Research Article

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Discrete approximate iterative method for fuzzy investment portfolio based on transaction cost threshold constraint

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Abstract: There are many non-probability factors affecting financial markets and the return on risk assets is fuzzy and uncertain. The authors propose new risk measurement methods to describe or measure the real investment risks. Currently many scholars are studying fuzzy asset portfolios. Based on previous research and in view of the threshold value constraint and entropy constraint of transaction costs and transaction volume, the multiple-period mean value -mean absolute deviation investment portfolio optimization model was proposed on a trial basis. This model focuses on a dynamic optimization problem with path dependence; solving using the discrete approximate iteration method certifies the algorithm is convergent. Upon the empirical research on 30 weighted stocks selected from Shanghai Stock Exchange and Shenzhen Stock Exchange, a multi-period investment portfolio optimum strategy was designed. Through the empirical research, it can be found that the multi-period investments dynamic optimization model has linear convergence and is more effective. This is of great value for investors to develop a multi-stage fuzzy portfolio investment strategy.

Keywords: Markowitz, mean-variance, fuzzy investment portfolio, optimization

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1 Introduction

There are many non-probability factors affecting financial markets and the return on risk assets is vague and uncertain. This paper proposes new risk measurement methods to describe or measure real investment risks. In the 1950s, Markowitz used a variance measure of investment risk and proposed the mean-variance single-period investment portfolio theory, which laid the basis of modern finance [1–5]. The M-V model takes the variance of asset income as the risk measure; it maximises the prospective earning of an asset portfolio for given risks, or goes after the investment portfolio strategy to minimise risk given the prospective earning of an asset portfolio [6–10]. Variance is widely used in the field of risk measures, but it has a number of limitations. Both low income and high income are undesirable in variance analysis since high income may also cause extreme value variance. If asset income is asymmetrical in distribution, the variance risk measurement method will also be imperfect. Consequently, other risk measures were proposed to overcome the limitations of mean-variance, such as: absolute deviation, semi-absolute variance, average absolute variance and VaR [11–15].

The aforementioned studies only considered single-period investment portfolios. However, in reality an investor can re-distribute their own assets and so maintain a multiple-period investment strategy. The single-period investment portfolio can definitely be expanded to a multiple-period one [16, 17, 17–21]. For instance, Mossin, Hakansson, Li, Chan and Ng, Li and Ng, Calafiore, Zhu etc., Wei and Ye, Gupinar and Rustem, Yu etc., Clikyurt and Ozekici. However, these studies used variance risk measurement; where the assets' income was distributed asymmetrically, variance risk measurement had the impact of sacrificing too much prospective earning to relieve extra-low earning or extra-high earning. In order to describe or measure the real investment risk of a financial market, scholars proposed new risk measures, such as Yan and Li using semi-variance instead of variance to measure risk in a multiple-period investment portfolio. Pinar proposed the lower-bound risk measure method.

Many non-probability factors affect a financial market, and asset earnings are fuzzy and uncertain. Currently many scholars are studying the fuzzy asset portfolio, such as: Watada, Leon *et al.*, Tanaka and Guo, Inuiguchi and Tanino, Wang and Zhu, Lai *et al.*, Giove *et al.*, Zhang and Nie *et al.*, Dubois and Prade, Carlsson and Fuller, Huang, Zhang *et al.*

Through their studies, Arnott and Wagner found neglect of transaction costs resulting in ineffective investment portfolios. Bertsimas and Pachamanova, Gulpinar introduced transaction cost to multiple-period investment portfolio selection. Considering the entropy and skewness of linear transaction costs in an investment portfolio, Zang and Liu *et al.* proposed the multiple-period fuzzy investment portfolio model.

Considering the entropy and skewness constraints of transaction cost and transaction volume, a multiple-period mean value – mean absolute deviation investment portfolio model was proposed. This model focused on dynamic optimization with path dependence. In this paper, a discrete approximate iteration method is proposed to solve this model and the algorithm is provento be convergent.

