$

$$\mathcal{G}_{\text{II},6}(x, y, t) = a_0 - \frac{a_1 l_3}{\exp \left(l_2 \left(t \left(-\frac{6a_0 l_3 l_2}{a_1} + \frac{6a_0^2 l_3^2}{a_1^2} + l_2^2 \right) - \eta + t - x - y \right) \right) + l_3}, \quad (18)$$

$$\mathcal{G}_{\text{III},4}(x, y, t) = -\frac{i\sqrt{6} l_3}{\sqrt{q} (1 + l_3 \exp(l_2(t(i\sqrt{6} a_0 l_2 \sqrt{q} + a_0^2 q - l_2^2 - 1) + \eta + x + y)))} + a_0 + \frac{i\sqrt{6} l_3}{\sqrt{q}}. \quad (19)$$

For $4l_1 l_3 > l_2^2$, we obtain

$$\mathcal{G}_{\text{I},7}(x, y, t) = a_0 - \frac{2a_{-1} l_3}{\sqrt{4l_1 l_3 - l_2^2} \tan \left(\frac{\sqrt{4l_1 l_3 - l_2^2} (a_{-1}^2 (-\eta + l_2^2 t + 2l_1 l_3 t + t - x - y) - 6a_0 a_{-1} l_1 l_2 t + 6a_0^2 l_1^2 t)}{2a_{-1}^2} \right) + l_2}, \quad (20)$$

$$\mathcal{G}_{\text{I},8}(x, y, t) = a_0 - \frac{2a_{-1} l_3}{\sqrt{4l_1 l_3 - l_2^2} \cot \left(\frac{\sqrt{4l_1 l_3 - l_2^2} (a_{-1}^2 (-\eta + l_2^2 t + 2l_1 l_3 t + t - x - y) - 6a_0 a_{-1} l_1 l_2 t + 6a_0^2 l_1^2 t)}{2a_{-1}^2} \right) + l_2}, \quad (21)$$

$$\mathcal{G}_{II,7}(x, y, t) = -\frac{a_1\sqrt{4l_1l_3 - l_2^2}}{2l_3} \tan\left(\frac{\sqrt{4l_1l_3 - l_2^2}}{2a_1^2}(a_1^2(-\eta + l_2^2t + 2l_1l_3t + t - x - y) - 6a_0a_1l_2l_3t + 6a_0^2l_3^2t)\right) - \frac{a_1l_2}{2l_3} + a_0, \quad (22)$$

$$\mathcal{G}_{II,8}(x, y, t) = -\frac{a_1\sqrt{4l_1l_3 - l_2^2}}{2l_3} \cot\left(\frac{\sqrt{4l_1l_3 - l_2^2}}{2a_1^2}(a_1^2(-\eta + l_2^2t + 2l_1l_3t + t - x - y) - 6a_0a_1l_2l_3t + 6a_0^2l_3^2t)\right) - \frac{a_1l_2}{2l_3} + a_0, \quad (23)$$

$$\begin{aligned} \mathcal{G}_{III,5}(x, y, t) = & -i\sqrt{\frac{3}{2q}}\sqrt{4l_1l_3 - l_2^2} \tan\left(\frac{1}{2}\sqrt{4l_1l_3 - l_2^2}(t(i\sqrt{6}a_0l_2\sqrt{q} + a_0^2q - l_2^2 + 4l_1l_3 - 1) + \eta + x + y)\right) \\ & + (2i\sqrt{6}l_1l_3)/\left(\sqrt{q}\left(l_2 - \sqrt{4l_1l_3 - l_2^2} \tan\left(\frac{1}{2}\sqrt{4l_1l_3 - l_2^2}(t(i\sqrt{6}a_0l_2\sqrt{q} + a_0^2q - l_2^2 + 4l_1l_3 - 1) + \eta + x + y)\right)\right.\right. \\ & \left.\left.+ y)\right)\right) + a_0 + \frac{i\sqrt{\frac{3}{2}}l_2}{\sqrt{q}}, \end{aligned} \quad (24)$$

$$\begin{aligned} \mathcal{G}_{III,6}(x, y, t) = & -i\sqrt{\frac{3}{2q}}\sqrt{4l_1l_3 - l_2^2} \cot\left(\frac{1}{2}\sqrt{4l_1l_3 - l_2^2}(t(i\sqrt{6}a_0l_2\sqrt{q} + a_0^2q - l_2^2 + 4l_1l_3 - 1) + \eta + x + y)\right) \\ & + (2i\sqrt{6}l_1l_3)/\left(\sqrt{q}\left(l_2 - \sqrt{4l_1l_3 - l_2^2} \cot\left(\frac{1}{2}\sqrt{4l_1l_3 - l_2^2}(t(i\sqrt{6}a_0l_2\sqrt{q} + a_0^2q - l_2^2 + 4l_1l_3 - 1) + \eta + x + y)\right)\right.\right. \\ & \left.\left.+ y)\right)\right) + a_0 + \frac{i\sqrt{\frac{3}{2}}l_2}{\sqrt{q}}, \end{aligned} \quad (25)$$

where η is the arbitrary constant [30,31].

