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Research Article

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Study on the applications of two analytical methods for the construction of traveling wave solutions of the modified equal width equation

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Abstract: In this article, the Sinh–Gordon function method and sub-equation method are used to construct traveling wave solutions of modified equal width equation. Thanks to the proposed methods, trigonometric soliton, dark soliton, and complex hyperbolic solutions of the considered equation are obtained. Common aspects, differences, advantages, and disadvantages of both analytical methods are discussed. It has been shown that the traveling wave solutions produced by both analytical methods with different base equations have different properties. 2D, 3D, and contour graphics are offered for solutions obtained by choosing appropriate values of the parameters. To evaluate the feasibility and efficacy of these techniques, a nonlinear evolution equation was investigated, and with the help of symbolic calculation, these methods have been shown to be a powerful, reliable, and effective mathematical tool for the solution of nonlinear partial differential equations.

Keywords: the Sinh–Gordon function method, exact solution, sub-equation method, nonlinear partial differential equation

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1 Introduction

Nonlinear partial differential equations (NPDEs) are commonly used to model complex physical phenomena arising in various fields of science such as mathematical physics, solid state physics, fluid mechanics, ocean engineering, quantum mechanics, hydrodynamics, and optical fibers. In the recent years, special concentration has been given to the modified equal width (MEW) equation that consists of the nonlinear medium by the dispersion process [1–3]. Thus, "exact solutions" is a trendy area of research for NPDEs.

Soliton has an important place in wave theory. There are many types of solitons in the literature. Some of these are dark soliton, bright soliton, singular soliton, bright-dark soliton, mixed dark-singular soliton, combined singular soliton, combined soliton, and so on [47]. In this study, the solitons that will contribute to wave theory with two different analytical methods have been discussed.

Because of their significant mathematical properties and vast applications, several techniques are offered to investigate various physical phenomena concerned to nonlinear wave equations.

It is considered that all these approaches depend on the problem; some techniques work well with the affected problems, but not applicable for others.

Analytical solutions of NPDEs play an important role to perfectly understand the qualitative characteristics and physical interpretation of a large number of phenomena. There are many methods that have been successfully developed and used in the literature for finding analytical solutions of NPDEs. Some of the techniques developed recently are the ansatz method [4], $(1/G')$-expansion method [5–9], $\left(m + \frac{G'}{G}\right)$-expansion method [10], decomposition method [11–13], auxiliary equation method [14], Clarkson–Kruskal (CK) direct method [15], meshless methods [16,17], (G'/G)-expansion method

[18,19], residual power series method [20], fractional iteration algorithm [21,22], modified $\exp(-\Omega(\xi))$-expansion function [23], new sub-equation method [24,25], and so on [26–35,48–55].

Consider the MEW equation [36]:

$$u_t + 3\alpha u^2 u_x - \beta u_{xxt} = 0, \tag{1}$$

where α, β are nonzero real parameters. This equation plays a significant role in fluid mechanics. Therefore, many studies have been conducted by scientists to investigate this equation for several physical phenomena. Some of these studies are as follows: Wazwaz [37] constructed different types of exact solutions of the MEW equation, whereas Lu [38] used variational iteration method for the numerical results of this type of equations. Solitary wave solutions have been obtained using finite difference method [39], whereas a lumped Galerkin method [40] has been used for the numerical treatment of the MEW equation. Traveling wave solutions of this equation were constructed using integral bifurcation technique [41], exact solutions of the MEW equation have been found via the (G'/G)-expansion method [42], numerical results of the MEW equation have been presented using homotopy perturbation method [43], numerical results have been obtained for the MEW equation using Fourier spectral method [44], solitonary solutions of the MEW equation using Exp-function method [45], and exact solutions of the MEW equation via the method of dynamical system [46].

In the current study, exact solutions for the MEW equation using the Sinh–Gordon function (ShGF) method and sub-equation method are obtained.

2 ShGF method

To illustrate the procedure, we consider an NPDE in two variables t, x.

$$\delta(u, uu_x, u^2u_t, uu_{tt}, \ldots) = 0. \tag{2}$$

Consider Sinh–Gordon equation [40]

$$u_{xt} = \lambda \sinh(u), \tag{3}$$

where u is the unknown function of the t, x and $\lambda \in R\backslash\{0\}$.

