Research Article

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Soliton structures in optical fiber communications with Kundu–Mukherjee–Naskar model

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Abstract: In the present work, we investigate soliton structures in optical fiber communications. The medium is described by the Kundu–Mukherjee–Naskar model. With the aid of the ansatz approach, the exact solutions are constructed. Consequently, distinct wave structures including W-shaped, bright and dark solitons are derived. These new soliton solutions are retrieved under certain parametric conditions. Besides, it is found that the bright soliton has two different types in a particular limit. Optical solitons are displayed graphically to shed light on their behaviors.

Keywords: optical solitons, Kundu–Mukherjee–Naskar model, ansatz approach, W-shaped, bright and dark solitons

1 Introduction

Information transmission in optical communication channels is based on soliton propagation [1–3]. Thus, the dynamic of solitons in the field of fiber optics has been under remarkable attention for over a decade [4,5]. One of the important models that describe the soliton in optics and optical fibers is the Kundu–Mukherjee–Naskar (KMN) equation. This model is considered as an extension to the nonlinear Schrödinger equation containing mixed types of nonlinear effect in reference to Kerr and non-Kerr law nonlinearities.

The model of KMN equation discussed in the present work is given by

$$i\Psi_t + a\Psi_{xy} + ib\Psi(\Psi\Psi_x^* - \Psi^*\Psi_x) = 0, \qquad (1)$$

where the dependent variable $\Psi(x, y, t)$ stands for the optical soliton profile. In equation (1), the first term indicates the temporal evolution of the pulse, while the second term with the coefficient of a stands for the dispersion term. The nonlinear effect given by the coefficient of b represents current-like nonlinearity arising from chirality.

Equation (1) was introduced by the Kundu and Mukherjee in 2013 [6]. It is basically originated as a two-dimensional nonlinear Schrödinger equation which is derived from the basic hydrodynamic equations. This model can be used to describe wave propagation in optical fiber and oceanic rogue waves as well as ion-acoustic wave in a magnetized plasma [7–10]. Unlike most two-dimensional models, it is classified as an integrable equation which can be solved by the inverse scattering transform. Moreover, many studies are devoted to equation (1) to examine soliton propagation into an optical fiber. Therefore, several mathematical tools are implemented to extract exact analytic solutions [11–15].

The present study is devoted to investigating new forms of optical solitons with nontrivial phase component possessing a form of nonlinear function, in contrast to previous studies in the literature which scrutinized KMN equation. The traveling wave reduction of the model is derived and then it is dealt with soliton ansatz having new functional form in terms of the hyperbolic secant and tangent functions.

2 Mathematical analysis of the model

Now, we intend to obtain the traveling wave reduction of equation (1) so as to derive the optical soliton solutions. Thus, we introduce the traveling wave transformation of the form

$$\Psi(x, y, t) = \psi(\xi)e^{i\phi(x,y,t)}, \qquad (2)$$

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where $\psi(\xi)$ accounts for the amplitude of the soliton while $\phi(x, y, t)$ denotes the phase component. The wave variable ξ is given by

$$\xi = \kappa x + \lambda y + \nu t, \tag{3}$$

and the phase component is presented as

$$\phi(x, y, t) = \alpha x + \beta y + \omega t + \theta(\xi), \tag{4}$$

where κ and λ are the inverse width of the soliton along x- and y-directions, respectively, whereas ν is the velocity of the soliton. The parameters α and β indicate the soliton frequencies in the x- and y-directions, respectively, and ω represents the soliton wave number. The function $\theta(\xi)$ is defined as a nonlinear phase shift.

Applying the transformation (2) to equation (1) and separating the real and imaginary parts lead to a pair of equations having the form

$$(\nu + a\beta\kappa + a\alpha\lambda)\psi' + 2a\kappa\lambda\psi'\theta' + a\kappa\lambda\psi\theta'' = 0, \tag{5}$$

$$\begin{aligned} &a\kappa\lambda\psi'' - (\omega + a\alpha\beta)\psi + 2ba\psi^3 - (\nu + a\beta\kappa + a\alpha\lambda)\psi\theta' \\ &\quad - a\kappa\lambda\psi\theta'^2 + 2b\kappa\psi^3\theta' = 0, \end{aligned} \tag{6}$$

where the prime denotes the derivative with respect to ξ. Once we multiply equation (5) by ψ and integrate it, we arrive at the first integral given by

$$\theta' = \frac{C\psi^{-2}}{a\kappa\lambda} - \frac{(\nu + a\beta\kappa + a\alpha\lambda)}{2a\kappa\lambda}, \tag{7}$$

where C is the constant of integration. Substituting equation (7) into equation (6) gives rise to

