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Urban road traffic flow control under incidental congestion as a function of accident duration

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Abstract: An abrupt increase in urban road traffic flow caused by incidental congestion is considered. The residual traffic capacity varies in different lanes after an accident, and the influence of accident duration on traffic flow is taken into account. The swallowtail catastrophe model was built based on catastrophe theory. The critical state of traffic congestion under incidental congestion was analyzed using this model, and a traffic flow control scheme is proposed with the goal of maximizing the traffic capacity. Finally, the operational state of traffic flow under different scenarios is analyzed through case study and the feasibility of the model is validated.

Keywords: Traffic flow control, incidental congestion, accident duration

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1 Introduction

Incidental congestion refers to the congestion caused by traffic accidents and emergency events on the road. The duration of incidental congestion has a large impact on traffic flow. Herein urban road traffic flow control is studied under incidental congestion as a function of accident duration. The findings can lead to improvement of operational efficiency in traffic flow [1, 2].

Among relevant studies, Zhang et al. [3] considered velocity as the major variable along with traffic flow and rate of passage during modeling. Tang and Huang [4] introduced time as another control variable into the swallowtail catastrophe model, which was used for traffic flow prediction. Ao et al. [5] considered mixed motorized traffic flow, and took the average velocity as a state variable and the mixed rate of large vehicles and traffic flow as control variables, and built the cusp catastrophe model. Hu et al. [6] introduced time as another control variable on this basis and built the swallowtail catastrophe model to analyze traffic flow congestion. Although the above studies on urban road traffic flow control generally produced realistic findings, the studies only considered single-lane situations. Under the actual multi-lane situation, the traffic capacity of different lanes varies as well [7] and the duration of accidents has a considerable impact on traffic flow.

This paper considers a multi-lane situation in urban roads and discusses traffic flow control problems under incidental congestion. The number of passable lanes will decrease once incidental congestion occurs, the actual traffic capacity of different lanes varies, and the traffic density increases. In addition, accident duration has an impact on traffic flow. In this study the occupancy rate of traffic volume in carrying capacity of the road is taken as a state variable. The variation rate of traffic flow and change in the rate of passage are examined as dynamic indicators. The swallowtail catastrophe model for traffic flow under incidental congestion is built. The critical state of traffic congestion under incidental congestion is analyzed by using this model. Finally, the traffic flow control scheme is proposed with the goal of maximizing the traffic capacity.

2 Catastrophe theory

The French mathematician, Thom, first described catastrophe theory in his book *Structural Stability and Morphogenesis*. Catastrophe theory usually consists of two parameters (state and control variables). The number of control variables determines the state of the system. The catastrophe theory attempts to identify the catastrophe occurring in the system by looking for the critical points of potential functions.

Depending on the number of state and control variables, the catastrophe model has 7 different forms, includ-

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ing cusp and swallowtail. Based on an analysis of index and state variables, modeling was conducted using the catastrophe function. With the theoretical model of evolution thus formed, the features of the system were investigated and the rules of catastrophe in the system were understood in theory. This model can aid the prevention of a catastrophe by predicting the time, location, and consequence of the catastrophe.

Catastrophe phenomena occurs in the urban road traffic system under incidental congestion. The catastrophe theory can characterize the changes in the system caused by different parameters. This study chose the swallowtail catastrophe model to analyze the critical state of congestion upon the occurrence of catastrophe (incidental congestion) in an urban road system. A scheme for traffic flow control under this condition is proposed.

3 State and control variables of the urban road traffic system under incidental congestion

The occupancy rate of traffic volume and the carrying capacity of the road is a state variable, denoted as $\eta(t)$:

$$\eta(t) = \frac{X(t)}{Y(t)} \times 100\% \tag{1}$$

where $X(t)$ is the traffic flow of the system and $Y(t)$ is the carrying capacity of the road.

Variation in the traffic flow rate, variation in the residual traffic capacity of the lane, and time-variation are the control variables.