With the use of wave transmutation

$$u = U(\xi), \quad \xi = (x + ct), \tag{4}$$

Using in equation (3), we obtain the beneath (NODE), the equation given with 4 here is the classical wave transformation.

$$U'' = \frac{\lambda}{c}\sinh(U), \tag{5}$$

where $U(\xi) = U$, and c is the velocity of the wave. The following relation can be obtained by integrating equation (5):

$$\left[\left(\frac{U}{2}\right)'\right]^2 = \frac{2\lambda}{c}\sinh^2\left(\frac{U}{2}\right) + \frac{\lambda q}{c}, \tag{6}$$

where q is the constant of integration. Here, we set $\varphi = \frac{U}{2}$ and $\sigma = \frac{2\lambda}{c} = \frac{\lambda q}{c}$, and equation (6) becomes

$$\varphi' = \sqrt{\sigma}\cosh(\varphi). \tag{7}$$

Equation (7) is known as variables separable equation. The below two significant equations can be obtained by simplifying equation (7):

$$\cosh(\varphi) = \tan(\sqrt{\sigma}(\xi + d)), \tag{8}$$

$$\sinh(\varphi) = \sec(\sqrt{\sigma}(\xi + d)), \tag{9}$$

where d is the integration constant. To find the new solutions to equation (2), we consider the following two equations:

$$U(\varphi) = \sum_{i=1}^{n}\cosh^{i-1}(\varphi)[B_i\sinh(\varphi) + A_i\cosh(\varphi)] + v, \tag{10}$$

$$\begin{aligned} U(\xi) = {} & \sum_{i=1}^{n}\tan^{i-1}(\sqrt{\sigma}(\xi + d))[B_i\sec(\sqrt{\sigma}(\xi + d)) \\ & + A_i\tan(\sqrt{\sigma}(\xi + d))] + v. \end{aligned} \tag{11}$$

The optimal value of n in equations (10) and (11) is found out with the help of the balancing method, so that the highest power nonlinear term and the highest derivative in the reduced NLODE are taken into consideration. A group of algebraic equations can be obtained by setting each sum of the coefficients of $\sinh^i(\varphi)\cosh^j(\varphi)$, $(0 \le i \le n,\ 0 \le j \le n)$ with equal to zero power. Simplifying these algebraic equations using computer package program gives the values of v, A_i, B_i, c, d, and using σ in equation (11) with the value of n gives the novel traveling wave solutions of the considering equation (2).

3 Sub-equation method

Consider the sub-equation method for the solving NPDEs. Regard the NPDEs as

$$\aleph(u, u_t, u_x, u_{tt}, u_{xx}, \ldots) = 0. \tag{12}$$

Applying the wave transmutation

$$\xi = x + ct, \quad u(x,t) = U(\xi) = U, \quad c \in R, \tag{13}$$

Equation (12) converts into ODE, whereas equation (13) is the classical wave transformation.

$$T(U, U', U'', \ldots) = 0, \tag{14}$$

where c is the arbitrary constant. In the obtained form supposed that equation (14) has a solution

$$U(\xi) = \sum_{i=0}^{n} a_i \phi^i(\xi), \quad a_n \neq 0, \tag{15}$$

in here $a_i, (0 \leq i \leq n)$ are constants to be find out, $n \in \{1, 2, 3, \ldots,\}$ which is going to be attained in equation (14) by balancing term is found according to the principle of balance and the solution of Riccati equation is $\phi(\xi)$

$$\phi'(\xi) = \mu + (\phi((\xi)))^2, \tag{16}$$

where μ is an arbitrary constant. Some exclusive solutions of the Riccati equation are given in equation (16) as follows:

$$\phi(\xi) = \begin{cases} -\sqrt{-\mu}\ \tanh(\sqrt{-\mu}\xi), & \mu < 0 \\ -\sqrt{-\mu}\ \coth(\sqrt{-\mu}\xi), & \mu < 0 \\ \sqrt{\mu}\ \tan(\sqrt{\mu}\xi), & \mu > 0 \\ -\sqrt{\mu}\ \cot(\sqrt{\mu}\xi), & \mu > 0 \\ -\dfrac{1}{\xi + r}, & \mu = 0\ (r \text{ is a cons.}) \end{cases} . \tag{17}$$

