Research Article

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Study on acoustic radiation response characteristics of sound barriers

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Abstract: The influence of acoustic radiation is considered in the prediction of noise attenuation effect of sound barrier, which provides a theoretical reference for further improving the insertion loss of sound barrier. Based on the theory of thin plate vibration, the vibration mode and natural frequencies of sound barrier under arbitrary boundary conditions are established by using two-dimensional beam function method, and the forced vibration response of the sound barrier is calculated based on the modal superposition method. MATLAB software (MathWorks Company, Natick, Massachusetts, USA) is used to calculate the natural frequencies and the radiated sound power level of the sound barrier, which indicated that the sound radiation caused by external excitation would significantly increase the sound pressure level at the received point, which should be considered as one of the influencing factors in the prediction of noise attenuation effect. The influence of diverse structural parameters on the radiated acoustic power is compared, providing an excellent reference for the design of sound barrier with low noise.

Keywords: sound barrier, two-dimensional beam function, acoustic radiation, sound power, insertion loss

1 Introduction

With the mass construction of expressway and high-speed railway elevated road, urban traffic noise has become the main source of environmental noise pollution, which has interfered with people's life and work, seriously affected their physical and mental health, and also influenced the economic development along the road. Due to the advantages of excellent noise reduction effect, convenient construction, long service life, and elegant appearance, sound barrier is one of the most significant measures of sound environment management [1].

At present, the research on sound barrier is mainly based on its sound absorption and insulation performance. Various sound absorption and insulation structures and diverse calculation methods are proposed and combined with various advanced software [2,3]. All these predictive analysis methods have achieved satisfactory results for the engineering design and further theoretical analysis of sound barrier. However, when external motivation is oversize, such as the sound barriers of highway, high-speed railway vehicles driving through the sound barrier generate large pulsating load of air and noise intensity, and are often considered in the design of the large load of mechanical properties [4,5]. Little consideration has been given to the acoustic radiation performance of sound barriers. The common sound barrier is mainly composed of steel structure columns and sound insulation plate, which is usually made of acrylic plate. Thin plate structure is a common structure in engineering. With advantages of light weight, energy saving, low cost, and environmental protection, thin plate structure has been widely used in various equipment [6]. However, thin plate structure is also an excellent radiation body that generates vibration and radiated noise in response to forced vibration [7]. Therefore, in the design of acoustic barriers, it is necessary to consider the acoustic radiation performance and the corresponding measures to recede the acoustic radiation performance, so as to effectively improve the actual noise reduction effect of sound barriers.

In this article, based on the vibration theory, the dynamic response of forced vibration of the sound barriers is calculated by the two-dimensional beam function. The effects of three structural parameters, such as material, aspect ratio, and thickness, were compared to provide an excellent reference for the low-noise design of sound barriers.

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2 Transverse vibration theory of rectangular thin plates

The vibration equation of isotropic and undamped plate is as follows [8]:

$$D\nabla^4 w(x,y,t) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = p(x,y,t), \tag{1}$$

where $\nabla^4 = \left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2} + \frac{\partial^4}{\partial y^4}\right)$, $D = \frac{Eh^3}{12(1-v^2)}$ is the bending rigidity of plate structure; E, v, ρ, h are young's modulus, poisson ratio, density, and thickness, respectively.

2.1 Free vibration

When $p(x,y,t) = 0$, equation (1) is the equation of free vibration of thin plate.

Set the main vibration mode as $w(x,y,t) = W(x,y)\sin(\omega t + \phi)$. After substituting into equation (1), the following equation can be obtained [8]:

$$\nabla^4 W - \beta^4 W = 0, \tag{2}$$

where $\beta^4 = \frac{\rho h}{D}\omega^2$. $W(x, y)$ is the mode function set according to the boundary condition, that is, the main mode of free vibration of thin plate.