2 Definitions and description

Firstly the definitions that will be used are introduced hereinafter. The fuzzy number A is the fuzzy set of the real number; the real number has normality and fuzzy convexity and continuity belonging to function boundedness. The fuzzy set is expressed in.

Carlsson and Fuller used γ level set to define the upper and lower possibilistic mean value, *i.e.*:

$$[A]^{\gamma} = \left[a_1(\gamma), a_2(\gamma)\right] (\gamma \in [0, 1])$$

$$M_{\star}(A) = \frac{\int_0^1 a(\gamma) Pos(A \le a(\gamma)) d\gamma}{\int_0^1 Pos(A \le a(\gamma)) d\gamma} = 2\int_0^1 \gamma a_1(\gamma) d\gamma$$

and

$$M^{\star}(A) = \frac{\int_0^1 a_2(\gamma) Pos(A \ge a_2(\gamma)) d\gamma}{\int_0^1 Pos(A \ge a_2(\gamma)) d\gamma} = 2\int_0^1 \gamma a_2(\gamma) d\gamma$$

Pos aforesaid means the probability.

$$Pos(A \le a_1(\gamma)) = \Pi(-\infty, a_1(\gamma)) = \sup_{u \le a_1(\gamma)} A(u) = \gamma$$

$$Pos(A \ge a_2(\gamma)) = \Pi(a_2(\gamma), +\infty) = \sup_{u \ge a_2(\gamma)} A(u) = \gamma$$

If $A, B \in, \lambda in \mathcal{R}$, so the references can be obtained as follows:

$$M_{\star}(A+B) = M_{\star}(A) + M_{\star}(B) \quad (1)$$
$$M^{\star}(A+B) = M^{\star}(A) + M^{\star}(B)$$
$$M_{\star}(\lambda A) = \begin{cases} \lambda M_{\star}(A), & \lambda \ge 0 \\ \lambda M^{\star}(A), & \lambda \le 0 \end{cases}$$
$$M^{\star}(\lambda A) = \begin{cases} \lambda M^{\star}(A), & \lambda \ge 0 \\ \lambda M_{\star}(A), & \lambda \le 0 \end{cases}$$

According to the results aforesaid, the following theorem can be obtained:

Theorem 1. 1, if $A_1 \in, \lambda_i \in \mathcal{R}_i$, $i = 1, \dots, n$, so:

$$M_{\star}\left(\sum_{i=1}^{n} \lambda_i A_i\right) = \sum_{i=1}^{n} |\lambda_i| M_{\star}\left(\varphi(\lambda_i) A_i\right),$$
$$M^{\star}\left(\sum_{i=1}^{n} \lambda_i A_i\right) = \sum_{i=1}^{n} |\lambda_i| M^{\star}\left(\varphi(\lambda_i) A_i\right)$$

$\varphi(\lambda_i)$ is the signal equation.

Definition 1. Carlsson and Fuller hypothesized the fuzzy number A had a relationship of $[A]^{\gamma} = \left[a_1(\gamma), a_2(\gamma)\right] (\gamma \in [0, 1])$, so the possibilistic mean value is:

$$M_{\star}\left(\sum_{i=1}^{n} \lambda_i A_i\right) = \sum_{i=1}^{n} |\lambda_i| M_{\star}\left(\varphi(\lambda_i) A_i\right),$$
$$M^{\star}\left(\sum_{i=1}^{n} \lambda_i A_i\right) = \sum_{i=1}^{n} |\lambda_i| M^{\star}\left(\varphi(\lambda_i) A_i\right)$$

Definition 2. The arbitrarily given fuzzy number A has a relationship of $[A]^{\gamma} = \left[a_1(\gamma), a_2(\gamma)\right] (\gamma \in [0, 1])$ and B has a relationship of $[B]^{\gamma} = \left[b_1(\gamma), b_2(\gamma)\right] (\gamma \in [0, 1])$, so the possibilistic mean absolute deviation between A and B is defined as follows:

$$\omega(A, B) = \frac{1}{2}\left(\overline{M}|A - \overline{M}(A)| + \overline{M}|B - \overline{M}(B)|\right) \quad (2)$$

