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Research Article

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# Multiplicative topological indices of honeycomb derived networks

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**Abstract:** Topological indices are the numerical values associated with chemical structures that correlate physico-chemical properties with structural properties. There are various classes of topological indices such as degree based topological indices, distance based topological indices and counting related topological indices. Among these classes, degree based topological indices are of great importance and play a vital role in chemical graph theory, particularly in chemistry. In this report, we have computed the multiplicative degree based topological indices of honeycomb derived networks of dimensions I, 2, 3 and 4.

**Keywords:** Honeycomb network, topological index, degree, chemical graph theory

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## 1 Introduction

Mathematical modeling of chemical reaction networks consists of a variety of methods for approaching questions about the dynamical behavior of chemical reactions arising in real world applications. After the invention of the law of mass action, dynamical properties of reaction networks have been extensively studied in both chemistry and physics. The essential steps for this study were the introduction of the detailed equilibrium of complex chemical reactions by Rudolf Wegscheider [1], the development of the quantitative theory of chemical chain reactions by Nikolay Semyonov [2], the development of catalytic reactions by Cyril Norman Hinshelwood [3] and many other results.

Three epochs of chemical dynamics can be observed in the flow of research and publications [4]. These times can be associated with leaders: the first is the Van't Hoff era, the second is the Semenov Hinshelwood era and the third is definitely the Aris era. The "times" can be distinguished on the basis of the priorities of scientific leaders:

- Van't Hoff looked for general laws of chemical reaction related to specific chemical properties. The term "chemical dynamics" belongs to van't Hoff.
- The goal of Hinshelwood Semenov, was to explore the critical phenomena observed in many chemical systems, especially in flames. The concept of chain reaction explored by these researchers influenced many sciences, particularly nuclear physics and engineering.
- Aris' activity was concentrated on the detailed systematization of mathematical ideas and approaches.

Rutherford Aris initiated the mathematical discipline called chemical reaction network theory. The work of R. Aris in the journal *Archive for Rational Mechanics and Analysis* has opened a series of works by other authors (informed by R. Aris). The most famous works of this series are the works of Frederick J. Krambeck [6], Roy Jackson, Friedrich Josef Maria Horn [7], Martin Feinberg [8] and others [9], which were published in the 1970s. In continuation of his early work in this area, R. Aris mentions the work of N.Z. Shapiro, L. S. Shapley [10], where a significant part of his scientific program was endorsed. Since then, a large number of researchers internationally [11–20] have further developed the chemical reaction network theory.

The honeycomb and hexagonal networks have been known as crucial for evolutionary biology, in particular, for the evolution of cooperation, where overlapping triangles are vital for the propagation of cooperation in social dilemmas. For relevant research, see [21, 22]. In the hexagonal network $HX(n)$, the parameter $n$ is the number of ver-

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tices on each side of the network [23], whereas for the honeycomb network $HC(n)$, $n$ is the number of hexagons between a boundary and central hexagon [23]. Due to the significance of topological indices in chemistry, a lot of research has been done in this area. For further studies of topological indices of various graph families, see [24–28].

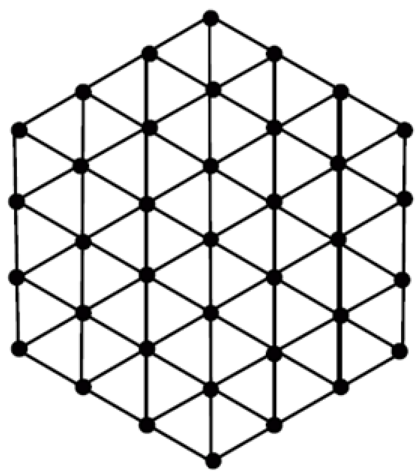

**Figure 1:** Hexagonal network

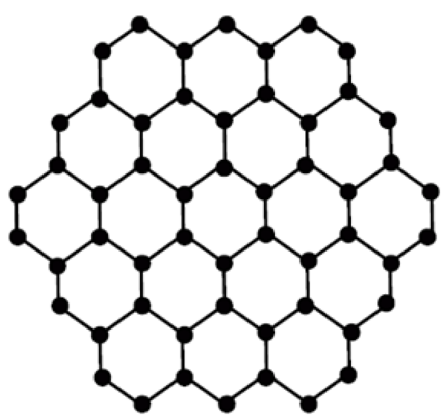

**Figure 2:** Honeycomb network

Let us consider a graph as shown in Figure 3.

The stellation of G is denoted by St(G) and can be obtained by adding a vertex in each face of G and then by join these vertices to all vertices of the respective face (see Figure 4).

The dual Du(G) of a graph G is a graph that has a vertex for each face of G. The graph has an edge whenever two faces of G are separated from each other by an edge, and a self-loop when the same face appears on both sides of an edge, see Figure 5. Hence the number of faces of a graph is equal to the number of edges of its dual.

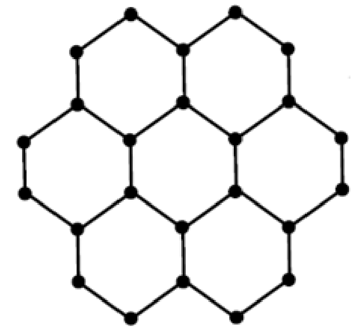

**Figure 3:** Graph G

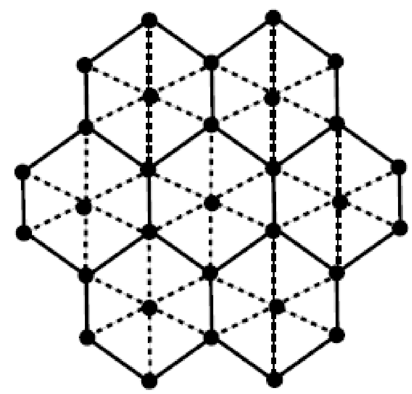

**Figure 4:** Stellation of G (dotted)

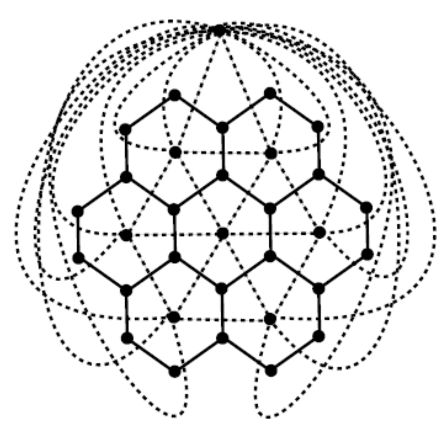

**Figure 5:** Dual of graph G (dotted)

In the dual graph, if we delete the vertex corresponding to the bounded face of the planar graph, which is unique, we get the bounded dual Bdu(G) (see Figure 6).

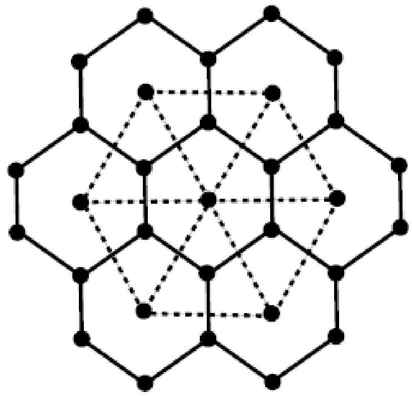

**Figure 6:** Bounded dual of graph G (dotted)

Given a connected plane graph G, its medial graph M(G) has a vertex for each edge of G and an edge between two vertices for each face of G in which their corresponding edges occur consecutively (see Figure 7).

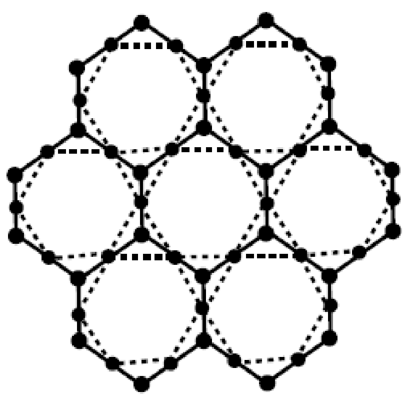

**Figure 7:** Medial of G (dotted)

In this report, we aim to compute multiplicative degree-based topological indices of networks derived from honeycomb networks by taking stellation, dual, bounded dual, and medial graphs of the honeycomb network.

## 2 Topological indices

A molecular graph is a simple graph in chemical graph theory in which atoms are represented by vertices and chemical bonds are represented by edges. A graph is connected if there is a connection between any pair of vertices. A network is a connected graph which has no multiple edge and no loop. The number of vertices which are connected to a fixed $v$ vertex is called the degree of $v$ and is denoted by $d_v$. The distance between two vertices is the length of shortest path between them. The concept of valence in chemistry and concept of degree are somewhat closely related. For details on the basics of graph theory, refer to the book [29]. Quantitative structure-activity and structure-property relationships predict the properties and biological activities of materials. In these studies, topological indices and some physicochemical properties are used to predict bioactivity of chemical compounds [30–33].

Throughout this paper, $G$ denotes a connected graph, $V$ and $E$ denote the vertex set and the edge set and $d_v$ denotes the degree of a vertex. The topological index of the graph of a chemical compound is a number which can be used to characterize the represented chemical compound and help to predict its physicochemical properties. Wiener laid the foundation of topological index in 1947. He approximated the boiling points of alkanes and introduced the Wiener index [34]. Up to now more than 140 topological indices have been defined but no single index is enough to determine all physicochemical properties; but these topological indices together can do this to some extent. Later, in 1975, Milan Randić introduced the Randić index, [35]. In 1998, Bollobas and Erdos [36] and Amic *et al.* [37] proposed the generalized Randić index which has been studied by both chemists and mathematicians [38]. The Randić index is one of the most popular, most studied and most applied topological indices. Many reviews, papers and books [39–44] are written on this simple graph invariant. Some indices related to Wiener's work are the first and second multiplicative Zagreb indices [45], respectively:

$$II_1(G) = \prod_{u \in V(G)} (d_u)^2$$

$$II_2(G) = \prod_{uv \in E(G)} d_u \cdot d_v$$

and the Narumi-Katayama index [46]:

$$NK(G) = \prod_{u \in V(G)} d_u$$

Like the Wiener index, these types of indices are the focus of considerable research in computational chemistry [47–50]. For example, in 2011 I. Gutman [47] characterized the multiplicative Zagreb indices for trees and determined the unique trees that obtained maximum and minimum values for M1(G) and M2(G), respectively. S. Wang and the last author [50] then extended Gutman's result to the following index for k-trees:

$$W_1^s(G) = \prod_{u \in V(G)} (d_u)^s.$$

Notice that $s = 1, 2$ correspond to the Narumi-Katayama and Zagreb index, respectively. Based on the successful consideration of multiplicative Zagreb indices, M. Eliasi *et al.* [51] continued to define a new multiplicative version of the first Zagreb index as

$$II_1^{\star}(G) = \prod_{uv \in E(G)} (d_u + d_v).$$

Furthering the concept of indexing with the edge set, the first author introduced the first and second hyper-Zagreb indices of a graph [52]. They are defined as

$$HII_1(G) = \prod_{uv \in E(G)} (d_u + d_v)^2$$

$$HII_2(G) = \prod_{uv \in E(G)} (d_u \cdot d_v)^2$$

In [53], Kulli *et al.* defined the first and second generalized Zagreb indices:

$$MZ_1^a(G) = \prod_{uv \in E(G)} (d_u + d_v)^a$$

$$MZ_2^a(G) = \prod_{uv \in E(G)} (d_u \cdot d_v)^a$$

Multiplicative sum connectivity and multiplicative product connectivity indices [54] are define as

$$SCII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}}$$

$$PCII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u \cdot d_v}}$$

Multiplicative atomic bond connectivity index and multiplicative Geometric arithmetic index are defined as

$$ABCII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}$$

$$GAII(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v}$$

$$GA^aII(G) = \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v}\right)^a$$

# 3 Computational results

In this section we give our computational results.

