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Research Article

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Competing Risks Model with Partially Step-Stress Accelerate Life Tests in Analyses Lifetime Chen Data under Type-II Censoring Scheme

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Abstract: The experiment design may need a stress level higher than use condition which is called accelerate life tests (ALTs). One of the most ALTs appears in different applications in the life testes experiment is partially step stress ALTs. Also, the experiment items is failure with several fatal risk factors, the only one is caused to failure which called competing risk model. In this paper, the partially step-stress ALTs based on Type-II censoring scheme is adopted under the different risk factors belong to Chen lifetime distributions. Under this assumption, we will estimate the model parameters of the different causes with the maximum likelihood method. The two, asymptotic distributions and the parametric bootstrap will be used to build each confidence interval of the model parameters. The precision results will be assessed through Monte Carlo simulation study.

Keywords: Accelerates life tests; Chen distribution; Computing risk model, Type-II censoring scheme, Maximum likelihood estimation, Bootstrap confidence interval

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1 Introduction

Under modern technology, products are becoming highly reliable, this makes life-testing under use conditions is difficult and the process of collecting sufficient information about the lifetime of the products is more expensive. The ALTs is the effective one solution to this problem, in which the test units are subjected to different stress levels than the use stress to cause rapid failures. The ALTs is applied to collect enough failure information in a shorter period of time as well as to discuss the effect of lifetime and the external stress variables. According to [1–4] there are different type of ALTs, the first one called constant stress ALTs, in this type the stress is still in a constant level with the life test product. Secondly, is a progressively stress ALTs, in which the stress applied to the product items in the test still increasing in time see Bai and Chung [5]. The final one is the step stress ALTs, in this test condition change for a given time or a specified number of failures, studied by some authors see Miller and Nelson [6] and Bai and Chung [7]. For more recent research on constant stress partially ALTs see Tahani and Soliman [8] and Abd-Elmougod and Emad [9]. Partially ALTs, the experiment run at use and stress conditions, such as constant and step-stress partially ALTs. In partially constant-stress ALTs, the experiment run simultaneously at use and stress condition, but in partially step-stress ALTs, the experiment run at use condition and stress change at a prefixed time or number through the experiment.

In a life testing experiment, commonly that the failure time of a product under consideration is record due to more than one causes. Our problem in such a situation, is assessed the effect of one cause in the presence of other causes. This problem known as the competing risks problem. Different work is discussed the problem of the analysis of competing risks model, see, Cox [10], David and Moeschberger [11], Crowder [12], Balakrishnan and Han [13] and Ganguly and Kundu [14]. In this paper, the model of the partially step-stress ALTs is applied when the units are fails due to two risk factor. The data will be collected from such experiment is used to estimate the parameters of the Chen failure time model under use stress level. For more detial of this problem see, Guan and Tang [15], Balakrishnan *et al.* [16], Lin and Chou [17], David and Kundu [18] and Abd-Elmougod and Abu-Zinadah [19].

Censoring is the common phenomenon in the life time experiments, it is applied for consideration of time and cost. The most common censoring scheme is Type-I and Type-II censoring schemes in life test experiments. In Type-I censoring terminate the experiment at a prior pre-fixed time point, but in Type-II censoring at a prior pre-fixed number of failure. The model of competing risks under consideration only two risk factors and Type-II censoring scheme is presented as the follows.

Let n independent items are put in life test experiment, and the prior m number of failure will be observe. The time X_i and cause of failure δ_i, where $i = 1, 2, ..., m$, $\delta_i \in \{1, 2\}$ and $m \leq n$ has record. The likelihood function of lifetime sample (X_i, δ_i), is presented by

$$L(\underline{\theta}|\underline{x}) = Q\prod_{i=1}^{m}\left[h_1(x_i)\right]^{\rho(\delta_i=1)}\left[h_2(x_i)\right]^{\rho(\delta_i=2)} \times \left[S_1(x_m)S_2(x_m)\right]^{(n-m)}, \tag{1}$$

where $Q = \frac{n!}{(n-m)!}$, $S(x) = 1 - F(x)$, $h(x) = \frac{f(x)}{F(x)}$, $\rho(\delta_i = j) = \begin{cases} 1, & \delta_i = j \\ 0, & \delta_i \neq j \end{cases}$ and

$$0 < x_1 < x_2 < \ldots < x_m < \infty.$$

The problem of analyzing the Type-II competing risks sample under partially step-stress ALTs model from Chen lifetime distribution is the main objective in this paper. The maximum likelihood estimation of the model parameters and accelerated factor is developed. Interval estimation with the two, approximate information matrix and bootstrap techniques are discussed. The performances of estimates are measured with average and mean squared error (MSE) for point estimation and mean length and probability coverage for interval estimation through Monte Carlo simulation.