The trapezoidal fuzzy number $A = (a_i, b_i, \alpha_1 \beta_i)$, and it has the subordinating degree function $\mu_A(x)$ as follows:

$$\mu_A(x) = \begin{cases} \frac{x-(a_l-\alpha_l)}{c}, & x \in [a_l - a_l, a_l] \\ 1, & x \in [a_l, b_l] \\ \frac{b_l+\beta_l-x}{\beta_l}, & x \in [b_l, b_l + \beta_l] \\ 0, & others \end{cases}$$

Where: α_1 and β_1 are positive numbers, and $\alpha_1, \beta_1 > 0$, therefore the γ level set of the trapezoidal fuzzy number $A = (a_l, b_l, \alpha_l, \beta_l)$ can be described as $[A_l]^{\gamma} = \left[a_l - (1-\gamma)\alpha_l, b_l + (1-\gamma)\beta_l\right]$, where all $\gamma \in [0, 1]$.

According to Definition 1, the upper and lower possibilitic mean value and the possibilitic mean value can be expressed as follows

$$M_*(A_i) = a_i - \frac{\alpha_i}{3} \tag{3}$$
$$M^*(A_i) = b_i + \frac{\beta_i}{3}$$
$$\overline{M}(A_i) = \frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}$$

According to Definition 2, the mean absolute deviation of $A_1 = (a_1, b_1, \alpha_1, \beta_1)$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2)$ is:

$$\omega(A_1, A_2) \tag{4}$$
$$= \frac{1}{2}\left[\left(\frac{b_1 - a_1}{2} + \frac{\beta_1 - \alpha_1}{6}\right) + \left(\frac{b_2 - a_2}{2} + \frac{\beta_2 - \alpha_2}{6}\right)\right]$$

3 Investment portfolio model

In this part, the first section sets out the problem and symbol descriptions; the second section describes the earning and risk of multiple-period investment portfolios; the final section introduces the entropy constraint of an investment portfolio.

3.1 Problem description and symbol description

According to the hypotheses, there are n kinds of risk assets for selection and risk asset earning is a fuzzy variable. If hypothesizing that an investor invests an initial wealth W_1 on n kinds of risk assets in a continuous way during the period T; in the following period T-1, the investor can re-assign the assets. For convenience, the symbols to be used in the following sections are listed as follows:
x_{it} is the investment proportion of risk asset i during the period t; x_{i0} is the investment proportion of the first risk asset i; x_t is the investment portfolio $x_i = (x_{1t}, x_{2t}, \ldots, x_{nt})$ during the period t; R_{it} is the earning of risk asset i during the period t; r_{pt} is the earning of investment portfolio x_t during the period t; u_{it} is the upper bound of x_{it}; r_{Nt} is the net earnings of the investment portfolio x_t during the period t; W_t is the initial wealth during the period t; c_{it} is the unit transaction cost of risk asset i during the period t.

3.2 Earning and risk of multiple-period investment portfolios

Hypothesizing the whole investment process as self-financing, namely there is no additional capital to invest during each period. The earning: $R_{it} = (a_{it}, b_{it}, \alpha_{it}, \beta_{it})(i = 1, 2, \ldots, n; t = 1, 2, \ldots, T)$, is a trapezoidal fuzzy number; according to Equation (3), the possibilistic mean value of the investment portfolio $x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})'$ during the period t can be obtained:

$$r_{pt} = \sum_{i=1}^{n} \overline{M}(R_{it})x_{it} = \sum_{i=1}^{n}\left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6}\right)x_{it}, \quad t = 1, \ldots, T \tag{5}$$