### Theorem 1

Let $HcDN1(n)$ be the honeycomb derived network of dimension 1. Then

1. $II_2(HcDN1(n)) = \frac{6^{42}}{5^{30}} \left(\frac{18^6 \times 15^{12} \times 30^{18} \times 36^{27n}}{3^{57}}\right)^n$
2. $II_1^{\star}(HcDN1(n)) = \frac{2^{30} \times 3^{36}}{11^{18}} \left(\frac{9^6 \times 8^{12} \times 11^{18} \times 12^{27n}}{12^{57}}\right)^n$
3. $HII_1(HcDN1(n)) = \frac{2^{60} \times 3^{72}}{11^{36}} \left(\frac{81^6 \times 64^{12} \times 121^8 \times 144^{27n}}{144^{57}}\right)^n$
4. $HII_2(HcDN1(n)) = \frac{6^{84}}{5^{60}} \left(\frac{5^{60} \times 6^{108n}}{2^{180} \times 3^{144}}\right)^n$
5. $MZ_1^a(HcDN1(n)) = \frac{2^{30a} \times 3^{36a}}{11^{18a}} \left(\frac{2^{36a} \times 3^{12a} \times 11^{18a} \times 12^{27an}}{12^{57a}}\right)^n$
6. $MZ_2^a(HcDN1(n)) = \frac{6^{42a}}{5^{30a}} \left(\frac{3^{42a} \times 5^{30a} \times 2^{24a} \times 36^{27an}}{36^{57a}}\right)^n$
7. $XII(HcDN1(n)) = \frac{11^9}{2^{15} \times 3^{18}} \left(\frac{6^{57} \times 3^{\frac{27}{2}n}}{6^{27n} \times 2^{18} \times 3^{\frac{93}{2}} \times 11^9}\right)^n$
8. $\chi II(HcDN1(n)) = \frac{5^{15}}{6^{21}} \left(\frac{2^{45} \times 3^{36}}{5^{15} \times 6^{27n}}\right)^n$
9. $ABCII(HcDN1(n)) = \frac{5^{30}}{2^6 \times 3^{45}} \left(\frac{2^{\frac{45}{2}} \times 3^{60} \times 7^3 \times 10^{\frac{27}{2}n}}{5^{\frac{87}{2}} \times 6^{27n}}\right)^n$
10. $GAII(HcDN1(n)) = \frac{11^{18}}{2^3 \times 6^{15}} \left(\frac{2^{12} \times 3^9 \times 5^{15}}{11^{18}}\right)^n$
11. $GA^aII(HcDN1(n)) = \frac{11^{18a}}{3^{15a} \times 5^{15a} \times 2^{3a}} \left(\frac{3^{9a} \times 5^{15a} \times 2^{12a}}{11^{18a}}\right)^n$

### Proof:

The honeycomb derived network of dimension one $HcDN1(n)$ is obtained by taking the union of the honeycomb network and its stellation, which is a planar graph. In the honeycomb derived network $HcDN1(n)$,

$$|V(HcDN1(n))| = 9n^2 - 3n + 1$$

$$|E(HcDN1(n))| = 27n^2 - 21n + 6$$

There are five types of edges in $E(HcDN1(n))$ based on the degree of end vertices, *i.e.*,

$$E_1(HcDN1(n)) = \{uv \in E(HcDN1(n)) : d_u = 3, d_v = 3\}$$
$$E_2(HcDN1(n)) = \{uv \in E(HcDN1(n)) : d_u = 3, d_v = 5\}$$
$$E_3(HcDN1(n)) = \{uv \in E(HcDN1(n)) : d_u = 3, d_v = 6\}$$
$$E_4(HcDN1(n)) = \{uv \in E(HcDN1(n)) : d_u = 5, d_v = 6\}$$
$$E_5(HcDN1(n)) = \{uv \in E(HcDN1(n)) : d_u = 6, d_v = 6\}$$

It can be observed from Figure 1 that

$$|E_1(HcDN1(n))| = 6$$
$$|E_2(HcDN1(n))| = 12(n-1)$$
$$|E_3(HcDN1(n))| = 6n$$
$$|E_4(HcDN1(n))| = 18(n-1)$$
$$|E_5(HcDN1(n))| = 27n^2 - 57n + 30$$

Now,

$$II_2(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} d_u \cdot d_v = (3\times 3)^6 \times (3\times 5)^{12(n-1)} \times (3\times 6)^{6n} \times (5\times 6)^{18(n-1)} \times (6\times 6)^{27n^2-57n+30} = \frac{6^{42}}{5^{30}}\left(\frac{18^6\times 15^{12}\times 30^{18}\times 36^{27n}}{3^{57}}\right)^n.$$

$$II_1^*(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} (d_u + d_v) = (3+3)^6 \times (3+5)^{12(n-1)} \times (3+6)^{6n} \times (5+6)^{18(n-1)} \times (6+6)^{27n^2-57n+30} = \frac{2^{30}\times 3^{36}}{11^{18}}\left(\frac{9^6\times 8^{12}\times 11^{18}\times 12^{27n}}{12^{57}}\right)^n.$$

$$HII_1(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} (d_u + d_v)^2 = \left[(3+3)^2\right]^6 \times \left[(3+5)^2\right]^{12(n-1)} \times \left[(3+6)^2\right]^{6n} \times \left[(5+6)^2\right]^{18(n-1)} \times \left[(6+6)^2\right]^{27n^2-57n+30} = \frac{2^{60}\times 3^{72}}{11^{36}}\left(\frac{81^6\times 64^{12}\times 121^8\times 144^{27n}}{144^{57}}\right)^n.$$

$$HII_2(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} (d_u \cdot d_v)^2 = \left[(3\times 3)^2\right]^6 \times \left[(3\times 5)^2\right]^{12(n-1)} \times \left[(3\times 6)^2\right]^{6n} \times \left[(5\times 6)^2\right]^{18(n-1)} \times \left[(6\times 6)^2\right]^{27n^2-57n+30} = \frac{6^{84}}{5^{60}}\left(\frac{5^{60}\times 6^{108n}}{2^{180}\times 3^{144}}\right)^n.$$

$$MZ_1^a(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} (d_u + d_v)^a = \left[(3+3)^a\right]^6 \times \left[(3+5)^a\right]^{12(n-1)} \times \left[(3+6)^a\right]^{6n} \times \left[(5+6)^a\right]^{18(n-1)} \times \left[(6+6)^a\right]^{27n^2-57n+30} = \frac{2^{30a}\times 3^{36a}}{11^{18a}}\left(\frac{2^{36a}\times 3^{12a}\times 11^{18a}\times 12^{27an}}{12^{57a}}\right)^n.$$

$$MZ_2^a(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} (d_u \cdot d_v)^a = \left[(3\times 3)^a\right]^6 \times \left[(3\times 5)^a\right]^{12(n-1)} \times \left[(3\times 6)^a\right]^{6n} \times \left[(5\times 6)^a\right]^{18(n-1)} \times \left[(6\times 6)^a\right]^{27n^2-57n+30} = \frac{6^{42a}}{5^{30a}}\left(\frac{3^{42a}\times 5^{30a}\times 2^{24a}\times 36^{27an}}{36^{57a}}\right)^n.$$

$$XII(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} \frac{1}{\sqrt{d_u + d_v}} = \left(\frac{1}{\sqrt{3+3}}\right)^6 \times \left(\frac{1}{\sqrt{3+5}}\right)^{12(n-1)} \times \left(\frac{1}{\sqrt{3+6}}\right)^{6n} \times \left(\frac{1}{\sqrt{5+6}}\right)^{18(n-1)} \times \left(\frac{1}{\sqrt{6+6}}\right)^{27n^2-57n+30} = \frac{11^9}{2^{15}\times 3^{18}}\left(\frac{6^{57}\times 3^{\frac{27}{2}n}}{6^{27n}\times 2^{18}\times 3^{\frac{93}{2}}\times 11^9}\right)^n.$$

$$\chi II(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} \frac{1}{\sqrt{d_u \cdot d_v}} = \left(\frac{1}{\sqrt{3\cdot 3}}\right)^6 \times \left(\frac{1}{\sqrt{3\cdot 5}}\right)^{12(n-1)} \times \left(\frac{1}{\sqrt{3\cdot 6}}\right)^{6n} \times \left(\frac{1}{\sqrt{5\cdot 6}}\right)^{18(n-1)} \times \left(\frac{1}{\sqrt{6\cdot 6}}\right)^{27n^2-57n+30} = \frac{5^{15}}{6^{21}}\left(\frac{2^{45}\times 3^{36}}{5^{15}\times 6^{27n}}\right)^n.$$

$$ABCII(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}$$

$$= \left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)^6 \times \left(\sqrt{\frac{3+5-2}{3\cdot 5}}\right)^{12(n-1)} \times \left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right)^{6n} \times \left(\sqrt{\frac{5+6-2}{5\cdot 6}}\right)^{18(n-1)} \times \left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{27n^2-57n+30}$$

$$= \frac{5^{30}}{2^6\times 3^{45}}\left(\frac{2^{\frac{45}{2}}\times 3^{60}\times 7^3\times 10^{\frac{27}{2}n}}{5^{\frac{87}{2}}\times 6^{27n}}\right)^n.$$

$$GAII(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} \frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v}$$

$$= \left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^6 \times \left(\frac{2\sqrt{3\cdot 5}}{3+5}\right)^{12(n-1)} \times \left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^{6n} \times \left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^{12(n-1)} \times \left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{27n^2-57n+30}$$

$$= \frac{11^{18}}{2^3\times 6^{15}}\left(\frac{2^{12}\times 3^9\times 5^{15}}{11^{18}}\right)^n.$$

$$GA^aII(HcDN1(n)) = \prod_{uv\in E(HcDN1(n))} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v}\right)^a$$

$$= \left[\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^a\right]^6 \times \left[\left(\frac{2\sqrt{3\cdot 5}}{3+5}\right)^a\right]^{12(n-1)} \times \left[\left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^a\right]^{6n} \times \left[\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^a\right]^{18(n-1)} \times \left[\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^a\right]^{27n^2-57n+30}$$

$$= \frac{11^{18a}}{3^{15a}\times 5^{15a}\times 2^{3a}}\left(\frac{3^{9a}\times 5^{15a}\times 2^{12a}}{11^{18a}}\right)^n.$$