The paper is organized as follows, in Section 2, the complete description of model formulation and the likelihood function for the partially step-stress ALTs of Type-II competing risks Chen lifetime sample. In Section 3, the MLEs of parameters and accelerated factor with the asymptotic confidence intervals. In Section 4, we discussed bootstrap confidence intervals. The quality points and interval estimates are assessed via Monte Carlo study in Section 5. Finally, some comment in Section 6.

2 Notation and Model Description

Some notations that used in this work under consideration failure items under independent two cause of failure and partially step-stress ALTs model, as given in notation table

Notations

δ_i	The cause of failure for i-th item.
$F(.)$:	The cumulative distribution function of X_i.
$f(.)$:	The probability density function function of X_i.
$F_j(.)$:	The cumulative distribution function *of* X_{ji}.
$f_j(.)$:	The probability density function *of* X_{ji}.
$S_j(.)$:	The survival density function *of* X_{ji}.
X_i :	The lifetime random variable presented by i-th item.
X_{ji} :	The lifetime random variable presented by i-th item and the cause j, $j = 1, 2$.

For given n identical items and the prior fixed number m and fixed time τ, the failure times X_i and cause of failure δ_i are recorded until fixed time τ is reached the items are tested under stress condition. The experiment is running to fixed number of failures m is observed. Under consideration that, the failure time has an independent Chen distribution and two cause of failure the model considered in the paper satisfies the following assumptions.

1. The random variable X_{ji} is Chen distribution with the parameter a, b_j, $j = 1, 2$ has the pdf

$$f_{j1}(x) = ab_jx^{a-1}\exp\left(x^a\right)\exp(b_j\left[1 - \exp(x^a)\right]) \quad ,x > 0,\ a, b_j > 0, \tag{2}$$

   and cdf, given by

$$F_{j1}(x)=1 - \exp(b_j\left[1 - \exp(x^a)\right]). \tag{3}$$

   Also, reliability $S_{j1}(t)$ and failure rate functions $h_{j1}(t)$ of Chen distribution for given time t, respectively presented by

$$S_{j1}(t) = \exp(b_j\left[1 - \exp(t^a)\right]),\ t > 0, \tag{4}$$

$$h_{j1}(t) = ab_jx^{a-1}\exp\left(t^a\right),\ t > 0. \tag{5}$$

2. Item has the lifetime denoted as X_i, $i = 1, 2, \cdots, m$ and the time at which the item i fails due to cause j is X_{ji}, and $X_i = \min\{X_{1i}, X_{2i}\}$.
3. Under consideration the total lifetime of items is multiply of inverse of the accelerated factor which

shorten the lifetime of test items. The total lifetime of a test items, denoted by Z, defined under, use and accelerated conditions. So, the lifetime of the item in partially step-stress ALTs, is presented by

$$Z = \begin{cases} X, & X < \tau \\ \tau + \beta^{-1}(X - \tau), & X > \tau, \end{cases} \tag{6}$$

where β is the accelerated factor, τ, is the chang time from use to higher stress level and the lifetime of the unit X computed at use condition. Under Chen lifetime distribution with parameters a, b_j, the lifetime distribution of Z has the pdf, given by.

$$f_j(z) = \begin{cases} f_{j2}(z), & z > \tau, \\ f_{j1}(z), & 0 < z \le \tau \\ 0, & z < 0, \end{cases} \tag{7}$$

where

$$\begin{aligned} f_{j2}(z) = ab_j\beta \exp(b_j\,[1 & \\ - \exp\left\{(\tau + \beta(z - \tau))^a\right\})] & \\ \times (\tau + \beta(z - \tau))^{a-1} \exp\left\{(\tau + \beta(z - \tau))^a\right\}, & \end{aligned} \tag{8}$$

and $f_{j1}(z)$, is given by (2). The cdf, $S_{j2}(z)$, and hazard rate function $h_{j2}(z)$, is given by

$$F_{j2}(z)=1 - \exp(b_j\left[1 - \exp\left\{(\tau + \beta(z - \tau))^a\right\})\right], \tag{9}$$

$$S_{j2}(z) = \exp(b_j\left[1 - \exp\left\{(\tau + \beta(z - \tau))^a\right\})\right], \tag{10}$$

and

$$\begin{aligned} h_{j2}(z) = ab_j\beta(\tau & \\ + \beta(z - \tau))^{a-1} \exp\left\{(\tau + \beta(z - \tau))^a\right\}. & \end{aligned} \tag{11}$$

The life test experiment is terminated with Type-II censoring scheme when the numbers of failure is reached to $m < n$. Then the random sample of the total lifetime Z is presented by $(Z_1, \delta_1) < (Z_2, \delta_2) < \ldots < (Z_J, \delta_J) < \tau < (Z_{J+1}, \delta_{J+1}) < \ldots < (Z_m, \delta_m)$ where J are the number of items failed under use conditions and $m - J$ at accelerated conditions.

