Research Article

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Fractional residual power series method for the analytical and approximate studies of fractional physical phenomena

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Abstract: In this article, analytical exact and approximate solutions for fractional physical equations are obtained successfully via efficient analytical method called fractional residual power series method (FRPSM). The fractional derivatives are described in the Caputo sense. Three applications are discussed, showing the validity, accuracy and efficiency of the present method. The solution via FRPSM shows excellent agreement in comparison with the solutions obtained from other established methods. Also, the FRPSM can be used to solve other nonlinear fractional partial differential equation problems. The final results are presented in graphs and tables, which show the effectiveness, quality and strength of the presented method.

Keywords: fractional calculus, fractional residual power series method, fractional physical equations

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1 Introduction

Fractional calculus nowadays has been growing vastly because it has versatile and unique properties. Fractional calculus is a very important and fruitful tool for describing many physical phenomena. Recently, fractional calculus is used for many purposes in several fields, such as chemistry, physics, dynamics systems, engineering and mathematical biology [1–10]. Multi-techniques are used to obtain the solutions for differential equations of fractional order such as fractional variational iteration method (VIM), Sumudu transform (ST) method, RBF meshless method, homotopy perturbation method (HPM), exp-function method, homotopy analysis method, quadrature tau method, variational Lyapunov method, adomian decomposition method (ADM), adaptive finite element method, sinc-collocation method, homotopy analysis transform method and adomian decomposition Sumudu transform method (ADSTM). Most common useful methods exist in refs. [11–45].

In this article, we present an ultra-modern way called the fractional residual power series method (FRPSM). The major target is to obtain analytical exact and approximate solutions for the fractional physical equations.

The FRPSM is constructed from the generalized Taylor series, which is a prevailing technique for solving nonlinear fractional partial differential equations [2,8]. The advantage of the current method is that it is not time-consuming and does not require large computer memory and also it is not affected by computational round off errors. Moreover, this method computes the coefficients of the power series by a chain of equations with more than one variables, which indicates a better convergence of the current method.

This article is organized as follows. In Section 2, the formulation of FRPSM is demonstrated briefly. Applications of the current method to study some fractional physical equations are shown in Section 3. A brief discussion and conclusions are provided in Section 4.

2 Proposed method

The main idea of the FRPSM is to look for the solutions related to physical phenomena in the form of power series in which the coefficients of the series should be evaluated. The basic definitions, properties and advantages of the residual power series method are given in ref. [12–16].

To demonstrate the basic idea of the FRPSM, we consider the beneath nonlinear differential equation:

$$D_t^{\alpha} W(x,t) - N(w) - R(w) = 0, \tag{1}$$

where $R(w)$ is a linear term and $N(w)$ is a nonlinear term.

With the initial condition:

$$W(x,0) = F(x). \tag{2}$$

FRPSM proposes the solution for equation (1) as a fractional power series at the initial point $t = 0$:

$$W(x,t) = \sum_{\infty}^{k=0} F_n(x) \frac{t^{k\alpha}}{\Gamma(\alpha k + 1)}, \quad 0 < \alpha \le 1, -\infty < x < \infty, o \le t < R. \tag{3}$$

Next, let $W_k(x,t)$ denote the kth truncated series of $W(x,t)$,

$$W_k(x,t) = \sum_{\infty}^{k=0} F_n(x) \frac{t^{k\alpha}}{\Gamma(\alpha k + 1)}. \tag{4}$$

The 0th FRPSM approximate solution of $W(x,t)$ is

$$W_0(x,t) = W(x,0) = F(x). \tag{5}$$

Equation (4) can be expressed as

$$W_k(x,t) = F(x) + \sum_{k=1}^{k} F_n(x) \frac{t^{k\alpha}}{\Gamma(\alpha k + 1)}, k = 1, 2, 3, \ldots \tag{6}$$