## Theorem 2

Let $HcDN2(n)$ be the honeycomb derived network of dimension 2. Then

1. $II_2(HcDN2(n)) = \frac{2^{84}\times 3^{228}}{5^{150}}\left(\frac{2^{108n}\times 3^{72n}\times 5^{90}}{2^{428}\times 3^{162}}\right)^n$
2. $II_1^\star(HcDN2(n)) = \frac{3^{138}\times 7^{18}\times 19^{12}}{2^{42}\times 5^{30}\times 11^{30}\times 13^{12}}\left(\frac{2^{63n}\times 5^{18}\times 11^{18}\times 3^{54n}\times 13^{6}}{2^{87}\times 3^{18}}\right)^n$
3. $HII_1(HcDN2(n)) = \frac{3^{276}\times 7^{36}\times 19^{24}}{2^{84}\times 5^{60}\times 11^{60}\times 13^{24}}\left(\frac{2^{126n}\times 11^{36}\times 3^{108n}\times 13^{12}\times 5^{36}}{2^{126}\times 3^{300}}\right)^n$
4. $HII_2(HcDN2(n)) = \frac{2^{168}\times 3^{456}}{5^{300}}\left(\frac{2^{216n}\times 3^{144n}\times 5^{180}}{2^{456}\times 3^{324}}\right)^n$
5. $MZ_1^a(HcDN2(n)) = \left(\frac{3^{97}\times 7^{18}\times 11^{30}\times 19^{12}}{2^{188}\times 5^{30}\times 13^{2}}\right)^a\left(\frac{2^{63n}\times 3^{54n}\times 5^{18}\times 11^{18}}{2^{63}\times 3^{150}}\right)^{an}$
6. $MZ_2^a(HcDN2(n)) = \left(\frac{2^{89}\times 3^{233}}{5^{145}}\right)^a\left(\frac{2^{72n}\times 3^{72n}\times 5^{85}}{2^{197}\times 3^{167}}\right)^{an}$
7. $XII(HcDN2(n)) = \frac{2^{21}\times 5^{15}\times 11^{15}\times 13^{6}}{3^{69}\times 7^{9}\times 19^{6}}\left(\frac{2^{32}\times 3^{\frac{151}{2}}}{5^{9}\times 11^{9}\times 13^{3}\times 2^{31n}\times 3^{\frac{-53}{2}}\times 6^{n(n+1)}}\right)^n$
8. $\chi II(HcDN2(n)) = \frac{5^{75}}{12^{54}\times 3^{114}}\left(\frac{2^{114}\times 3^{81}}{5^{45}\times 2^{54n}\times 3^{36n}}\right)^n$
9. $ABCII(HcDN2(n)) = \frac{2^{30}\times 3^{102}\times 5^{75}\times 11^{9}\times 17^{6}\times 19^{3}}{13^{6}\times 7^{18}}\left(\frac{2^{18}\times 3^{99}\times 7^{9}\times 13^{6}\times 2^{18n}\times 5^{\frac{9}{2}n}\times 11^{\frac{9}{2}n}}{5^{\frac{99}{2}}\times 11^{\frac{27}{2}}\times 3^{36n}}\right)^n$
10. $GAII(HcDN2(n)) = \frac{2^{96}\times 11^{30}\times 13^{12}}{3^{24}\times 5^{45}\times 7^{18}\times 19^{12}}\left(\frac{2^{12}\times 3^{69}\times 5^{27}}{11^{18}\times 13^{6}\times 2^{72n}\times 3^{18n}}\right)^n$
11. $GA^aII(HcDN2(n)) = \left(\frac{2^{96}\times 11^{30}\times 13^{12}}{3^{24}\times 5^{45}\times 7^{18}\times 19^{12}}\right)^a\left(\frac{2^{12}\times 3^{69}\times 5^{27}}{11^{18}\times 13^{6}\times 2^{72n}\times 3^{18n}}\right)^{an}$

### 3.1 Proof:

The honeycomb derived network of dimension 2 $HcDN2(n)$ is obtained by taking the union of the honeycomb network, its stellation and its bounded dual, which is a non-planar graph. In the honeycomb derived network $HcDN2(n)$,

$$|V(HcDN2(n))| = 9n^2 - 3n + 1$$

$$|E(HcDN2(n))| = 27n^2 - 21n + 6$$

There are sixteen types of edges in $E(HcDN2(n))$ based on the degree of end vertices, *i.e,*

$$E_1(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 3, d_v = 3\}$$
$$E_2(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 3, d_v = 5\}$$
$$E_3(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 3, d_v = 9\}$$
$$E_4(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 3, d_v = 10\}$$
$$E_5(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 5, d_v = 6\}$$
$$E_6(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 5, d_v = 9\}$$
$$E_7(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 5, d_v = 10\}$$
$$E_8(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 6, d_v = 6\}$$
$$E_9(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 6, d_v = 9\}$$
$$E_{10}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 6, d_v = 10\}$$
$$E_{11}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 6, d_v = 12\}$$
$$E_{12}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 9, d_v = 10\}$$
$$E_{13}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 9, d_v = 12\}$$
$$E_{14}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 10, d_v = 10\}$$
$$E_{15}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 10, d_v = 12\}$$
$$E_{16}(HcDN2(n)) = \{uv \in E(HcDN2(n)) : d_u = 12, d_v = 12\}$$

It can be observed from Figure 2 that

$$|E_1(HcDN2(n))| = 6$$
$$|E_2(HcDN2(n))| = 12(n-1)$$
$$|E_3(HcDN2(n))| = 12$$
$$|E_4(HcDN2(n))| = 6(n-2)$$
$$|E_5(HcDN2(n))| = 6(n-1)$$
$$|E_6(HcDN2(n))| = 12$$
$$|E_7(HcDN2(n))| = 12(n-2)$$
$$|E_8(HcDN2(n))| = 9n^2 - 21n + 12$$
$$|E_9(HcDN2(n))| = 12$$
$$|E_{10}(HcDN2(n))| = 18(n-2)$$
$$|E_{11}(HcDN2(n))| = 18n^2 - 54n + 42$$
$$|E_{12}(HcDN2(n))| = 12$$
$$|E_{13}(HcDN2(n))| = 6$$
$$|E_{14}(HcDN2(n))| = 6(n-3)$$
$$|E_{15}(HcDN2(n))| = 12(n-2)$$
$$|E_{16}(HcDN2(n))| = 9n^2 - 33n + 30$$

Now,

$$II_2(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} d_u \cdot d_v = (3\times 3)^6 \times (3\times 5)^{12(n-1)} \times (3\times 9)^{12} \times (3\times 10)^{6(n-2)} \times (5\times 6)^{6(n-1)} \times (5\times 9)^{12} \times (5\times 10)^{12(n-2)} \times (6\times 6)^{9n^2-21n+12} \times (6\times 9)^{12} \times (6\times 10)^{18(n-2)} \times (6\times 12)^{18n^2-54n+42} \times (9\times 10)^{12} \times (9\times 12)^{6} \times (10\times 10)^{6(n-3)} \times (10\times 12)^{12(n-2)} \times (12\times 12)^{9n^2-33n+30} = \frac{2^{84}\times 3^{228}}{5^{150}}\left(\frac{2^{108n}\times 3^{72n}\times 5^{90}}{2^{428}\times 3^{162}}\right)^n.$$

$$II_1^\star(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} d_u + d_v = (3+3)^6 \times (3+5)^{12(n-1)} \times (3+9)^{12} \times (3+10)^{6(n-2)} \times (5+6)^{6(n-1)} \times (5+9)^{12} \times (5+10)^{12(n-2)} \times (6+6)^{9n^2-21n+12}$$

$$\times (6+9)^{12} \times (6+10)^{18(n-2)} \times (6+12)^{18n^2-54n+42} \times (9+10)^{12} \times (9+12)^{6} \times (10+10)^{6(n-3)} \times (10+12)^{12(n-2)} \times (12+12)^{9n^2-33n+30} = \frac{3^{138} \times 7^{18} \times 19^{12}}{2^{42} \times 5^{30} \times 11^{30} \times 13^{12}} \left(\frac{2^{63n} \times 5^{18} \times 11^{18} \times 3^{54n} \times 13^{6}}{2^{87} \times 3^{18}}\right)^{n}.$$

$$HII_1(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} (d_u+d_v)^2 = \left[(3+3)^2\right]^6 \times \left[(3+5)^2\right]^{12(n-1)} \times \left[(3+9)^2\right]^{12} \times \left[(3+10)^2\right]^{6(n-2)} \times \left[(5+6)^2\right]^{6(n-1)} \times \left[(5+9)^2\right]^{12} \times \left[(5+10)^2\right]^{12(n-2)} \times \left[(6+6)^2\right]^{9n^2-21n+12} \times \left[(6+9)^2\right]^{12} \times \left[(6+10)^2\right]^{18(n-2)} \times \left[(6+12)^2\right]^{18n^2-54n+42} \times \left[(9+10)^2\right]^{12} \times \left[(9+12)^2\right]^{6} \times \left[(10+10)^2\right]^{6(n-3)} \times \left[(10+12)^2\right]^{12(n-2)} \times \left[(10+12)^2\right]^{9n^2-33n+30} = \frac{3^{276} \times 7^{36} \times 19^{24}}{2^{84} \times 5^{60} \times 11^{60} \times 13^{24}} \left(\frac{2^{126n} \times 11^{36} \times 3^{108n} \times 13^{12} \times 5^{36}}{2^{126} \times 3^{300}}\right)^{n}.$$

$$HII_2(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} (d_u \cdot d_v)^2 = \left[(3\times 3)^2\right]^6 \times \left[(3\times 5)^2\right]^{12(n-1)} \times \left[(3\times 9)^2\right]^{12} \times \left[(3\times 10)^2\right]^{6(n-2)} \times \left[(5\times 6)^2\right]^{6(n-1)} \times \left[(5\times 9)^2\right]^{12} \times \left[(5\times 10)^2\right]^{12(n-2)} \times \left[(6\times 6)^2\right]^{9n^2-21n+12} \times \left[(6\times 9)^2\right]^{12} \times \left[(6\times 10)^2\right]^{18(n-2)} \times \left[(6\times 12)^2\right]^{18n^2-54n+42} \times \left[(9\times 10)^2\right]^{12} \times \left[(9\times 12)^2\right]^{6} \times \left[(10\times 10)^2\right]^{6(n-3)} \times \left[(10\times 12)^2\right]^{12(n-2)} \times \left[(12\times 12)^2\right]^{9n^2-33n+30} = \frac{2^{168} \times 3^{456}}{5^{300}} \left(\frac{2^{216n} \times 3^{144n} \times 5^{180}}{2^{456} \times 3^{324}}\right)^{n}.$$