Hypothesizing the transaction cost is a V-shaped function of the investment portfolio $x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})$ during the period t and the investment portfolio $x_{t-1} = (x_{1-1}, x_{2-1}, \ldots, x_{n-1})$ during the period t-1, namely the transaction cost of asset i during the period t is $c_{it}|x_{it} - x_{it-1}|$. The total transaction cost of the investment portfolio $x_t = (x_{1t}, x_{2t}, \ldots, x_{nt})$ during the period t is:

$$C_t = \sum_{i=1}^{n} c_{it}|x_{it} - x_{it-1}|, t = 1, \ldots, T \tag{6}$$

The net earnings of the investment portfolio x_t during the period t are:

$$r_{Nt} = \sum_{i=1}^{n}\sum_{i=1}^{n}\left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6}\right)x_{it} - \sum_{i=1}^{n} c_{it}|x_{it} - x_{it-1}|, t = 1, \ldots, T \tag{7}$$

The equation of transfer of wealth during the period t+1 is:

$$W_{t+1} = W_t(1 + r_{Nt}) = W_t \left(1 + \sum_{i=1}^{n}\left(\frac{a_{it} + b_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6}\right)x_{it} - \sum_{i=1}^{n} c_{it}|x_{it} - x_{it-1}|\right), \quad t = 1, \ldots, T \tag{8}$$

Therefore, according to Equation (4), the mean absolute deviation of the investment portfolio is:

$$\omega_t(x_t) = \frac{1}{n}\sum_{i=1}^{n} \overline{M}\left(|\overline{M}(R_{it}) - R_{it}|\right)x_{it} \tag{9}$$
$$= \frac{1}{n}\sum_{i=1}^{n}\left(\frac{b_{it} - a_{it}}{2} + \frac{\beta_{it} - \alpha_{it}}{6}\right)x_{it}$$

In order to meet the requirements of investment diversification, the diversification of the investment portfolio is measured by proportion entropy. Proportion entropy was firstly used by Fang *et al.*, Kapur and Jana *et al.* in the single-period investment portfolio. The entropy of the investment portfolio x_t can be expressed as follows:

$$En(x_t) = -\sum_{i=1}^{n} x_{it} \ln x_{it} \tag{10}$$

Where $x_{it} \geq 0 (i = 1, 2, \dots, n)$, so short selling is not allowed. When $x_{1t} = x_{2t} = \dots = 1/n$, equation (11) obtains the maximum value. At this moment, the diversification of the investment portfolio is at the highest level. However, in the actual investment process, if the estimated return rate of an asset $i : R_{it}$ is less than the return rate of a risk-free asset, the investor will abandon the investment in this asset, *i.e.*: $x_{it} = 0$.

A rational investor considers not only expected revenue maximization, but also risk minimization. Therefore, an investor tries to balance expected revenue and risk. If $\theta(0 \leq \theta \leq 1)$ is the preference coefficient of an investor, the objective function of the investor can be expressed as follows:

$$\begin{aligned} U_t(r_{Nt}, \omega_t(x_t)) &= (1-\theta) \\ &\left(\sum_{i=1}^{n} \left(\frac{a_{it}+b_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} \right) x_{it} - \sum_{i=1}^{n} c_{it} |x_{it} - x_{it-1}| \right) \\ &- \theta \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{b_{it}-a_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} \right) x_{it} \right) \end{aligned} \tag{11}$$

Where different θ means a different preference to mean value and mean absolute deviation. If $\theta = 1$, it means the investor only considers the minimized mean absolute deviation, namely the investor dislikes the concentrated investment strategy; if $\theta = 0.5$, it means the investor prefers the two objectives similarly. If $\theta = 0$, it means the investor takes the maximized investment portfolio mean as the objective.