Due to the limitation of boundary conditions, all rectangular thin plates with boundary conditions have no analytical solutions except rectangular thin plates with simply supported four sides. By two-dimensional beam function method, the mode of vibration of thin plate can be expressed as the product of two mode functions satisfying quasi-orthogonal and boundary conditions:

$$W(x,y) = X_m(x)\cdot Y_n(y). \tag{3}$$

By using the orthogonal relation, the natural frequency can be obtained as follows [8]:

$$\omega_{mn} = \sqrt{\frac{D}{\rho h}}\cdot\sqrt{\frac{I_1 I_2 + 2I_3 I_4 + I_5 I_6}{I_2 I_6}}, \tag{4}$$

where [8]

$$\begin{aligned}
I_1 &= \int_0^{L_x} \frac{\partial^4 X_m(x)}{\partial x^4} X_m(x)\mathrm{d}x\\
I_2 &= \int_0^{L_y} (Y_n(x))^2 \mathrm{d}y\\
I_3 &= \int_0^{L_x} \frac{\partial^2 X_m(x)}{\partial x^2} X_m(x)\mathrm{d}x\\
I_4 &= \int_0^{L_y} \frac{\partial^2 Y_n(y)}{\partial y^2} Y_n(y)\mathrm{d}y\\
I_5 &= \int_0^{L_y} \frac{\partial^4 Y_m(y)}{\partial y^4} Y_m(y)\mathrm{d}y\\
I_6 &= \int_0^{L_x} (X_n(x))^2 \mathrm{d}x.
\end{aligned} \tag{5}$$

Simply supported boundary conditions are

$$W = 0, \quad \frac{\partial W}{\partial s} = 0. \tag{6}$$

Fixed supported boundary conditions are

$$W = 0, \quad \frac{\partial W}{\partial s} = \frac{\partial W}{\partial n} = 0. \tag{7}$$

The mode function satisfying the boundary conditions of the four-sided fixed support is as follows [8]:

$$\begin{aligned}
X_m(y) = {} & \cosh\left(\frac{\lambda_m y}{L_y}\right) - \cos\left(\frac{\lambda_m y}{L_y}\right)\\
& - \beta_m\left[\sinh\left(\frac{\lambda_m y}{L_y}\right) - \sin\left(\frac{\lambda_m y}{L_y}\right)\right]
\end{aligned} \tag{8}$$

$$\begin{aligned}
Y_n(y) = {} & \cosh\left(\frac{\lambda_n y}{L_y}\right) - \cos\left(\frac{\lambda_n y}{L_y}\right)\\
& - \beta_n\left[\sinh\left(\frac{\lambda_n y}{L_y}\right) - \sin\left(\frac{\lambda_n y}{L_y}\right)\right],
\end{aligned} \tag{9}$$

where $\beta_x = \frac{\sinh(\lambda_x l) + \sin(\lambda_x l)}{\cosh(\lambda_x l) - \cos(\lambda_x l)}$ $(x = m, n)$, $\lambda_x l$ is the root of equation $\cosh(\lambda_x l)\cos(\lambda_x l) + 1 = 0$ $(x = m, n)$.

2.2 Forced vibration

The response w of thin plate is expanded to W according to the regular mode as the following double series [8]:

$$w(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W(x, y)\eta(t), \tag{10}$$

where $\eta(t)$ is the regular coordinate. Its expression is

$$\eta(t) = \frac{q(t)}{\omega_{mn}^2 - \omega^2}, \tag{11}$$

where $q(t)$ is the regular generalized force, and its expression is

$$q(t) = \iint_{\Omega} p(x, y, t)W(x, y)\mathrm{d}x\mathrm{d}y. \tag{12}$$

If subjected to the concentrated harmonic force, then

$$p(x, y, t) = F_0 \sin \omega t\delta(x - x_0, y - y_0). \tag{13}$$

If the harmonic force is uniformly distributed, then

$$p(x, y, t) = F_0 \sin \omega t. \tag{14}$$

Then take the derivative of time t to obtain the velocity distribution of the plate

$$v(x, y, t) = \frac{\mathrm{d}w(x, y, t)}{\mathrm{d}t}. \tag{15}$$

3 Acoustic radiation theory of sound barriers

Assuming that the structure is in an infinite baffle plate, each element on the rectangular plate can be regarded as a simple point sound source radiating outward sound, and the radiation sound pressure of the whole rectangular plate can be obtained. The sound pressure can be expressed by velocity with Rayleigh integration [9,10]:

$$p(r_n) = \frac{j\omega\rho_0}{2\pi}\iint_S v(r_n)\frac{e^{(-jkR)}}{R}\mathrm{d}S, \tag{16}$$

where $R = |r - r_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2 + z^2}$ is the distance between any point r on the rectangular plate and observation point, r_n. According to the definition of sound intensity, the sound intensity of the observation point can be expressed as follows: $I(r_n) = \frac{1}{2}\mathrm{Re}[p(r_n)v * (r_n)]$.