The residual function of equation (1) can be defined as follows:

$$\mathrm{Res}_w(x,t) = D_t^{\alpha} W(x,t) - N(w) - R(w). \tag{7}$$

Consequently, the kth residual function $\mathrm{Res}_{w,k}$ is

$$\mathrm{Res}_{w,k}(x,t) = D_t^{\alpha} W_k(x,t) - N(w_k) - R(w_k). \tag{8}$$

As in refs. [24,25], to determine $F_1(x), F_2(x), F_3(x), \ldots$ we consider $k = 1, 2, 3, \ldots$ in equation (6) and substitute in equation (8). Applying the fractional derivative $D_t^{(k-1)\alpha}$ on both sides for $k = 1, 2, 3, \ldots$ and finally, we solve

$$D_t^{(k-1)\alpha} \mathrm{Res}_{w,k}(x,0) = 0, k = 1, 2, 3, \ldots \tag{9}$$

3 Applications

To illustrate the accuracy, simplicity, efficiency and quality of the FRPSM, we introduce three applications related to physical phenomena.

3.1 Application 1

We consider the radioactive decay fractional order differential equation:

$$D_t^{\alpha} W(t) = -\lambda W(t), 0 < \alpha \le 1, \tag{10}$$

with the initial condition:

$$W(0) = W_0. \tag{11}$$

The exact solution at $\alpha \to 1$ is

$$W(t) = W_0 e^{-\lambda t}. \tag{12}$$

The residual function for equation (10) is defined as:

$$\mathrm{Res}_w(t) = D_t^{\alpha} W(t) + \lambda W(t). \tag{13}$$

Therefore, the kth residual function $\mathrm{Res}_{w,k} W(t)$ is

$$\mathrm{Res}_{W,k}(t) = D_t^{\alpha} W_k(t) + \lambda W_k(t). \tag{14}$$

To define $F_1(x)$, we consider ($k = 1$) in equation (14) as:

$$\mathrm{Res}_{w,1}(t) = D_t^{\alpha} W_1(t) + \lambda W_1(t). \tag{15}$$

But from equation (6) at $k = 1$, we get

$$W_1 = F(x) + F_1(x) \frac{t^{\alpha}}{\Gamma(\alpha + 1)}, \tag{16}$$

$$\mathrm{Res}_{w,1}(t) = F_1(x) + \lambda F(x) + \lambda F_1(x) \frac{t^{\alpha}}{\Gamma(\alpha + 1)}. \tag{17}$$

Now based on the result of equation (8) for $k = 1$, we have $\mathrm{Res}_{w,1}(0) = 0$,

$$F_1(x) = -\lambda F(x), \tag{18}$$

i.e.,

$$F_1(x) = -\lambda W_0. \tag{19}$$

To find out $W_2(t)$, we consider ($k = 2$) in equation (14).

$$\mathrm{Res}_{w,2}(t) = D_t^{\alpha} W_2(t) + \lambda W_2(t). \tag{20}$$

But from equation (6) at ($k = 2$) we obtain

$$W_2 = F(x) + F_1(x) \frac{t^{\alpha}}{\Gamma(\alpha + 1)} + F_2(x) \frac{t^{2\alpha}}{\Gamma(2\alpha + 1)}, \tag{21}$$

$$\mathrm{Res}_{w,2}(t) = F_1(x) + \lambda F(x) + (F_2(x) + \lambda F_1(x)) \times \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \lambda F_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}. \tag{22}$$

Applying D_t^{α} on both sides and solving the equations $D_t^{\alpha}\mathrm{Res}_{w,2}(0) = 0$, then we get

$$F_2(x) = -\lambda F_1(x), \tag{23}$$

i.e.,

$$F_2(x) = \lambda^2 W_0. \tag{24}$$

To determine W_3, we consider $(k = 3)$ in equation (14)

$$\mathrm{Res}_{w,3}(t) = D_t^{\alpha} W_3(t) + \lambda W_3(t). \tag{25}$$

But from equation (6) at $(k = 3)$ we obtain

$$W_3 = F(x) + F_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + F_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + F_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}, \tag{26}$$

$$\mathrm{Res}_{w,3}(t) = F_1(x) + (\lambda F_1(x) + F_2(x))\frac{t^{\alpha}}{\Gamma(\alpha+1)} + (\lambda F_2(x) + F_3(x))\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + (\lambda F(x) + \lambda F_3(x))\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}. \tag{27}$$

