

Research Article

Precious Sibanda* and Osman Adam Ibrahim Noreldin

Thermo-convective instability in a rotating ferromagnetic fluid layer with temperature modulation

<https://doi.org/10.1515/phys-2018-0109>

Received Dec 18, 2017; accepted Sep 27, 2018

Abstract: We study the thermoconvective instability in a rotating ferromagnetic fluid confined between two parallel infinite plates with temperature modulation at the boundaries. We use weakly nonlinear stability theory to analyze the stationary convection in terms of critical Rayleigh numbers. The influence of parameters such as the Taylor number, the ratio of the magnetic force to the buoyancy force and the magnetization on the flow behaviour and structure are investigated. The heat transfer coefficient is analyzed for both the in-phase and the out-of-phase modulations. A truncated Fourier series is used to obtain a set of ordinary differential equations for the time evolution of the amplitude of convection for the ferromagnetic fluid flow. The system of differential equations is solved using a recent multi-domain spectral collocation method that has not been fully tested on such systems before. The solutions sets are presented as sets of trajectories in the phase plane. For some supercritical values of the Rayleigh number, spiralling trajectories that transition to chaotic solutions are obtained. Additional results are presented in terms of streamlines and isotherms for various Rayleigh numbers.

Keywords: Thermal instability, Ferromagnetic Fluids, Weakly nonlinear stability, Rotation, Multi-domain spectral collocation method

PACS: 47.11.Kb; 47.20.Bp

***Corresponding Author: Precious Sibanda:** School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X01, Scottsville, Pietermaritzburg 3209, South Africa; Email: sibandaP@ukzn.ac.za

Osman Adam Ibrahim Noreldin: School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Private Bag X01, Scottsville, Pietermaritzburg 3209, South Africa; Email: noreldino@ukzn.ac.za

1 Introduction

Ferromagnetic fluids are colloids consisting of nanometer-sized magnetic particles suspended in a fluid carrier. The magnetization of a ferromagnetic fluid depends on the temperature, the magnetic field, and the density of the fluid. The magnetic force and the thermal state of the fluid may give rise to convection currents. Studies on the flow of ferromagnetic fluids include, for example, Finlayson [1] who studied instabilities in a ferromagnetic fluid using free-free and rigid-rigid boundaries conditions. He used the linear stability theory to predict the critical Rayleigh number for the onset of instability when both a magnetic and a buoyancy force are present. The generalization of Rayleigh Benard convection under various assumption is reported by Chandrasekhar [2]. In the last few decades the study of heat transfer in ferromagnetic fluids has attracted many researchers due to the potential application of these fluids in industry, such as in the sealing of rotating shafts, ink, and so on. An authoritative introduction to research on magnetic fluids is given by Rosensweig [3].

Schwab *et al.* [4] studied the Finlayson problem experimentally in the case of a strong magnetic field and determined the parameters for the onset of convection. Their results were shown to be in good agreement with those of Finlayson [1]. Stiles and Kagan [5] extended the experimental problem reported by Schwab *et al.* [4] by introducing a strong magnetic field. A weakly nonlinear stability analysis was used by Russell *et al.* [6] for magnetized ferrofluids heated from above with the Rayleigh number as the control parameter for the onset of convection. They showed that heat transfer depends on the temperature difference between the bounding surfaces.

The rotation of fluids is an interesting topic that has been studied by, for example, Greenspan [7]. The classical Rayleigh-Benard problem when the fluid layer is rotating is well known in the case of ordinary viscous fluids and has been reported by Chandrasekhar [2]. However, ferromagnetic fluids are known to exhibit very peculiar characteristics when set to rotate. Demonstrating the effect of

rotation on convection in ferromagnetic fluids is scientifically important to researchers. Gupta and Gupta [8] examined the onset of convection in a ferromagnetic fluid heated from below and rotating about a vertical axis subject to a uniform magnetic field. They concluded that overstability may not occur for Prandtl numbers smaller than unity. The thermo-convective instability in a rotating ferrofluid was further analyzed by Venkatasubramanian and Kaloni [9]. They presented both analytical and numerical results for free and rigid boundary conditions. Their results were in good agreement with those of Finlayson [1] and Chandrasekhar [2] for some limiting cases. Thermo-convection in a ferromagnetic fluid has been studied by other researchers, for instance, [10, 12].