Then, $*$ stands for conjugate, the radiated sound power is

$$W = \frac{1}{2}\mathrm{Re}\left[\iint_S p(r_n)v * (r_n)\mathrm{d}S\right]. \tag{17}$$

The flat plate is divided into N units with equal area ΔS, and the sound power is expressed as

$$W = \frac{\omega\rho_0}{4\pi}\sum_{m=1}^{N}\sum_{n=1}^{N} v_m\frac{\sin(kR)}{R}v_n * \Delta S \cdot \Delta S, \tag{18}$$

where v_m, v_n are the velocities of unit m, n.

Formula (18) is expressed as a matrix:

$$W = V^H RV, \tag{19}$$

where the (m, n) unit of matrix R is [9]

$$R_{mn} = \frac{\omega^2\rho_0(\Delta S)^2}{4\pi c_0}\frac{\sin(kr_{mn})}{kr_{mn}}. \tag{20}$$

If $m = n$, $r_{mn} = 0$, and $\frac{\sin(kr_{mn})}{kr_{mn}} \to 1$, then $R_{mn} = \frac{\omega^2\rho_0(\Delta S)^2}{4\pi c_0}$. The matrix R is a real symmetric positive definite matrix.

Assuming that the sound point is 5 m away from a sound barrier, the sound pressure level is calculated by using the following formula:

$$L_p = L_w - 10\lg S - 10\lg(400/\rho_0 c_0), \tag{21}$$

where $L_w = 10\lg(W/W_0)$ is the sound power level; $W_0 = 10^{-12}(W)$ is the reference sound power; $S = 2\pi r^2$ is the radiation area of semi-free field sound source; ρ_0 is the air density and c_0 is the air sound velocity.

4 Numerical calculation

Consider the sound barrier in Figure 1 as an example. The middle transparent plate is acrylic plate, the four sides are fixed by the aluminum alloy profile frame, and the

Figure 1: Real shot image of a sound barrier.

Table 1: The first ten order natural frequencies of the four-sided fixed supported sound barrier/Hz

Modal order number	1(1,1)	2(2,1) (1,2)	3(2,2)	4(3,1) (1,3)	5(3,2) (2,3)
MATLAB solution	7.30	14.88	21.91	26.67	33.39
ANSYS simulation results	7.32	14.95	22.09	26.84	33.74
Modal order number	6(1,4) (4,1)	7 (3,3)	8(4,2) (2,4)	9(4,3) (3,4)	10(5,1) (1,5)
MATLAB solution	42.68	44.46	48.99	59.83	62.62
ANSYS simulation results	43.03	45.13	49.64	60.95	63.30

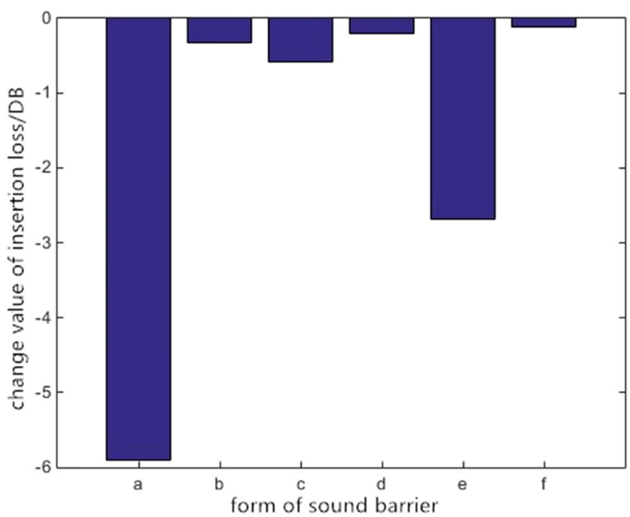

Figure 2: Change of insertion loss caused by different forms of sound barrier radiation.