Variation in traffic density is a control variable of the traffic system. According to Greenshields velocity function, the mathematical expressions involving velocity $u(t)$, traffic flow $V(t)$, and density $K(t)$ are $u(t) = aK(t) + b$ and $V(t) = K(t)u(t) = aK(t)^2 + bK(t)$, where a and b are coefficients obtained by regression analysis. Because the traffic flow is linearly related to velocity, the traffic flow rate is measured by traffic density. Thus, the variation in traffic flow is $M(t)$:

$$M(t) = \frac{V(i) - V(t)}{V(i)} \times 100\% \tag{2}$$

where $V(i)$ is the traffic flow rate at the moment of the catastrophe; $V(t)$ is the traffic flow rate at time t, and $V(t) = aK(t)^2 + bK(t)$.

Variation in the residual traffic capacity is another control variable of the system, denoted as $N(t)$:

$$N(t) = \frac{C(i) - C(t)}{C(i)} \times 100\% \tag{3}$$

$C(i)$ is the traffic capacity at the moment of catastrophe, $C(t)$ is the traffic capacity at time t, and $C(t) = C\alpha_j P$. C is the design traffic capacity of the lane. P is the lane loss coefficient, which reflects the decrease in traffic capacity caused by accident. P is expressed as the ratio of the number of unoccupied, passable lanes to the original number of lanes. $P = \frac{N-n}{N} \times 100\%$, where n is the number of occupied lanes, and N is the original number of lanes [8, 9].

The traffic capacity varies for different lanes. The traffic state of each lane is an important factor influencing the traffic capacity of the lane. Multi-lane design is generally adopted in urban road planning, and the stop and overtaking of vehicles in one lane will cause disturbance to the traveling of vehicles in another lane. This will further influence the overall traffic capacity. Generally, the center lane is the least disturbed by accidents [10]. When there is no separation zone in the lanes of the same direction, the traffic capacity of the center lane is the highest. Thus, the center lane is defined as lane 1. The lane on the rightmost side has the lowest traffic capacity, and it is defined as lane 3.

The degree of influence is determined by the lane utilization coefficient, α_q. Starting from the center line of the road, the coefficient for each lane on two sides of the center line is 1.00, 0.8-0.89 (0.87), and 0.65-0.78 (0.73). The traffic capacity of a lane is calculated by multiplying the traffic capacity of one lane (usually the inner lane) by the lane utilization coefficient of the corresponding lane. Because accidents may occur in any lane, let $\alpha_j = 1 - \frac{\sum \alpha_{q_i^*}}{\sum \alpha_q} (i = 1, 2, 3)$ be the traffic capacity loss coefficient of the lane, where $\frac{\sum \alpha_{q_i^*}}{\sum \alpha_q} (i = 1, 2, 3)$ is the weight of the utilization coefficient of the lane in which the accident occurs to the sum of lane utilization coefficients. $\alpha_{q_i^*} (i = 1, 2, 3)$ is the utilization coefficient of the lane occupied by the accident.

Accident duration is the length of time from the moment of an accident to restoration of normal traffic flow. Accident duration has a direct impact on traffic capacity [11]. The longer the accident duration, the longer the duration of the reduction in traffic capacity and the more severe the consequence of the accident. The normal traffic flow will not be restored until the accidents are cleared. Therefore, the influence of accident duration on traffic flow is considered, and time-variation is chosen as the third control variable of the system, denoted as $T(t)$:

$$T(t) = \frac{T_z(t) - T_s(t)}{T_z(t)} \tag{4}$$

where $T_s(t)$ is the accident duration. $T_z(t)$ is the time required for a vehicle to pass through this lane without incidental congestion.