The problem associated with convection in ferromagnetic fluids is both relevant and mathematically challenging. The unmodulated Rayleigh Benard problem of a ferromagnetic fluid has been extensively studied. The effect of a magnetic modulation on the stability of a magnetic liquid layer heated from above was studied by Aniss *et al.* [13]. They used the Floquet theory for their study of the onset of convection. The study showed the possibility of a competitive interaction between harmonic and subharmonic modes at the onset of convection. Convective instability in a ferromagnetic fluid layer with time-periodic modulation in the temperature field was investigated by Singh and Bajaj [14] using the linear stability theory and the classical Floquet theory. Their result agrees with those of Aniss *et al.* [13].

Convection in a rotating horizontal fluid layer confined in a porous medium with temperature modulation at the boundary was studied by Bhadauria [19]. He investigated the stability of the flow using the Galerkin method and the Floquet theory. In this study we analyze thermoconvective instability in a rotating ferromagnetic fluid layer with time periodic temperature boundary conditions. The fluid layer is heated from below and rotates about the vertical axis subject to a uniform magnetic field. We assume two stress free and two rigid boundary conditions. The Ginzburg Landau equation is obtained, see [20] for details on the relevance of the Ginzburg Landau equation. Nonlinear ordinary differential equations of the Lorenz type are obtained and solved numerically using the multi-domain spectral collocation method [16–18]. This method has not been fully tested before on evolution equations of this nature, hence the accuracy of solutions obtained using this method is also a matter of concern in this study. Heat transfer in the rotating horizontal fluid layer is discussed.

2 Mathematical formulation

Consider a ferromagnetic fluid confined between two infinite horizontal plates at $z = -h/2$ and $z = h/2$. The layer is heated from below and cooled from above, and is rotating uniformly about the vertical axis with constant angular velocity Ω . The lower and upper plates are subjected to an oscillatory temperature $T_0 + \Delta T[1 + \epsilon^2 \cos(\omega t + \varphi)]$ where ω is the modulation frequency and φ is the phase angle. The Oberbeck-Boussinesq approximation is assumed to be applicable. The magnetization \mathbf{M} of the ferrofluid is assumed to be parallel to the magnetic field \mathbf{H} . The equations describing the fluid motion under these assumptions are the continuity equation, modified momentum equation, energy equation and Maxwell's equations (Finlayson [1] and Gupta and Gupta [8]):

$$\nabla \cdot \mathbf{V} = 0, \quad (1)$$

$$\rho_0 \frac{D\mathbf{V}}{Dt} = -\nabla P' + \mu \nabla^2 \mathbf{V} + \rho \mathbf{g}$$

$$+ \nabla \cdot (\mathbf{H}\mathbf{B}) + 2\rho_0 \mathbf{V} \times \Omega + \frac{\rho_0}{2} \nabla(|\Omega \times \mathbf{r}|), \quad (2)$$

$$\left[\rho_0 C_{V,H} - \mu_0 \mathbf{H} \cdot \left(\frac{\partial \mathbf{M}}{\partial T} \right)_{V,H} \right] \frac{DT}{Dt} + \mu_0 T \left(\frac{\partial \mathbf{M}}{\partial T} \right)_{V,H} \cdot \frac{D\mathbf{H}}{Dt} = \kappa \nabla^2 T + \Phi, \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0; \quad \nabla \times \mathbf{H} = 0, \quad (4)$$

where \mathbf{V} is the velocity field, ρ_0 is the density at the ambient temperature, $P' = P + \frac{\mu_0}{2} \mathbf{H}^2$ is the pressure, μ is the viscosity, \mathbf{g} is the gravitational body force, \mathbf{B} is the magnetic induction, μ_0 is the magnetic permeability, T is the temperature, κ is the thermal conductivity, $C_{V,H}$ is the heat capacity at constant volume and magnetic field, α is the thermal expansion coefficient and Φ is the viscous dissipation. The magnetization and magnetic field are related by the formula

