Research Article

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Closed-form wave structures of the space-time fractional Hirota–Satsuma coupled KdV equation with nonlinear physical phenomena

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Abstract: The present paper applies the variation of (G'/G)-expansion method on the space-time fractional Hirota–Satsuma coupled KdV equation with applications in physics. We employ the new approach to receive some closed form wave solutions for any nonlinear fractional ordinary differential equations. First, the fractional derivatives in this research are manifested in terms of Riemann–Liouville derivative. A complex fractional transformation is applied to transform the fractional-order ordinary and partial differential equation into the integer order ordinary differential equation. The reduced equations are then solved by the method. Some novel and more comprehensive solutions of these equations are successfully constructed. Besides, the intended approach is simplistic, conventional, and able to significantly reduce the size of computational work associated with other existing methods.

Key words: analytical method, the space-time fractional Hirota–Satsuma coupled KdV equation, Riemann–Liouville derivative

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1 Introduction

Nonlinear dynamical systems play an important role in physics. New ideas in physics including astrophysics, computational physics, fluid dynamic, mathematical physics, biological physics and medical physics often explain the fundamental mechanisms studied by other sciences and suggest new avenues of research in academic disciplines such as mathematics. Nature and research problems and most of the life and compound phenomena are explained and invented through fractional wave models. For example, the fluid-dynamic traffic model [1] could succeed the deficiency based upon the hypothesis of continuum traffic flow, the principles of fractional calculus could model the dynamical methods in fluids and porous structures [2] and fractional derivatives can form the nonlinear oscillation of earthquake [3]. For all the attention, it is an indispensable task to attain the closed-form wave structures of the fractional differential equations (FDEs). Various meaningful and truthful approaches have been interpolated for achieving the closed-form wave structures of FDEs, including the -expansion approach [4,5], extended Jacobi elliptic function expansion method [6], improved sub-equation scheme [7], modified fractional reduced differential transform method [8], sub-equation method [9], singular manifold method [10], fractional homotopy method [11], fractional reduced differential transform method [12], modified (G'/G)-expansion approach [13], extended modified mapping method [14], Sine–Gordon expansion method [15], extended trial equation method [16], iterative method [17], simplest equation method [18], ansatz scheme [19], F-expansion method [20], modified Kudryashov method [21], extended mapping method [22], homo separation analysis method [23], modified simple equation method [24], reduced differential transform scheme [25], modified extended mapping method [26], functional variable method [27], extended direct algebraic method [28], Darcy's law rule [29], function

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transformation method [30], the variation of (G'/G)-expansion method [31], differential transform method [32], unified scheme, (G'/G)-expansion approach [33,34], Kudryashov method [35], new extended Kudryashov process [36], Sine-cosine approach [37], auxiliary equation scheme [38] and many other techniques [39–50].

Here, we introduce a new method [31] for nonlinear FDEs based on the homogenous balancing method employing wave transformation. In this research, through the transformation $\xi = \frac{kx^{\beta}}{\Gamma(1+\beta)} + \frac{\lambda t^{\beta}}{\Gamma(1+\alpha)}$ for ordinary differential equation and $\xi = x + y + z - V\frac{t^{\alpha}}{\alpha}$ for partial differential equation, a given fractional ordinary and a partial differential equation turn into a fractional ordinary differential equation. Suppose the solutions have the form $V(\xi) = \sum_{i=0}^{M} A_i L^i + \sum_{i=1}^{M} B_i L^{i-1} H$, where $L = \frac{G'}{G}$, $H = \frac{N'}{N}$, where $G = G(\xi)$ and $N = N(\xi)$ represent the solution of the coupled Riccati equation $G'(\xi) = -G(\xi)N(\xi)$ and $N'(\xi) = 1 - N(\xi)^2$. This coupled Riccati equation gives us four types of hyperbolic function solutions, such as sech, tanh, csch and coth. We have manipulated the unique method for obtaining more general closed form wave structures of the nonlinear FDEs, such as the space-time fractional Hirota–Satsuma coupled KdV equation [36]. The improvement of this process over the existing rule is that it presents a few novel closed-form wave solutions. Apart from the physical importance, the closed-form wave structures of the nonlinear FDEs might be helpful to the analytical solvers to compare the exactness of their outcomes and also develop the stability analysis of the nonlinear FDEs. In Section 2, some definitions, characters, properties, theorems and fundamental facts of fractional derivatives are introduced. Section 3 shows that the current scheme of the closed-form wave solutions of the proposed models is obtained in Section 4. Discussion and future works are given in Section 5.

2 Preliminaries

Herein, we acquaint a few definitions, characters, properties, theorems and fundamental facts, which are implemented throughout this article.

2.1 The modified Riemann–Liouville derivative

Fractional calculus possesses several ways to generalize the concept of differential derivatives to fractional derivatives [51,52].

Definition 2.1. A real function $g(t)$, $t > 0$ is called in the space C_k where $k \in \mathbb{R}$ if there exists a real number $p > k$, for example, $g(t) = t^p g_1(t)$, where $g_1(t) \in C(0, \infty)$ and called in the space C_k^n if $g^n \in C_k$, where n is a positive integer.