3.3 Multiple-period investment portfolio model

The multiple-period investment portfolio selection is described as follows:

$$\begin{aligned} \max \Bigg(\sum_{t=1}^{T} (1-\theta) \Bigg(\sum_{i=1}^{n} \left(\frac{a_{it}+b_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} \right) y_{it} \\ - \sum_{i=1}^{n} c_{it} \left(|x_{it} - x_{it-1}| \right) \Bigg) \\ - \theta \left(\frac{1}{n} \sum_{i=1}^{n} \left(\frac{b_{it}-a_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} \right) x_{it} \right) \Bigg) \end{aligned}$$

$$s.t. \begin{cases} W_{t+1} = \left(1 + \left[\sum_{i=1}^{n} \left(\frac{a_{it}+b_{it}}{2} + \frac{\beta_{it}-\alpha_{it}}{6} \right) x_{it} \right. \right. & (a) \\ \left. \left. - \sum_{i=1}^{n} c_{it} \left(|y_{it} - x_{it}| \right) \right] \right) W_t & \\ \sum_{i=1}^{n} x_{it} = 1 & (b) \\ \sum_{i=1}^{n} -x_{it} \ln x_{it} \geq H_t, t = 1, \dots, T & (c) \\ l_{it} \leq x_{it} \leq u_{it}, i = 1, \dots, n; t = 1, \dots, T & (d) \end{cases} \tag{12}$$

The constraint condition (12) (a) is the wealth accumulation constraint. The constraint condition (12) (b) means that the total sum of the asset investment proportion during every period is 1; the constraint condition (12) (c) states that the entropy of every investment portfolio during every period reaches or exceeds the given minimized earning constraint; the constraint condition (12) (d) is the threshold value constraint of x_{it}.

4 Discrete approximate iteration method

In this second, a discrete approximate iteration method is proposed to solve the model (12).

The discrete approximate iteration method was proposed in the 1980s. It has unique advantages in the control of nonlinear, unknown models and other systems. It has a very good application prospect in the fields of industrial robots, CNC machine tools and so on. Of course, as a young discipline, discrete approximate iteration has many aspects to be further studied and improved. The design of discrete approximate iteration algorithms is always the focus of iterative learning control. Based on the analysis of the causality of input and output variables, a new P-type causal iterative learning algorithm is proposed. The new algorithm does not need the derivative information of the system output error, and can well reflect the causality between the system input and output. Focusing on linear discrete systems, a concrete iterative learning law is given. Simulation results also show that the proposed iterative learning algorithm has better convergence characteristics than the ordinary P-type iterative learning algorithm. Secondly, two kinds of optimal iterative learning algorithm design problems are considered: 1) iterative learning algorithm design for quadratic performance function optimization in the time domain; 2) optimal iterative learning law design for deterministic systems in an iterative domain and guaranteed cost iterative learning law design for uncertain systems. In this paper, we use this algorithm to solve the multistage portfolio problem.

5 Empirical study

Hypothesizing an investor selects 30 weight stocks from the Shanghai Stock Exchange and Shenzhen Stock Exchange, *i.e.*: S1(001896), S2(600100), S3(002787), S4(002399), S5(000626), S6(000767), S7(002353), S8(600758), S9(600519), S10(300442), S11(300011), S12(000516), S13(600805), S14(600726), S15(002669), S16(000020), S17(000816), S18(300017), S19(600565), S20(002006), S21(002070), S22(300360), S23(300267), S24(300377), S25(000002), S26(601388), S27(000672), S28(600385), S29(002208), S30(600122). The investor invests the initial wealth for 5 consecutive periods, so his wealth will start adjustment when every period starts. We collected data from April 2010 to December 2016 (every three month period was a cycle) and the simple estimate method proposed by Vercher *et al.* was used to process this data. If the earning, cost and turnover rate of every stock during every period is a trapezoid fuzzy number, the unit transaction cost $c_{it} = 0.003(i = 1, \dots, 30; t = 1, \dots, 5)$, the lower bound constraint $l_{it} = 0$, and the upper bound constraint $u_{it} = 0.6(i = 1, \dots, 30; t = 1, \dots, 5)$. H_t takes the maximum value when 30 risk assets are invested on the basis of equal proportion, *i.e.:* $H_t = -\sum_{i=1}^{30} \frac{1}{30} \ln \frac{1}{30} = 3.401$ and when the investor invests all wealth in one risk asset, H_t takes the minimum value, *i.e.:* $H_t = 0$. When the investment preference $\theta = 0.5$, the possible entropy $H_t = 0.6$ or $H_t = 1.6(t = 1, \dots, 5)$, the optimal strategy of multiple-period investment portfolio is shown as follows (see Table 1 and 2 respectively).