$$MZ_1^a(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} (d_u+d_v)^a = \left[(3+3)^a\right]^6 \times \left[(3+5)^a\right]^{12(n-1)} \times \left[(3+9)^a\right]^{12} \times \left[(3+10)^a\right]^{6(n-2)} \times \left[(5+6)^a\right]^{6(n-1)} \times \left[(5+9)^a\right]^{12} \times \left[(5+10)^a\right]^{12(n-2)} \times \left[(6+6)^a\right]^{9n^2-21n+12} \times \left[(6+9)^a\right]^{12} \times \left[(6+10)^a\right]^{18(n-2)} \times \left[(6+12)^a\right]^{18n^2-54n+42} \times \left[(9+10)^a\right]^{12} \times \left[(9+12)^a\right]^{6} \times \left[(10+10)^a\right]^{6(n-3)} \times \left[(10+12)^a\right]^{12(n-2)} \times \left[(10+12)^a\right]^{9n^2-33n+30} = \left(\frac{3^{97} \times 7^{18} \times 11^{30} \times 19^{12}}{2^{188} \times 5^{30} \times 13^{2}}\right)^{a} \left(\frac{2^{63n} \times 3^{54n} \times 5^{18} \times 11^{18}}{2^{63} \times 3^{150}}\right)^{an}.$$

$$MZ_2^a(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} (d_u \cdot d_v)^a = \left[(3\times 3)^a\right]^6 \times \left[(3\times 5)^a\right]^{12(n-1)} \times \left[(3\times 9)^a\right]^{12} \times \left[(3\times 10)^a\right]^{6(n-2)} \times \left[(5\times 6)^a\right]^{6(n-1)} \times \left[(5\times 9)^a\right]^{12} \times \left[(5\times 10)^a\right]^{12(n-2)} \times \left[(6\times 6)^a\right]^{9n^2-21n+12} \times \left[(6\times 9)^a\right]^{12} \times \left[(6\times 10)^a\right]^{18(n-2)} \times \left[(6\times 12)^a\right]^{18n^2-54n+42} \times \left[(9\times 10)^a\right]^{12} \times \left[(9\times 12)^a\right]^{6} \times \left[(10\times 10)^a\right]^{6(n-3)} \times \left[(10\times 12)^a\right]^{12(n-2)} \times \left[(12\times 12)^a\right]^{9n^2-33n+30} = \left(\frac{2^{89} \times 3^{233}}{5^{145}}\right)^{a} \left(\frac{2^{72n} \times 3^{72n} \times 5^{85}}{2^{197} \times 3^{167}}\right)^{an}.$$

$$XII(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} \frac{1}{\sqrt{d_u+d_v}} = \left(\frac{1}{\sqrt{3+3}}\right)^6 \times \left(\frac{1}{\sqrt{3+5}}\right)^{12(n-1)} \times \left(\frac{1}{\sqrt{3+9}}\right)^{12} \times \left(\frac{1}{\sqrt{3+10}}\right)^{6(n-2)} \times \left(\frac{1}{\sqrt{5+6}}\right)^{6(n-1)} \times \left(\frac{1}{\sqrt{5+9}}\right)^{12} \times \left(\frac{1}{\sqrt{5+10}}\right)^{12(n-2)} \times \left(\frac{1}{\sqrt{6+6}}\right)^{9n^2-21n+12} \times \left(\frac{1}{\sqrt{6+9}}\right)^{12} \times \left(\frac{1}{\sqrt{6+10}}\right)^{18(n-2)} \times \left(\frac{1}{\sqrt{6+12}}\right)^{18n^2-54n+42} \times \left(\frac{1}{\sqrt{9+10}}\right)^{12} \times \left(\frac{1}{\sqrt{9+12}}\right)^{6} \times \left(\frac{1}{\sqrt{10+10}}\right)^{6(n-3)} \times \left(\frac{1}{\sqrt{10+12}}\right)^{12(n-2)} \times \left(\frac{1}{\sqrt{12+12}}\right)^{9n^2-33n+30} = \frac{2^{21} \times 5^{15} \times 11^{15} \times 13^{6}}{3^{69} \times 7^{9} \times 19^{6}} \left(\frac{2^{32} \times 3^{\frac{151}{2}}}{5^{9} \times 11^{9} \times 13^{3} \times 2^{31n} \times 3^{\frac{-53}{2}} \times 6^{n(n+1)}}\right)^{n}.$$

$$\chi II(HcDN2(n)) = \prod_{uv\in E(HcDN2(n))} \frac{1}{\sqrt{d_u.d_v}} = \left(\frac{1}{\sqrt{3.3}}\right)^6 \times \left(\frac{1}{\sqrt{3.5}}\right)^{12(n-1)} \times \left(\frac{1}{\sqrt{3.9}}\right)^{12} \times \left(\frac{1}{\sqrt{3.10}}\right)^{6(n-2)} \times \left(\frac{1}{\sqrt{5.6}}\right)^{6(n-1)} \times \left(\frac{1}{\sqrt{5.9}}\right)^{12} \times \left(\frac{1}{\sqrt{5.10}}\right)^{12(n-2)} \times \left(\frac{1}{\sqrt{6.6}}\right)^{9n^2-21n+12} \times \left(\frac{1}{\sqrt{6+9}}\right)^{12} \times \left(\frac{1}{\sqrt{6+10}}\right)^{18(n-2)}$$

$$\times\left(\frac{1}{\sqrt{6+12}}\right)^{18n^2-54n+42}\times\left(\frac{1}{\sqrt{9+10}}\right)^{12}\times\left(\frac{1}{\sqrt{9+12}}\right)^{6}$$
$$\times\left(\frac{1}{\sqrt{10+10}}\right)^{6(n-3)}\times\left(\frac{1}{\sqrt{10+12}}\right)^{12(n-2)}$$
$$\times\left(\frac{1}{\sqrt{12+12}}\right)^{9n^2-33n+30}=\frac{5^{75}}{12^{54}\times 3^{114}}$$
$$\left(\frac{2^{114}\times 3^{81}}{5^{45}\times 2^{54n}\times 3^{36n}}\right)^{n}.$$

$$ABCII\,(HcDN2\,(n))=\prod_{uv\in E(HcDN2(n))}\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}$$
$$=\left(\sqrt{\frac{3+3-2}{3\cdot 3}}\right)^{6}\times\left(\sqrt{\frac{3+5-2}{3\cdot 5}}\right)^{12(n-1)}$$
$$\times\left(\sqrt{\frac{3+9-2}{3\cdot 9}}\right)^{12}\times\left(\sqrt{\frac{3+10-2}{3\cdot 10}}\right)^{6(n-2)}$$
$$\times\left(\sqrt{\frac{5+6-2}{5\cdot 6}}\right)^{6(n-1)}\times\left(\sqrt{\frac{5+9-2}{5\cdot 9}}\right)^{12}$$
$$\times\left(\sqrt{\frac{5+10-2}{5\cdot 10}}\right)^{12(n-2)}\times\left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{9n^2-21n+12}$$
$$\times\left(\sqrt{\frac{6+9-2}{6\cdot 9}}\right)^{12}\times\left(\sqrt{\frac{6+10-2}{6\cdot 10}}\right)^{18(n-2)}$$
$$\times\left(\sqrt{\frac{6+12-2}{6\cdot 12}}\right)^{18n^2-54n+42}\times\left(\sqrt{\frac{9+10-2}{9\cdot 10}}\right)^{12}$$
$$\times\left(\sqrt{\frac{9+12-2}{9\cdot 12}}\right)^{6}\times\left(\sqrt{\frac{10+10-2}{10\cdot 10}}\right)^{6(n-3)}$$
$$\times\left(\sqrt{\frac{10+12-2}{10\cdot 12}}\right)^{12(n-2)}\times\left(\sqrt{\frac{12+12-2}{12\cdot 12}}\right)^{9n^2-33n+30}$$
$$=\frac{2^{30}\times 3^{102}\times 5^{75}\times 11^{9}\times 17^{6}\times 19^{3}}{13^{6}\times 7^{18}}$$
$$\left(\frac{2^{18}\times 3^{99}\times 7^{9}\times 13^{6}\times 2^{18n}\times 5^{\frac{9}{2}n}\times 11^{\frac{9}{2}n}}{5^{\frac{99}{2}}\times 11^{\frac{27}{2}}\times 3^{36n}}\right)^{n}.$$

$$GAII\,(HcDN2\,(n))=\prod_{uv\in E(HcDN2(n))}\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}$$
$$=\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{6}\times\left(\frac{2\sqrt{3\cdot 5}}{3+5}\right)^{12(n-1)}\times\left(\frac{2\sqrt{3\cdot 9}}{3+9}\right)^{12}$$
$$\times\left(\frac{2\sqrt{3\cdot 10}}{3+10}\right)^{6(n-2)}\times\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^{6(n-1)}\times\left(\frac{2\sqrt{5\cdot 9}}{5+9}\right)^{12}$$
$$\times\left(\frac{2\sqrt{5\cdot 10}}{5+10}\right)^{12(n-2)}\times\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{9n^2-21n+12}$$
$$\times\left(\frac{2\sqrt{6\cdot 9}}{6+9}\right)^{12}\times\left(\frac{2\sqrt{6\cdot 10}}{6+10}\right)^{18(n-2)}$$
$$\times\left(\frac{2\sqrt{6\cdot 12}}{6+12}\right)^{18n^2-54n+42}\times\left(\frac{2\sqrt{9\cdot 10}}{9+10}\right)^{12}$$
$$\times\left(\frac{2\sqrt{9\cdot 12}}{9+12}\right)^{6}\times\left(\frac{2\sqrt{10\cdot 10}}{10+10}\right)^{6(n-3)}$$
$$\times\left(\frac{2\sqrt{10\cdot 12}}{10+12}\right)^{12(n-2)}\times\left(\frac{2\sqrt{12\cdot 12}}{12+12}\right)^{9n^2-33n+30}$$
$$=\frac{2^{96}\times 11^{30}\times 13^{12}}{3^{24}\times 5^{45}\times 7^{18}\times 19^{12}}\left(\frac{2^{12}\times 3^{69}\times 5^{27}}{11^{18}\times 13^{6}\times 2^{72n}\times 3^{18n}}\right)^{n}.$$