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \quad (5)$$

The magnetization is dependent on the temperature and magnitude of magnetic field, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H} \mathbf{M}(H, T). \quad (6)$$

Equation (6) is linearized using the Taylor expansion

$$\mathbf{M} = \mathbf{M}_0 + \chi(\mathbf{H} - \mathbf{H}_0) - K(T - T_1), \quad (7)$$

where $\chi \equiv (\partial \mathbf{M} / \partial \mathbf{H})_{H_0, T_1}$ is the magnetic susceptibility and $K \equiv -(\partial \mathbf{M} / \partial T)_{H_0, T_1}$ is pyromagnetic coefficient, H_0 is the uniform magnetic field and $T_1 = (T_\infty + T_0)/2$, T_∞ and T_0 are the temperatures at $h/2$ and $-h/2$, respectively. The study is restricted to the case when magnetization induced by the temperature variation is much smaller than that induced by the external magnetic field. The density varies linearly with temperature as

$$\rho = \rho_0(1 - \alpha(T - T_1)). \quad (8)$$

3 Weakly nonlinear stability

In this section we use weakly nonlinear stability analysis to study the evolution of disturbances in a ferromagnetic fluid with two free and two rigid boundary conditions and temperature modulation. Using Eqs. (4) and (5) and assuming that the magnetic field \mathbf{H} is collinear with the magnetic induction \mathbf{B} , Eq. (2) reduces to

$$\begin{aligned} \rho_0 \frac{DV}{Dt} = -\nabla P + \mu \nabla^2 V + \rho \mathbf{g} + \mu_0 \mathbf{M} \cdot \nabla \mathbf{H} \\ + 2\rho_0 V \times \Omega + \frac{\rho_0}{2} \nabla(|\Omega \times \mathbf{r}|). \end{aligned} \quad (9)$$

The basic state solution of Eqs. (1)–(4) with (9) is obtained as

$$\begin{aligned} V = 0, \\ T_b(z, t) = T_1 + \frac{\Delta T}{2} - \beta z + \epsilon^2(F(z, t)), \end{aligned} \quad (10)$$

where

$$\begin{aligned} F(z, t) = \text{Re} \left\{ \frac{\Delta T}{\sinh \lambda} [e^{i\varphi} \sinh \lambda(1/2 - z/h) \right. \\ \left. + \sinh \lambda(1/2 - z/h)] e^{i\omega t} \right\}, \\ \lambda^2 = i\omega \rho_0 C h^2 / \kappa, \quad \beta = \frac{\Delta T}{h}. \end{aligned}$$

Following [1, 3] we define

$$\mathbf{M}_b + \mathbf{H}_b = \text{constant}, \quad (11)$$

and

$$\begin{aligned} \mathbf{H}_b = \left(\mathbf{H}_0 - \frac{K(T_b - T_1)}{1 + \chi} \right) \hat{e}_z, \\ \mathbf{M}_b = \left(\mathbf{M}_0 + \frac{K(T_b - T_1)}{1 + \chi} \right) \hat{e}_z. \end{aligned} \quad (12)$$