4 Swallowtail catastrophe model for urban road traffic under incidental congestion

The occupancy rate of traffic volume in the carrying capacity of the road is the dynamic variable. The passage rate of vehicles, traffic capacity, and time of passage are determined through the swallowtail catastrophe model for urban road traffic under incidental congestion:

Potential function:

$$E(\eta) = \eta(t)^5 + M(t)\eta(t)^3 + N(t)\eta(t)^2 + T(t)\eta(t) \quad (5)$$

Equilibrium surface:

$$5\eta(t)^4 + 3M(t)\eta(t)^2 + 2N(t)\eta(t) + T(t) = 0 \quad (6)$$

The bifurcation set must simultaneously satisfy

$$5\eta(t)^4 + 3M(t)\eta(t)^2 + 2N(t)\eta(t) + T(t) = 0$$

$$20\eta(t)^3 + 6M(t)\eta(t) + 2N(t) = 0 \quad (7)$$

Simultaneous equations are built using formulae (6) and (7). The bifurcation set of the swallowtail catastrophe model is obtained by eliminating $\eta(t)$; however, this method is complicated, so the bifurcation set is obtained by the equilibrium section method [12]. As shown in Figure 1, the bifurcation set is a curved surface in 3D space composed of $M(t)$, $N(t)$, and $T(t)$.

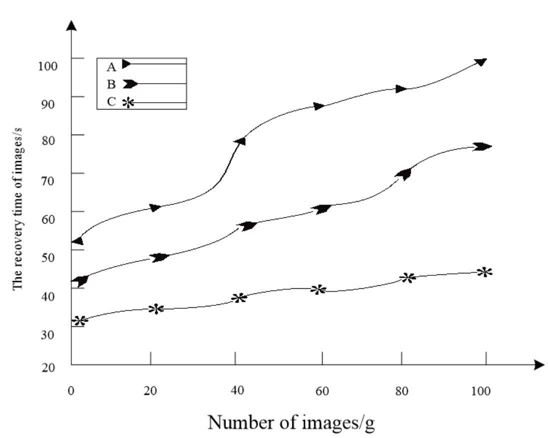

Figure 1: Bifurcation set of the swallowtail catastrophe model

5 Critical state of catastrophe and stability analysis

5.1 Catastrophe analysis

According to catastrophe theory, the bifurcation set has features of points in a set. The bifurcation set is composed of a series of critical values of the catastrophe, and the points on the surface may be the critical points leading to catastrophe in the traffic flow [13, 14]. When the value ranges of the three control variables are not located within the range of the bifurcation set, the traffic flow changes instantaneously. This time point corresponds to the critical point of the catastrophe. The occurrence of a catastrophe depends on whether or not the equilibrium positions of the bifurcation set have changed. Because the curve of the bifurcation set is multi-dimensional in space, the state variable $\eta(t)$ is considered a parameter. With one control variable fixed, the plane constituted by the other two control variables is discussed. This method is used to analyze the critical state of traffic congestion.

5.1.1 Because $M(t)$ is constant, the $N(t) - T(t)$ plane is discussed. Because the bifurcation sets with $M(t) \geq 0$ or $M(t) < 0$ are different, classified discussion is conducted.

When $M(t) \geq 0$, the plane is divided into two parts, as shown in Figure 2:

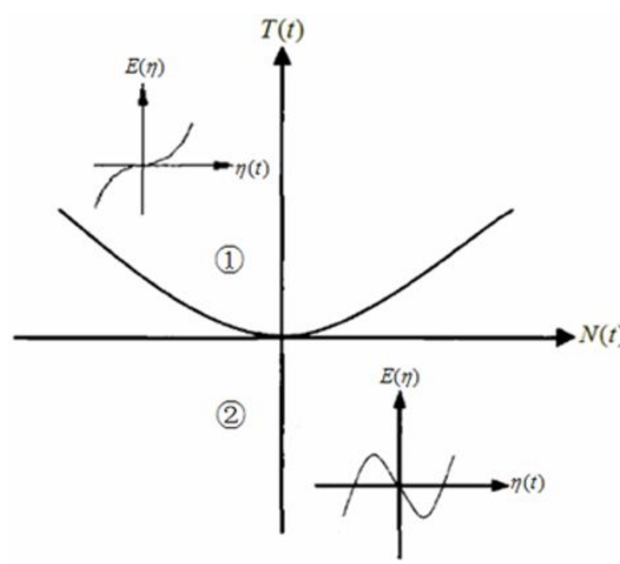

Figure 2: Bifurcation set of the swallowtail catastrophe model and the potential function ($M(t) \geq 0$)