2.1.1 Semi-analytical solutions

Applying the VI method [34] for equation (1) with the following initial condition $\mathcal{G}_I(x, y, 0) = 0.5 \tanh(0.4x + 0.4y) + 0.01$, based on the VI method, the correction function of equation (10) is given by

$$\begin{aligned} \mathcal{G}_{n+1}(x, y, t) = & \mathcal{G}_n(x, y, t) + \int_0^t \lambda((\mathcal{G}_t + 0.0612245\mathcal{G}\mathcal{G}_x \\ & - 3.06122\mathcal{G}^2\mathcal{G}_x + \mathcal{G}_{xxx})_x + \mathcal{G}_{yy})ds, \end{aligned} \quad (26)$$

where $1 + \lambda|_{s=t} = 0$, $\lambda' = 0$. Thus, the Lagrangian multiplier gives $(\lambda = -1)$. Consequently, the semi-analytical solution of equation (10) is given by:

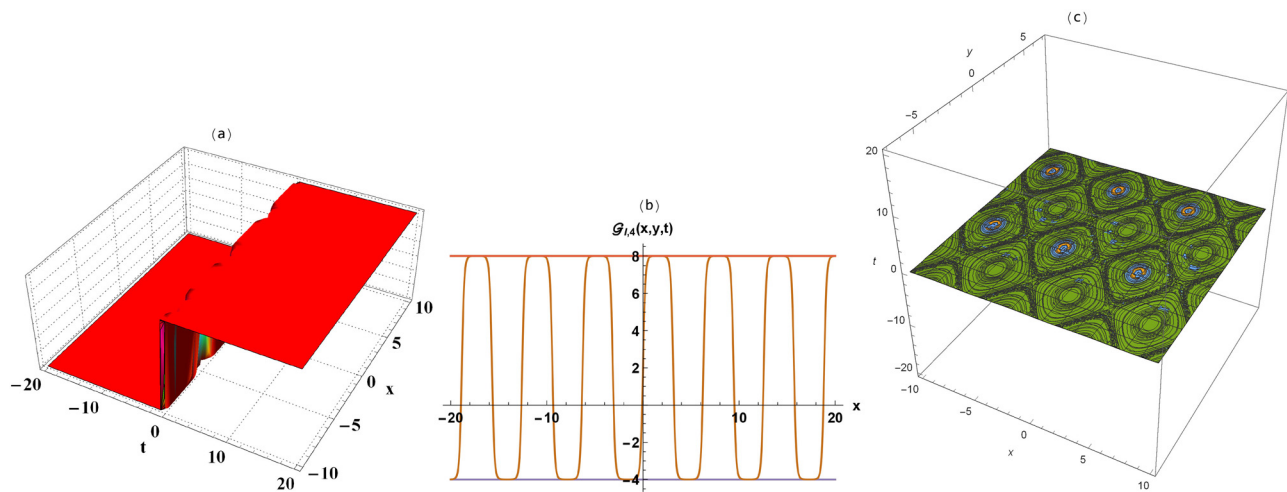


Figure 1: Solitary wave solution equation (10) in (a) three, (b) two, and (c) contour plots.

$$\begin{aligned} \mathcal{G}_{1,1}(x, y, t) = & t \operatorname{sech}^6(0.4x + 0.4y)(0.1 \sinh(0.7x \\ & + 0.7y) + 0.004 \sinh(1.4x + 1.4y) \\ & + 0.1 \cosh(0.7x + 0.7y) \\ & - 0.005 \cosh(1.4x + 1.4y) - 0.2) \\ & + 0.5 \tanh(0.4x + 0.4y) + 0.01. \end{aligned} \quad (27)$$

Using the same techniques, it is easy to obtain $\mathcal{G}_i(x, y, t)$, where $i = 2, 3, 4, \dots$

2.2 MKud method's solutions

Calculating the above mentioned parameters through the MKud method's framework gives:

$$\begin{aligned} a_0 &\rightarrow -\frac{i(\sqrt{6}\sqrt{q}\log(k) - ip)}{2q}, \quad a_1 \rightarrow \frac{i\sqrt{6}\log(k)}{\sqrt{q}}, \\ r &\rightarrow \frac{2q\log^2(k) + p^2 - 4q}{4q}, \quad \text{where } q < 0. \end{aligned}$$

Thus, the computational solutions of the positive non-linear (2 + 1)-D GKP equation are constructed as follows:

$$\begin{aligned} \mathcal{G}(x, y, t) = & \frac{-1}{2q} \left(\frac{-2i\sqrt{6}\sqrt{q}\log(k)}{1 \pm k^{\frac{t(2q\log^2(k) + p^2 - 4q)}{4q} + x + y}} \right. \\ & \left. + i\sqrt{6}\sqrt{q}\log(k) + p \right). \end{aligned} \quad (28)$$