(1) The likelihood function of observed values is reduced to (1) if $\tau > z_m$ but, under consideration the observed values $(z_1, \delta_1) < (z_2, \delta_2) < \ldots < (z_J, \delta_J) < \tau < (z_{J+1}, \delta_{J+1}) < \ldots < (z_m, \delta_m)$ and the time $\tau < z_m$, can be presented by

$$\begin{aligned} L(\underline{\theta}|\underline{z}) = Q\left[S_{21}(z_m)S_{22}(z_m)\right]^{(n-m)} & \\ \times \prod_{i=1}^{J} \left[h_{11}(z_i)\right]^{\rho(\delta_i=1)} \left[h_{21}(z_i)\right]^{\rho(\delta_i=2)} & \\ \times \prod_{i=J+1}^{m} \left[h_{12}(z_i)\right]^{\rho(\delta_i=1)} \left[h_{22}(z_i)\right]^{\rho(\delta_i=2)} & \end{aligned} \tag{12}$$

where $0 < (Z_1, \delta_1) < (Z_2, \delta_2) < \ldots < (Z_J, \delta_J) < \tau < (Z_{J+1}, \delta_{J+1}) < \ldots < (Z_m, \delta_m) < \infty$.

3 Maximum Likelihood Estimation

In this section, we adopted the point and interval maximum likelihood estimates of model parameters and accelerate factor under consideration that two cause of failure and items is failure under only one cause of failure.

3.1 The point MLEs

With the consideration that, observed random sample $(Z_1, \delta_1) < (Z_2, \delta_2) < \ldots < (Z_J, \delta_J) < \tau < (Z_{J+1}, \delta_{J+1}) < \ldots < (Z_m, \delta_m)$ has life time Chen distribution with parameters a, b_j, $j = 1, 2$. The likelihood function in (12) without normalized conatant is reduced to

$$\begin{aligned} L(a, b_1, b_2, \beta|\underline{z}) = a^m b_1^{s_1} b_2^{s_2} \beta^{m-J} \exp\left\{(n - m)(b_1\right. & \\ \left. + b_2)\left[1 - \exp\left\{(\tau + \beta(z_m - \tau))^a\right\}\right]\right\} & \\ \times \prod_{i=1}^{J} z_i^{a-1} \prod_{i=J+1}^{m} (\tau + \beta(z_i - \tau))^{a-1} & \\ \times \exp\left\{\sum_{i=1}^{J} z_i^a + \sum_{i=J+1}^{m} (\tau + \beta(z_i - \tau))^a\right\}, & \end{aligned} \tag{13}$$

where J is the number of items failure at the use condition, s_1 and s_1 are the number of items failure under causes (δ_1, δ_2) and m is the total failure time. Then after taken the natural likelihood function, we obtain

$$\begin{aligned} \ell(a, b_1, b_2, \beta|\underline{z}) = m\log a + s_1 \log b_1 + s_2 \log b_2 & \\ + (m - J)\log\beta + (n - m)(b_1 + b_2) & \\ \times \left[1 - \exp\left\{(\tau + \beta(z_m - \tau))^a\right\}\right] + (a - 1) & \\ \times \left\{\sum_{i=1}^{J} \log z_i + \sum_{i=J+1}^{m} \log(\tau + \beta(z_i - \tau))\right\} & \\ + \sum_{i=1}^{J} z_i^a + \sum_{i=J+1}^{m} (\tau + \beta(z_i - \tau))^a. & \end{aligned} \tag{14}$$

The likelihood equations can be obtain from (14) by taken the first partial derivatives to a, b_1, b_2 and β as follows

$$\begin{aligned} \frac{\partial\ell(a, b_1, b_2, \beta|\underline{z})}{\partial b_1} = \frac{s_1}{b_1} + (n - m) & \\ \times \left[1 - \exp\left\{(\tau + \beta(z_m - \tau))^a\right\}\right] = 0, & \end{aligned} \tag{15}$$

$$\frac{\partial \ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_2} = \frac{s_2}{b_2} + (n-m) \times \left[1-\exp\left\{(\tau+\beta(z_m-\tau))^a\right\}\right] = 0. \tag{16}$$