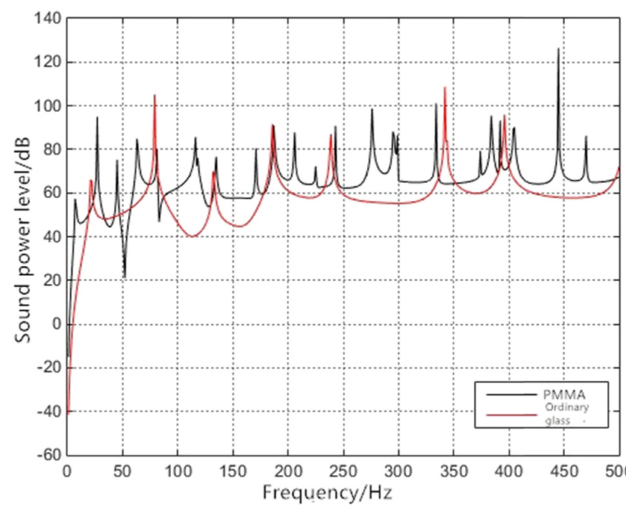

Figure 3: Radiated sound power levels of sound barriers of two materials.

gaps are filled with glass silicone and other sealing materials. Therefore, the middle part can be treated as an ideal isotropic plate with four fixed edges. Since the vibration response is dominated by low-order modes, only the frequencies of first ten modes are calculated, and the natural frequency and acoustic radiation power are obtained by the above formula, so as to discuss the influences of the acoustic radiation power of the sound barrier with its parameters.

The thin plate in the middle of the sound barrier has a length of $a = 2\,\text{m}$, width of $b = 2\,\text{m}$ and thickness of $h = 0.01\,\text{m}$. Young's modulus is $E = 3.35 \times 10^9\ \text{N/m}^2$, Poisson ratio is $\nu = 0.3$, density is $\rho = 1{,}180\ \text{kg/m}^3$, sound velocity is $c_0 = 343\ \text{m/s}$, and air density is $\rho_0 = 1.22\ \text{kg/m}^3$. Each sound barrier plate is subjected to 1 Pa of uniformly distributed harmonic load. The first ten natural frequencies are shown in Table 1.

As can be observed from Table 1, the natural frequency calculated by the two-dimensional beam function method is very similar to the ANSYS simulation result, indicating that this method is feasible to calculate the natural frequency of the thin plate. The material is acrylic plate. The sound barrier with other parameters unchanged is denoted as a, and the sound barrier with glass is denoted as b. When the length-to-width ratio is 0.5:1, the sound barrier with other parameters unchanged is denoted as c, and the sound barrier with the length-to-width ratio of 0.5:1 is denoted as d. When the thickness is 20 mm, the sound barrier with other parameters unchanged is denoted as e, and the sound barrier with a thickness of 30 mm is denoted as f. The change in the insertion loss of the sound barrier caused by acoustic radiation is obtained by using the formula, as shown in Figure 2.

4.1 Influence of flat plate materials on acoustic radiation characteristics of sound barriers

It can be observed from Figure 3 that the excitation frequency has a significant influence on the acoustic radiation power of the sound barrier, especially when it is close to the natural frequency of the acoustic barrier, a peak value will be generated. Moreover, since the external load is assumed to be uniformly distributed, the forced vibration response of the thin plate is only odd–odd terms. Therefore, the suppression of acoustic radiation of the sound barrier is focused on the natural frequency point of odd–odd terms. Due to the neglect of damping, the actual radiated acoustic power is relatively small. Compared with the two materials, the acoustic power radiated by the plate is also diverse. In the frequency range of 0–500 Hz, the radiated sound power of polymethyl methacrylate board with small stiffness is almost all greater than that of ordinary glass, except for a small number of common vibration points. In connection with a and b in Figure 2, the acoustic radiation characteristics of acrylic plate decrease the insertion loss more than that of ordinary glass. It can be concluded that diverse materials have an excellent influence on the radiation characteristics of the sound barrier, and other characteristics such as light transmittance, load resistance and sound insulation should also be considered.