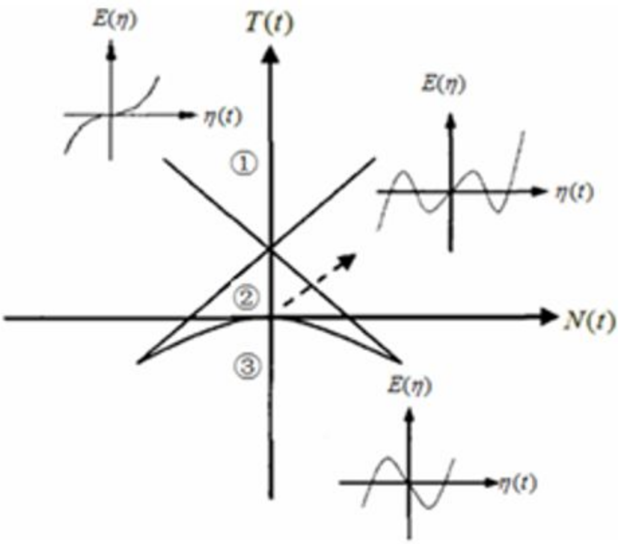

Figure 3: Bifurcation set of the swallowtail catastrophe model and the potential function ($M(t) < 0$)

Given the symmetry of the set, we only need to analyze the situation when $N(t) = 0$. The equilibrium surface is expressed as $\eta^2(t) = \frac{1}{10}\left[-3M(t) \pm \sqrt{9M^2(t) - 20T(t)}\right]$.

When $T(t) > 0$, it corresponds to region I in Figure 1. At this time $\eta^2(t) < 0$ and $\eta(t)$ has no real root; the equation for the equilibrium surface has no real solutions. That is, the potential function has no singularity, and there is no equilibrium position.

When $T(t) < 0$, it corresponds to region III in Figure 1. At this time, $\eta(t)$ has two real roots (one positive and the other negative). That is, the equation for the equilibrium surface has symmetric solutions, and the curve is symmetric as well. The function has different singularities. When the solution is positive, $E^{'}(\eta) < 0, E^{\star}(\eta) > 0$, corresponds to the point of minimum value and the point of stable equilibrium. When the solution is negative, $E^{'}(\eta) > 0, E^{\star}(\eta) < 0$, corresponds to the point of maximum value and the point of unstable equilibrium.

When $M(t) < 0$, the plane is divided into three parts, as shown in Figure 3.

When $T(t) > 0.45M^2(t)$, it corresponds to region I in Figure 1. The equation for the equilibrium surface has no real solutions, and the corresponding potential function has no singularities.

When $0 < T(t) < 0.45M^2(t)$, it corresponds to region II in Figure 1. At this time, the range of $\eta^2(t)$ is both positive and negative. By solving the equation, four positive or negative solutions are obtained, indicating that the function has four singularities, which are represented as four extreme values on the curve. This is verified through equation $E^{\star}(\eta) = 20\eta(t)^3 + 6M(t)\eta(t) + 2N(t)$. The extreme values correspond to the catastrophe points of the curve, and exhibit a symmetric distribution.