Definition 2.2. Suppose that $g : x \to g(x)$. It denotes a continuous but not significantly differentiable function, then the fractional derivatives of order α are represented through the following expression [51,52]:

$$D_x^{\alpha} = \begin{cases} \frac{1}{\Gamma(-\alpha)} \int_0^x (x-\xi)^{-\alpha-1}[G(\xi) - G(0)]\,\mathrm{d}\xi, \alpha < 0, \\ \frac{1}{\Gamma(-\alpha)} \frac{d}{\mathrm{d}x} \int_0^x (x-\xi)^{-\alpha}[G(\xi) - G(0)]\,\mathrm{d}\xi, 0 < \alpha < 1, \\ (G^{(n)}(x))^{\alpha-m}, m \le \alpha \le m+1, m \ge 1, \end{cases} \quad (1)$$

in which $\Gamma(.)$ is the Gamma function illustrated by [53].

$$\Gamma(\alpha) = \lim_{m\to\infty} \frac{m! m^{\alpha}}{\alpha(\alpha+1)(\alpha+2)\ldots(\alpha+m)} \quad (2)$$

or

$$\Gamma(x) = \int_0^{\infty} \mathrm{e}^{-t} t^{x-1} \mathrm{d}x \quad (3)$$

and

$$D_x^{\alpha} = \lim_{h\downarrow 0} h^{-\alpha} \sum_{k=0}^{\infty} (-1)^k G[x + (\alpha - k)h]. \quad (4)$$

Definition 2.3. The Mittag–Leffler function including two parameters is represented as follows [54]

$$E_{\alpha,\beta}(x) = \sum_{i=0}^{\infty} \frac{x^i}{\Gamma(\alpha i + \beta)}, \mathrm{Re}(\alpha) > 0, \beta, x \in C. \quad (5)$$

Some notable characteristics for the fractional derivative are as follows:

1. $D_x^{\alpha} x^{\gamma} = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} x^{\gamma-\alpha}, \gamma > 0.$
2. $D_x^{\alpha}(cG(x)) = cD_x^{\alpha}(G(x)).$
3. $D_x^{\alpha}(aG(x) + bH(x)) = aD_x^{\alpha} G(x) + bD_x^{\alpha} H(x).$

2.2 The fractional complex transformation

This section performs the complex fractional transformation for the fractional-order ordinary differential equation (ODE). First, we explore the nonlinear fractional ODE:

$$P(u, D_t^{\alpha} u, D_x^{\beta} u, D_t^{\alpha} D_t^{\alpha} u, D_t^{\alpha} D_x^{\beta} u, D_x^{\beta} D_x^{\beta} u, \ldots) = 0, \quad (6)$$

where $0 < \alpha \leq 0$, $0 < \beta \leq 0$, $D_t^{\alpha} u$ and $D_x^{\beta} u$ are the fractional derivatives of u with respect to t and x.

We investigate equation (6) through the transformation $u = u(x, t) = u(\xi)$, $\xi = \frac{kx^{\beta}}{\Gamma(1+\beta)} + \frac{\lambda t^{\beta}}{\Gamma(1+\alpha)}$, where k and λ are nonzero arbitrary constants, then equation (6) becomes

$$Q = \left(u, \frac{\partial u}{\partial \xi}, \frac{\partial^2 u}{\partial \xi^2}, \frac{\partial^3 u}{\partial \xi^3}, \ldots\right) = 0. \tag{7}$$

3 Glimpse of the method

At this moment, the general scheme of the variation of (G'/G)-expansion method [31] is shortened as follows:

- **Step 1:** Calculate N through rule of the homogeneous balance in equation (7).
- **Step 2:** Considering the method can be expressed as the form:

$$V(\xi) = \sum_{i=0}^{M} A_i L^i + \sum_{i=1}^{M} B_i L^{i-1} H, \tag{8}$$

  where $L = \frac{G'}{G}$, $H = \frac{N'}{N}$ and $G = G(\xi)$ and $N = N(\xi)$ represent the solution of the coupled Riccati equations

$$G'(\xi) = -G(\xi)N(\xi), \tag{9}$$

$$N'(\xi) = 1 - N(\xi)^2. \tag{10}$$

  These coupled Riccati equations give us four types of hyperbolic function solutions including sech, tanh, csch and coth such as

$$G(\xi) = \pm\text{sech}(\xi), N(\xi) = \tanh(\xi), \tag{11}$$

$$G(\xi) = \pm\text{csch}(\xi), N(\xi) = \coth(\xi). \tag{12}$$

- **Step 3:** A polynomial in L or N is accomplished plugging equation (8) into equation (7). Determining the coefficients of the equivalent power of L or N produces a system of algebraic equations, which can be determined to construct the values of A_i and B_i using MAPLE. Turning the over measured values of A_i and B_i in 12, the general solutions of equation (16) and (17) complete the calculation of the result of equation (6).