We superimpose small perturbations on the basic state. The perturbed quantities are defined as

$$T = T_b + T', \quad \mathbf{H}_i = \mathbf{H}'_i, \quad \mathbf{M}_i = \mathbf{M}'_i \quad \text{for } i=1,2; \quad (13)$$

$$\mathbf{H}_3 = \mathbf{H}_b + \mathbf{H}'_3, \quad \mathbf{M}_3 = \mathbf{M}_b + \mathbf{M}'_3,$$

where the prime represents a perturbed quantity. The linearization of Eqs. (6) and (7) gives

$$\mathbf{H}'_i + \mathbf{M}'_i = \left(1 + \frac{\mathbf{M}_0}{\mathbf{H}_0} \right) \mathbf{H}'_i, \quad i = 1, 2. \quad (14)$$

$$\mathbf{H}'_3 + \mathbf{M}'_3 = (1 + \chi) \mathbf{H}' - K T'. \quad (15)$$

We assume that $K \Delta T \ll (1 + \chi) \mathbf{H}_0$. For more details see [1]. Substituting Eq. (13) and using the curl operator on Eq. (9) we obtain the vorticity equation

$$\rho_0 \frac{\partial \zeta'}{\partial t} = \mu \nabla^2 \zeta' + \rho_0 \Omega \frac{\partial w'}{\partial z}. \quad (16)$$

Substituting Eqs. (12), (14) and (15) in Eq. (9) and using $\mathbf{H}' = \nabla \phi'$ where ϕ is the magnetic potential, the z -component of the resulting equation can be written as,

$$\begin{aligned} \rho_0 \frac{\partial}{\partial t} \nabla^2 w' - \mu \nabla^4 w' = \rho_0 g \alpha \nabla_l^2 T' \\ + \frac{\mu_0 K^2 \beta}{1 + \chi} \nabla_l^2 T' - \mu_0 K \beta \frac{\partial}{\partial z} \nabla_l^2 \phi' \\ - 2\rho_0 \Omega \frac{\partial \zeta'}{\partial z}. \end{aligned} \quad (17)$$

On using Eq. (13) in Eq. (3) and linearizing we obtain

$$\begin{aligned} \rho_0 C \frac{\partial T'}{\partial t} + \rho_0 C w' \frac{\partial T'}{\partial z} - \mu_0 T_0 K \left(\frac{\partial}{\partial t} \left(\frac{\partial \phi'}{\partial z} \right) \right. \\ \left. + w' \frac{\partial^2 \phi'}{\partial z^2} \right) + \left(\rho_0 C \beta - \frac{\mu_0 T_0 K^2 \beta}{1 + \chi} \right) \left(-1 + \epsilon \frac{\partial F}{\partial z} \right) w \\ = \nabla^2 T'. \end{aligned} \quad (18)$$

Finally, substituting Eqs. (14) and (15) into Eq. (4) we have

$$(1 + \chi) \frac{\partial^2 \phi'}{\partial z^2} - \left(1 + \frac{\mathbf{M}_0}{\mathbf{H}_0} \right) \nabla_l^2 \phi' - K \frac{\partial T'}{\partial z} = 0, \quad (19)$$

where $\rho_0 C = \rho_0 C_{V,H} + \mu_0 K \mathbf{H}_0$ and $\nabla_l^2 = \partial^2 / \partial x^2 + \partial^2 / \partial y^2$ is the Laplace operator in two dimension. For the clarity we drop the prime from the perturbed quantities and introduce the following dimensionless variables

$$\begin{aligned} (x^*, y^*, z^*) = (x, y, z)/h, \quad w^* = w'h/\nu, \\ t^* = \mu t / \rho_0 h^2, \\ \zeta^* = \zeta' h^2 / \nu, \quad \theta^* = (\kappa R a^{\frac{1}{2}} T') / (\rho_0 C \beta \nu h), \\ \phi^* = ((1 + \chi) \kappa R a^{\frac{1}{2}} \phi') / (\rho_0 C \beta \nu h^2). \end{aligned} \quad (20)$$