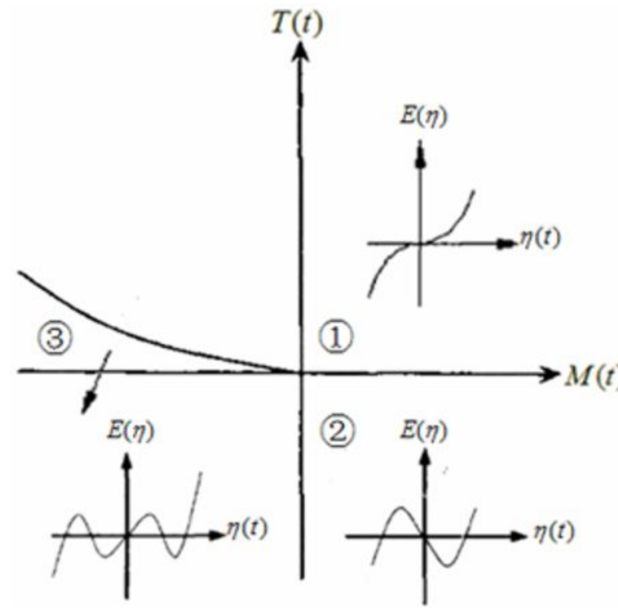

Figure 4: Bifurcation set of the swallowtail catastrophe model and the potential function ($N(t) = 0$)

By solving the equation, we obtain two different solutions for the equilibrium surface, which correspond to two extreme points and to the catastrophe points on the curve.

5.1.2 When $N(t) = 0$, the $M(t) - T(t)$ plane is discussed. As shown in Figure 4, the plane is divided into three parts.

When $M(t) > 0$, $T(t) > 0$ or $M(t) < 0$, $T(t) > 0.45M^2(t)$, it corresponds to region I in Figure 1. This equation has no solutions; thus, the curve has no singularities.

When $T(t) < 0$, it corresponds to region III in Figure 1. We obtained four positive or negative solutions for this equation, indicating that the function has two singularities. They are represented as two extreme points on the curve, corresponding to two different catastrophe points.

When $M(t) < 0$, $0 < T(t) < 0.45M^2(t)$, it corresponds to region II in Figure 1. The equilibrium surface has four real solutions (two positive and two negative). Thus, the potential function has four equilibrium positions, two of which are stable and two of which are unstable. The equilibrium positions correspond to two minimum points and two extreme points among the four singularities.

5.1.3 When $T(t) = 0$, the $M(t) - N(t)$ plane is discussed. The plane is divided into four parts, as shown in Figure 5.

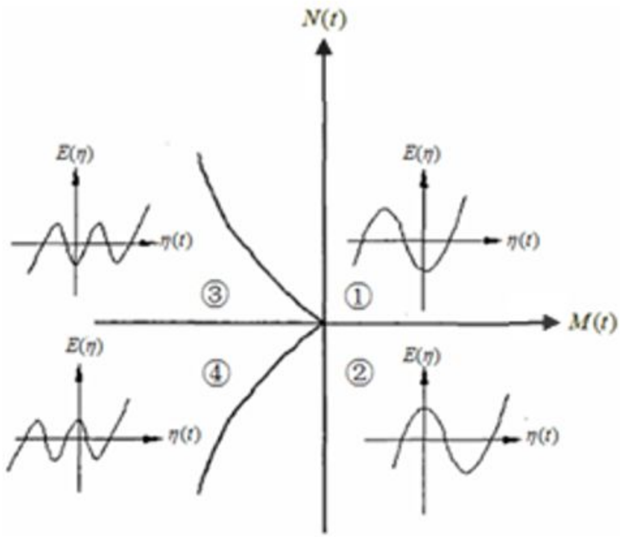

Figure 5: Bifurcation set of swallowtail catastrophe model and the potential function ($T(t) = 0$)

At this time, the solution for the equilibrium surface contains $\sqrt{5M^3(t) + 25N^2(t)}$.

When $M(t) > 0, N(t) > 0$ or $M(t) < 0, N(t) > \sqrt{-M^3(t)/5}$, it corresponds to region V in Figure 1. The equation for the equilibrium surface has different solutions. Zero and negative solutions are represented by the equilibrium points on the curve.