4 Implementation of the method

To illustrate the idea of the new method, we implement the process on the space-time fractional Hirota–Satsuma coupled KdV equation. We implement the new method to the considered model, which models the intercommunication between two long waves that have well-defined dispersion connection:

$$\begin{aligned} D_t^{\alpha} u &= \frac{1}{4} u_{xxx} + 3uu_x + 3(-v^2 + w)_x \\ D_t^{\alpha} v &= -\frac{1}{2} v_{xxx} - 3uv_x \\ D_t^{\alpha} w &= -\frac{1}{2} w_{xxx} - 3uw_x, \end{aligned} \tag{13}$$

where $u = u(x, t)$, $u = u(x, t)$ and $u = u(x, t)$, $t > 0$ and $0 < \alpha \leq 0$. For our goal, we utilize the transformations

$$\begin{aligned} u(x, t) &= \frac{1}{\lambda} u(\xi)^2, \\ v(x, t) &= -\lambda + u(\xi), \\ w(x, t) &= 2\lambda^2 - 2\lambda u(\xi), \end{aligned}$$

where $\xi = x - \frac{\lambda t^{\beta}}{\Gamma(1+\alpha)}$. Then equation (13) reduced to the ODE

$$\lambda \frac{\partial^2 u}{\partial \xi^2} + 2u^3 - 2\lambda^2 u. \tag{14}$$

Applying the homogeneous balance rule on equation (14) yields $M = 2$. Therefore,

$$u(\xi) = (a_0 - b_2) + \frac{b_1}{N} - (a_1 + b_1)N + (a_2 + b_2)N^2, \tag{15}$$

where a_0, a_1, a_2, b_1 and b_2 are free parameters. Plugging equation (15) into equation (14) and calculating each coefficient of L or N to zeros, we have:

- **Case 1:** Let $\lambda = -1$, $a_0 = b_2$, $a_1 = \pm 1$, $a_2 = -b_2$, $b_1 = 0$, and insert the values of Case 1 into equation (15), we find:

$$u_1(x, t) = -1 + \text{sech}^2\left(2x + \frac{2t^{\alpha}}{\Gamma(1+\alpha)}\right), \tag{16}$$

$$u_2(x, t) = -1 - \text{csch}^2\left(2x + \frac{2t^{\alpha}}{\Gamma(1+\alpha)}\right), \tag{17}$$

$$v_1(x, t) = 1 \mp \tanh\left(2x + \frac{2t^{\alpha}}{\Gamma(1+\alpha)}\right), \tag{18}$$

$$v_2(x, t) = 1 \mp \coth\left(2x + \frac{2t^{\alpha}}{\Gamma(1+\alpha)}\right), \tag{19}$$

$$w_1(x, t) = 2 \mp 2\tanh\left(2x + \frac{2t^{\alpha}}{\Gamma(1+\alpha)}\right), \tag{20}$$

$$w_2(x, t) = 2 \mp 2\coth\left(2x + \frac{2t^{\alpha}}{\Gamma(1+\alpha)}\right). \tag{21}$$

- **Case 2:** Let $\lambda = -4$, $a_0 = b_2$, $a_1 = -4$, $a_2 = -b_2$, $b_1 = 2$ and put the values of Case 2 into equation (15), we get:

$$u_3(x,t) = -\frac{\left(\tanh^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) + 1\right)^2}{\tanh^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)}, \tag{22}$$

$$u_4(x,t) = -\frac{\left(\coth^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) + 1\right)^2}{\coth^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)}, \tag{23}$$

$$v_3(x,t) = 4 + 2\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) + \frac{2}{\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}, \tag{24}$$

$$v_4(x,t) = 4 + 2\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) + \frac{2}{\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}, \tag{25}$$

$$w_3(x,t) = 32 + 16\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) + \frac{16}{\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}, \tag{26}$$

$$w_4(x,t) = 32 + 16\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) + \frac{16}{\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}. \tag{27}$$

- **Case 3:** Let $\lambda = -1$, $a_0 = b_2$, $a_1 = \mp 1$, $a_2 = -b_2$, $b_1 = \pm 1$ and substitute the values of Case 3 into equation (15), we achieve:

$$u_5(x,t) = -\coth^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right), \tag{28}$$

$$u_6(x,t) = -\tanh^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right), \tag{29}$$