Applying $D_t^{2\alpha}$ on both sides and solving the equations $D_t^{2\alpha}\mathrm{Res}_{w,2}(0) = 0$, then we get:

$$F_3(x) = -\lambda F_2(x). \tag{28}$$

The solution in a series form can be obtained as

$$W(t) = F(x) + F_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + F_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + F_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \ldots, \tag{29}$$

$$W(t) = W_0\left[1 - \frac{\lambda}{\Gamma(\alpha+1)}t^{\alpha} + \frac{\lambda^2}{\Gamma(2\alpha+1)}t^{2\alpha} - \frac{\lambda^3}{\Gamma(3\alpha+1)}t^{3\alpha} + \ldots\right]. \tag{30}$$

i.e.,

$$W(t) = W_0 E_{\alpha}(-\lambda t^{\alpha}), \tag{31}$$

where $E_{\alpha}(-\lambda t^{\alpha}) = \sum_{k=0}^{\infty}\frac{(-\lambda t^{\alpha})^k}{\Gamma(\alpha k+1)}$ is the Mittag–Leffler function.

For $\alpha = 1$, the following solution can be obtained from equation (31):

$$W(t) = W_0 e^{-\lambda t}, \tag{32}$$

which is the exact solution of equation (10) obtained via ST and VIM [26] as in Figures 1 and 2.

3.2 Application 2

Consider the backward Kolmogorov equation:

$$D_t^{\alpha} W(x,t) - (x+1)W_x - x^2 e^t W_{xx} = 0, \tag{33}$$

having the initial condition:

$$W_0(x,t) = x + 1, \; x \in R, \tag{34}$$

and the exact solution at $\alpha \to 1$:

$$W(x,t) = (x+1)e^t. \tag{35}$$

Now, applying the procedures of the FRPSM as in application 1, we get

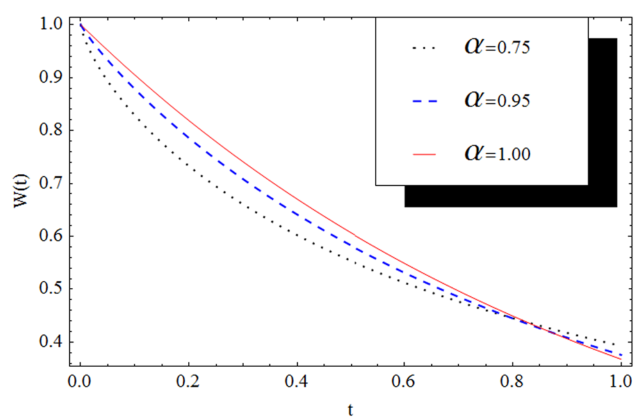

Figure 1: FRPSM solution plot of equation (10) for different values of α with fixed $W_0 = 1$ and $\lambda = 1$.

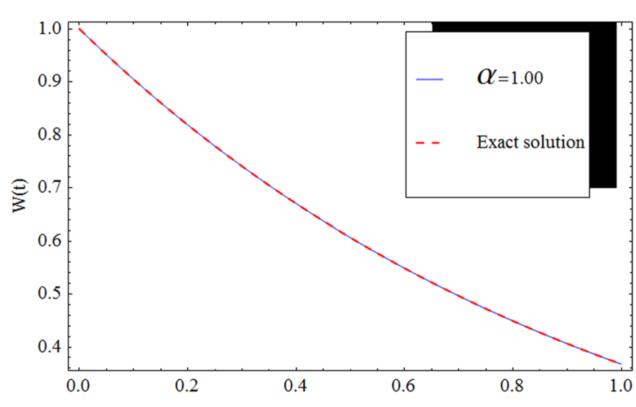

Figure 2: FRPSM solution plot of equation (10) at $\alpha = 1$.

$$F_1(x) = F_2(x) = F_3(x) = (x+1), \tag{36}$$

$$W(t) = F(x) + F_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + F_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + F_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \ldots, \tag{37}$$

$$W(t) = (x+1)\left[1 + \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \ldots\right], \tag{38}$$

$$W(x,t) = (x+1)E_{\alpha}(t^{\alpha}). \tag{39}$$

At $\alpha \to 1$ from equation (39), we gain

$$W(t) = (x+1)e^{t}, \tag{40}$$

which is the exact solution and is the same as obtained via HPM [27], ADM [28], VIM [29] and ADSTM [30] as in Figures 3–5.