Equations (15) and (16) are reduce to the MLE of b_1 and b_2 as follows

$$\hat{b}_1(a,\beta) = \frac{s_1}{(n-m)\left[\exp\left\{(\tau+\beta(z_m-\tau))^a\right\}-1\right]}. \tag{17}$$

and

$$\hat{b}_2(a,\beta) = \frac{s_2}{(n-m)\left[\exp\left\{(\tau+\beta(z_m-\tau))^a\right\}-1\right]}. \tag{18}$$

The likelihood equations respected to a and β, are given by

$$\begin{aligned}\frac{\partial \ell(a,b_1,b_2,\beta|\underline{z})}{\partial a} &= \frac{m}{a} - (n-m)(b_1+b_2)\exp\left\{(\tau+\beta \times (z_m-\tau))^a\right\}\log\left\{\tau+\beta(z_m-\tau)\right\} \\ &+ \sum_{i=1}^{J}\log z_i + \sum_{i=1}^{J} z_i^a \log z_i + \sum_{i=J+1}^{m}\log(\tau+\beta(z_i-\tau)) \\ &+ \sum_{i=J+1}^{m}(\tau+\beta(z_i-\tau))^a \times \log(\tau+\beta(z_i-\tau)) = 0.\end{aligned} \tag{19}$$

and

$$\begin{aligned}\frac{\partial \ell(a,b_1,b_2,\beta|\underline{z})}{\partial \beta} &= \frac{(m-J)}{\beta} - a(n-m)(b_1+b_2) \times (z_m-\tau)(\tau+\beta(z_m-\tau))^{a-1} \\ &\times \left[\exp\left\{(\tau+\beta(z_m-\tau))^a\right\}\right] + (a-1)\left\{\sum_{i=J+1}^{m}\frac{(z_i-\tau)}{\tau+\beta(z_i-\tau)}\right\} \\ &+ a\sum_{i=J+1}^{m}(z_i-\tau) \times (\tau+\beta(z_i-\tau))^{a-1} = 0.\end{aligned} \tag{20}$$

Equations (19) and (20) presented the two non-linear equations of a and β, solved numerical with the iteration method such as Newton Raphson method hence the MLE $\hat{a}$ and $\hat{\beta}$ of a and β are obtained. The MLE of b_1 and b_2 are obtained from (17) and (18) after replace a and β by $\hat{a}$ and $\hat{\beta}$.

3.2 Approximate information matrix and interval estimation

The second partial derivatives of (14) with respect to a, β, b_1 and b_2 presented by

$$\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1^2} = \frac{-s_1}{b_1^2} \tag{21}$$

$$\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_2^2} = \frac{-s_2}{b_2^2}. \tag{22}$$

$$\begin{aligned}\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial a^2} &= \frac{-m}{a^2} - (n-m)(b_1+b_2)\exp\left\{(\tau + \beta(z_m-\tau))^a\right\}\log^2\left(\tau+\beta(z_m-\tau)\right) \\ &+ \sum_{i=1}^{J} z_i^a(\log z_i)^2 + \sum_{i=J+1}^{m}(\tau+\beta \times (z_i-\tau))^a\log^2(\tau+\beta(z_i-\tau)),\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial \beta^2} &= \frac{-(m-J)}{\beta^2} - a(n-m)(b_1+b_2) \times (z_m-\tau)^2\exp\left\{(\tau+\beta(z_m-\tau))^a\right\} \\ &\times (\tau+\beta(z_m-\tau))^{a-2}\left\{(a-1) + a(\tau+\beta(z_m-\tau))^a\right\} + (a-1) \\ &\times \left\{\sum_{i=J+1}^{m}\frac{-(z_i-\tau)^2}{\left[\tau+\beta(z_i-\tau)\right]^2}\right\} + a(a-1)\sum_{i=J+1}^{m}(z_i-\tau)^2 \times (\tau+\beta(z_i-\tau))^{a-2}\end{aligned} \tag{24}$$

$$\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1 \partial b_2} = \frac{\partial^2 \ell\left(a,b_1,b_2,\beta|\underline{y}\right)}{\partial b_2 \partial b_1} = 0 \tag{25}$$