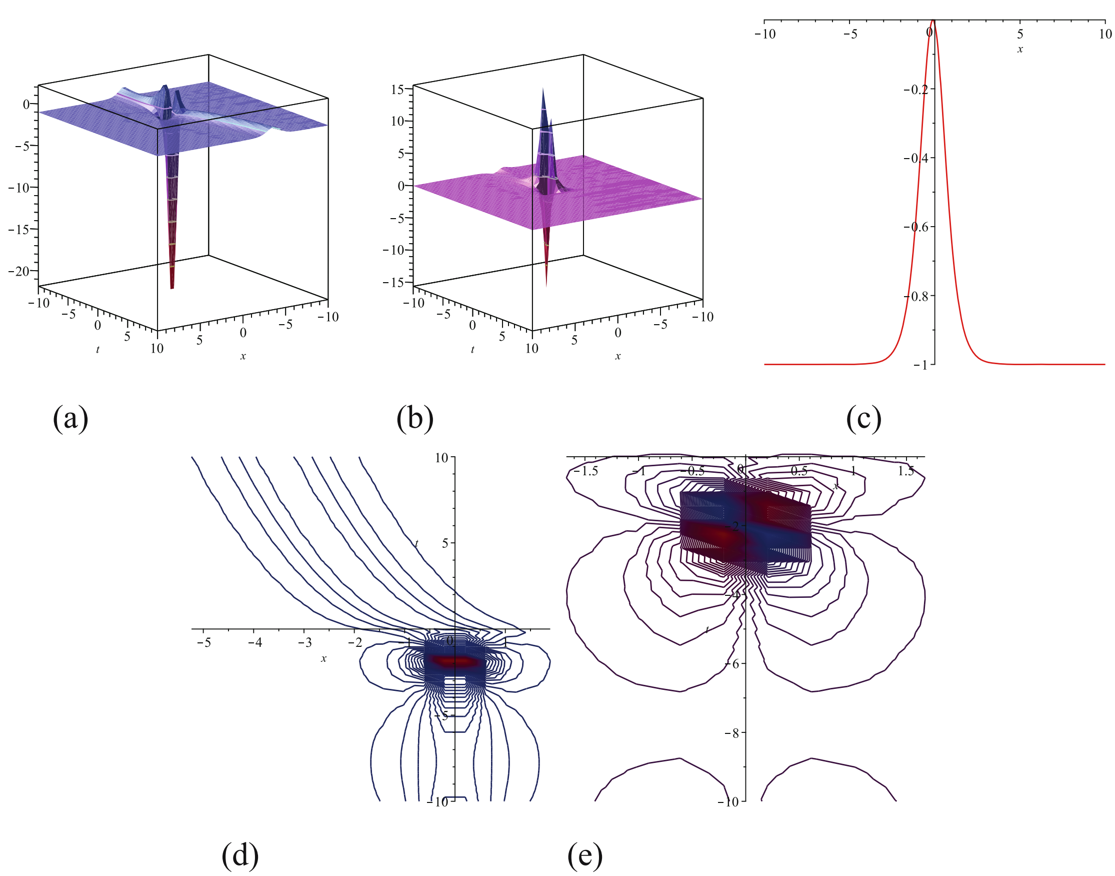
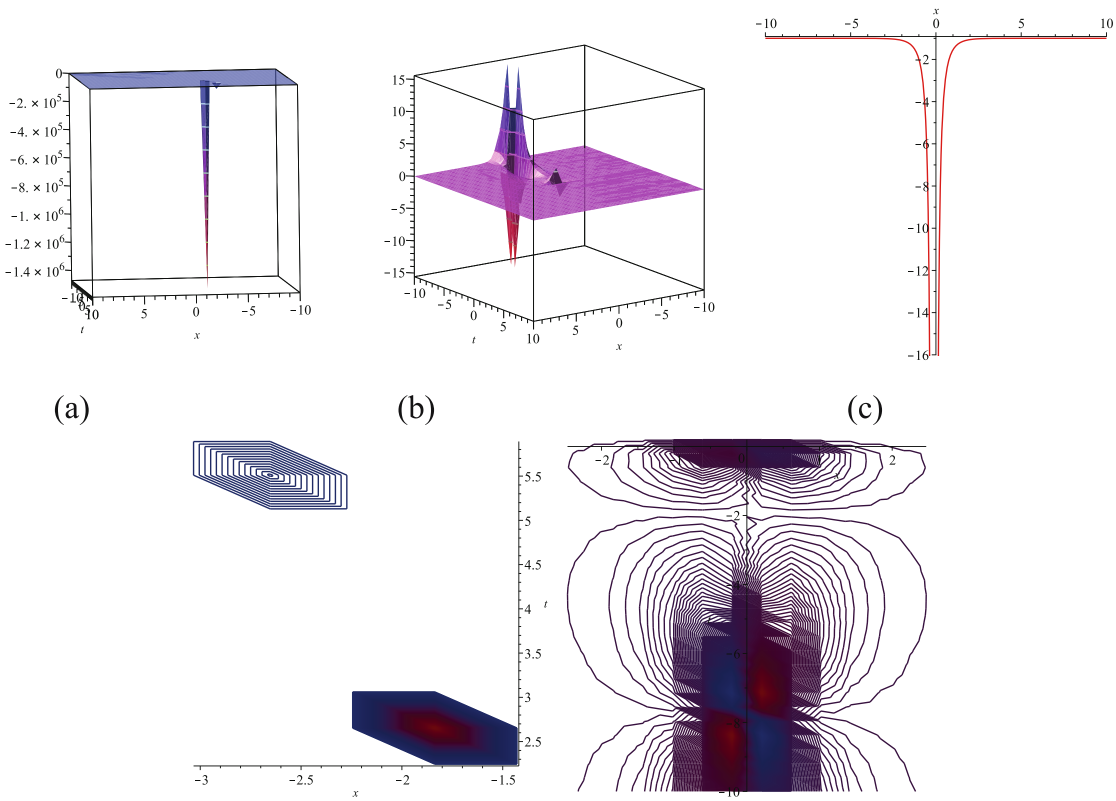