$$\psi'' + A_1\psi + A_2\psi^3 - A_3\psi^{-3} = 0. \tag{8}$$

Multiplying equation (8) by ψ' and integrating with respect to ξ, we find

$$\psi'^2 + A_1\psi^2 + \frac{A_2}{2}\psi^4 + A_3\psi^{-2} + 2A_0 = 0, \tag{9}$$

where A_0 is an arbitrary constant of integration and the rest of constants are defined by

$$A_1 = \frac{8b\kappa C - 4a\kappa\lambda(\omega + a\alpha\beta) + (\nu + a\beta\kappa + a\alpha\lambda)^2}{4a^2\kappa^2\lambda^2}, \tag{10}$$

$$A_2 = \frac{2ab\alpha\kappa\lambda - b\kappa(\nu + a\beta\kappa + a\alpha\lambda)}{a^2\kappa^2\lambda^2}, \tag{11}$$

$$A_3 = \frac{C^2}{a^2\kappa^2\lambda^2}. \tag{12}$$

To avoid the complexity, we make use of the variable transformation given by

$$\psi^2 = V. \tag{13}$$

Hence, equation (9) reduces to

$$V'^2 + 4A_1V^2 + 2A_2V^3 + 4A_3 + 8A_0V = 0. \tag{14}$$

Differentiating equation (14), for convenience, yields

$$V'' + 4A_0 + 4A_1V + 3A_2V^2 = 0. \tag{15}$$

The solution to equation (15) along with the relations (2) and (13) constructs the general form of exact solutions for equation (1) addressed as

$$\Psi(x, y, t) = V^{1/2}e^{i[\alpha x + \beta y + \omega t + \theta(\xi)]}, \tag{16}$$

where the phase variable $\theta(\xi)$ can be turned up through integrating equation (7) with respect to ξ as

$$\theta(\xi) = \frac{C}{a\kappa\lambda}\int\frac{d\xi}{V} - \frac{(\nu + a\beta\kappa + a\alpha\lambda)}{2a\kappa\lambda}(\kappa x + \lambda y + \nu t) + \theta_0, \tag{17}$$

where θ_0 is a constant phase.

3 Optical soliton solutions

In this section, soliton ansatz is applied to extract the exact solutions of equation (1). Various solutions describing W-shaped, bright and dark solitons are retrieved.

We assume that equation (15) has an exact optical soliton solution in the form

$$\begin{aligned} V(\xi) = l &+ \frac{m\,\mathrm{sech}^2(p\xi)}{4 - [1 - \tanh(p\xi)]^2} \\ &+ \frac{n\,\mathrm{sech}^4(p\xi)}{(4 - [1 - \tanh(p\xi)]^2)^2}, \end{aligned} \tag{18}$$

where l, m, n and p are constants to be determined. As one can see that the expression (18) has a constant term and two terms in the hyperbolic secant and tangent functions with degree two and four. This combination of terms may enable us to generate different forms of soliton solutions.

Substituting equation (18) into equation (15) and equating all coefficients having the same order of $\mathrm{sech}(p\xi)\tanh(p\xi)$ to zero give us a set of algebraic equations that induce different cases of solutions presented as follows.

Case I. *W-shaped soliton*

$$l = -\frac{4A_1}{3A_2},\ m = \frac{16A_1}{A_2},\ n = -\frac{32A_1}{A_2},\ p = \sqrt{A_1},$$

with neglecting integration constant in equation (15), i.e., $A_0 = 0$. Making use of these findings leads to the formation of optical soliton solution to equation (1) given as

$$\Psi(x,y,t)=\left\{-\frac{4A_1}{3A_2}\left[1-\frac{12\,\text{sech}^2(\sqrt{A_1}\,\xi)}{4-[1-\tanh(\sqrt{A_1}\,\xi)]^2}+\frac{24\,\text{sech}^4(\sqrt{A_1}\,\xi)}{(4-[1-\tanh(\sqrt{A_1}\,\xi)]^2)^2}\right]\right\}^{\frac{1}{2}}e^{i\phi(x,y,t)}, \tag{19}$$

provided that $A_1 > 0$ and $A_2 < 0$. It is clear from Figure 1(a) that the plot of solution (19) takes the shape of W.

Case II. *Bright soliton* (*type I*)

$$l=0,\ \ m=-\frac{16A_1}{A_2},\ \ n=\frac{32A_1}{A_2},\ \ p=\sqrt{-A_1}.$$

with the constraint $A_0 = 0$. These results give rise to exact soliton solution to equation (1) of the form

$$\Psi(x,y,t)=\left\{-\frac{16A_1}{A_2}\left[\frac{\text{sech}^2(\sqrt{-A_1}\,\xi)}{4-[1-\tanh(\sqrt{-A_1}\,\xi)]^2}-\frac{2\,\text{sech}^4(\sqrt{-A_1}\,\xi)}{(4-[1-\tanh(\sqrt{-A_1}\,\xi)]^2)^2}\right]\right\}^{\frac{1}{2}}\times e^{i\phi(x,y,t)}, \tag{20}$$

which demands $A_1 < 0$ and $A_2 > 0$. The behavior of solution (20) describes a bright soliton pulse as shown in Figure 1(b).