$$GA^{\alpha}II\,(HcDN2\,(n))=\prod_{uv\in E(HcDN2(n))}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{\alpha}$$
$$=\left[\left(\frac{2\sqrt{3\cdot 3}}{3+3}\right)^{\alpha}\right]^{6}\times\left[\left(\frac{2\sqrt{3\cdot 5}}{3+5}\right)^{\alpha}\right]^{12(n-1)}$$
$$\times\left[\left(\frac{2\sqrt{3\cdot 9}}{3+9}\right)^{\alpha}\right]^{12}\times\left[\left(\frac{2\sqrt{3\cdot 10}}{3+10}\right)^{\alpha}\right]^{6(n-2)}$$
$$\times\left[\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^{\alpha}\right]^{6(n-1)}\times\left[\left(\frac{2\sqrt{5\cdot 9}}{5+9}\right)^{\alpha}\right]^{12}$$
$$\times\left[\left(\frac{2\sqrt{5\cdot 10}}{5+10}\right)^{\alpha}\right]^{12(n-2)}\times\left[\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{\alpha}\right]^{9n^2-21n+12}$$
$$\times\left[\left(\frac{2\sqrt{6\cdot 9}}{6+9}\right)^{\alpha}\right]^{12}\times\left[\left(\frac{2\sqrt{6\cdot 10}}{6+10}\right)^{\alpha}\right]^{18(n-2)}$$
$$\times\left[\left(\frac{2\sqrt{6\cdot 12}}{6+12}\right)^{\alpha}\right]^{18n^2-54n+42}\times\left[\left(\frac{2\sqrt{9\cdot 10}}{9+10}\right)^{\alpha}\right]^{12}$$
$$\times\left[\left(\frac{2\sqrt{9\cdot 12}}{9+12}\right)^{\alpha}\right]^{6}\times\left[\left(\frac{2\sqrt{10\cdot 10}}{10+10}\right)^{\alpha}\right]^{6(n-3)}$$
$$\times\left[\left(\frac{2\sqrt{10\cdot 12}}{10+12}\right)^{\alpha}\right]^{12(n-2)}\times\left[\left(\frac{2\sqrt{12\cdot 12}}{12+12}\right)^{\alpha}\right]^{9n^2-33n+30}$$
$$=\left(\frac{2^{96}\times 11^{30}\times 13^{12}}{3^{24}\times 5^{45}\times 7^{18}\times 19^{12}}\right)^{\alpha}$$
$$\left(\frac{2^{12}\times 3^{69}\times 5^{27}}{11^{18}\times 13^{6}\times 2^{72n}\times 3^{18n}}\right)^{\alpha n}.$$

## Theorem 3

Let $HcDN3\,(n)$ be the honeycomb derived network of dimension 3. Then

1. $II_2\,(HcDN3\,(n))=\frac{2^{30}\times 3^{78}}{5^{30}}\left(\frac{2^{108n}\times 3^{108n}\times 5^{30}}{3^{84}\times 3^{162}}\right)^{n}$
2. $II_1^{\star}\,(HcDN3\,(n))=\frac{2^{96}\times 3^{30}}{5^{12}\times 11^{18}}\left(\frac{2^{108n}\times 3^{54n}\times 5^{12}\times 7^{12}\times 11^{18}}{2^{186}\times 3^{72}}\right)^{n}$
3. $HII_1\,(HcDN3\,(n))=\frac{2^{192}\times 3^{60}}{5^{24}\times 11^{36}}\left(\frac{2^{216n}\times 3^{108n}\times 5^{24}\times 7^{24}\times 11^{36}}{2^{372}\times 2^{144}}\right)^{n}$

4. $HII_2\left(HcDN3\left(n\right)\right)=\frac{2^{60}\times3^{156}}{5^{60}}\left(\frac{2^{216n}\times3^{216n}\times5^{60}}{2^{168}\times3^{324}}\right)^{n}$
5. $MZ_1^a\left(HcDN3\left(n\right)\right)=\frac{2^{96a}\times3^{30a}}{5^{12a}\times11^{18a}}\left(\frac{2^{108n}\times3^{54n}\times5^{12}\times7^{12}\times11^{18}}{2^{186}\times3^{72}}\right)^{an}$
6. $MZ_2^a\left(HcDN3\left(n\right)\right)=\frac{2^{30a}\times3^{78a}}{5^{30a}}\left(\frac{2^{108n}\times3^{108n}\times5^{30}}{3^{84}\times3^{162}}\right)^{an}$
7. $XII\left(HcDN3\left(n\right)\right)=\frac{5^{6}\times11^{9}}{2^{48}\times3^{15}}\left(\frac{2^{93}\times3^{36}}{2^{54n}\times3^{27n}\times5^{6}\times7^{6}\times11^{9}}\right)^{n}$
8. $\chi II\left(HcDN3\left(n\right)\right)=\frac{5^{15}}{2^{15}\times3^{39}}\left(\frac{2^{42}\times3^{81}}{2^{54n}\times3^{54n}\times5^{15}}\right)^{n}$
9. $ABCII\left(HcDN3\left(n\right)\right)=\frac{5^{42}}{2^{6}\times3^{57}\times7^{6}}\left(\frac{2^{27n}\times2^{12}\times3^{105}\times7^{6}}{2^{27n}\times3^{54n}\times5^{63}}\right)^{n}$
10. $GAII\left(HcDN3\left(n\right)\right)=\frac{3^{9}\times11^{18}}{2^{69}\times5^{3}}\left(\frac{2^{102}\times5^{3}}{3^{9}\times7^{12}\times11^{18}}\right)^{n}$
11. $GA^aII\left(HcDN3\left(n\right)\right)=\frac{3^{9a}\times11^{18a}}{2^{69a}\times5^{3a}}\left(\frac{2^{102}\times5^{3}}{3^{9}\times7^{12}\times11^{18}}\right)^{an}$

## Proof

The honeycomb derived network of dimension 3 $HcDN3\left(n\right)$ is obtained by taking the union of the honeycomb network, its stellation and its medial, which is a non-planar graph.

In the honeycomb derived network $HcDN3\left(n\right)$,

$$\left|V\left(HcDN3\left(n\right)\right)\right|=18n^2-6n+1$$

$$\left|E\left(HcDN3\left(n\right)\right)\right|=54n^2-42n+12$$

There are seven types of edges in $E\left(HcDN3\left(n\right)\right)$ based on the degree of end vertices, *i.e.*,

$$E_1\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=3,d_v=4\}$$
$$E_2\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=3,d_v=6\}$$
$$E_3\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=4,d_v=4\}$$
$$E_4\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=4,d_v=5\}$$
$$E_5\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=4,d_v=6\}$$
$$E_6\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=5,d_v=6\}$$
$$E_7\left(HcDN3\left(n\right)\right)=\{uv\in E\left(HcDN3\left(n\right)\right):d_u=6,d_v=6\}$$

$$\left|E_1\left(HcDN3\left(n\right)\right)\right|=12n$$
$$\left|E_2\left(HcDN3\left(n\right)\right)\right|=6n$$
$$\left|E_3\left(HcDN3\left(n\right)\right)\right|=6n$$
$$\left|E_4\left(HcDN3\left(n\right)\right)\right|=12\left(n-1\right)$$
$$\left|E_5\left(HcDN3\left(n\right)\right)\right|=12\left(n-1\right)$$
$$\left|E_6\left(HcDN3\left(n\right)\right)\right|=18\left(n-1\right)$$
$$\left|E_7\left(HcDN3\left(n\right)\right)\right|=54n^2-108n+54$$

Now,

$$II_2\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}d_u\cdot d_v=\left(3\times4\right)^{12n}\times\left(3\times6\right)^{6n}\times\left(4\times4\right)^{6n}\times\left(4\times5\right)^{12(n-1)}\times\left(4\times6\right)^{12(n-1)}\times\left(5\times6\right)^{18(n-1)}\times\left(6\times6\right)^{54n^2-108n+54}=\frac{2^{30}\times3^{78}}{5^{30}}\left(\frac{2^{108n}\times3^{108n}\times5^{30}}{3^{84}\times3^{162}}\right)^{n}.$$

$$II_1^{\star}\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\left(d_u+d_v\right)=\left(3+4\right)^{12n}\times\left(3+6\right)^{6n}\times\left(4+4\right)^{6n}\times\left(4+5\right)^{12(n-1)}\times\left(4+6\right)^{12(n-1)}\times\left(5+6\right)^{18(n-1)}\times\left(6+6\right)^{54n^2-108n+54}=\frac{2^{96}\times3^{30}}{5^{12}\times11^{18}}\left(\frac{2^{108n}\times3^{54n}\times5^{12}\times7^{12}\times11^{18}}{2^{186}\times3^{72}}\right)^{n}.$$

$$HII_1\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\left(d_u+d_v\right)^2=\left[\left(3+4\right)^2\right]^{12n}\times\left[\left(3+6\right)^2\right]^{6n}\times\left[\left(4+4\right)^2\right]^{6n}\times\left[\left(4+5\right)^2\right]^{12(n-1)}\times\left[\left(4+6\right)^2\right]^{12(n-1)}\times\left[\left(5+6\right)^2\right]^{18(n-1)}\times\left[\left(6+6\right)^2\right]^{54n^2-108n+54}=\frac{2^{192}\times3^{60}}{5^{24}\times11^{36}}\left(\frac{2^{216n}\times3^{108n}\times5^{24}\times7^{24}\times11^{36}}{2^{372}\times2^{144}}\right)^{n}.$$

$$HII_2\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\left(d_u\cdot d_v\right)^2=\left[\left(3\times4\right)^2\right]^{12n}\times\left[\left(3\times6\right)^2\right]^{6n}\times\left[\left(4\times4\right)^2\right]^{6n}\times\left[\left(4\times5\right)^2\right]^{12(n-1)}\times\left[\left(4\times6\right)^2\right]^{12(n-1)}\times\left[\left(5\times6\right)^2\right]^{18(n-1)}\times\left[\left(6\times6\right)^2\right]^{54n^2-108n+54}=\frac{2^{60}\times3^{156}}{5^{60}}\left(\frac{2^{216n}\times3^{216n}\times5^{60}}{2^{168}\times3^{324}}\right)^{n}.$$

$$MZ_1^a\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\left(d_u+d_v\right)^a=\left[\left(3+4\right)^a\right]^{12n}\times\left[\left(3+6\right)^a\right]^{6n}\times\left[\left(4+4\right)^a\right]^{6n}\times\left[\left(4+5\right)^a\right]^{12(n-1)}\times\left[\left(4+6\right)^a\right]^{12(n-1)}\times\left[\left(5+6\right)^a\right]^{18(n-1)}\times\left[\left(6+6\right)^a\right]^{54n^2-108n+54}=\frac{2^{96a}\times3^{30a}}{5^{12a}\times11^{18a}}\left(\frac{2^{108n}\times3^{54n}\times5^{12}\times7^{12}\times11^{18}}{2^{186}\times3^{72}}\right)^{an}.$$