When $M(t) > 0, N(t) < 0$ or $M(t) < 0, N(t) < -\sqrt{-M^3(t)/5}$, it corresponds to region IV in Figure 1. The equation for the equilibrium surface has different solutions. The zero and positive solutions are represented by different equilibrium points on the curve. The positive solutions are unstable.

When $M(t) < 0, 0 < N(t) < \sqrt{-M^3(t)/5}$, it corresponds to region II in Figure 1. The equation for the equilibrium surface has two different groups of solutions, among which the positive solutions are unstable and the negative solutions are stable.

When $M(t) < 0, -\sqrt{-M^3(t)/5} < N(t) < 0$, it corresponds to region II in Figure 1. The equation for the equilibrium surface has two different groups of solutions, among which the positive and negative solutions are stable and the zero solution is unstable.

Based on the above discussion on potential functions of the bifurcation set for each plane, the potential functions and equilibrium points for each region in Figure 6

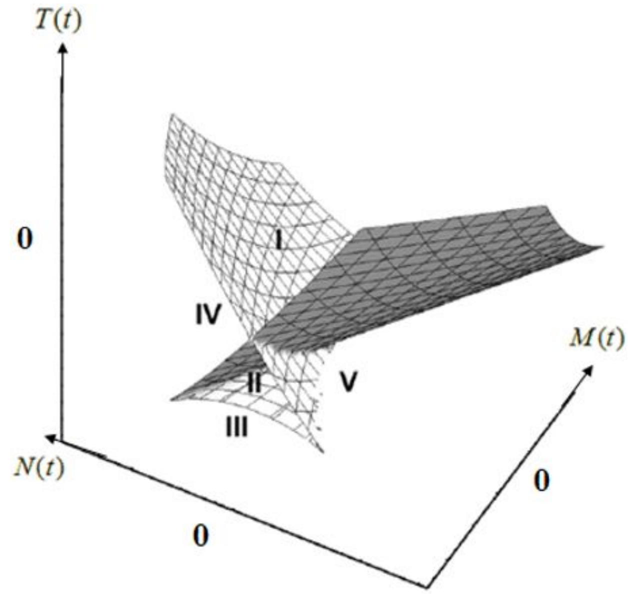

Figure 6: Bifurcation set of the swallowtail catastrophe model in the 3D space

are summarized below. In region I, the potential function has no singularities, and therefore no equilibrium positions. In region II, the potential function has more singularities, with two different groups of extreme points which display stability or instability. On the corresponding curves, the extreme points are alternately arranged, indicating that different equilibrium states may be mutually transformable. Potential functions in region III, IV and V have disparate extremum points, which are alternately arranged, indicating the risk of catastrophe.

The stable state of the system is the state corresponding to the minimum of the potential function. When $E^{\star}(\eta) > 0$, the potential function has a minimum point (the stable equilibrium point). The stable singularity indicates that the traffic flow is in a stable state and that the system is stable and not prone to catastrophe. When $E^{\star}(\eta) < 0$, the potential function has a maximum point (the unstable equilibrium point), corresponding to the unstable state of the system. Therefore, the system is likely to change from the current state to another state. Specifically, an unstable system is susceptible to catastrophe. There is a greater probability for the system to change state when there are two unstable singularities. When $E^{\star}(\eta) = 0$, we cannot determine the state of the traffic flow. Without another imposed disturbance, the system under this state will not change to another state and the catastrophe will not occur.