Case III. *Bright soliton* (*type II*)

$$l=-\frac{6A_1}{11A_2},\ \ m=-\frac{32A_1}{11A_2},\ \ n=\frac{64A_1}{11A_2},\ \ p=\sqrt{-\frac{2A_1}{11}},$$

under the constraint condition $121A_0A_2 = 39A_1^2$. By virtue of the obtained outcomes, an exact optical soliton solution to equation (1) is acquired as

$$\Psi(x,y,t)=\left\{-\frac{2A_1}{11A_2}\left[3+\frac{16\,\text{sech}^2\left(\sqrt{-\frac{2A_1}{11}}\,\xi\right)}{4-\left[1-\tanh\left(\sqrt{-\frac{2A_1}{11}}\,\xi\right)\right]^2}-\frac{32\,\text{sech}^4\left(\sqrt{-\frac{2A_1}{11}}\,\xi\right)}{\left(4-\left[1-\tanh\left(\sqrt{-\frac{2A_1}{11}}\,\xi\right)\right]^2\right)^2}\right]\right\}^{\frac{1}{2}}\times e^{i\phi(x,y,t)}, \tag{21}$$

provided that $A_1 < 0$ and $A_2 > 0$. As one can see, Figure 1(c) illustrates the evolution of solution (21) which characterizes a bright soliton wave.

Case IV. *Dark soliton*

$$l=-\frac{26A_1}{33A_2},\ \ m=\frac{32A_1}{11A_2},\ \ n=-\frac{64A_1}{11A_2},\ \ p=\sqrt{\frac{2A_1}{11}},$$

under the restriction $121A_0A_2 = 39A_1^2$. From these results, one can secure exact soliton solution to equation (1) of the form

$$\Psi(x,y,t)=\left\{-\frac{2A_1}{33A_2}\left[13-\frac{48\,\text{sech}^2\left(\sqrt{\frac{2A_1}{11}}\,\xi\right)}{4-\left[1-\tanh\left(\sqrt{\frac{2A_1}{11}}\,\xi\right)\right]^2}+\frac{96\,\text{sech}^4\left(\sqrt{\frac{2A_1}{11}}\,\xi\right)}{\left(4-\left[1-\tanh\left(\sqrt{\frac{2A_1}{11}}\,\xi\right)\right]^2\right)^2}\right]\right\}^{\frac{1}{2}}\times e^{i\phi(x,y,t)}, \tag{22}$$

provided that $A_1 > 0$ and $A_2 < 0$. The behavior of this soliton solution is depicted in Figure 1(d) and it obviously presents the pulse profile for dark soliton.

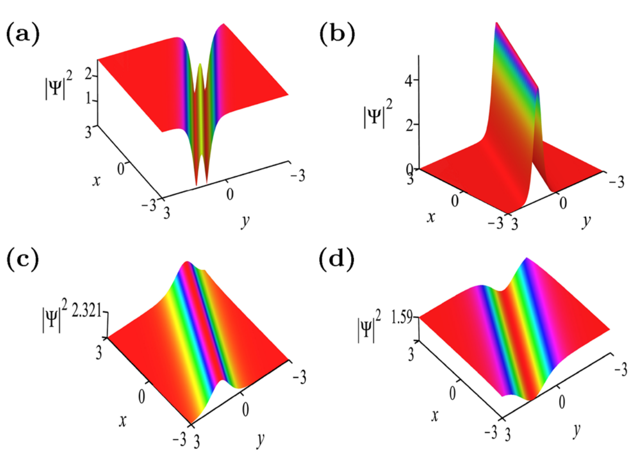

Figure 1: W-shaped, bright (type I), bright (type II) and dark soliton solutions given by (19)–(22) with $a = b = \beta = v = \omega = \kappa = 0.5$, $\alpha = -0.5$, $\lambda = C = t = 1$; for (b) and (c) $b = -0.5$.

4 Discussion and conclusion

The current study has dealt with the soliton solutions of KMN equation. Based on soliton ansatz method, new types of soliton structures are revealed with phase component having a form of nonlinear function which is different from the one addressed in the previous studies. Among them, the W-shaped and bright (type I) optical solitons are derived due to the absence of integration constant in equation (15). Additionally, bright (type II) and dark solitons are extracted under specific relation between the physical parameters. The validity conditions for the existence of all optical solitons are given. The structures of obtained solitons are clearly illustrated by selecting suitable values of the model parameters. The results obtained for the KMN model are new and can be benefited in the field of optical fiber.

As the KMN model is in its infancy, it can be studied further in future to examine its applications in various physical areas such as optical couplers, meta-optics and magneto-optic waveguides. Hence, there are still many powerful analytical and numerical techniques to be implemented so as to generate new forms of solutions.

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