Figure 1: The picture of the result in $u_1(x,t)$ with the values $\alpha = 0.35$, $b_2 = 0.5$ and $t = 0.01$ for 2D graphics. (a) Real 3D surface, (b) complex 3D surface, (c) 2D shape, (d) real contour shape, (e) complex contour shape.



(a) (b) (c)

(d) (e)

Figure 2: The picture of the result in $u_2(x,t)$ with the values $\alpha = 0.35$, $b_2 = 0.5$ and $t = 0.01$ for 2D graphics. (a) Real 3D surface, (b) complex 3D surface, (c) 2D shape, (d) real contour shape, (e) complex contour shape.

$$v_5(x,t) = 1 \mp \frac{1}{\tanh\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)}, \quad (30)$$

$$v_6(x,t) = 1 \mp \frac{1}{\coth\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)}, \quad (31)$$

$$w_5(x,t) = 2 \mp \frac{2}{\tanh\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)}, \quad (32)$$

$$w_6(x,t) = 2 \mp \frac{2}{\coth\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)}. \quad (33)$$

- **Case 4:** Let $\lambda = -4$, $a_0 = b_2$, $a_1 = 4$, $a_2 = -b_2$, $b_1 = -2$ and use the values of Case 4 into equation (15), we find:

$$u_7(x,t) = -4 + \operatorname{sech}^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) - \operatorname{csch}^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right), \quad (34)$$

$$u_8(x,t) = -4 + \operatorname{sech}^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) - \operatorname{csch}^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right), \quad (35)$$

$$v_7(x,t) = 4 - 2\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) - \frac{2}{\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}, \quad (36)$$

$$v_8(x,t) = 4 - 2\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) - \frac{2}{\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}, \quad (37)$$

$$w_7(x,t) = 32 - 16\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) - \frac{16}{\tanh\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}, \quad (38)$$