The linearized perturbed Eqs. (16)–(19) in the dimensionless form can be written as,

$$\frac{\partial}{\partial t^*} \nabla^2 w^* - \nabla^4 w^* + T a^{\frac{1}{2}} \frac{\partial \zeta^*}{\partial z} \quad (21)$$

$$-Ra^{\frac{1}{2}}(1+M_1)\nabla_l^2\theta^* + Ra^{\frac{1}{2}}M_1\frac{\partial}{\partial z}\nabla_l^2\phi^* = 0,$$

$$-Ta^{\frac{1}{2}}\frac{\partial w^*}{\partial z^*} + \frac{\partial \zeta^*}{\partial t^*} - \nabla^2\zeta^* = 0, \quad (22)$$

$$\begin{aligned} (Ra^{\frac{1}{2}} - Ra^{\frac{1}{2}}M_2)\left(-1 + \epsilon^2\frac{\partial F}{\partial z^*}\right)w^* + Pr\frac{\partial \theta^*}{\partial t^*} \\ + Prw^*\frac{\partial \theta^*}{\partial z^*} - \nabla^2\theta^* - PrM_2\frac{\partial}{\partial t^*}\left(\frac{\partial \phi^*}{\partial z^*}\right) \\ - PrM_2w^*\frac{\partial^2\phi^*}{\partial z^{*2}} = 0, \end{aligned} \quad (23)$$

$$-\frac{\partial \theta^*}{\partial z^*} + \frac{\partial^2\phi^*}{\partial z^2} + M_3\nabla_l^2\phi^* = 0, \quad (24)$$

where $Ta = \frac{4Qh^4}{\nu^2}$ is the Taylor number, $Ra = \frac{\rho_0 C \beta \alpha g h^4}{\nu \kappa}$ is the Rayleigh number, $M_1 = \frac{\mu_0 K^2 \beta}{(1+\chi)\rho_0 \alpha g}$ is the ratio of the magnetic force to the buoyancy force, $M_2 = \frac{\mu_0 T_0 K^2}{(1+\chi)\rho_0 C}$ is a nondimensional parameter, $Pr = \frac{\mu C}{\kappa}$ is the Prandtl number and $M_3 = (1 + \frac{M_0}{H_0})/(1 + \chi)$ is a measure of nonlinearity of the magnetization. The magnetic Rayleigh number can be obtained from the formula $N = RaM_1$. Hereafter the asterisk will be dropped from Eqs. (21)-(24).

The associated boundary conditions for the system of Eqs. (21)-(24) are

– Free boundary conditions

$$\begin{aligned} w = 0, \quad D^2w = 0, \quad D\zeta = 0, \quad \theta = 0, \quad D\phi = 0, \quad (25) \\ \text{at } z = \pm \frac{1}{2}. \end{aligned}$$

– Rigid boundary conditions

$$\begin{aligned} w = 0, \quad Dw = 0, \quad \zeta = 0, \quad \theta = 0, \quad (26) \\ \text{at } z = \pm \frac{1}{2}, \end{aligned}$$

where

$$D = \partial/\partial z.$$

3.1 The solution for stress free boundaries

The solution for stress free boundaries has been discussed in [1, 3, 9]. Here we only emphasize the solution aspects which have not been discussed before. We solve the eigenvalue problem with two stress free boundaries to study the onset of instability in the ferromagnetic fluid. We consider a small variation in time scale $\tau = \epsilon^2 t$ such that stationary convection occurs at lower orders of ϵ and introduce the following asymptotic expansions

$$Ra^{\frac{1}{2}} = (Ra^\alpha)^{\frac{1}{2}} + \epsilon^2 Ra^{\frac{1}{2}} + \epsilon^4 Ra^{\frac{1}{2}} + \dots, \quad (27)$$

$$w = \epsilon w_1 + \epsilon^2 w_2 + \epsilon^3 w_3 + \dots, \quad (28)$$

$$\zeta = \epsilon \zeta_1 + \epsilon^2 \zeta_2 + \epsilon^3 \zeta_3 + \dots, \quad (29)$$