2.2.1 Semi-analytical solutions

Applying the VI method for equation (1) with the following initial condition $\mathcal{G}(x, y, 0) = \frac{1}{8} \left(\frac{4\sqrt{6}}{1 \pm e^{x+y}} - 2\sqrt{6} + 1 \right)$, based on the VI method, the correction function of equation (10) is given by

$$\begin{aligned} \mathcal{G}_{n+1}(x, y, t) = & \mathcal{G}_n(x, y, t) + \int_0^s \lambda ((\mathcal{G}_t + \mathcal{G}\mathcal{G}_x - 4\mathcal{G}^2\mathcal{G}_x \\ & + \mathcal{G}_{xxx})_x + \mathcal{G}_{yy}) ds, \end{aligned} \quad (29)$$

where $1 + \lambda|_{s=t} = 0$, $\lambda' = 0$. Thus, the Lagrangian multiplier gives ($\lambda = -1$). Consequently, the semi-analytical solution of equation (10) is given by:

$$\begin{aligned} \mathcal{G}_1(x, y, t) = & \frac{1}{16(e^{x+y} + 1)^6} \left(e^{3(x+y)} \left(\right. \right. \\ & -16(26\sqrt{6} + 9)t \cosh(x + y) \\ & + 8(2\sqrt{6} + 9)t \cosh(2(x + y)) \\ & - 2\sqrt{6} \sinh(x + y)((45t + 32) \cosh(x \\ & + y) - 51t + 8 \cosh(2(x + y)) + 24) \\ & \left. \left. + 24(22\sqrt{6} - 9)t + 128 \cosh^6\left(\frac{x + y}{2}\right) \right) \right). \end{aligned} \quad (30)$$

Using the same techniques, it is easy to obtain $\mathcal{G}_i(x, y, t)$, where $i = 2, 3, 4, \dots$

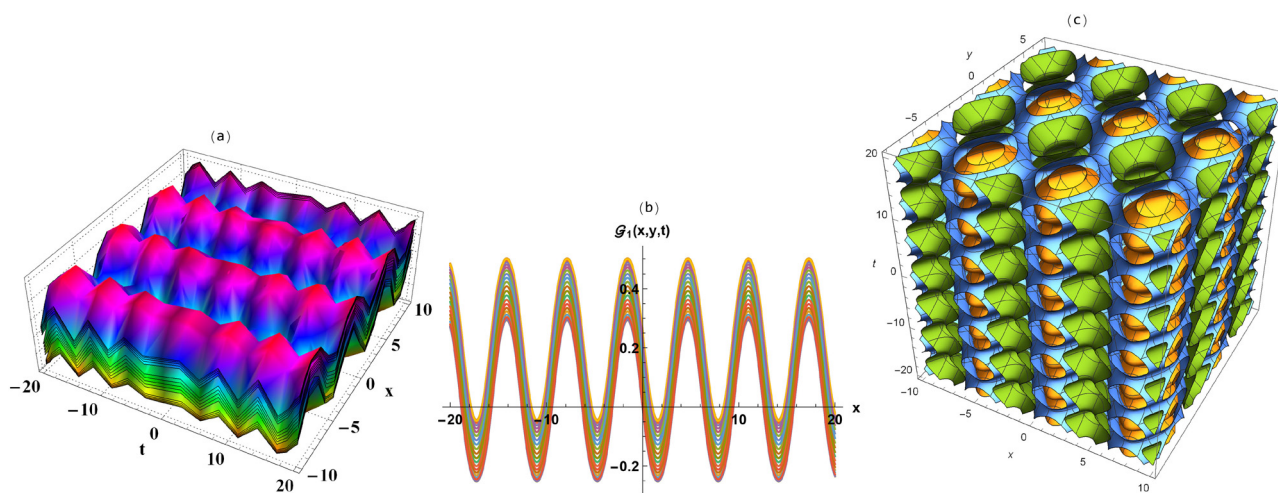


Figure 2: Solitary wave solution equation (28) in (a) three, (b) two, and (c) contour plots.

Table 1: Absolute error between analytical and semi-analytical solutions based on ESE and VI schemes