$$\begin{aligned}\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1 \partial a} &= \frac{\partial^2 \ell\left(a,b_1,b_2,\beta|\underline{y}\right)}{\partial a \partial b_1} = \frac{\partial^2 \ell\left(a,b_1,b_2,\beta|\underline{y}\right)}{\partial b_2 \partial a} = \frac{\partial^2 \ell\left(a,b_1,b_2,\beta|\underline{y}\right)}{\partial a \partial b_2} \\ &= -(n-m)(\tau+\beta(z_m-\tau))^a \times \log(\tau+\beta(z_m-\tau)) \times \exp\left\{(\tau+\beta(z_m-\tau))^a\right\},\end{aligned} \tag{26}$$

$$\frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial a \partial \beta} = \frac{\partial^2 \ell(a,b_1,b_2,\beta|\underline{z})}{\partial \beta \partial a} \tag{27}$$

$$
\begin{aligned}
&= -(n-m)(b_1+b_2)\frac{(z_m-\tau)}{\tau+\beta(z_m-\tau)} \\
&\times \exp\left\{(\tau+\beta(z_m-\tau))^a\right\} \\
&\times \left\{a(\tau+\beta(z_m-\tau))^a\right. \\
&\times \left.\log\left\{\tau+\beta(z_m-\tau)\right\}+1\right\} \\
&+ \sum_{i=J+1}^{m}\frac{(z_i-\tau)}{\tau+\beta(z_i-\tau)} \\
&+ \sum_{i=J+1}^{m}(z_i-\tau)(\tau+\beta(z_i-\tau))^{a-1} \\
&\times \left\{a\log(\tau+\beta(z_i-\tau))+1\right\}
\end{aligned}
$$

The expected information matrix $I(a,\beta,b_1,b_2)$ of parameters a, β, b_1 and b_2 is negative expectation of second derivative of log likelihood function. Practice, $I^{-1}(a,\beta,b_1,b_2)$ is estimated by $I^{-1}\left(\hat{a},\beta,\hat{b}_1,\hat{b}_2\right)$. Hence, the normal approximation is used as follows

$$
\begin{aligned}
&(\hat{a},\beta,\hat{b}_1,\hat{b}_2) \\
&\to N\left((a,\beta,b_1,b_2), I_0^{-1}(\hat{a},\beta,\hat{b}_1,\hat{b}_2)\right),
\end{aligned}
\tag{28}
$$

where $I_0^{-1}(a,\beta,b_1,b_2)$ is the inverse of observed information matrix, presented by

$$
I_0(\hat{a},\beta,\hat{b}_1,\hat{b}_2) = \begin{bmatrix}
-\frac{\partial\ell(a,b_1,b_2,\beta|\underline{z})}{\partial a^2} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial a\partial\beta} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial a\partial b_1} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial a\partial b_1} \\
-\frac{\partial\ell(a,b_1,b_2,\beta|\underline{z})}{\partial\beta\partial\beta} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial\beta^2} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial\beta\partial b_1} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial\beta\partial b_2} \\
-\frac{\partial\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1\partial a} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1\partial\beta} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1^2} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_1\partial b_2} \\
-\frac{\partial\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_2\partial a} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_2\partial\beta} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_2\partial b_1} & -\frac{\partial^2\ell(a,b_1,b_2,\beta|\underline{z})}{\partial b_2^2}
\end{bmatrix}
\tag{29}
$$

distribution with mean (a,β,b_1,b_2) and variance covariance matrix $I_0^{-1}\left(\hat{a},\beta,\hat{b}_1,\hat{b}_2\right)$ is used to present the aproximate confidence intervals of a,β,b_1 and b_2. Hence, the 100(1-2γ)% approximate confidence intervals of a,β,b_1 and b_2 presented by

$$
\begin{aligned}
&\left(\hat{a}\mp z_\gamma\sqrt{V_{11}}\right), \ \left(\hat{\beta}\mp z_\gamma\sqrt{V_{22}}\right), \\
&\left(\hat{b}_1\mp z_\gamma\sqrt{V_{33}}\right) \text{ and } \left(\hat{b}_2\mp z_\gamma\sqrt{V_{44}}\right)
\end{aligned}
\tag{30}
$$

respectively, where value V_{11}, V_{22} , V_{33} and V_{44} are the elements of the diagonal of $I_0^{-1}\left(\hat{a},\beta,\hat{b}_1,\hat{b}_2\right)$ and z_γ is the percentile right-tail with probable of γ standard normal distribution.

4 Bootstrap Confidence Intervals

Bootstrap technique is the important methods not only in estimations of confidence intervals but can be used to measures of accuracy (defined in terms of bias, variance, prediction error or some other such measure) to sample estimates, can be easily obtained with the bootstrap technique. Parametric and nonparametric bootstrap technique are the important two types of bootstrap technique are available see Davison and Hinkley [20] and Efron and Tibshirani [21]. In the following, we adopted to the parametric bootstrap technique to obtaining interval bootstrap estimations follows.