In equation (14) if we use the equations (16) and (15), we attained the new polynomial with respect $\varphi(\xi)$ a nonlinear algebraic equation system in a_i, $(i = 0, 1, \ldots, n)$ setting all the coefficients of to zero yields $\phi^i(\xi)$, $(i = 0, 1, \ldots, n)$. To find solutions in nonlinear algebraic equations to we determine constants μ, c, r, a_i, $(i = 0, 1, \ldots, n)$. Substituting attained constants from this system and by aid of the formulas (17) the solutions of equation (16) into equation (15). Then, we get the analytic solutions for equation (12).

4 Application of the ShGF method

In this section, the procedure of the proposed technique is presented. Considering the equation (1) and utilizing the transmutation $u(x, t) = U(\xi)$, $\quad \xi = x + ct$,

$$cU' + 3\alpha U^2 U' - c\beta U''' = 0. \tag{18}$$

Integrated once in equation (18)

$$cU + \alpha U^3 - \beta c U'' + W = 0, \tag{19}$$

where W is the integral constant. In equation (19), $n = 1$ is obtained according to the homogeneous balance principle between U'' and U^3. In equation (10),

$$U(\varphi) = v + A_1 \cosh[\varphi] + B_1 \sinh[\varphi]. \tag{20}$$

If equation (20) is substituted in equation (19) and some necessary modifications are made, the following system of equations can be obtained:

$$\begin{aligned} &\text{Const: } cv + W + v^3\alpha - 3v\alpha B_1^2 = 0, \\ &\cosh[w] : cA_1 + 3v^2\alpha A_1 + cp\beta A_1 - 3\alpha A_1 B_1^2 = 0, \\ &\cosh[w]^2 : 3v\alpha A_1^2 + 3v\alpha B_1^2 = 0, \\ &\cosh[w]^3 : -2cp\beta A_1 + \alpha A_1^3 + 3\alpha A_1 B_1^2 = 0, \\ &\sinh[w] : cB_1 + 3v^2\alpha B_1 - \alpha B_1^3 = 0, \\ &\cosh[w]^2 \sinh[w] : -2cp\beta B_1 + 3\alpha A_1^2 B_1 + \alpha B_1^3 = 0. \end{aligned} \tag{21}$$

A_1, B_1, B_2, W and α, p, c, v constants are obtained from equation (21) system using package program.

If

$$A_1 = \mp B_1, \quad c = \alpha B_1^2, \quad W = 0, \quad v = 0, \quad \beta = -\frac{2}{p}, \tag{22}$$

substituting values from equation (22) into equation (19), trigonometric soliton solution for equation (1) can be obtained as

$$\begin{aligned} u_1(x,t) &= \sec[\sqrt{p}(d + x + t\alpha B_1^2)] B_1 \\ &\mp B_1\ \tan[\sqrt{p}(d + x + t\alpha B_1^2)]. \end{aligned} \tag{23}$$

The trigonometric soliton solution of equation (23) produced using the ShGF method can be seen in Figure 1.

5 Application of the sub-equation method

From equation (19), we found the balancing term $n = 1$, and from equation (5), the below equations are obtained:

$$U(\xi) = a_0 + a_1 \phi(\xi). \tag{24}$$

If equation (24) is substituted in equation (19) and some modifications are made, the below system of equations can be obtained:

$$\left.\begin{aligned} &(\phi(\xi))^0 : \quad W + ca_0 + \alpha a_0^3 = 0, \\ &(\phi(\xi))^1 : \quad ca_1 - 2c\beta\mu a_1 + 3\alpha a_0^2 a_1 = 0, \\ &(\phi(\xi))^2 : \quad 3\alpha a_0 a_1^2 = 0, \\ &(\phi(\xi))^3 : \quad -2c\beta a_1 + \alpha a_1^3 = 0. \end{aligned}\right\} \tag{25}$$