Value of x	$t = 2$	$t = 4$	$t = 6$	$t = 8$	$t = 10$	$t = 12$	$t = 14$	$t = 16$	$t = 18$	$t = 20$
0	7.5233×10^{-5}	0.00015047	0.0002257	0.00030093	0.00037616	0.0004514	0.00052663	0.00060186	0.000677095	0.000752327
1	3.998×10^{-5}	7.9959×10^{-5}	0.00011994	0.00015992	0.0001999	0.00023988	0.00027986	0.00031984	0.000359816	0.000399796
2	2.0417×10^{-5}	4.0835×10^{-5}	6.1252×10^{-5}	8.1669×10^{-5}	0.00010209	0.0001225	0.00014292	0.00016334	0.000183756	0.000204473
3	1.0239×10^{-5}	2.0478×10^{-5}	3.0717×10^{-5}	4.0955×10^{-5}	5.1194×10^{-5}	6.1433×10^{-5}	7.1672×10^{-5}	8.1911×10^{-5}	9.21496×10^{-5}	0.000102388
4	5.0902×10^{-6}	1.018×10^{-5}	1.5271×10^{-5}	2.0361×10^{-5}	2.5451×10^{-5}	3.0541×10^{-5}	3.5632×10^{-5}	4.0722×10^{-5}	4.58122×10^{-5}	5.09024×10^{-5}
5	2.52×10^{-6}	5.04×10^{-6}	7.56×10^{-6}	1.008×10^{-5}	1.26×10^{-5}	1.512×10^{-5}	1.764×10^{-5}	2.016×10^{-5}	2.26801×10^{-5}	2.52001×10^{-5}
6	1.245×10^{-6}	2.49×10^{-6}	3.735×10^{-6}	4.98×10^{-6}	6.225×10^{-6}	7.4701×10^{-6}	8.7151×10^{-6}	9.9601×10^{-6}	1.12051×10^{-5}	1.24501×10^{-5}
7	6.1448×10^{-7}	1.229×10^{-6}	1.8434×10^{-6}	2.4579×10^{-6}	3.0724×10^{-6}	3.6869×10^{-6}	4.3013×10^{-6}	4.9158×10^{-6}	5.53028×10^{-6}	6.14476×10^{-6}
8	3.0312×10^{-7}	6.0625×10^{-7}	9.0937×10^{-7}	1.2125×10^{-6}	1.5156×10^{-6}	1.8187×10^{-6}	2.1219×10^{-6}	2.425×10^{-6}	2.72812×10^{-6}	3.03125×10^{-6}
9	1.495×10^{-7}	2.9899×10^{-7}	4.4849×10^{-7}	5.9799×10^{-7}	7.4748×10^{-7}	8.9698×10^{-7}	1.0465×10^{-6}	1.196×10^{-6}	1.34547×10^{-6}	1.49497×10^{-6}
10	7.3721×10^{-8}	1.4744×10^{-7}	2.2116×10^{-7}	2.9488×10^{-7}	3.686×10^{-7}	4.4233×10^{-7}	5.1605×10^{-7}	5.8977×10^{-7}	6.63488×10^{-7}	7.37209×10^{-7}
11	3.6352×10^{-8}	7.2703×10^{-8}	1.0905×10^{-7}	1.4541×10^{-7}	1.8176×10^{-7}	2.1811×10^{-7}	2.5446×10^{-7}	2.9081×10^{-7}	3.27164×10^{-7}	3.63516×10^{-7}
12	1.7924×10^{-8}	3.5849×10^{-8}	5.3773×10^{-8}	7.1697×10^{-8}	8.9622×10^{-8}	1.0755×10^{-7}	1.2547×10^{-7}	1.4339×10^{-7}	1.61319×10^{-7}	1.79243×10^{-7}
13	8.8381×10^{-9}	1.7676×10^{-8}	2.6514×10^{-8}	3.5352×10^{-8}	4.419×10^{-8}	5.3028×10^{-8}	6.1866×10^{-8}	7.0704×10^{-8}	7.95425×10^{-8}	8.83805×10^{-8}
14	4.3578×10^{-9}	8.7156×10^{-9}	1.3073×10^{-8}	1.7431×10^{-8}	2.1789×10^{-8}	2.6147×10^{-8}	3.0505×10^{-8}	3.4862×10^{-8}	3.92202×10^{-8}	4.3578×10^{-8}
15	2.1487×10^{-9}	4.2974×10^{-9}	6.4461×10^{-9}	8.5948×10^{-9}	1.0744×10^{-8}	1.2892×10^{-8}	1.5041×10^{-8}	1.719×10^{-8}	1.93383×10^{-8}	2.1487×10^{-8}