$$XII\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\frac{1}{\sqrt{d_u+d_v}}=\left(\frac{1}{\sqrt{3+4}}\right)^{12n}\times\left(\frac{1}{\sqrt{3+6}}\right)^{6n}\times\left(\frac{1}{\sqrt{4+4}}\right)^{6n}$$

$$\times\left(\frac{1}{\sqrt{4+5}}\right)^{12(n-1)}\times\left(\frac{1}{\sqrt{4+6}}\right)^{12(n-1)}$$

$$\times\left(\frac{1}{\sqrt{5+6}}\right)^{18(n-1)}\times\left(\frac{1}{\sqrt{6+6}}\right)^{54n^2-108n+54}$$

$$=\frac{5^6\times 11^9}{2^{48}\times 3^{15}}\left(\frac{2^{93}\times 3^{36}}{2^{54n}\times 3^{27n}\times 5^6\times 7^6\times 11^9}\right)^n.$$

$$\chi II\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\frac{1}{\sqrt{d_u\cdot d_v}}=\left(\frac{1}{\sqrt{3\cdot 4}}\right)^{12n}$$

$$\times\left(\frac{1}{\sqrt{3\cdot 6}}\right)^{6n}\times\left(\frac{1}{\sqrt{4\cdot 4}}\right)^{6n}\times\left(\frac{1}{\sqrt{4\cdot 5}}\right)^{12(n-1)}$$

$$\times\left(\frac{1}{\sqrt{4\cdot 6}}\right)^{12(n-1)}\times\left(\frac{1}{\sqrt{5\cdot 6}}\right)^{18(n-1)}$$

$$\times\left(\frac{1}{\sqrt{6\cdot 6}}\right)^{54n^2-108n+54}=\frac{5^{15}}{2^{15}\times 3^{39}}\left(\frac{2^{42}\times 3^{81}}{2^{54n}\times 3^{54n}\times 5^{15}}\right)^n.$$

$$ABCII\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}$$

$$=\left(\sqrt{\frac{3+4-2}{3\cdot 4}}\right)^{12n}\times\left(\sqrt{\frac{3+6-2}{3\cdot 6}}\right)^{6n}$$

$$\times\left(\sqrt{\frac{4+4-2}{4\cdot 4}}\right)^{6n}\times\left(\sqrt{\frac{4+5-2}{4\cdot 5}}\right)^{12(n-1)}$$

$$\times\left(\sqrt{\frac{4+6-2}{4\cdot 6}}\right)^{12(n-1)}\times\left(\sqrt{\frac{5+6-2}{5\cdot 6}}\right)^{18(n-1)}$$

$$\times\left(\sqrt{\frac{6+6-2}{6\cdot 6}}\right)^{54n^2-108n+54}=\frac{5^{42}}{2^6\times 3^{57}\times 7^6}$$

$$\left(\frac{2^{27n}\times 2^{12}\times 3^{105}\times 7^6}{2^{27n}\times 3^{54n}\times 5^{63}}\right)^n.$$

$$GAII\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}$$

$$=\left(\frac{2\sqrt{3\cdot 4}}{3+4}\right)^{12n}\times\left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^{6n}\times\left(\frac{2\sqrt{4\cdot 4}}{4+4}\right)^{6n}$$

$$\times\left(\frac{2\sqrt{4\cdot 5}}{4+5}\right)^{12(n-1)}\times\left(\frac{2\sqrt{4\cdot 6}}{4+6}\right)^{12(n-1)}$$

$$\times\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^{18(n-1)}\times\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{54n^2-108n+54}$$

$$=\frac{3^9\times 11^{18}}{2^{69}\times 5^3}\left(\frac{2^{102}\times 5^3}{3^9\times 7^{12}\times 11^{18}}\right)^n.$$

$$GA^aII\left(HcDN3\left(n\right)\right)=\prod_{uv\in E(HcDN3(n))}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^a$$

$$=\left[\left(\frac{2\sqrt{3\cdot 4}}{3+4}\right)^a\right]^{12n}\times\left[\left(\frac{2\sqrt{3\cdot 6}}{3+6}\right)^a\right]^{6n}$$

$$\times\left[\left(\frac{2\sqrt{4\cdot 4}}{4+4}\right)^a\right]^{6n}\times\left[\left(\frac{2\sqrt{4\cdot 5}}{4+5}\right)^a\right]^{12(n-1)}$$

$$\times\left[\left(\frac{2\sqrt{4\cdot 6}}{4+6}\right)^a\right]^{12(n-1)}\times\left[\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^a\right]^{18(n-1)}$$

$$\times\left[\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^a\right]^{54n^2-108n+54}$$

$$=\frac{3^{9a}\times 11^{18a}}{2^{69a}\times 5^{3a}}\left(\frac{2^{102}\times 5^3}{3^9\times 7^{12}\times 11^{18}}\right)^{an}.$$

## Theorem 4

Let $HcDN4\left(n\right)$ be the honeycomb derived network of dimension 4. Then

1. $II_1^{\star}\left(HcDN4\left(n\right)\right)=\frac{3^{156}\times 19^{12}}{2^{276}\times 5^{18}\times 7^{282}\times 11^{30}\times 13^{12}}\left(\frac{2^{99n}\times 3^{45n}\times 5^{18}\times 7^{156}\times 11^{18}\times 13^6}{2^9\times 3^{153}}\right)^n$
2. $HII_1\left(HcDN4\left(n\right)\right)=\frac{3^{312}\times 19^{24}}{2^{552}\times 5^{36}\times 7^{564}\times 11^{60}\times 13^{24}}\left(\frac{2^{198n}\times 3^{90n}\times 5^{36}\times 7^{312}\times 11^{36}\times 13^{12}}{2^{18}\times 3^{306}}\right)^n$
3. $HII_2\left(HcDN4\left(n\right)\right)=\frac{2^{192}}{3^{672}\times 5^{804}}\left(\frac{2^{324n}\times 3^{252n}\times 3^{72}\times 5^{420}}{2^{468}}\right)^n$
4. $MZ_2^a\left(HcDN4\left(n\right)\right)=\left(\frac{2^{96}}{3^{336}\times 5^{402}}\right)^a\left(\frac{2^{162n}\times 3^{162n}\times 3^{36}\times 5^{210}}{2^{234}}\right)^{an}$
5. $XII\left(HcDN4\left(n\right)\right)=\frac{2^{138}\times 5^9\times 7^{141}\times 11^{15}\times 13^6}{3^{78}\times 19^6}\left(\frac{2^{\frac{9}{2}}\times 3^{\frac{153}{2}}}{2^{\frac{99}{2}n}\times 3^{\frac{45}{2}n}\times 5^9\times 7^{78}\times 11^9\times 13^3}\right)^n$
6. $\chi II\left(HcDN4\left(n\right)\right)=\frac{3^{168}\times 5^{201}}{2^{48}}\left(\frac{2^{117}}{2^{81n}\times 3^{63n}\times 3^{18}\times 5^{105}}\right)^n$
7. $ABCII\left(HcDN4\left(n\right)\right)=\frac{5^{213}\times 11^9\times 13^6\times 17^6\times 19^3}{2^{282}\times 7^{24}}\left(\frac{2^{\frac{483}{2}n}\times 3^{57n}\times 5^{36n}\times 2^{\frac{399}{2}}\times 3^{273}\times 5^{123}\times 7^{15}\times 11^{\frac{15}{2}}}{11^{\frac{33}{2}}}\right)^n$
8. $GAII\left(HcDN4\left(n\right)\right)=\frac{2^{30}\times 11^{30}\times 13^{12}}{5^{33}\times 7^{18}\times 19^{12}}\left(\frac{2^{27n}\times 2^3\times 3^{51}\times 5^{15}}{3^{18n}\times 7^{12}\times 11^{18}\times 13^6}\right)^n$
9. $GA^aII\left(HcDN4\left(n\right)\right)=\left(\frac{2^{30}\times 11^{30}\times 13^{12}}{5^{33}\times 7^{18}\times 19^{12}}\right)^a\left(\frac{2^{27n}\times 2^3\times 3^{51}\times 5^{15}}{3^{18n}\times 7^{12}\times 11^{18}\times 13^6}\right)^{an}$

## Proof

The honeycomb derived network of dimension 4 $HcDN4\left(n\right)$ is obtained by taking the union of the honeycomb network, its stellation, its bounded dual and its medial, which is a non-planar graph.

In the honeycomb derived network $HcDN4\left(n\right)$,

$$\left|V\left(HcDN4\left(n\right)\right)\right|=18n^2-6n+1$$

$$\left|E\left(HcDN4\left(n\right)\right)\right|=63n^2-57n+18$$

There are sixteen types of edges in $E(HcDN4(n))$ based on the degree of end vertices, *i.e.*,

$$E_1(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 3, d_v = 4\}$$
$$E_2(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 3, d_v = 9\}$$
$$E_3(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 3, d_v = 10\}$$
$$E_4(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 4, d_v = 4\}$$
$$E_5(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 4, d_v = 5\}$$
$$E_6(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 4, d_v = 6\}$$
$$E_7(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 5, d_v = 6\}$$
$$E_8(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 5, d_v = 9\}$$
$$E_9(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 5, d_v = 10\}$$
$$E_{10}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 6, d_v = 6\}$$
$$E_{11}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 6, d_v = 9\}$$
$$E_{12}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 6, d_v = 10\}$$
$$E_{13}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 6, d_v = 12\}$$
$$E_{14}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 9, d_v = 10\}$$
$$E_{15}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 9, d_v = 12\}$$
$$E_{16}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 10, d_v = 10\}$$
$$E_{17}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 10, d_v = 12\}$$
$$E_{18}(HcDN4(n)) = \{uv \in E(HcDN4(n)) : d_u = 12, d_v = 12\}$$

$$|E_1(HcDN4(n))| = 12n$$
$$|E_2(HcDN4(n))| = 12$$
$$|E_3(HcDN4(n))| = 6(n-2)$$
$$|E_4(HcDN4(n))| = 6n$$
$$|E_5(HcDN4(n))| = 12(n-1)$$
$$|E_6(HcDN4(n))| = 12(n-1)$$
$$|E_7(HcDN4(n))| = 6(n-1)$$
$$|E_8(HcDN4(n))| = 12$$
$$|E_9(HcDN4(n))| = 12(n-2)$$
$$|E_{10}(HcDN4(n))| = 36n^2 - 72n + 36$$
$$|E_{11}(HcDN4(n))| = 12$$
$$|E_{12}(HcDN4(n))| = 18(n-2)$$
$$|E_{13}(HcDN4(n))| = 18n^2 - 54n + 42$$
$$|E_{14}(HcDN4(n))| = 12$$
$$|E_{15}(HcDN4(n))| = 6$$
$$|E_{16}(HcDN4(n))| = 6(n-3)$$
$$|E_{17}(HcDN4(n))| = 12(n-2)$$
$$|E_{18}(HcDN4(n))| = 9n^2 - 33n + 30$$