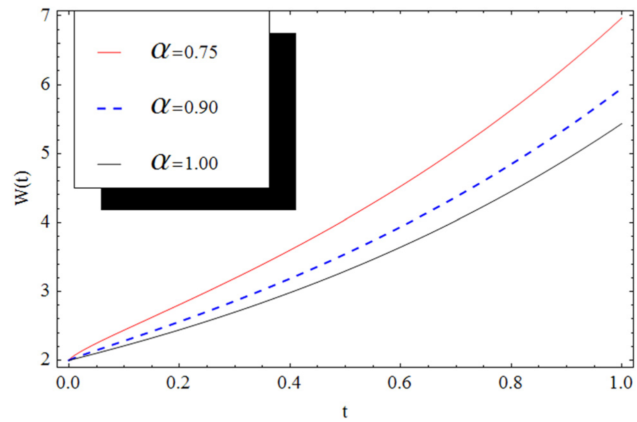

Figure 4: FRPSM solution of equation (33) for different values of α with fixed $x = 1$.

3.3 Application 3

Consider the time fractional Rosenau–Hyman equation:

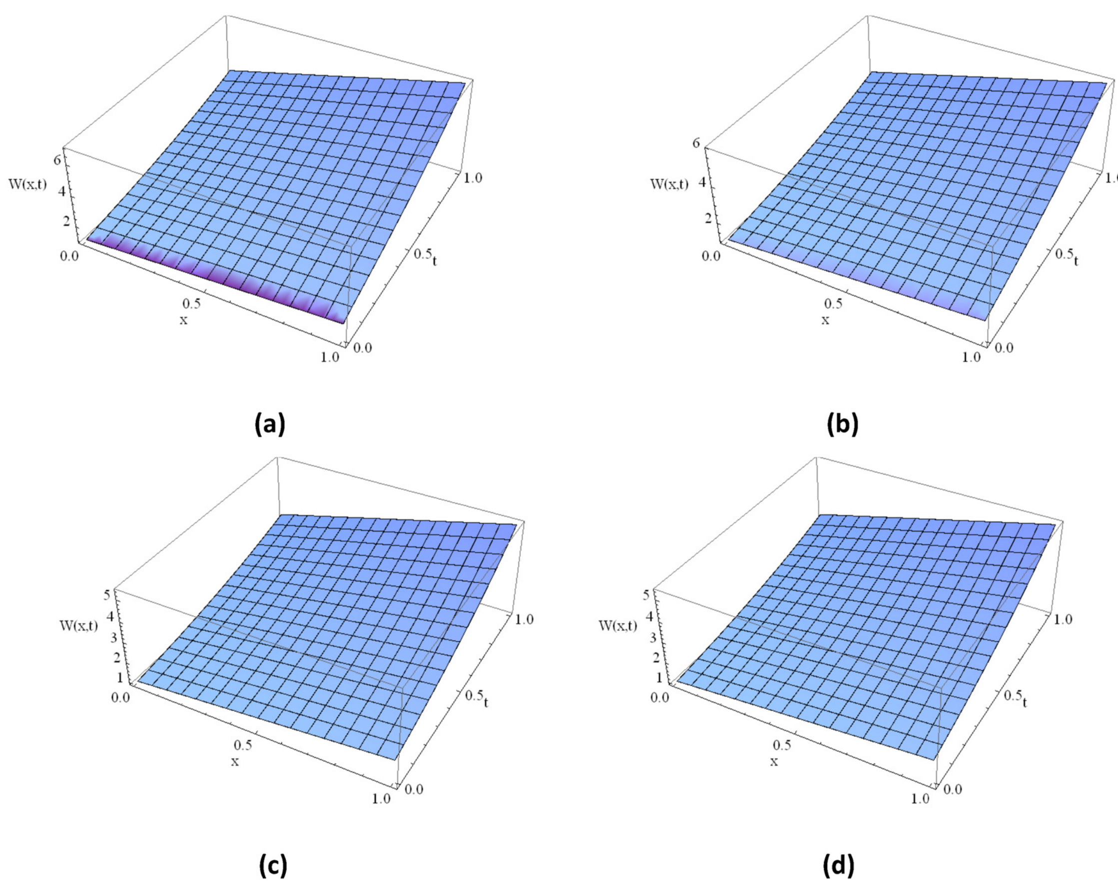

Figure 3: FRPSM solution of equation (33) with fixed $x = 1$ for different values of α at (a) $\alpha = 0.75$, (b) $\alpha = 0.90$, (c) $\alpha = 1.00$ and the exact solution in (d).

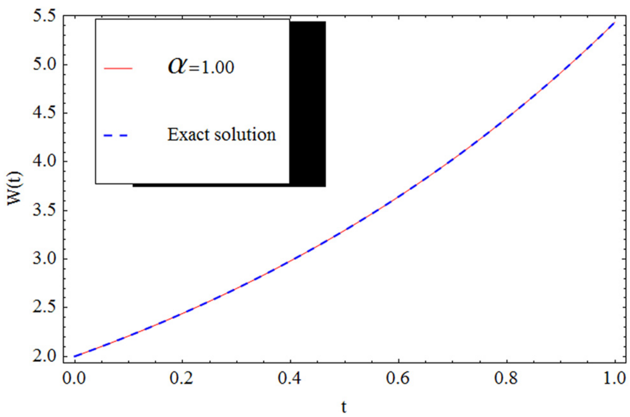

Figure 5: FRPSM solution plot of equation (33) at $\alpha = 1$.

$$D_t^{\alpha} W(x,t) - WW_{xxx} - WW_x - 3W_x W_{xx} = 0,\ t > 0,\ 0 < \alpha \le 1, \tag{41}$$

having the initial condition:

$$W(x,0) = -\frac{8c}{3}\cos^2\left(\frac{x}{4}\right) \tag{42}$$

and the exact solution at $\alpha \to 1$:

$$W(x,t) = -\frac{8c}{3}\cos^2\left(\frac{x-ct}{4}\right). \tag{43}$$

Similar to the previous applications via the FRPSM, we have

$$F_1(x) = -\frac{2c^2}{3}\sin\left(\frac{x}{2}\right), \tag{44}$$

$$F_2(x) = \frac{c^3}{3}\cos\left(\frac{x}{2}\right). \tag{45}$$

The following solution in a series form can be obtained:

$$\begin{aligned} W(t) &= F(x) + F_1(x)\frac{t^{\alpha}}{\Gamma(\alpha+1)} + F_2(x)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} \\ &+ F_3(x)\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + \ldots, \end{aligned} \tag{46}$$

$$\begin{aligned} W(x,t) &= -\frac{8c}{3}\cos^2\left(\frac{x}{4}\right) - \frac{2c^2}{3}\sin\left(\frac{x}{2}\right)\frac{t^{\alpha}}{\Gamma(\alpha+1)} \\ &+ \frac{c^3}{3}\cos\left(\frac{x}{2}\right)\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + \cdots \end{aligned} \tag{47}$$

In the case $\alpha \to 1$ from equation (47), we have

$$\begin{aligned} W(x,t) &= -\frac{8c}{3}\cos^2\left(\frac{x}{4}\right) - \frac{2c^2}{3}\sin\left(\frac{x}{2}\right)t \\ &+ \frac{c^3}{6}\cos\left(\frac{x}{2}\right)t^2 + \cdots, \end{aligned} \tag{48}$$

which is the exact solution and is also entirely confirmed with HPM [31], ADM [26] and VIM [25] as in Figures 6 and 7 (Tables 1 and 2).