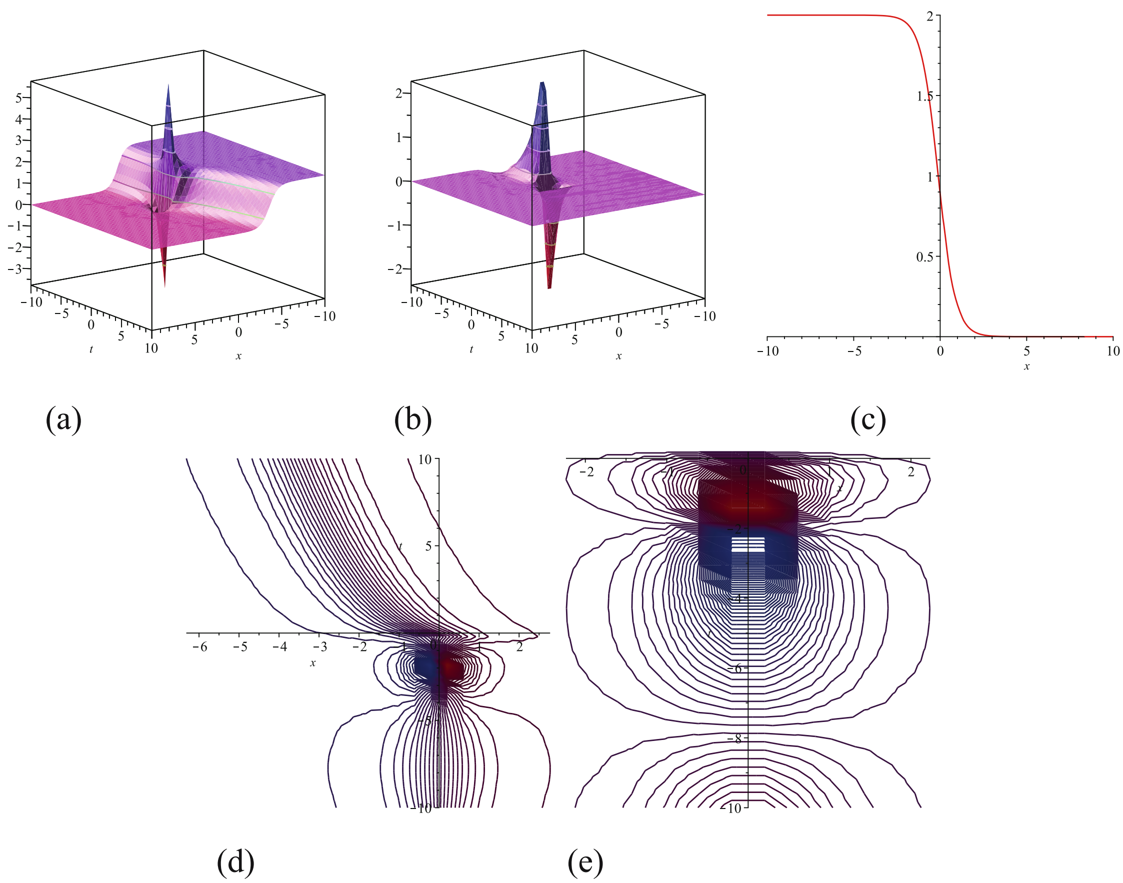


Figure 3: The picture of the result in $v_1(x,t)$ with the values $\alpha = 0.35$, $b_2 = 0.5$ and $t = 0.01$ for 2D graphics. (a) Real 3D surface, (b) complex 3D surface, (c) 2D shape, (d) real contour shape, (e) complex contour shape.

$$w_8(x,t) = 32 - 16\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right) - \frac{16}{\coth\left(x + \frac{4t^\alpha}{\Gamma(1+\alpha)}\right)}. \tag{39}$$

- **Case 5:** Let $\lambda = 2$, $a_0 = b_2$, $a_1 = 0$, $a_2 = -b_2$, $b_1 = \pm\sqrt{-2}$ with b_2 being a free parameter and plug the values of Case 5 into equation (15), we obtain:

$$u_9(x,t) = \tanh^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) - \coth^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right), \tag{40}$$

$$u_{10}(x,t) = \coth^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) - \tanh^2\left(2x + \frac{2t^\alpha}{\Gamma(1+\alpha)}\right), \tag{41}$$

$$v_9(x,t) = -2 \pm \sqrt{-2}\left(\frac{1}{\tanh\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)} - \tanh\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)\right), \tag{42}$$

$$v_{10}(x,t) = -2 \pm \sqrt{-2}\left(\frac{1}{\coth\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)} - \coth\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)\right), \tag{43}$$

$$w_9(x,t) = 8 \mp 4\sqrt{-2}\left(\tanh\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right) - \tanh\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)\right) \tag{44}$$

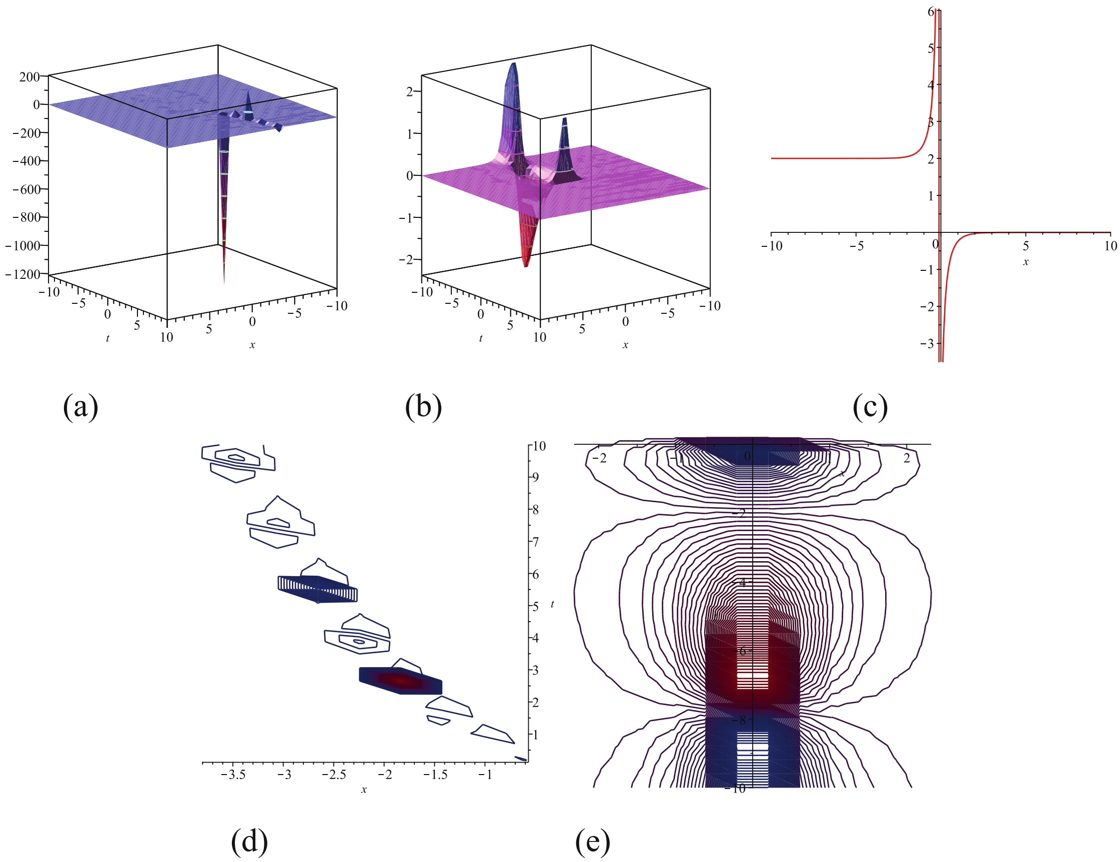


Figure 4: The picture of the result in $v_2(x, t)$ with the unknown parameter values $\alpha = 0.35$, $b_2 = 0.5$ and $t = 0.01$ for 2D graphics. (a) Real 3D surface, (b) complex 3D surface, (c) 2D shape, (d) real contour shape, (e) complex contour shape.