16	1.0595×10^{-9}	2.1189×10^{-9}	3.1784×10^{-9}	4.2378×10^{-9}	5.2973×10^{-9}	6.3568×10^{-9}	7.4162×10^{-9}	8.4757×10^{-9}	9.53513×10^{-9}	1.05946×10^{-8}
17	5.2239×10^{-10}	1.0448×10^{-9}	1.5672×10^{-9}	2.0895×10^{-9}	2.6119×10^{-9}	3.1343×10^{-9}	3.6567×10^{-9}	4.1791×10^{-9}	4.70148×10^{-9}	5.22386×10^{-9}
18	2.5757×10^{-10}	5.1514×10^{-10}	7.7272×10^{-10}	1.0303×10^{-9}	1.2879×10^{-9}	1.5454×10^{-9}	1.803×10^{-9}	2.0606×10^{-9}	2.31815×10^{-9}	2.57572×10^{-9}
19	1.27×10^{-10}	2.54×10^{-10}	3.81×10^{-10}	5.08×10^{-10}	6.35×10^{-10}	7.6201×10^{-10}	8.8901×10^{-10}	1.016×10^{-9}	1.14301×10^{-9}	1.27001×10^{-9}
20	6.262×10^{-11}	1.2524×10^{-10}	1.8786×10^{-10}	2.5048×10^{-10}	3.131×10^{-10}	3.7572×10^{-10}	4.3834×10^{-10}	5.0096×10^{-10}	5.63582×10^{-10}	6.26202×10^{-10}
21	3.0876×10^{-11}	6.1752×10^{-11}	9.2628×10^{-11}	1.235×10^{-10}	1.5438×10^{-10}	1.8526×10^{-10}	2.1613×10^{-10}	2.4701×10^{-10}	2.77885×10^{-10}	3.08761×10^{-10}
22	1.5224×10^{-11}	3.0448×10^{-11}	4.5672×10^{-11}	6.0896×10^{-11}	7.612×10^{-11}	9.1344×10^{-11}	1.0657×10^{-10}	1.2179×10^{-10}	1.37016×10^{-10}	1.5224×10^{-10}
23	7.5064×10^{-12}	1.5013×10^{-11}	2.2519×10^{-11}	3.0026×10^{-11}	3.7532×10^{-11}	4.5039×10^{-11}	5.2545×10^{-11}	6.0052×10^{-11}	6.75584×10^{-11}	7.50648×10^{-11}
24	3.7012×10^{-12}	7.4024×10^{-12}	1.1104×10^{-11}	1.4805×10^{-11}	1.8506×10^{-11}	2.2207×10^{-11}	2.5908×10^{-11}	2.961×10^{-11}	3.33109×10^{-11}	3.70121×10^{-11}
25	1.825×10^{-12}	3.6499×10^{-12}	5.4748×10^{-12}	7.2998×10^{-12}	9.1248×10^{-12}	1.095×10^{-11}	1.2775×10^{-11}	1.46×10^{-11}	1.64245×10^{-11}	1.82495×10^{-11}
26	8.9984×10^{-13}	1.7997×10^{-12}	2.6995×10^{-12}	3.5993×10^{-12}	4.4992×10^{-12}	5.399×10^{-12}	6.2987×10^{-12}	7.1986×10^{-12}	8.09841×10^{-12}	8.99825×10^{-12}
27	4.4365×10^{-13}	8.874×10^{-13}	1.331×10^{-12}	1.7747×10^{-12}	2.2183×10^{-12}	2.6621×10^{-12}	3.1057×10^{-12}	3.5494×10^{-12}	3.99314×10^{-12}	4.43678×10^{-12}
28	2.1871×10^{-13}	4.3743×10^{-13}	6.5625×10^{-13}	8.7497×10^{-13}	1.0938×10^{-12}	1.3125×10^{-12}	1.5313×10^{-12}	1.75×10^{-12}	1.96876×10^{-12}	2.18758×10^{-12}
29	1.0791×10^{-13}	2.1572×10^{-13}	3.2363×10^{-13}	4.3143×10^{-13}	5.3935×10^{-13}	6.4715×10^{-13}	7.5506×10^{-13}	8.6298×10^{-13}	9.70779×10^{-13}	1.07869×10^{-12}
30	5.318×10^{-14}	1.0636×10^{-13}	1.5954×10^{-13}	2.1272×10^{-13}	2.659×10^{-13}	3.1908×10^{-13}	3.7226×10^{-13}	4.2544×10^{-13}	4.78617×10^{-13}	5.31797×10^{-13}