Table 1: Optimal solution when

t	Asset i—	Optimal Investment Percentage	
1	Asset 13 0. 6	Asset 18 0. 4	Others 0
2	Asset 13 0. 6	Asset 18 0. 4	Others 0
3	Asset 13 0. 6	Asset 18 0. 4	Others 0
4	Asset 13 0. 6	Asset 18 0. 4	Others 0
5	Asset 13 0. 6	Asset 18 0. 4	Others 0

If $H_t = 0.6$, the optimal investment strategy during the period 1 is $x_{131} = 0.6$, $x_{181} = 0.4$, so the investor invests in the Asset 13 and 18 at the rate of 60% and 40%, without investment in other assets. According to Table 1, the optimal investment strategy during the period 2, 3, 4 and 5 can be respectively obtained. The final-value wealth is 1.9601.

The final-value wealth is 1. 9295.

According to Table 1 and 2, when $H_t = 1.6$ and $H_t = 0.6$, the asset with the larger investment percentage among the optimal investment strategy of investment portfolio during every period is same, it is the Asset 13 and 18.

When $\theta = 0.5$, so H_t is the equal-space value of (0, 3. 40), so the discrete approximate dynamic planning method can be used to solve the final-value wealth, see Table 3.

According to Table 3, it can be seen that when $0 < H_t \leq 3.4$, W_6 does not reduce as H_t increases; when $0.6 < H_t \leq 3.4$, W_6 reduces as H_t increases. At this moment, the larger the value of H_t is, the more discrete the investment in investment portfolio is, and the smaller the final wealth is.

6 Conclusions

In the 1950s, Markowitz used a variance measure of investment risk and proposed the mean-variance single-period investment portfolio theory, which laid the basis of the modern finance. However, using the variance as the risk measure method is imperfect. A financial market is effected by many non-probability factors and the risk assets' income is fuzzy and uncertain. Currently, many scholars are studying the fuzzy asset portfolio. On the basis of previous research and in view of the threshold value constraint and entropy constraint of transaction costs and transaction volume, the multiple-period mean value -mean absolute deviation investment portfolio optimization model was proposed on a trial basis. This model focuses on a dynamic optimization problem with path dependence; using the discrete approximate iteration method to solve the model certifies the algorithm is convergent. Upon the empirical research of 30 weighting stocks selected from Shanghai Stock Exchange and Shenzhen Stock Exchange, a multi-period investment portfolio optimum strategy was designed. Through the empirical research, it can be found that the multi-period investments dynamic optimization model has linear convergence and is more effective.This provides new thinking for multi-period investment portfolio optimization.