4.2 The influence of the ratio of length to width on the acoustic radiation characteristics of the sound barrier

With other parameters of the plate unchanged, the aspect ratio of the sound insulation board of the sound barrier is 1:1, 0.5:1, and 0.25:1, and the unit length is 2 m. According to the above formula, the natural frequency is calculated by MATLAB programming, as shown in Table 2.

As can be observed from Table 2 and Figures 2 and 4, when the aspect ratio is 1:1 to 0.5:1 and then 0.5:1, the modal density gradually decreases, and the overall radiated sound power level also reveals a decreasing trend. When the plate shape is square, the modal density is the highest, and the peak value of the radiated sound power level is the highest. When the aspect ratio is away from 1, the peak value of the radiated sound power level decreases, and the overall radiated sound power level is also smaller. It is mainly due to the square thin plate, whose natural frequencies of order (m, n) and order (n, m) are equal, and the actual mode of vibration generated by force has two orders, so the noise radiation capability is relatively strong, as can be observed from Figure 4. When the aspect ratio is 1:1, the radiated sound power of the first peak value (1,1) is less than that of the second peak value (3,1) and that of the second peak value (1,3).

Table 2: The natural frequency is calculated by MATLAB programming

Modal order number	1	2	3	4	5
Aspect ratio 1:1	7.30	14.88	21.91	26.67	33.39
Aspect ratio 0.5:1	19.88	25.60	35.89	50.73	51.78
Aspect ratio 0.25:1	73.83	77.58	84.49	95.23	110.30
Modal order number	6	7	8	9	10
Aspect ratio 1:1	42.68	44.46	48.99	59.83	62.62
Aspect ratio 0.5:1	57.08	66.27	69.91	79.62	93.30
Aspect ratio 0.25:1	129.91	154.14	182.94	202.16	211.86

4.3 Influence of plate thickness on acoustic radiation characteristics of sound barriers

One of the measures to control the acoustic radiation from a sound barrier is how much the acoustic effect can be improved by simply changing the thickness of the board. The first ten natural frequencies and radiated acoustic power of the sound barriers with five thicknesses are compared, as revealed in Table 3 and Figure 5.

As can be observed from Table 3 and Figure 5, as the thickness increases, the peak point of radiated sound power within 100 Hz decreases. In other words, the frequency range covered by the same number of peak points increases, and the modal density decreases, thereby affecting the natural frequency. The tenth natural frequency of the plate with h = 10 mm is 62.62 Hz, and the tenth natural frequency of the plate with h = 30 mm is

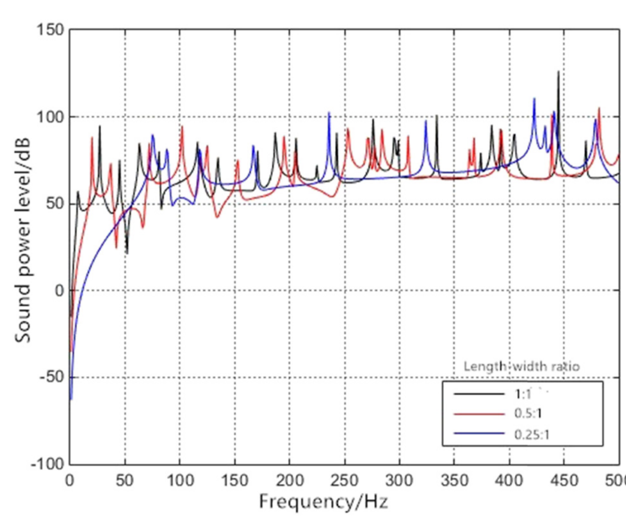

Figure 4: Radiated sound power levels of three aspect ratio of sound barriers.

Table 3: The first ten natural frequencies of three thickness barriers/Hz

Modal order number	**1**	**2**	**3**	**4**	**5**
h = 10 mm	7.30	14.88	21.91	26.67	33.39
h = 20 mm	14.56	29.68	43.57	53.21	66.29
h = 30 mm	21.85	44.52	65.35	79.81	99.43
Modal order number	6	7	8	9	10
h = 10 mm	42.68	44.46	48.99	59.83	62.62
h = 20 mm	85.16	87.92	97.64	118.03	125.00
h = 30 mm	127.74	131.88	145.80	177.05	187.49

187.49 Hz. In general, the increase of plate thickness can significantly decrease the radiated sound power, mainly because the bending stiffness D of thin plate is proportional to the cubic of plate thickness. It can also be seen from the comparison of the change value of insertion loss in Figure 2.