Table 2: Absolute error between analytical and semi-analytical solutions through MKud and VI schemes

Value of x	$t = 2$	$t = 4$	$t = 6$	$t = 8$	$t = 10$	$t = 12$	$t = 14$	$t = 16$	$t = 18$	$t = 20$
0	2.61928×10^{-6}	5.238×10^{-6}	7.857×10^{-6}	1.047×10^{-5}	1.309×10^{-5}	1.571×10^{-5}	1.833×10^{-5}	2.095×10^{-5}	2.357×10^{-5}	2.619×10^{-5}
1	9.91091×10^{-7}	1.982×10^{-6}	2.973×10^{-6}	3.964×10^{-6}	4.955×10^{-6}	5.946×10^{-6}	6.937×10^{-6}	7.928×10^{-6}	8.919×10^{-6}	9.910×10^{-6}
2	3.68327×10^{-7}	7.366×10^{-7}	1.104×10^{-6}	1.473×10^{-6}	1.841×10^{-6}	2.209×10^{-6}	2.578×10^{-6}	2.946×10^{-6}	3.314×10^{-6}	3.683×10^{-6}
3	1.36004×10^{-7}	2.720×10^{-7}	4.080×10^{-7}	5.440×10^{-7}	6.80×10^{-7}	8.160×10^{-7}	9.520×10^{-7}	1.088×10^{-6}	1.224×10^{-6}	1.360×10^{-6}
4	5.01013×10^{-8}	1.002×10^{-7}	1.503×10^{-7}	2.004×10^{-7}	2.505×10^{-7}	3.006×10^{-7}	3.507×10^{-7}	4.008×10^{-7}	4.509×10^{-7}	5.010×10^{-7}
5	1.844×10^{-8}	3.68×10^{-8}	5.532×10^{-8}	7.376×10^{-8}	9.220×10^{-8}	1.106×10^{-7}	1.290×10^{-7}	1.475×10^{-7}	1.659×10^{-7}	1.844×10^{-7}
6	6.785×10^{-9}	1.357×10^{-8}	2.035×10^{-8}	2.714×10^{-8}	3.392×10^{-8}	4.071×10^{-8}	4.749×10^{-8}	5.42×10^{-8}	6.106×10^{-8}	6.785×10^{-8}
7	2.496×10^{-9}	4.992×10^{-9}	7.488×10^{-9}	9.985×10^{-9}	1.248×10^{-8}	1.497×10^{-8}	1.747×10^{-8}	1.997×10^{-8}	2.246×10^{-8}	2.496×10^{-8}
8	9.183×10^{-10}	1.83×10^{-9}	2.755×10^{-9}	3.67×10^{-9}	4.59×10^{-9}	5.51×10^{-9}	6.42846×10^{-9}	7.346×10^{-9}	8.26×10^{-9}	9.183×10^{-9}
9	3.37×10^{-10}	6.756×10^{-10}	1.013×10^{-9}	1.351×10^{-9}	1.689×10^{-9}	2.027×10^{-9}	2.364×10^{-9}	2.702×10^{-9}	3.040×10^{-9}	3.378×10^{-9}
10	1.242×10^{-10}	2.485×10^{-10}	3.728×10^{-10}	4.971×10^{-10}	6.214×10^{-10}	7.457×10^{-10}	8.700×10^{-10}	9.942×10^{-10}	1.118×10^{-9}	1.242×10^{-9}
11	4.572×10^{-11}	9.14×10^{-11}	1.371×10^{-10}	1.82×10^{-10}	2.28×10^{-10}	2.74×10^{-10}	3.201×10^{-10}	3.65×10^{-10}	4.11×10^{-10}	4.572×10^{-10}
12	1.68×10^{-11}	3.36×10^{-11}	5.046×10^{-11}	6.728×10^{-11}	8.410×10^{-11}	1.009×10^{-10}	1.177×10^{-10}	1.345×10^{-10}	1.513×10^{-10}	1.682×10^{-10}
13	6.188×10^{-12}	1.23×10^{-11}	1.85×10^{-11}	2.475×10^{-11}	3.093×10^{-11}	3.712×10^{-11}	4.331×10^{-11}	4.950×10^{-11}	5.569×10^{-11}	6.187×10^{-11}
14	2.27635×10^{-12}	4.55×10^{-12}	6.82×10^{-12}	9.105×10^{-12}	1.138×10^{-11}	1.365×10^{-11}	1.593×10^{-11}	1.821×10^{-11}	2.048×10^{-11}	2.27×10^{-11}
15	8.37×10^{-13}	1.675×10^{-12}	2.512×10^{-12}	3.35×10^{-12}	4.18×10^{-12}	5.02×10^{-12}	5.86×10^{-12}	6.69×10^{-12}	7.53×10^{-12}	8.374×10^{-12}
16	3.08×10^{-13}	6.16×10^{-13}	9.24×10^{-13}	1.23×10^{-12}	1.54×10^{-12}	1.84×10^{-12}	2.16×10^{-12}	2.46×10^{-12}	2.772×10^{-12}	3.080×10^{-12}
17	1.13×10^{-13}	2.263×10^{-13}	3.39×10^{-13}	4.53138×10^{-13}	5.66×10^{-13}	6.79×10^{-13}	7.92977×10^{-13}	9.06×10^{-13}	1.019×10^{-12}	1.133×10^{-12}
18	4.15×10^{-14}	8.31×10^{-14}	1.24×10^{-13}	1.66×10^{-13}	2.082×10^{-13}	2.500×10^{-13}	2.916×10^{-13}	3.334×10^{-13}	3.751×10^{-13}	4.168×10^{-13}
19	1.537×10^{-14}	3.069×10^{-14}	4.607×10^{-14}	6.12843×10^{-14}	7.677×10^{-14}	9.20×10^{-14}	1.0747×10^{-13}	1.227×10^{-13}	1.381×10^{-13}	1.533×10^{-13}
20	5.71765×10^{-15}	1.132×10^{-14}	1.698×10^{-14}	2.253×10^{-14}	2.831×10^{-14}	3.402×10^{-14}	3.969×10^{-14}	4.524×10^{-14}	5.095×10^{-14}	5.639×10^{-14}
21	2.220×10^{-15}	4.163×10^{-15}	6.161×10^{-15}	8.326×10^{-15}	1.049×10^{-14}	1.249×10^{-14}	1.465×10^{-14}	1.659×10^{-14}	1.876×10^{-14}	2.076×10^{-14}
22	6.661×10^{-16}	1.443×10^{-15}	2.164×10^{-15}	2.942×10^{-15}	3.663×10^{-15}	4.385×10^{-15}	5.273×10^{-15}	5.995×10^{-15}	6.716×10^{-15}	7.549×10^{-15}
23	2.22×10^{-16}	2.22×10^{-16}	7.216×10^{-16}	9.992×10^{-16}	1.221×10^{-15}	1.498×10^{-15}	1.720×10^{-15}	2.109×10^{-15}	2.498×10^{-15}	2.609×10^{-15}
24	-5.55×10^{-17}	-5.55×10^{-17}	1.665×10^{-16}	3.331×10^{-16}	3.331×10^{-16}	3.331×10^{-16}	5.551×10^{-16}	7.21645×10^{-16}	7.216×10^{-16}	9.43×10^{-16}
25	1.11×10^{-16}	1.11×10^{-16}	1.11×10^{-16}	2.22×10^{-16}	3.886×10^{-16}	2.224×10^{-16}	3.886×10^{-16}	3.886×10^{-16}	3.885×10^{-16}	5.551×10^{-16}
26	-1.11×10^{-16}	-2.22×10^{-16}	-2.22×10^{-16}	-2.22×10^{-16}	-1.11×10^{-16}	-2.22×10^{-16}	-1.11×10^{-16}	-1.11×10^{-16}	-1.110×10^{-16}	-1.110×10^{-16}
27	0	0	-1.11×10^{-16}	-1.11×10^{-16}	-1.11×10^{-16}	-1.11×10^{-16}	-1.11×10^{-16}	-1.11×10^{-16}	0	0
28	0	-1.110×10^{-16}	-1.110×10^{-16}	0	0	-1.110×10^{-16}	-1.110×10^{-16}	-1.110×10^{-16}	-1.110×10^{-16}	-1.110×10^{-16}
29	-5.55×10^{-17}	-3.33×10^{-16}	-2.775×10^{-16}	-2.775×10^{-16}	-2.775×10^{-16}	-5.551×10^{-17}	-2.775×10^{-16}	-2.775×10^{-16}	-2.775×10^{-16}	-3.330×10^{-16}
30	2.775×10^{-16}	1.665×10^{-16}	1.665×10^{-16}	1.665×10^{-16}	5.551×10^{-17}	1.665×10^{-16}	1.665×10^{-16}	1.665×10^{-16}	1.665×10^{-16}	1.665×10^{-16}