$$\theta = \epsilon \theta_1 + \epsilon^2 \theta_2 + \epsilon^3 \theta_3 + \dots, \quad (30)$$

$$\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \dots. \quad (31)$$

Substituting Eqs. (27)–(31) into Eqs. (21)–(24), at the lowest order of ϵ we obtain

$$BZ_1 = R_1, \quad (32)$$

where

$$\begin{aligned} B = \begin{pmatrix} -\nabla^4 & \sqrt{Ta}\frac{\partial}{\partial z} & -\sqrt{Ra^\alpha}(1+M_1)\nabla_l^2 & \sqrt{Ra^\alpha}M_1\frac{\partial}{\partial z}\nabla_l^2 \\ -\sqrt{Ta}\frac{\partial}{\partial z} & -\nabla^2 & 0 & 0 \\ -\sqrt{Ra^\alpha}(1-M_2) & 0 & -\nabla^2 & 0 \\ 0 & 0 & -\frac{\partial}{\partial z} & \frac{\partial^2}{\partial z^2} + M_3\nabla_l^2 \end{pmatrix}, \\ Z_1 = \begin{pmatrix} w_1 \\ \zeta_1 \\ \theta_1 \\ \phi_1 \end{pmatrix} \text{ and } R_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \end{aligned}$$

This equation corresponds to the linear equations in [1, 9]. Solving Eq. (32) we obtain the solution

$$w_1 = A(\tau) \sin ax \cos \pi z, \quad (33)$$

$$\zeta_1 = -\frac{Ta^{\frac{1}{2}}\pi}{\pi^2 + a^2}A(\tau) \sin ax \sin \pi z, \quad (34)$$

$$\theta_1 = \frac{(Ra^\alpha)^{\frac{1}{2}}(1-M_2)}{\pi^2 + a^2}A(\tau) \sin ax \cos \pi z, \quad (35)$$

$$\begin{aligned} \phi_1 = -\left(\frac{(Ra^\alpha)^{\frac{1}{2}}(1-M_2)\pi}{(\pi^2 + a^2)(\pi^2 + a^2M_3)}\right) \\ A(\tau) \sin ax \sin \pi z, \end{aligned} \quad (36)$$

where a is a dimensionless wave number. Thus, the stationary Rayleigh number is given as

$$Ra^\alpha = \frac{((\pi^2 + a^2)^3 + \pi^2 Ta)(\pi^2 + a^2 M_3)}{(1-M_2)(a^2 \pi^2 M_1 + a^2(M_1+1)(\pi^2 + a^2 M_3))}. \quad (37)$$

To find the critical wave number and the corresponding critical Rayleigh number we set $a^2 = \pi^2 x$. Then the stationary Rayleigh number can be written as

$$Ra^\alpha = \frac{(\pi^4(1+x)^3 + Ta)(1+xM_3)}{(1-M_2)x(1+xM_3(M_1+1))}. \quad (38)$$

This result agrees with [1, 3, 9]. Since M_2 is very small as indicated in [1, 9] it can be neglected in the subsequent analysis.

The critical wave number and the corresponding critical Rayleigh number are obtained from

$$\frac{\partial Ra^\alpha}{\partial x} = 0,$$

then we have

$$\begin{aligned} & -Ta - 2Ta(xM_3 + xM_1M_3) \\ & -Ta(x^2M_3^2 + x^2M_1M_3^2) \\ & + \pi^4(1+x)^2 \left(-1 + 2x - 2xM_3 - 2xM_1M_3 - 4x^2M_3 \right. \\ & \left. + x^2M_3^2M_1 + 2x^3M_3^2 + 2x^3M_1M_3^2 \right) = 0. \end{aligned} \quad (39)$$