1 The original sample, $(Z_1,\delta_1) < (Z_2,\delta_2) < \ldots < (Z_J,\delta_J) < \tau < (Z_{J+1},\delta_{J+1}) < \ldots < (Z_m,\delta_m)$ are used to estimates $\hat{a}$, $\hat{\beta}$, $\hat{b}_1$ and $\hat{b}_2$.

2 The independent bootstrap sample $(z_1^\star,\delta_1^\star) < (z_2^\star,\delta_2^\star) < \ldots < (z_J^\star,\delta_J^\star) < \tau < (z_{J+1}^\star,\delta_{J+1}^\star) < \ldots < (z_m^\star,\delta_m^\star)$. generate from Chen distribution with parameters values given by estimates $\hat{a}$, $\hat{\beta}$, $\hat{b}_1$ and $\hat{b}_2$ with prior value m and τ.

3 The bootstrap sample estimates $\hat{a}^\star$, $\hat{\beta}^\star$, $\hat{b}_1^\star$ and $\hat{b}_2^\star$ are computed of $\hat{a}$, β, $\hat{b}_1$ and $\hat{b}_2$ as in step 1.

4 Steps 2 and 3 are repeted N times, then N different bootstrap samples are represented.

5 Let the bootstrap sample estimates vector $\Sigma^\star = (\hat{a}^\star,\hat{\beta}^\star,\hat{b}_1^\star,\hat{b}_2^\star)$ put in assiding order $(\Sigma_k^{\star[1]},\Sigma_k^{\star[2]},\ldots,\Sigma_k^{\star[N]})$, $k=1,2,3,4$.

6 The cdf of $\Sigma_k^\star$ is given by $G(x) = P(\hat{\Sigma}_k^\star \leqslant x)$ for any given x. Then $\hat{\Sigma}_{k-boot}^\star = G^{-1}(x)$ for given z. Then percentile bootstrap confidence intervals $100(1-2\gamma)$ confidence interval of $\hat{\Sigma}_k^\star$ is given by

$$
\left[\hat{\Sigma}_{k-boot}^\star(\gamma), \hat{\Sigma}_{k-boot}^\star(1-\gamma)\right]. \tag{31}
$$

7 Let us define statistic $\Omega_k^{\star[j]}$ by

$$
\Omega_k^{\star(j)} = \frac{\hat{\Sigma}_k^{\star(i)} - \hat{\Sigma}_k}{\sqrt{\operatorname{var}\left(\hat{\Sigma}_k^{\star(i)}\right)}}, \tag{32}
$$

the order values of statstics $\Omega_k^{\star(j)}$ defined by $\Omega_k^{\star[j]}$, we define the cmulative distribution $G(x) = P(\Omega_k^\star < x)$ for any given x, define

$$
\hat{\Sigma}_{k\text{-boot-t}}^\star = \hat{\Sigma}_k + \sqrt{\operatorname{Var}(\hat{\Sigma}_k)}G^{-1}(z). \tag{33}
$$

Then bootstrap-t confidence intervals (BTCI) of $100(1-2\gamma)$ approximate confidence intervals of $\hat{\Sigma}_k$ is given by

$$
\left(\hat{\Sigma}_{k\text{-boot-t}}^\star(\gamma, \hat{\Sigma}_{k\text{-boot-t}}^\star(1-\gamma)\right). \tag{34}
$$

Table 1: The mean estimates and MSEs for the parameters (a, β, b_1, b_2) at (1, 1.5, 1.5, 2.0)