4 Conclusion

In this article, we applied successfully a novel approach cold FRPSM for finding out the exact solutions of the physical phenomena in the fractional order at $\alpha \to 1$. The obtained results are in excellent agreement with other known methods and with the exact ones. The performance of the FRPSM shows the power as well as accuracy and the ability for finding analytical as well as numerical solutions to many various fractional physical phenomena arising in engineering and physics. We can

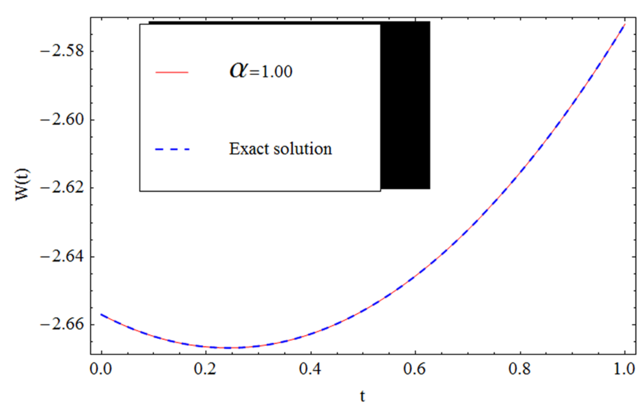

Figure 6: FRPSM solution of equation (41) at $\alpha = 1$, which is the exact solution obtained by ADM [26], VIM [25], HPM [31] and the exact solution.

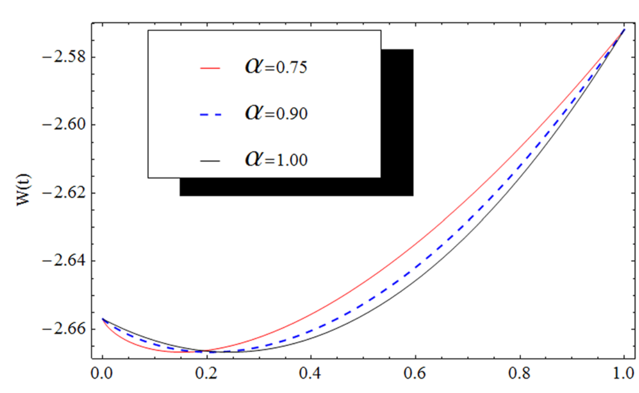

Figure 7: FRPSM solution plot of equation (41) for different values of α with fixed $x = 1$.

Table 1: Comparison between approximate solution in equation (41) using fifth terms of VIM [31], fifth terms of HPM [31] and two terms of FRPSM at $\alpha = 1$ when $c = 1$

x	t	VIM	HPM	FRPSM	Exact
$\frac{\pi}{4}$	0.2	−2.6099581	−2.6099581	−2.6100380	−2.6099581
	0.6	−2.6609433	−2.6609420	−2.6628133	−2.6609420
	1.0	−2.6590258	−2.6589984	−2.6664151	−2.6589984
$\frac{\pi}{2}$	0.2	−2.3655561	−2.3655561	−2.3653571	−2.3655561
	0.6	−2.5126533	−2.5126533	−2.2165581	−2.5126533
	1.0	−2.6127547	−2.6127328	−2.6296950	−2.6127328

Table 2: Comparison between approximate solution in equation (41) using two terms of FRPSM in different values of α with the exact solution when $c = 1$

x	t	$\alpha = 0.75$	$\alpha = 0.90$	$\alpha = 1$	Exact
$\frac{\pi}{2}$	0.2	−2.4065842	−2.3803824	−2.3657092	−2.3655561
	0.6	−2.5427414	−2.5268188	−2.5165586	−2.5126533
	1.0	−2.6296958	−2.6296958	−2.6296958	−2.6127328
π	0.2	−1.5327132	−1.4899491	−1.4666666	−1.4665554
	0.6	−1.6235043	−1.7542974	−1.7333333	−1.7273602
	1.0	−2.0000000	−2.0000000	−2.0000000	−1.9725674

see that the solutions obtained by the FRPSM and other methods are consistent and efficient for solving fractional partial differential equations.

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