Now,

$$II_2(HcDN4(n)) = \prod_{uv\in E(HcDN4(n))} d_u \cdot d_v = (3\times 4)^{12n}$$
$$\times (3\times 9)^{12} \times (3\times 10)^{6(n-2)} \times (4\times 4)^{6n} \times (4\times 5)^{12(n-1)}$$
$$\times (4\times 6)^{12(n-1)} \times (5\times 6)^{6(n-1)} \times (5\times 9)^{12} \times (5\times 10)^{12(n-2)}$$
$$\times (6\times 6)^{36n^2-72n+36} \times (6\times 9)^{12} \times (6\times 10)^{18(n-2)}$$
$$\times (6\times 12)^{18n^2-54n+42} \times (9\times 10)^{12} \times (9\times 12)^{6}$$
$$\times (10\times 10)^{6(n-3)} \times (10\times 12)^{12(n-2)} \times (12\times 12)^{9n^2-33n+30}$$
$$= \frac{2^{96}}{3^{336}\times 5^{402}} \left(\frac{2^{162n}\times 3^{162n}\times 3^{36}\times 5^{210}}{2^{234}}\right)^n.$$

$$II_1^{\star}(HcDN4(n)) = \prod_{uv\in E(HcDN4(n))} d_u + d_v = (3+4)^{12n}$$
$$\times (3+9)^{12} \times (3+10)^{6(n-2)} \times (4+4)^{6n} \times (4+5)^{12(n-1)}$$
$$\times (4+6)^{12(n-1)} \times (5+6)^{6(n-1)} \times (5+9)^{12} \times (5+10)^{12(n-2)}$$
$$\times (6+6)^{36n^2-72n+36} \times (6+9)^{12} \times (6+10)^{18(n-2)}$$
$$\times (6+12)^{18n^2-54n+42} \times (9+10)^{12} \times (9+12)^{6}$$
$$\times (10+10)^{6(n-3)} \times (10+12)^{12(n-2)} \times (12+12)^{9n^2-33n+30}$$
$$= \frac{3^{156}\times 19^{12}}{2^{276}\times 5^{18}\times 7^{282}\times 11^{30}\times 13^{12}}$$
$$\left(\frac{2^{99n}\times 3^{45n}\times 5^{18}\times 7^{156}\times 11^{18}\times 13^{6}}{2^{9}\times 3^{153}}\right)^n.$$

$$HII_1(HcDN4(n)) = \prod_{uv\in E(HcDN4(n))} (d_u + d_v)^2 = \left[(3+4)^2\right]^{12n}$$
$$\times \left[(3+9)^2\right]^{12} \times \left[(3+10)^2\right]^{6(n-2)} \times \left[(4+4)^2\right]^{6n}$$
$$\times \left[(4+5)^2\right]^{12(n-1)} \times \left[(4+6)^2\right]^{12(n-1)} \times \left[(5+6)^2\right]^{6(n-1)}$$
$$\times \left[(5+9)^2\right]^{12} \times \left[(5+10)^2\right]^{12(n-2)} \times \left[(6+6)^2\right]^{36n^2-72n+36}$$
$$\times \left[(6+9)^2\right]^{12} \times \left[(6+10)^2\right]^{18(n-2)} \times \left[(6+12)^2\right]^{18n^2-54n+42}$$
$$\times \left[(9+10)^2\right]^{12} \times \left[(9+12)^2\right]^{6} \times \left[(10+10)^2\right]^{6(n-3)}$$
$$\times \left[(10+12)^2\right]^{12(n-2)} \times \left[(10+12)^2\right]^{9n^2-33n+30}$$
$$= \frac{3^{312}\times 19^{24}}{2^{552}\times 5^{36}\times 7^{564}\times 11^{60}\times 13^{24}}$$
$$\left(\frac{2^{198n}\times 3^{90n}\times 5^{36}\times 7^{312}\times 11^{36}\times 13^{12}}{2^{18}\times 3^{306}}\right)^n.$$

$$HII_2(HcDN4(n)) = \prod_{uv\in E(HcDN4(n))} (d_u \cdot d_v)^2 = \left[(3\times 4)^2\right]^{12n}$$
$$\times \left[(3\times 9)^2\right]^{12} \times \left[(3\times 10)^2\right]^{6(n-2)} \times \left[(4\times 4)^2\right]^{6n}$$
$$\times \left[(4\times 5)^2\right]^{12(n-1)} \times \left[(4\times 6)^2\right]^{12(n-1)} \times \left[(5\times 6)^2\right]^{6(n-1)}$$
$$\times \left[(5\times 9)^2\right]^{12} \times \left[(5\times 10)^2\right]^{12(n-2)} \times \left[(6\times 6)^2\right]^{36n^2-72n+36}$$

$$\times\left[(6\times9)^2\right]^{12}\times\left[(6\times10)^2\right]^{18(n-2)}\times\left[(6\times12)^2\right]^{18n^2-54n+42}$$
$$\times\left[(9\times10)^2\right]^{12}\times\left[(9\times12)^2\right]^{6}\times\left[(10\times10)^2\right]^{6(n-3)}$$
$$\times\left[(10\times12)^2\right]^{12(n-2)}\times\left[(12\times12)^2\right]^{9n^2-33n+30}$$
$$=\frac{2^{192}}{3^{672}\times5^{804}}\left(\frac{2^{324n}\times3^{252n}\times3^{72}\times5^{420}}{2^{468}}\right)^{n}.$$

$$MZ_1^a\left(HcDN4\left(n\right)\right)=\prod_{uv\in E(HcDN4(n))}\left(d_u+d_v\right)^a=\left[(3+4)^a\right]^{12n}$$
$$\times\left[(3+9)^a\right]^{12}\times\left[(3+10)^a\right]^{6(n-2)}\times\left[(4+4)^a\right]^{6n}$$
$$\times\left[(4+5)^a\right]^{12(n-1)}\times\left[(4+6)^a\right]^{12(n-1)}\times\left[(5+6)^a\right]^{6(n-1)}$$
$$\times\left[(5+9)^a\right]^{12}\times\left[(5+10)^a\right]^{12(n-2)}\times\left[(6+6)^a\right]^{36n^2-72n+36}$$
$$\times\left[(6+9)^a\right]^{12}\times\left[(6+10)^a\right]^{18(n-2)}\times\left[(6+12)^a\right]^{18n^2-54n+42}$$
$$\times\left[(9+10)^a\right]^{12}\times\left[(9+12)^a\right]^{6}\times\left[(10+10)^a\right]^{6(n-3)}$$
$$\times\left[(10+12)^a\right]^{12(n-2)}\times\left[(10+12)^a\right]^{9n^2-33n+30}$$
$$=\left(\frac{3^{156}\times19^{12}}{2^{276}\times5^{18}\times7^{282}\times11^{30}\times13^{12}}\right)^{a}$$
$$\left(\frac{2^{99n}\times3^{45n}\times5^{18}\times7^{156}\times11^{18}\times13^{6}}{2^{9}\times3^{153}}\right)^{an}.$$

$$MZ_2^a\left(HcDN4\left(n\right)\right)=\prod_{uv\in E(HcDN4(n))}\left(d_u\cdot d_v\right)^a=\left[(3\times3)^a\right]^{6}$$
$$\times\left[(3\times5)^a\right]^{12(n-1)}\times\left[(3\times9)^a\right]^{12}\times\left[(3\times10)^a\right]^{6(n-2)}$$
$$\times\left[(5\times6)^a\right]^{6(n-1)}\times\left[(5\times9)^a\right]^{12}\times\left[(5\times10)^a\right]^{12(n-2)}$$
$$\times\left[(6\times6)^a\right]^{9n^2-21n+12}\times\left[(6\times9)^a\right]^{12}\times\left[(6\times10)^a\right]^{18(n-2)}$$
$$\times\left[(6\times12)^a\right]^{18n^2-54n+42}\times\left[(9\times10)^a\right]^{12}\times\left[(9\times12)^a\right]^{6}$$
$$\times\left[(10\times10)^a\right]^{6(n-3)}\times\left[(10\times12)^a\right]^{12(n-2)}$$
$$\times\left[(12\times12)^a\right]^{9n^2-33n+30}=\left(\frac{2^{96}}{3^{336}\times5^{402}}\right)^{a}$$
$$\left(\frac{2^{162n}\times3^{162n}\times3^{36}\times5^{210}}{2^{234}}\right)^{an}.$$

$$XII\left(HcDN4\left(n\right)\right)=\prod_{uv\in E(HcDN4(n))}\sqrt{\frac{1}{d_u+d_v}}=\left(\frac{1}{\sqrt{3+4}}\right)^{12n}$$
$$\times\left(\frac{1}{\sqrt{3+9}}\right)^{12}\times\left(\frac{1}{\sqrt{3+10}}\right)^{6(n-2)}\times\left(\frac{1}{\sqrt{4+4}}\right)^{6n}$$
$$\times\left(\frac{1}{\sqrt{4+5}}\right)^{12(n-1)}\times\left(\frac{1}{\sqrt{4+6}}\right)^{12(n-1)}\times\left(\frac{1}{\sqrt{5+6}}\right)^{6(n-1)}$$
$$\times\left(\frac{1}{\sqrt{5+9}}\right)^{12}\times\left(\frac{1}{\sqrt{5+10}}\right)^{12(n-2)}\times\left(\frac{1}{\sqrt{6+6}}\right)^{36n^2-72n+36}$$
$$\times\left(\frac{1}{\sqrt{6+9}}\right)^{12}\times\left(\frac{1}{\sqrt{6+10}}\right)^{18(n-2)}\times\left(\frac{1}{\sqrt{6+12}}\right)^{18n^2-54n+42}$$
$$\times\left(\frac{1}{\sqrt{9+10}}\right)^{12}\times\left(\frac{1}{\sqrt{9+12}}\right)^{6}\times\left(\frac{1}{\sqrt{10+10}}\right)^{6(n-3)}$$
$$\times\left(\frac{1}{\sqrt{10+12}}\right)^{12(n-2)}\times\left(\frac{1}{\sqrt{12+12}}\right)^{9n^2-33n+30}$$
$$=\frac{2^{138}\times5^{9}\times7^{141}\times11^{15}\times13^{6}}{3^{78}\times19^{6}}$$
$$\left(\frac{2^{\frac{9}{2}}\times3^{\frac{153}{2}}}{2^{\frac{99}{2}n}\times3^{\frac{45}{2}n}\times5^{9}\times7^{78}\times11^{9}\times13^{3}}\right)^{n}.$$