		MLE							
τ	(n, m))	AV				MSE			
		a	β	b_1	b_1	a	β	b_1	b_1
0.5	(30,15)	0.81	1.307	1.34	1.72	0.231	0.336	0.421	0.721
	(30,25)	0.81	1.319	1.34	1.74	0.221	0.161	0.321	0.621
	(50,25)	0.84	1.33	1.37	1.79	0.204	0.131	0.231	0.654
	(50,40)	0.84	1.36	1.40	1.81	0.199	0.153	0.203	0.521
	(75,50)	0.87	1.38	1.38	1.82	0.136	0.136	0.188	0.521
1.0	(30,15)	0.85	1.33	1.40	1.78	0.230	0.161	0.175	0.421
	(30,25)	0.80	1.38	1.41	1.79	0.205	0.134	0.124	0.521
	(50,25)	0.92	1.43	1.57	1.80	0.174	0.161	0.133	0.621
	(50,40)	0.93	1.43	1.42	1.99	0.170	0.131	0.109	0.421
	(75,50)	0.93	1.64	1.54	1.92	0.102	0.153	0.111	0.321
1.5	(30,15)	0.94	1.64	1.54	1.92	0.201	0.137	0.100	0.754
	(30,25)	0.99	1.65	1.58	1.93	0.105	0.163	0.090	0.451
	(50,25)	0.98	1.69	1.61	1.95	0.097	0.120	0.101	0.452
	(50,40)	0.97	1.69	1.62	1.96	0.097	0.140	0.107	0.421
	(75,50)	0.99	1.74	1.40	1.82	0.084	0.114	0.094	0.399
		Bootstrap							
τ	(n, m)	AV				MSE			
		a	β	b_1	b_1	a	β	b_1	b_1
0.5	(30,15)	0.91	1.88	1.88	1.99	0.421	0.542	0.621	0.942
	(30,25)	0.89	1.45	1.82	1.75	0.401	0.512	0.584	0.7541
	(50,25)	0.84	1.354	1.81	1.94	0.365	0.499	0.542	0.771
	(50,40)	0.78	1.42	1.84	1.89	0.321	0.475	0.511	0.612
	(75,50)	0.84	1.30	1.38	1.93	0.213	0.421	0.421	0.600
1.0	(30,15)	0.75	1.17	1.60	1.96	0.421	0.411	0.521	0.599
	(30,25)	0.82	1.75	1.77	1.85	0.321	0.385	0.478	0.583
	(50,25)	0.94	1.40	1.64	1.81	0.210	0.375	0.402	0.531
	(50,40)	0.99	1.43	1.69	1.90	0.199	0.321	0.339	0.466
	(75,50)	0.92	1.77	1.91	1.92	0.189	0.321	0.310	0.399
1.5	(30,15)	1.04	1.64	1.88	1.99	0.210	0.199	0.254	0.623
	(30,25)	100	1.62	1.78	1.88	0.23	0.189	0.213	0.509
	(50,25)	0.92	1.77	1.62	1.91	0.120	0.179	0.213	0.520
	(50,40)	0.94	1.69	1.69	1.92	0.110	0.160	0.188	0.499
	(75,50)	0.92	1.85	1.74	1.87	0.109	0.158	0.142	0.421

5 Monte Carlo Simulations

In this section, we adopted the simulation study to assess and compare the developed results in this paper. Some numerical experiments performed for sample of sizes n, the effective sample sizes m, accelerated time τ and model parameters (a, β, b_1, b_2). In the simulation stuides,we adopted the case, the model parameters $(a, \beta, b_1, b_1) = (1, 1.5, 1.5, 2.0)$ and accelerate time $\tau = (0.5, 1.0, 1.5)$. Essentially, the simulation results are computed to compare the MLEs and bootstrap estimators, mainly are compared in terms of their average (AV) and MSE. The confidence intervals are compared in terms of their average lengths (AL) and the probability coverage (CP). The results in this paper is computed with Mathematica version 9 and reported in Tables 1-2.

Table 2: The AL and (CP), respectivly of MLEPBCIs and PTCIs of 95% CIs for the parameters (α, β, b_1, b_2) at (1, 1.5, 1.5, 2.0)