Because of the heterogeneity of the three control variables, there are bifurcation sets for the traffic flow in different ranges. Different forms of potential functions lead to changes of the state variables in different state spaces, which may result in a catastrophe. When the unstable points cross over the bifurcation set, the properties of the unstable singularities will change. The unstable points either change from a stable state to an unstable state, or the stable state vanishes completely, leading to a catastrophe in traffic flow. The greater the number of unstable points in the region, the greater the probability of the catastrophe. b. If the stable points are alternately arranged with the unstable points, the region with more stable points will evolve towards the direction with fewer stable points. When the stable points cross over the bifurcation set, the extreme points in the function will cause catastrophe of the system (i.e., traffic congestion). When the stable points change to an unstable state or vanish completely, there will be a catastrophe in the traffic flow. In contrast, the region with fewer stable equilibrium points will evolve towards the direction with more stable points. When the stable points cross over the bifurcation set, the extreme points in the function will no longer cause catastrophe of the system. The entire system is still in a stable state.

5.2 Analysis of catastrophe features

Swallowtail catastrophe has some basic features, as follows: sudden jump; hysteresis; divergence; multimodality; and inaccessibility.

5.2.1 Sudden jump

Sudden jump is the most commonly observed and most frequently used feature in catastrophe theory. Sudden jump refers to a large change in the value of the potential function within a short period of time as a result of the changes in the control variables. As the traffic flow increases continuously, the value of the potential function also increases. When the traffic flow suddenly changes to a degree that exceeds traffic capacity, the traffic flow will instantaneously exceed the critical point of catastrophe. Hence, the value of the potential function decreases suddenly and traffic congestion occurs.

5.2.2 Hysteresis

The occurrence of catastrophe is related to the direction of control variable change. The position of the control variable changes when it jumps from the first local minimum to the second local minimum, as compared with the situation when the control variable jumps from the second local minimum to the first local minimum. That is, as the direction of changing is reversed, the state of catastrophe changes as well. This phenomenon is referred to as hysteresis. The traffic flow system obeys the Maxwell convention and there is no hysteresis.

5.2.3 Divergence

Changes in the state variables are caused by control variable changes. Under most situations, even extremely small changes in the dependent variables will cause oscillation of the system state. The influence of the variable is more prominent in the region of the bifurcation set. Consequently, the system will change into a completely different state. Such instability caused by the disturbance of the control variables is known as divergence. An example of this is the severe traffic congestion after the sudden jump.

5.2.4 Multimodality

Under the action of different conditions and factors, the system will exhibit different states. Within the range of the model there are usually two unequal corresponding state variables. The potential function may have two or more minima. With respect to traffic flow, either unobstructed traffic flow or traffic congestion may correspond to the minima. This feature represents the multimodality of a swallowtail catastrophe.

5.2.5 Inaccessibility

A system has an unstable equilibrium position. When the system jumps from one stable state to another stable state, a catastrophe occurs. It is impossible for an actual system to directly cross over an unstable equilibrium position. As the state of traffic flow evolves from stable to catastrophe, there is no intermediate state. Thus, the intermediate state is inaccessible.

6 Urban road traffic flow control scheme under incidental congestion

The duration of a traffic accident consists of the time to discover the accident. Following an accident, the responsible driver will report to the traffic administrative department. If the message is transmitted accurately and in a timely, the time to discover the accident is designated as zero time [15], which is followed by the time to respond to the accident, time to clear the accident, and time to restore normal traffic. The fourth stage is the restoration of traffic flow and disappearance of congestion after handling the traffic accident. Traffic congestion is closely related to the accident. More importantly, the scale, nature, and time of the traffic accident and traffic flow are uncontrollable variables. The duration of the traffic accident can be reduced by increasing the efficiency of the traffic accident response and handling [16–34].