$$\chi II\left(HcDN4\left(n\right)\right)=\prod_{uv\in E(HcDN4(n))}\frac{1}{\sqrt{d_u.d_v}}$$
$$\left(\frac{1}{\sqrt{3.4}}\right)^{12n}\times\left(\frac{1}{\sqrt{3.9}}\right)^{12}\left(\frac{1}{\sqrt{3.10}}\right)^{6(n-2)}\times\left(\frac{1}{\sqrt{4.4}}\right)^{6n}$$
$$\times\left(\frac{1}{\sqrt{4.5}}\right)^{12(n-1)}\times\left(\frac{1}{\sqrt{4.6}}\right)^{12(n-1)}\times\left(\frac{1}{\sqrt{5.6}}\right)^{6(n-1)}$$
$$\times\left(\frac{1}{\sqrt{5.9}}\right)^{12}\times\left(\frac{1}{\sqrt{5.10}}\right)^{12(n-2)}\times\left(\frac{1}{\sqrt{6.6}}\right)^{36n^2-72n+36}$$
$$\times\left(\frac{1}{\sqrt{6.9}}\right)^{12}\times\left(\frac{1}{\sqrt{6.10}}\right)^{18(n-2)}\times\left(\frac{1}{\sqrt{6.12}}\right)^{18n^2-54n+42}$$
$$\times\left(\frac{1}{\sqrt{9.10}}\right)^{12}\times\left(\frac{1}{\sqrt{9.12}}\right)^{6}\times\left(\frac{1}{\sqrt{10.10}}\right)^{6(n-3)}$$
$$\times\left(\frac{1}{\sqrt{10.12}}\right)^{12(n-2)}\times\left(\frac{1}{\sqrt{12.12}}\right)^{9n^2-33n+30}$$
$$=\frac{3^{168}\times5^{201}}{2^{48}}\left(\frac{2^{117}}{2^{81n}\times3^{63n}\times3^{18}\times5^{105}}\right)$$

$$ABCII(HcDN4(n))=\prod_{uv\in E(HcDN4(n))}\sqrt{\frac{d_u+d_v-2}{d_u\cdot d_v}}$$
$$=\left(\sqrt{\frac{3+4-2}{3\cdot4}}\right)^{12n}\times\left(\sqrt{\frac{3+9-2}{3\cdot9}}\right)^{12}$$
$$\times\left(\sqrt{\frac{3+10-2}{3\cdot10}}\right)^{6(n-2)}\times\left(\sqrt{\frac{4+4-2}{4\cdot4}}\right)^{6n}$$
$$\times\left(\sqrt{\frac{4+5-2}{4\cdot5}}\right)^{12(n-1)}\times\left(\sqrt{\frac{4+4-2}{4\cdot6}}\right)^{12(n-1)}$$
$$\times\left(\sqrt{\frac{5+6-2}{5\cdot6}}\right)^{6(n-1)}\times\left(\sqrt{\frac{5+9-2}{5\cdot9}}\right)^{12}$$
$$\times\left(\sqrt{\frac{5+10-2}{5\cdot10}}\right)^{12(n-2)}\times\left(\sqrt{\frac{6+6-2}{6\cdot6}}\right)^{36n^2-72n+36}$$
$$\times\left(\sqrt{\frac{6+9-2}{6\cdot9}}\right)^{12}\times\left(\sqrt{\frac{6+10-2}{6\cdot10}}\right)^{18(n-2)}$$
$$\times\left(\sqrt{\frac{6+12-2}{6\cdot12}}\right)^{18n^2-54n+42}\times\left(\sqrt{\frac{9+10-2}{9\cdot10}}\right)^{12}$$

$$\times\left(\sqrt{\frac{9+12-2}{9\cdot 12}}\right)^{6}\times\left(\sqrt{\frac{10+10-2}{10\cdot 10}}\right)^{6(n-3)}$$

$$\times\left(\sqrt{\frac{10+12-2}{10\cdot 12}}\right)^{12(n-2)}\times\left(\sqrt{\frac{12+12-2}{12\cdot 12}}\right)^{9n^2-33n+30}$$

$$=\frac{5^{213}\times 11^{9}\times 13^{6}\times 17^{6}\times 19^{3}}{2^{282}\times 7^{24}}$$

$$\left(\frac{2^{\frac{483}{2}n}\times 3^{57n}\times 5^{36n}\times 2^{\frac{399}{2}}\times 3^{273}\times 5^{123}\times 7^{15}\times 11^{\frac{15}{2}}}{11^{\frac{33}{2}}}\right)^{n}.$$

$$GAII\,(HcDN4\,(n))=\prod_{uv\in E(HcDN4(n))}\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}$$

$$=\left(\frac{2\sqrt{3\cdot 4}}{3+4}\right)^{12n}\times\left(\frac{2\sqrt{3\cdot 9}}{3+9}\right)^{12}\times\left(\frac{2\sqrt{3\cdot 10}}{3+10}\right)^{6(n-2)}$$

$$\times\left(\frac{2\sqrt{4\cdot 4}}{4+4}\right)^{6n}\times\left(\frac{2\sqrt{4\cdot 5}}{4+5}\right)^{12(n-1)}\times\left(\frac{2\sqrt{4\cdot 6}}{4+6}\right)^{12(n-1)}$$

$$\times\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^{6(n-1)}\times\left(\frac{2\sqrt{5\cdot 9}}{5+9}\right)^{12}\times\left(\frac{2\sqrt{5\cdot 10}}{5+10}\right)^{12(n-2)}$$

$$\times\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{36n^2-72n+36}\times\left(\frac{2\sqrt{6\cdot 9}}{6+9}\right)^{12}$$

$$\times\left(\frac{2\sqrt{6\cdot 10}}{6+10}\right)^{18(n-2)}\times\left(\frac{2\sqrt{6\cdot 12}}{6+12}\right)^{18n^2-54n+42}$$

$$\times\left(\frac{2\sqrt{9\cdot 10}}{9+10}\right)^{12}\times\left(\frac{2\sqrt{9\cdot 12}}{9+12}\right)^{6}\times\left(\frac{2\sqrt{10\cdot 10}}{10+10}\right)^{6(n-3)}$$

$$\times\left(\frac{2\sqrt{10\cdot 12}}{10+12}\right)^{12(n-2)}\times\left(\frac{2\sqrt{12\cdot 12}}{12+12}\right)^{9n^2-33n+30}$$

$$=\frac{2^{30}\times 11^{30}\times 13^{12}}{5^{33}\times 7^{18}\times 19^{12}}\left(\frac{2^{27n}\times 2^{3}\times 3^{51}\times 5^{15}}{3^{18n}\times 7^{12}\times 11^{18}\times 13^{6}}\right)^{n}.$$

$$GA^{a}II\,(HcDN4\,(n))=\prod_{uv\in E(HcDN4(n))}\left(\frac{2\sqrt{d_u\cdot d_v}}{d_u+d_v}\right)^{a}$$

$$=\left[\left(\frac{2\sqrt{3\cdot 4}}{3+4}\right)^{a}\right]^{12n}\times\left[\left(\frac{2\sqrt{3\cdot 9}}{3+9}\right)^{a}\right]^{12}$$

$$\times\left[\left(\frac{2\sqrt{3\cdot 10}}{3+10}\right)^{a}\right]^{6(n-2)}\times\left[\left(\frac{2\sqrt{4\cdot 4}}{4+4}\right)^{a}\right]^{6n}$$

$$\times\left[\left(\frac{2\sqrt{4\cdot 5}}{4+5}\right)^{a}\right]^{12(n-1)}\times\left[\left(\frac{2\sqrt{4\cdot 6}}{4+6}\right)^{a}\right]^{12(n-1)}$$

$$\times\left[\left(\frac{2\sqrt{5\cdot 6}}{5+6}\right)^{a}\right]^{6(n-1)}\times\left[\left(\frac{2\sqrt{5\cdot 9}}{5+9}\right)^{a}\right]^{12}$$

$$\times\left[\left(\frac{2\sqrt{5\cdot 10}}{5+10}\right)^{a}\right]^{12(n-2)}\times\left[\left(\frac{2\sqrt{6\cdot 6}}{6+6}\right)^{a}\right]^{36n^2-72n+36}$$

$$\times\left[\left(\frac{2\sqrt{6\cdot 9}}{6+9}\right)^{a}\right]^{12}\times\left[\left(\frac{2\sqrt{6\cdot 10}}{6+10}\right)^{a}\right]^{18(n-2)}$$

$$\times\left[\left(\frac{2\sqrt{6\cdot 12}}{6+12}\right)^{a}\right]^{18n^2-54n+42}\times\left[\left(\frac{2\sqrt{9\cdot 10}}{9+10}\right)^{a}\right]^{12}$$

$$\times\left[\left(\frac{2\sqrt{9\cdot 12}}{9+12}\right)^{a}\right]^{6}\times\left[\left(\frac{2\sqrt{10\cdot 10}}{10+10}\right)^{a}\right]^{6(n-3)}$$

$$\times\left[\left(\frac{2\sqrt{10\cdot 12}}{10+12}\right)^{a}\right]^{12(n-2)}\times\left[\left(\frac{2\sqrt{12\cdot 12}}{12+12}\right)^{a}\right]^{9n^2-33n+30}$$

$$=\left(\frac{2^{30}\times 11^{30}\times 13^{12}}{5^{33}\times 7^{18}\times 19^{12}}\right)^{a}\left(\frac{2^{27n}\times 2^{3}\times 3^{51}\times 5^{15}}{3^{18n}\times 7^{12}\times 11^{18}\times 13^{6}}\right)^{an}.$$

## 4 Conclusions

In recent years, the computational physics problem on special molecular network structures has gained attention in theoretical physics. From the standpoint of graph theory, one learns a real-valued function that assigns scores to molecular graphs. What is important is the relative physicochemical characters of the corresponding compounds induced by those scores. This topic is distinct from both classical graph theory and physical computing problems, and it is natural to ask what kinds topological index and molecular structure generalizations hold for this problem. Although there have been several recent advances in developing results for various settings of the molecular calculation problem, the study of topological indices of special molecular networks has been largely limited to the special mathematical setting. In this manuscript, the authors study the multiplicative degree based topological indices of honeycomb derived networks of dimension 1, 2, 3 and 4, and their specific expressions are obtained. These results can also play a vital part in the determination of the significance of honeycomb derived networks. For example, it has been experimentally verified that the first Zagreb index is directly related to total $\pi$-electron energy. The Randić index is useful for determining physicochemical properties of alkanes as noticed by the chemist Melan Randić in 1975. He noticed the correlation between the Randic index R and several physicochemical properties of alkanes like enthalpies of formation, boiling points, chromatographic retention times, vapor pressure and surface areas. Calculations of distance-based and surface-based topological indices of these networks are an open problem in this area of research. Also, the construction of new networks by taking line graphs and para-line graphs of honeycomb networks is an interesting problem.

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