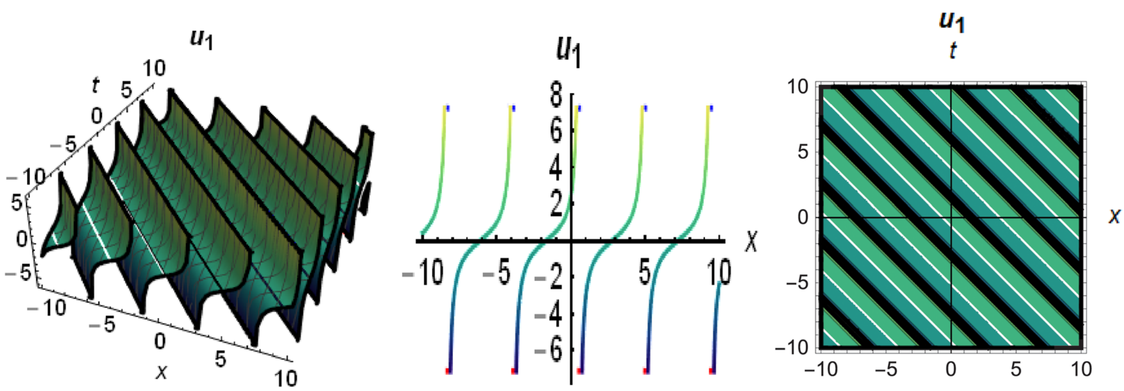

Figure 1: 3D (left), 2D (middle), and contour graphics (right) of equation (23) for $B_1 = 1$, $\alpha = 1$, $p = 2$, $d = 4$.

a_0, a_1, β, c and α constants are obtained from equation (25) system using packet program.

Case 1: If $\mu < 0$,

$$a_0 = 0, \quad a_1 = -\frac{\sqrt{2}\sqrt{c}\sqrt{\beta}}{\sqrt{\alpha}}, \quad \mu = \frac{1}{2\beta}, \quad W = 0. \tag{26}$$

Substituting the values of equation (26) into equation (19), the complex hyperbolic traveling wave solution for equation (1) can be obtained (Figure 2).

$$u_2(x,t) = \frac{i\sqrt{c}}{\sqrt{\alpha}}\tanh\left[\frac{(ct+x)}{\sqrt{-2\beta}}\right]. \tag{27}$$

Case 2: If $\mu < 0$,

$$a_0 = 0, \quad a_1 = -\frac{\sqrt{2}\sqrt{c}\sqrt{\beta}}{\sqrt{\alpha}}, \quad \mu = \frac{1}{2\beta}, \quad W = 0. \tag{28}$$

Substituting the values of equation (28) into equation (19), the complex hyperbolic traveling wave solution for equation (1) can be obtained (Figure 3).

$$u_3(x,t) = \frac{i\sqrt{c}}{\sqrt{\alpha}}\coth\left[\frac{(ct+x)}{\sqrt{-2\beta}}\right]. \tag{29}$$

Case 3: If $\mu > 0$,

$$a_0 = 0, \quad a_1 = -\frac{\sqrt{2}\sqrt{c}\sqrt{\beta}}{\sqrt{\alpha}}, \quad \mu = \frac{1}{2\beta}, \quad W = 0. \tag{30}$$

Substituting the values of equation (30) into equation (19), the dark soliton for equation (1) can be obtained (Figure 4).

$$u_4(x,t) = -\frac{\sqrt{c}}{\sqrt{\alpha}}\tan\left[\frac{(ct+x)}{\sqrt{2\beta}}\right]. \tag{31}$$

Case 4: If $\mu > 0$,

$$a_0 = 0, \quad a_1 = -\frac{\sqrt{2}\sqrt{c}\sqrt{\beta}}{\sqrt{\alpha}}, \quad \mu = \frac{1}{2\beta}, \quad W = 0. \tag{32}$$

Substituting the values of equation (32) into equation (19), the dark soliton for equation (1) can be obtained (Figure 5).

$$u_5(x,t) = \frac{\sqrt{c}}{\sqrt{\alpha}}\cot\left[\frac{(ct+x)}{\sqrt{2\beta}}\right]. \tag{33}$$