The principle of traffic flow control is to determine the state of traffic flow based on the catastrophe model and to control catastrophe in traffic flow. The values of the three control variables can be obtained from relevant data and the region of the bifurcation set where the traffic flow is located can be determined. By analyzing the properties of sets in different regions, we can gain a thorough understanding of the stability of the entire system and the influence of different variables on the system. Unfavorable changes can be avoided by taking countermeasures against the inducing factors to reduce system risk and to make the system more stable. The catastrophe model, as described above, can aid the response and handling of the traffic accidents. The goal is to maximize the traffic capacity following the incidental accidents. Region II of the bifurcation set corresponds to the most unstable traffic flow. If the traffic flow system is located in region II, a catastrophe will occur regardless of the direction of change in the system. To control the urban road traffic flow under incidental congestion, it is necessary to determine the system state from the model, then decide on the possible measures to maintain a desired state. Apparently, if the system is currently congested, it is necessary to take certain measures to reverse the unfavorable state. If the traffic flow is unobstructed, specific measures can be taken to maintain the system in the current region of the bifurcation set to a prevent catastrophe, so as to achieve the maximum traffic capacity.

When there is no incidental congestion, changing the traffic capacity must be realized by changing the system variables. This is important for keeping the traffic flow unobstructed and for avoiding catastrophe, congestion, and traffic accidents. If the points do not cross over the bifurcation set and are maintained in the original region, the system will not undergo a catastrophe. Alternatively, the system can be made to evolve from the region of potential catastrophe to the more stable and safer region to prevent a catastrophe.

When there is incidental congestion and the traffic flow catastrophe has already occurred, the control variables can be changed to enter the bifurcation set. The traffic flow system will be made to evolve towards the direction of a catastrophe to facilitate the catastrophe.

After a traffic accident, measures should be immediately taken to handle the accident, control the traffic flow rate, and ensure the diversion of the traffic flow. During the peak traffic hours, some coercive measures should be taken to maintain an unobstructed traffic flow. Once the traffic accident occurs, the emergency response plan must be activated immediately. The nature, features, and scope of the accident are determined, and effective measures are taken to ensure normal traffic flow.

7 Case study

According to 2012 Urban Road Design Code, the design velocity of this road section is 60 km/h, and the design traffic capacity is 1400 pcu/h. The correction coefficients of traffic capacity of the 3 lanes are 2.6, and the design traffic capacity of this road section is 3640 pcu/h.

8 Conclusions and outlook

Considering the influence of accident duration on traffic flow, the problem of urban road traffic flow control under incidental congestion is discussed. Based on the features of incidental congestion, the swallowtail catastrophe model was built for traffic flow as a function of accident duration. The critical state of congestion was analyzed and a traffic flow control scheme is proposed. The feasibility of the model was verified through case studies. The swallowtail catastrophe model can predict a traffic flow catastrophe.

The proposed model had some limitations because of the complexity of the actual traffic flow. Choosing the appropriate state and control variables to characterize incidental congestion and to reproduce the traffic flow state more realistically is worthy of further study. The curve

Table 1: Measured traffic data of Chang, an overpass in the south second ring road of Xi' an city from 17:15-18:05 on 4 August 2009

Lane	1	2	3
Velocity km/h	16.4	15.4	15.65
Traffic flow rate veh/h	1504.3	1475.6	1422.2

(Data source: Measured traffic flow video and data sharing platform of Fudan University)

Table 2: Case study under different scenarios

Indicator \ Scenario	No incidental congestion $a = -0.042$ $b = 27.948$	Lane 3 occupied $a = -0.28$ $b = 70.52$	Lanes 2 and 3 simultaneously occupied $a = -0.266$ $b = 69.215$
$M(t)$	−16.88%	−65.41%	−91.83%
$N(t)$	3.78%	28.8%	47.9%

Table 3: Case study under different scenarios and at different times

	No incidental congestion	Lane 3 occupied	Lanes 2 and 3 simultaneously occupied
5 min	Region I	Region I	Region I
10 min	Region V	Region V	Region V
15 min	Region III	Region III	Region III

of the swallowtail catastrophe model is more complex than the cusp catastrophe model. Further investigation is needed to establish accurate correspondence between the five regions of the bifurcation set and different states of traffic flow. Some researchers believe that the swallowtail catastrophe model considers more factors than the cusp catastrophe model, and therefore should be more accurate in theory. The actual findings are in contrast, and the reasons for this finding have not been fully clarified.

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