5 Conclusion

In the prediction of noise reduction effect of acoustic barriers, the influence of acoustic radiation is analyzed. The sound barrier model is simplified as a four-sided plate with fixed support. The free vibration forced vibration and acoustic radiation characteristics of the sound barrier are analyzed by means of the theory of vibration and acoustic radiation of the plate. The results reveal that when the external excitation is conspicuously large, the acoustic radiation characteristics of the sound barrier have a significant impact on the sound pressure level of the affected sound points, which should be included as one of the influencing factors in the design of noise reduction effect prediction of the sound barrier. The influence of three parameters, namely material, aspect ratio and thickness, on the radiated acoustic power was compared, and the following conclusions were drawn:

1. Diverse materials have diverse radiative capacity.
2. The closer the plate is to the square, the larger the radiation sound power is, mainly because the natural frequency of the square plate of order (m, n) and order (n, m) is the same, the mode of vibration produced by the same order is of two orders, leading to a strong radiation ability.
3. The larger the thickness of the plate, the smaller the radiation sound power, mainly because the bending stiffness D of the plate is proportional to the third square of the plate thickness, the plate surface vibration speed is small, and the radiation capacity is weak.

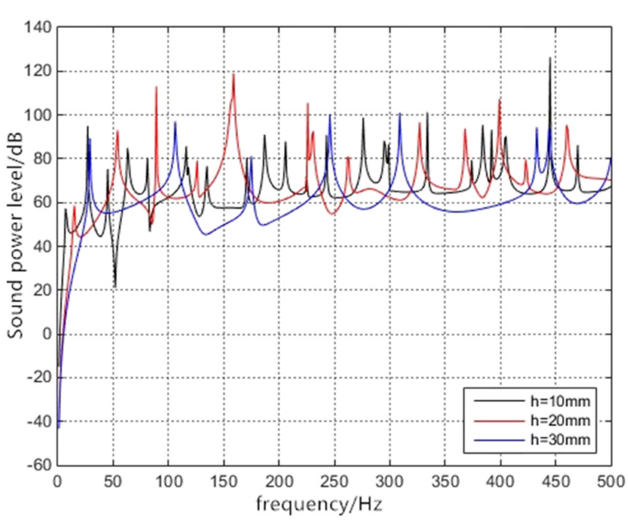

Figure 5: Radiated sound power levels of three thickness barriers.

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References

[1] Zhou JQ, Wu SQ, Wu GQ. Review of new city viaduct noise barrier. Adv Mater Res. 2014;3530(1044):65–8.
[2] Zhou X, Xiao XB, He B. Influential factors and rules for insertion loss of high-speed railway noise barriers. J Southwest Jiaotong Univ. 2014;49(60):1024–31.
[3] Chen JH. Study on influence of sound barrier structure on its noise reduction performance with numerical simulation. Noise Vib Control. 2010;19(4):63–6.
[4] He B, Xiao XB, Zhou X. Numerical study on acoustic performance of high-speed railway noise barriers with different geometric shape. J Mech Eng. 2016;52(2):99–107.
[5] Long LP, Zhao LB, Liu L. Research on the air turbulent force loaded on noise barrier caused by train. Eng Mech. 2010;27(3):246–50.

[6] Peiró-Torres MP, Redondo J, Bravo JM, Pérez JS. Open noise barriers based on sonic crystals. Advances in noise control in transport infrastructures. Trans Res Proc. 2016;18:392–8.
[7] Zhang YY, Shen HM. Research on the thin plate's vibration and acoustic radiation based on matlab software. J Chongqing Univ Technol (Nat Sci). 2014;8:34–8.
[8] Cao ZY. Plate and shell vibration theory. Beijing, China: China Railway Publishing House; 1989.
[9] Liu C, Jiang Z. Study of radiated sound power and its sensitivity analysis of plate. Noise Vib Control. 2012;4:116–20.
[10] Zhang B, Shen HM, Zhi W. Sound radiation analysis of variable thickness plate based on FEM and BEM. Sci Technol Eng. 2010;10(36):8963–7.