Case 5: If $\mu = 0$,
for

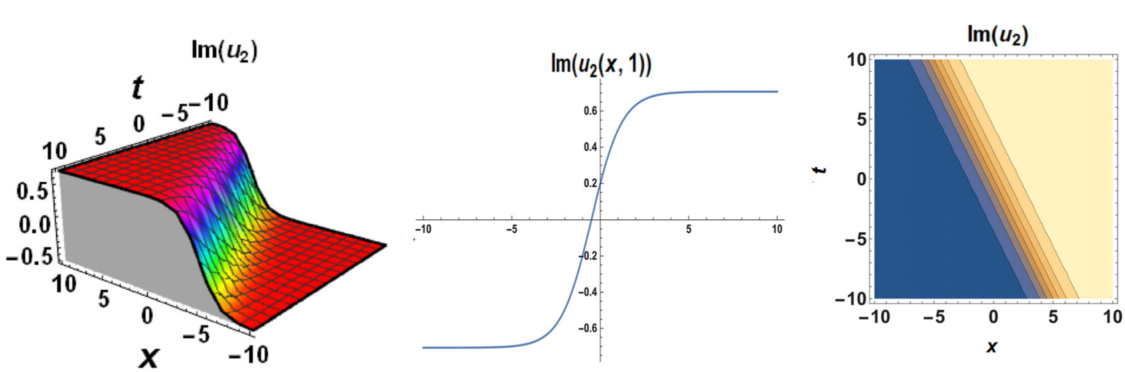

Figure 2: Imaginary parts of 3D (left), 2D (middle), and contour graphics (right) of equation (27) for $c = 0.5$, $\beta = -1.5$, $\alpha = 1$.

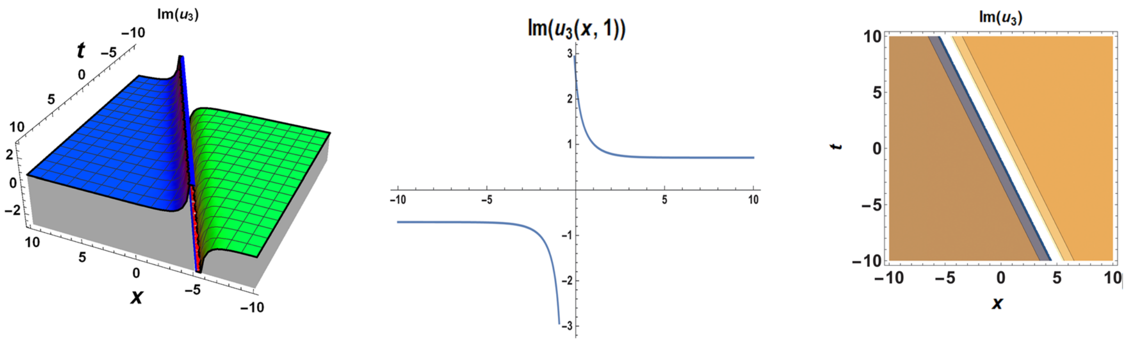

Figure 3: Imaginary parts of 3D (left), 2D (middle), and contour graphics (right) of equation (29) for $c = 0.5$, $\beta = -1.5$, $\alpha = 1$.

$$a_0 = 0, \quad a_1 = -\frac{\sqrt{2}\sqrt{c}\sqrt{\beta}}{\sqrt{\alpha}}, \quad \mu = \frac{1}{2\beta}, \quad W = 0.$$

μ is not zero and therefore algebraic solution cannot be written.

6 Results and discussion

In this study, two analytical methods, which are used to get analytical solutions of differential equations and which are important instruments in mathematics, have been analyzed. The reliability, usefulness, applicability, and validity of both methods have been tested. In addition, both methods have some advantages and disadvantages. First, let us present the common aspects of both analytical methods.

- Both methods produce solutions of NPDEs.
- Classical wave solution is applied in both methods.
- The term balancing is used in both methods.
- In both methods, algebraic equation system is obtained.
- Both techniques produce a traveling wave solution.

Second, let us present the different aspects of the two methods.

- In the ShGF method the base equation is a partial differential equation, whereas in the sub-equation method it is an ordinary differential equation.
- As the base equations are different, they have different properties in the solutions produced by both methods.