$$w_{10}(x,t) = 8 \mp 4\sqrt{-2}\left(\frac{1}{\coth\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)} - \coth\left(x - \frac{2t^\alpha}{\Gamma(1+\alpha)}\right)\right). \quad (45)$$

A picture is an indispensable tool for conversation and to demonstrate the results to the difficulties lucidly. When performing the computation in daily life, we require the fundamental understanding of constructing the use of pictures. Therefore, the graphical presentations of few obtained results are depicted in Figures 1–6.

5 Discussions and future work

In this research, the suggested method has been triumphantly displayed to attain new closed-form wave solutions of equation (13). It has been determined that the complex fractional transformation and the advanced method are an essential and significant mathematical device in analyzing closed-form wave structures of a whole class of fractional FDEs. Consequently, a few new closed-form wave answers of the equation are determined.

The compensations and legality of the recommended approach over the extended Kudryashov approach are as follows. The critical compensation of the proposed method over the extended Kudryashov approach is that the proposed method implements numerous general and plentiful new closed-form wave structures. The closed-form wave solutions of nonlinear FDE have its vital importance to show the complicated physical aspects. For example, [36] extends the Kudryashov method to solve equation $Q_\xi(\xi) = Q^3(\xi) - Q(\xi)$ as an auxiliary equation and the closed-form solutions presented by $u(\xi) = \sum_{i=0}^{n} \alpha_i Q^i(\xi)$, where $Q(\xi) = \frac{\pm 1}{\sqrt{1 \pm e^{2\xi}}}$. Our solutions u_1, v_1, w_1, u_2, v_2, w_2, u_5 and u_6 are similar to solutions of [36] and solutions u_3, v_3, w_3, u_4, v_4, w_4, v_5, w_5, v_6, w_6, u_7, v_7, w_7, u_8, v_8, w_8, u_9, v_9, w_9, u_{10}, v_{10} and w_{10} are all new exact