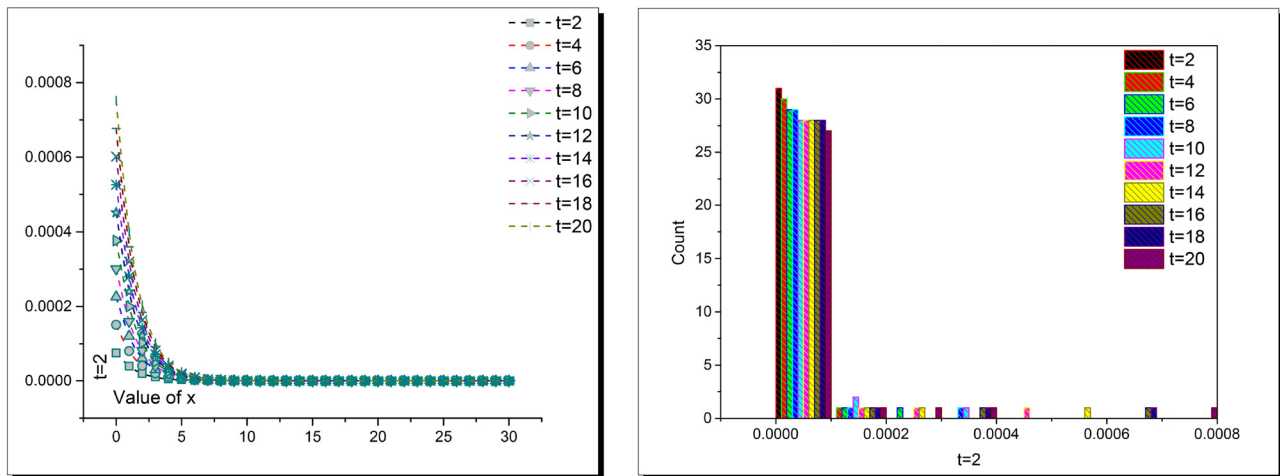


Figure 3: Absolute error between analytical and semi-analytical solutions along ESE and VI methods.