As the operation of both methods consists of different steps, the sub-equation method using the ordinary differential equation as the base equation has some advantages. The main of these advantages is the low processing complexity and intensity. On the contrary, in ShGF method, whose base equation is a partial differential equation, the process complexity and density are high. We can see this processing density in the obtained (21) and (25) equation systems. While ShGF method produces equation (11) type solution, sub-equation method produces equation (17) type solution. Most of the solutions produced in this study contain singular points. Solutions containing singular points can shed light on the shock wave event. In addition, we can say that the wave is broken at the singular point. In this study, an

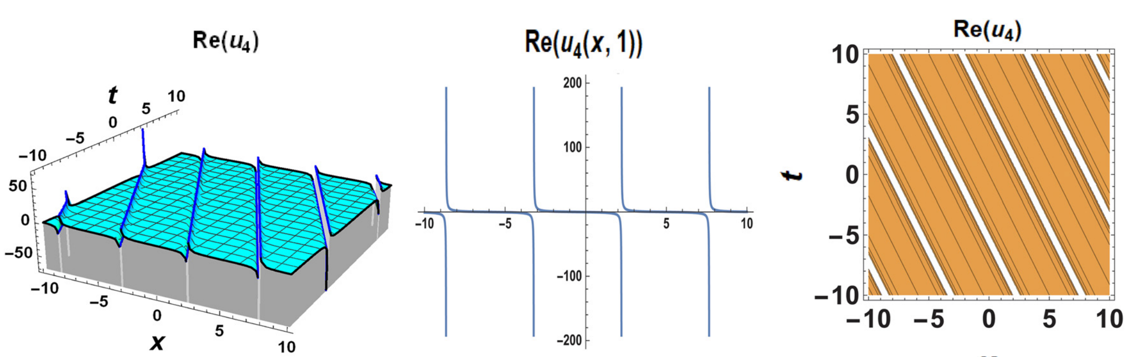

Figure 4: Reel parts of 3D (left), 2D (middle), and contour graphics (right) of equation (31) for $c = 0.5$, $\beta = 1.5$, $\alpha = 1$.

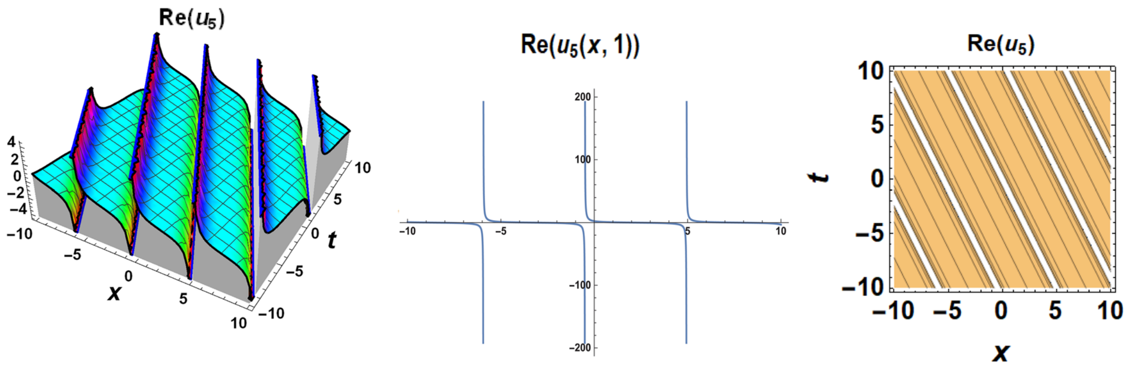

Figure 5: Reel parts of 3D (left), 2D (middle), and contour graphics (right) of equation (33) for $c = 0.5, \quad \beta = 1.5, \quad \alpha = 1$.

analytical solution has been generated by two different methods for the MEW equation, which is one of the important models in fluid dynamics. When the constants in these solutions gain physical meaning, they can explain the physical phenomena in fluid dynamics. This will be more valuable for scientists studying fluid dynamics.

7 Conclusions

In this paper, we have achieved exact solutions for the MEW equation with the help of the ShGF method and sub-equation method. 3D, 2D, and contour graphics of the solutions obtained are drawn by giving arbitrary values to the parameters. Computer technology was used in the construction of these solutions. Advantages and disadvantages of both methods are discussed. The common and different aspects of the two analytical methods are presented in the "Results and discussion" (Section 6). It can be said that both analytical methods can be used reliably to obtain traveling wave solutions of NPDEs in the future. In addition, a higher version of the method can be developed by expanding the accepted solutions for both analytical methods. The MEW equation that plays an important role in mathematical physics is tested by the effectiveness and reliability of the method.

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