τ	(n, m)	MLE				PBCIs				PTCIs			
		α	β	b_1	b_2	α	β	b_1	b_1	α	β	b_1	b_1
0.5	(30,15)	2.323	3.543	3.421	4.410	3.355	4.223	4.001	5.433	2.122	3.421	3.339	4.254
		(0.89)	(0.89)	(0.88)	(0.89)	(0.87)	(0.88)	(0.89)	(0.89)	(0.89)	(0.90)	(0.89)	(0.90)
	(30,25)	2.311	3.42	3.326	4.214	3.322	4.111	4.421	5.338	2.218	3.321	3.214	4.051
		(0.89)	(0.90)	(0.91)	(0.90)	(0.89)	(0.89)	(0.90)	(0.90)	(0.91)	(0.91)	(0.92)	(0.91)
	(50,25)	2.302	3.214	3.300	4.124	3.344	4.252	4.305	5.121	2.270	3.154	3.221	4.021
		(0.88)	(0.91)	(0.91)	(0.92)	(0.90)	(0.90)	(0.90)	(0.90)	(0.91)	(0.93)	(0.94)	(0.97)
	(50,40)	2.198	3.109	3.002	3.950	3.125	3.999	3.998	5.050	2.100	3.025	2.902	3.741
		(0.90)	(0.92)	(0.90)	(0.93)	(0.91)	(0.91)	(0.89)	(0.91)	(0.93)	(0.95)	(0.94)	(0.93)
	(75,50)	2.082	2.998	2.977	3.258	3.074	3.991	3.901	5.008	2.005	2.742	2.900	3.133
		(0.91)	(0.93)	(0.92)	(0.94)	(0.92)	(0.91)	(0.90)	(0.91)	(0.91)	(0.93)	(0.95)	(0.94)
1.0	(30,15)	2.301	3.520	3.400	4.3854	3.332	4.223	3.932	5.404	2.002	3.362	3.329	4.226
		(0.87)	(0.88)	(0.89)	(0.90)	(0.88)	(0.88)	(0.89)	(0.88)	(0.89)	(0.91)	(0.89)	(0.91)
	(30,25)	2.295	3.385	3.311	4.212	3.309	4.101	4.414	5.322	2.189	3.285	3.197	4.031
		(0.90)	(0.89)	(0.90)	(0.91)	(0.889)	(0.88)	(0.91)	(0.91)	(0.91)	(0.93)	(0.92)	(0.93)
	(50,25)	2.288	3.190	3.277	4.101	3.322	4.253	4.312	5.100	2.257	3.142	3.207	4.018
		(0.91)	(0.90)	(0.91)	(0.93)	(0.90)	(0.91)	(0.91)	(0.90)	(0.92)	(0.94)	(0.92)	(0.93)
	(50,40)	2.171	3.111	3.011	3.938	3.114	3.987	3.975	5.041	2.099	3.019	2.910	3.738
		(0.92)	(0.90)	(0.92)	(0.91)	(0.97)	(0.90)	(0.91)	(0.97)	(0.94)	(0.95)	(0.95)	(0.96)
	(75,50)	2.100	2.892	2.962	3.241	3.064	3.977	3.842	5.011	2.011	2.726	2.904	3.121
		(0.921)	(0.92)	(0.94)	(0.93)	(0.91)	(0.98)	(0.93)	(0.92)	(0.93)	(0.94)	(0.95)	(0.93)
1.5	(30,15)	2.338	3.550	3.445	4.432	3.355	4.247	4.031	5.451	2.144	3.445	3.350	4.260
		(0.89)	(0.88)	(0.89)	(0.87)	(0.86)	(0.88)	(0.89)	(0.88)	(0.89)	(0.91)	(0.89)	(0.91)
	(30,25)	2.325	3.436	3.341	4.225	3.339	4.124	4.442	5.339	2.241	3.335	3.227	4.063
		(0.88)	(0.91)	(0.90)	(0.92)	(0.89)	(0.89)	(0.910)	(0.91)	(0.91)	(0.93)	(0.91)	(0.91)
	(50,25)	2.314	3.227	3.313	4.137	3.350	4.211	4.314	5.128	2.269	3.166	3.240	4.017
		(0.92)	(0.93)	(0.92)	(0.93)	(0.91)	(0.91)	(0.90)	(0.91)	(0.96)	(0.95)	(0.95)	(0.93)
	(50,40)	2.204	3.121	3.019	3.960	3.141	3.989	4.091	5.062	2.111	3.035	2.918	3.752
		(0.92)	(0.92)	(0.91)	(0.93)	(0.91)	(0.90)	(0.90)	(0.92)	(0.93)	(0.94)	(0.93)	(0.95)
	(75,50)	2.097	3.018	2.987	3.270	3.079	3.999	3.923	5.011	2.014	2.763	2.911	3.108
		(0.92)	(0.93)	(0.92)	(0.92)	(0.93)	(0.91)	(0.91)	(0.92)	(0.94)	(0.93)	(0.94)	(0.96)

6 Conclusions

Type-II censoring competing risks model is discussed in this paper in the presence of partially step-stress ALTs under consideration failure times of the competing risks follow independent Chen distributions. The MLEs of the unknown model parameters are derived. The asymptotic distribution of MLEs and bootstrap method for constructing CIs. Results from Tables 1-2 of the simulation study reported as follows

(1): From the Table 2, we observe that PTCIs performs the best as its CIs has a small length and coverage probabilities to be much closer to the nominal levels than ACs and PBCIs.

(2): From the Table 1, we observe the point estimate of MLE performs the best than bootstrap estimates.

(3): From all tables, we observe that results for the value of accelerate change time τ performs the best for the vale of τ narrow of distribution mean

(4): For the effective m sample increases, the MSEs and the AL of different estimators are reduced.

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