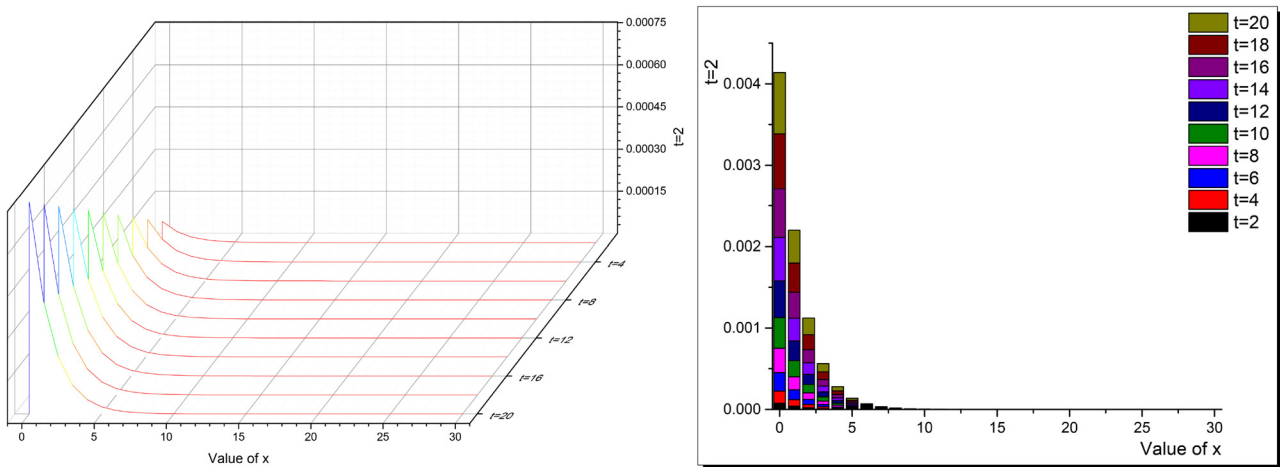


Figure 4: Absolute error between analytical and semi-analytical solutions along MKud and VI methods.

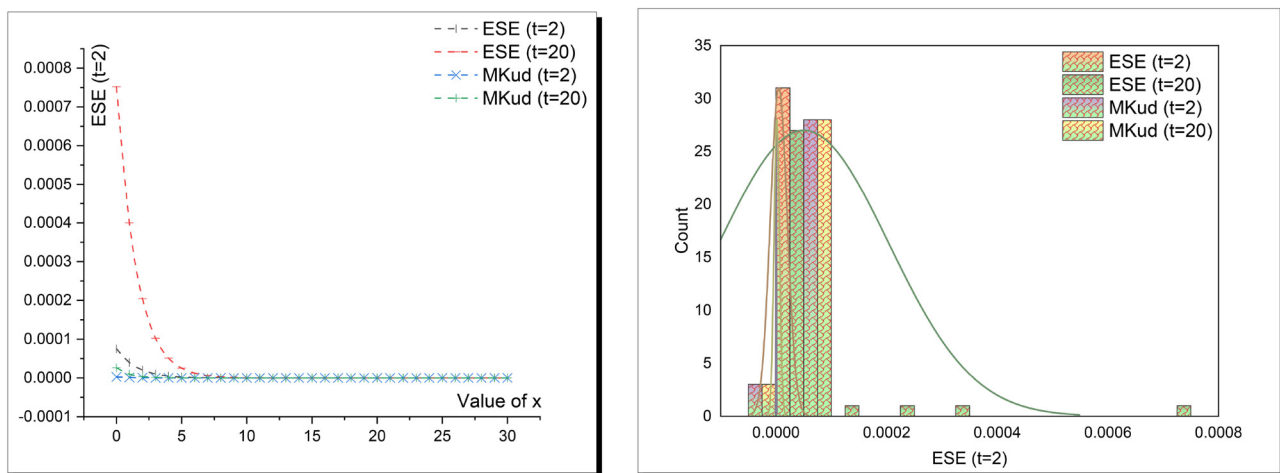


Figure 5: Matching between the calculated absolute error along ESE and MKud analytical methods and VI semi-analytical technique.

3 Results' interpretation

Here, the analytical and semi-analytical obtained results are explained and discussed to show the objectives of this manuscript. Two computational schemes have been successfully implemented to the positive nonlinear $(2 + 1)$ D-GKP equation, and many novel solitary wave solutions in various forms such as exponential, trigonometric, hyperbolic have been constructed. The physical characterization of the evolution of nonlinear waves where the effect of the surface tension and the viscosity is negligible is explained through Figures 1 and 2 for the, respectively, values of the parameters ($a_0 = 2$, $a_{-1} = 3$, $\eta = 1$, $l_1 = -2$, $l_3 = 8$ and $k = e$, $p = 1$, $q = -4$). Comparing our obtained solutions with those obtained by Kalim U Tariq *et al.*, who have used the auxiliary equation method, finds just one match between both solutions (equations (5) and (33), [23]

when $\left[\sqrt{p} \lambda_1 = 2a_1 a_0, \sqrt{\frac{3(4\omega - 5)}{l_1 l_3}} = \frac{2a_1 a_{-1}}{l_1} \right]$ and all our solutions are new and different from their solutions. The evaluated analytical solutions have been used to calculate the initial condition for the studied model. Thus, the VI method has been productively handling the semi-analytical solutions that have been used to calculate the absolute error between the exact and semi-analytical solutions, which show the match between them (Tables 1, 2 and Figures 3, 4). The superiority of the MKud method has been shown over the ESE method (Figure 5).

4 Conclusion

Three analytical and semi-analytical methods for solving the positive nonlinear $(2 + 1)$ -D GKP problem have been developed. Numerous new computational solutions have been found. Some have been sketched in 2D, three-dimensional, and contour plots to illustrate the physical structure of the shallow oceans' internal solitary waves. The semi-analytical solutions have been described using the VI technique to demonstrate the power and efficacy of the analytical methods employed. The uniqueness of our study is shown by comparing our answers to those found in previously published studies.

Funding information: This research was supported by Taif University Researchers Supporting Project Number (TURSP-2020/52), Taif University, Taif, Saudi Arabia.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

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