For the case $Ta = 0$, $M_1 = 0$ and $M_3 = 0$, the classical critical wave number is

$$a_c = \frac{\pi}{\sqrt{2}}$$

with corresponding classical critical Rayleigh number

$$Ra_c^\alpha = \frac{27}{4}\pi^4.$$

The magnetic Rayleigh number is also of an interest and can be expressed as

$$N^\alpha = Ra^\alpha M_1 = \frac{(\pi^4(1+x)^3 + Ta)M_1(1+xM_3)}{x(1+xM_3(M_1+1))}. \quad (40)$$

For large values of M_1 the magnetic Rayleigh number in the absence of buoyancy effects is obtained as

$$N^\alpha = \frac{(\pi^4(1+x)^3 + Ta)(1+xM_3)}{x^2M_3}. \quad (41)$$

The critical wave number and corresponding critical magnetic Rayleigh number are obtained from solving the equation

$$-2Ta + M_3x + \pi^4(1+x)^2(-2+x - M_3x + 2M_3x^2) = 0. \quad (42)$$

The critical wave number and corresponding Rayleigh numbers are given for different values of the Taylor number Ta in Tables 1 and 2.

At second order $O(\epsilon^2)$ we obtain the following equations:

$$BZ_2 = R_2 \quad (43)$$

$$\text{where } Z_2 = \begin{pmatrix} w_2 \\ \zeta_2 \\ \theta_2 \\ \phi_2 \end{pmatrix} \text{ and } R_2 = \begin{pmatrix} R_{21} \\ R_{22} \\ R_{23} \\ R_{24} \end{pmatrix} \text{ with}$$

$$R_{21} = 0, \quad R_{22} = 0, \quad R_{24} = 0, \quad (44)$$

Table 1: Comparison of the critical wave number and corresponding Rayleigh number for $M_1 = 1$ and $M_3 = 5$.

Present study			Ref [9]	
Ta	a_c	Ra_c^α	a_c	Ra_c^α
1	2.11212	269.6382	2.7348	461.5368
10	2.15849	275.0998	3.7827	883.4022
100	2.49953	322.9328	5.7215	2727.2022
1000	3.65242	608.4028	8.6174	10619.856
10000	5.66466	1862.582	12.8255	45606.430

Table 2: Comparison of the critical wave number and corresponding Magnetic Rayleigh number for $M_1 \rightarrow \infty$ and $M_3 = 3$.

Present study			Ref [9]	
Ta	a_c	N_c^α	a_c	N_c^α
1	2.6084	880.7959	2.7348	1037.8896
10	2.62069	897.4541	3.8770	1885.6592
100	2.72574	1050.072	5.7736	5617.3574
1000	3.2225	2083.258	8.6487	21520.978
10000	4.28305	7315.093	12.8455	91759.273

$$R_{23} = -\frac{Pr\pi(Ra^\alpha)^{\frac{1}{2}}}{2(\pi^2 + a^2)}A^2(\tau)\sin^2 ax \sin 2\pi z.$$

The solution at the second order is

$$w_2 = 0, \quad \zeta_2 = 0, \quad (45)$$

$$\begin{aligned} \theta_2 = & -\left(\frac{Pr\pi(Ra^\alpha)^{\frac{1}{2}}}{4(\pi^2 + a^2)(2\pi^2 + a^2)} \right) \\ & A^2(\tau)\sin^2 ax \sin 2\pi z, \end{aligned} \quad (46)$$

$$\begin{aligned} \phi_2 = & -\left(\frac{Pr\pi^2(Ra^\alpha)^{\frac{1}{2}}}{4(2\pi^2 + a^2)(\pi^2 + a^2)(2\pi^2 + a^2M_3)} \right) \\ & A^2(\tau)\sin^2 ax \cos 2\pi z. \end{aligned} \quad (47)$$