Table 2: Optimal solution when $H_t = 1.6$

t	Asset i	Optimal Investment Percentage								
1		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6	Asset 7	Asset 8	Asset 9
		0. 0602	0. 0012	0. 0030	0. 0085	0. 0005	0. 0011	0. 0019	0. 0157	0. 0022
		Asset 10	Asset 11	Asset 12	Asset 13	Asset 14	Asset 15	Asset 17	Asset 18	Asset 19
		0. 0003	0. 0003	0. 0290	0. 59	0. 0002	0. 0720	0. 0439	0. 0849	0. 0059
		Asset 20	Asset 21	Asset 22	Asset 24	Asset 25	Asset 26	Asset	Asset 29	Asset 30
		0. 0087	0. 0005	0. 0102	0. 0008	0. 0019	0. 0184	283	0. 0040	0. 0019
								0. 0350		
2		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6	Asset 7	Asset 8	Asset 9
		0. 0429	0. 0019	0. 0038	0. 0084	0. 0009	0. 0015	0. 0035	0. 0135	0. 0008
		Asset 11	Asset 12	Asset 13	Asset 14	Asset 15	Asset 17	Asset 18	Asset 19	Asset 20
		0. 0006	0. 0294	0. 6	0. 0004	0. 0451	0. 0510	0. 1228	0. 0020	0. 0079
		Asset 21	Asset 22	Asset 23	Asset 24	Asset 25	Asset 26	Asset 28	Asset 29	Asset 30
		0. 0079	0. 0076	0. 0003	0. 0021	0. 0019	0. 0174	0. 0271	0. 0045	0. 0027
3		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6	Asset 7	Asset 8	Asset 9
		0. 0340	0. 0025	0. 0043	0. 0090	0. 0014	0. 0021	0. 0049	0. 0119	0. 0013
		Asset 11	Asset 12	Asset 13	Asset 14	Asset 15	Asset 16	Asset 17	Asset 18	Asset 19
		0. 0009	0. 0354	0. 6	0. 0005	0. 0334	0. 0018	0. 0380	0. 1470	0. 0016
		Asset 20	Asset 21	Asset 22	Asset 24	Asset 25	Asset 26	Asset 28	Asset 29	Asset 30
		0. 0076	0. 0015	0. 0075	0. 0018	0. 0030	0. 0164	0. 0235	0. 0043	0. 0026
4		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6	Asset 7	Asset 8	Asset 9
		0. 0340	0. 0029	0. 0039	0. 0074	0. 0013	0. 0027	0. 0046	0. 0163	0. 0012
		Asset 12	Asset 13	Asset 15	Asset 16	Asset 17	Asset 18	Asset 19	Asset 20	Asset 21
		0. 0380	0. 6	0. 0369	0. 0027	0. 0297	0. 1449	0. 0017	0. 0069	0. 0017
		Asset 22	Asset 23	Asset 24	Asset 25	Asset 26	Asset 27	Asset 28	Asset 29	Asset 30
		0. 0059	0. 0006	0. 0018	0. 0031	0. 0201	0. 0004	0. 0237	0. 0045	0. 0024
5		Asset 1	Asset 2	Asset 3	Asset 4	Asset 5	Asset 6	Asset 7	Asset 8	Asset 9
		0. 0262	0. 0034	0. 0034	0. 0064	0. 0012	0. 0024	0. 0038	0. 0148	0. 0011
		Asset 12	Asset 13	Asset 15	Asset 16	Asset 17	Asset 18	Asset 19	Asset 20	Asset 21
		0. 0350	0. 5440	0. 0305	0. 0025	0. 0275	0. 2330	0. 0015	0. 0070	0. 0015
		Asset 22	Asset 23	Asset 24	Asset 25	Asset 26	Asset 27	Asset 28	Asset 29	Asset 30
		0. 0057	0. 0008	0. 0016	0. 0031	0. 0157	0. 0059	0. 0210	0. 0040	0. 0019

Table 3: Corresponding final-value wealth of different H_t in multiple-period mean value – mean absolute deviation fuzzy investment portfolio model

H_t	0	0. 2	0. 4	0. 6	0. 8	1	1. 2	1. 4	1. 6	1. 8
W_6	1. 9589	1. 9589	1. 9589	1. 9589	1. 9594	1. 9568	1. 9548	1. 9485	1. 9384	1. 9186
H_t	2	2. 2	2. 4	2. 6	2. 8	3. 0	3. 2	3. 4		
W_6	1. 8951	1. 8659	1. 8375	1. 8028	1. 7676	1. 7298	1. 6737	1. 5728		

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