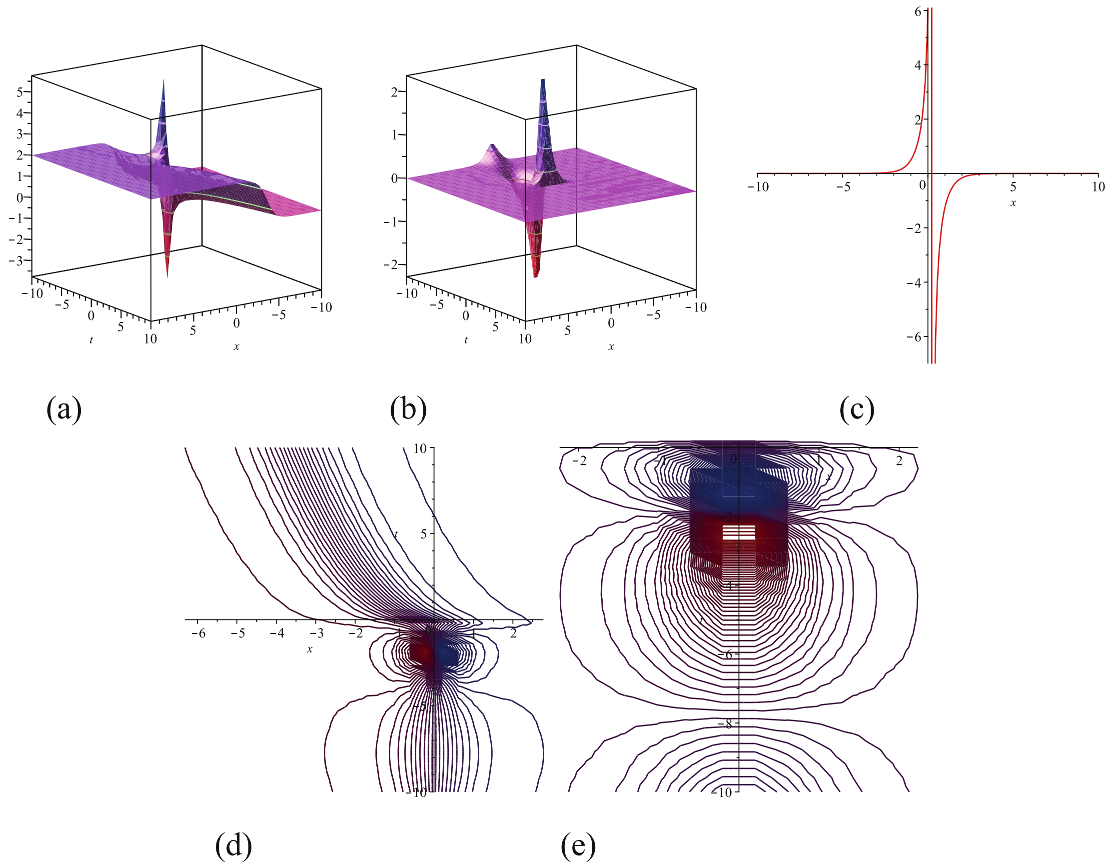


Figure 5: The picture of the result in $v_6(x, t)$ with the unknown parameter values $\alpha = 0.35$ and $b_2 = 0.5$ and $t = 0.01$ for 2D graphics. (a) Real 3D surface, (b) complex 3D surface, (c) 2D shape, (d) real contour shape, (e) complex contour shape.

solutions obtained in this paper, which validates our proposed methods. Although we have presented diverse closed-form wave solutions by performing the recommended method, our obtained solutions show that our proposed methods are more helpful and extraordinary in contributing much more general and many new closed-form wave solutions at the same time. To state the effectiveness and present the insight of processes and the comparison among our proposed method, the new ansatz method is much more proficient than the new extended Kudryashov method. We can underline from our understanding that the method can be performed in other nonlinear FDEs and can reduce the amount of computational work. Therefore, the investigation of closed-form wave structures of other nonlinear FDEs deserves further analysis.

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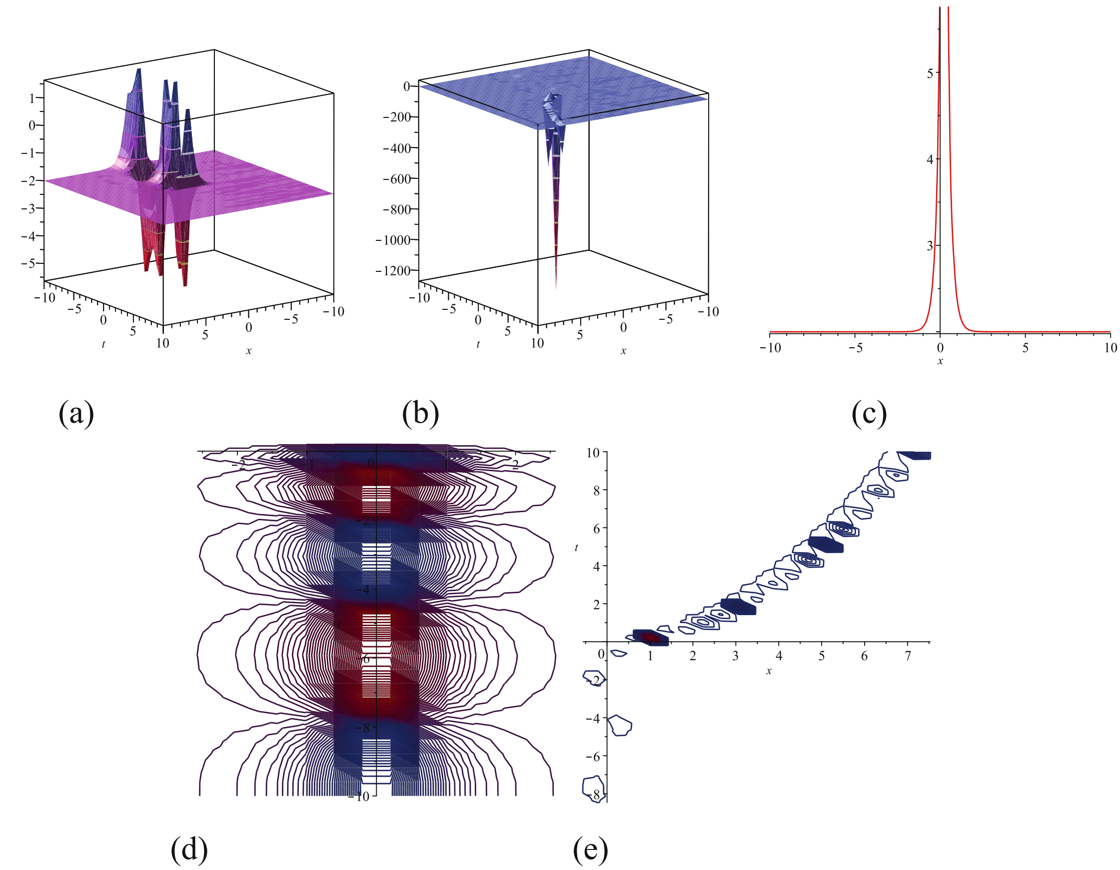

Figure 6: The picture of the result in $v_{10}(x, t)$ with the unknown parameter values $\alpha = 0.35$, $b_2 = 0.5$ and $t = 0.01$ for 2D graphics. (a) Real 3D surface, (b) complex 3D surface, (c) 2D shape, (d) real contour shape, (e) complex contour shape.

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