At the third order, we obtain

$$BZ_3 = R_3, \quad (48)$$

$$\text{where } Z_3 = \begin{pmatrix} w_3 \\ \zeta_3 \\ \theta_3 \\ \phi_3 \end{pmatrix} \text{ and } R_3 = \begin{pmatrix} R_{31} \\ R_{32} \\ R_{33} \\ R_{34} \end{pmatrix} \text{ with}$$

$$\begin{aligned} R_{31} = & \left\{ -\gamma^2 \frac{dA}{d\tau} + \frac{a^2(Ra^\alpha)^{\frac{1}{2}}R_{a2}^{\frac{1}{2}}(1+M_1)}{\gamma^2(\pi^2 + a^2M_3)} \right. \\ & \left. ((1+M_1)\pi^2 + a^2M_3)A(\tau) \right\} \sin ax \cos \pi z, \end{aligned} \quad (49)$$

$$R_{32} = -\frac{Ta^{\frac{1}{2}}\pi^2}{\gamma^2} \frac{dA}{d\tau} \sin ax \sin \pi z, \quad (50)$$

$$R_{33} = -\left((Ra^\alpha)^{\frac{1}{2}} \frac{\partial F}{\partial z} - Ra^{\frac{1}{2}} \right) A(\tau) + \frac{Pr(Ra^\alpha)^{\frac{1}{2}}}{\gamma^2} \frac{dA}{d\tau} \sin ax \cos \pi z + \frac{Pr^2\pi^2(Ra^\alpha)^{\frac{1}{2}}}{2\gamma^2(2\pi^2 + a^2)} A^3(\tau) \sin^3 ax \cos \pi z \cos 2\pi z, \quad (51)$$

$$R_{34} = 0. \quad (52)$$

Here, $\gamma^2 = a^2 + \pi^2$. To obtain the Ginzburg Landau equation we applied the Fredholm solvability condition [11, 18]

$$\int_0^{\frac{1}{2}} \int_0^{\frac{2\pi}{a}} \left[\hat{w}_1 R_{31} + \hat{\zeta}_1 R_{32} + \hat{\theta}_1 R_{33} \right] dx dz = 0, \quad (53)$$

where \hat{w}_1 , $\hat{\zeta}_1$ and $\hat{\theta}_1$ are the solutions of the adjoint system of the first order. This gives

$$\left[\frac{\pi\gamma^2}{4a} + \frac{PrRa^\alpha\pi}{4a\gamma^4} - \frac{Ta\pi^4}{4a\gamma^4} \right] \frac{dA}{d\tau} = \left\{ \frac{\pi Ra^{\frac{1}{2}} Ra^{\frac{1}{2}}}{4a\gamma^2} - \frac{\pi Ra^\alpha}{a\gamma^2} \mathbf{I} + \frac{a\pi Ra^{\frac{1}{2}} Ra^{\frac{1}{2}} (1 + M_1)}{4\gamma^2(\pi^2 + a^2 M_3)} \right. \\ \left. \left((1 + M_1)\pi^2 + a^2 M_3 \right) \right\} A - \frac{3Pr^2\pi^3 Ra^\alpha}{64a\gamma^4(2\pi^2 + a^2)} A^3. \quad (54)$$

The above equation reduces to

$$\frac{dA}{d\tau} = \Delta_1 A - \Delta_2 A^3, \quad (55)$$

where $\Delta_1 = \gamma^2 / (\gamma^6 + PrRa^\alpha - Ta\pi^3)$
 $\left(Ra^* - 4Ra^\alpha \mathbf{I} + \frac{a^2 Ra^* (1 + M_1) ((1 + M_1)\pi^2 + a^2 M_3)}{\pi^2 + a^2 M_3} \right),$

$Ra^* = Ra^{\frac{1}{2}} Ra^{\frac{1}{2}}, \Delta_2 = 3Pr^2\pi^3 Ra^\alpha / (16(2\pi^2 + a^2)),$
 and

$$\mathbf{I} = \int_0^{\frac{1}{2}} \frac{dF}{dz} \cos^2(\pi z) dz.$$

In this study we are also interested in heat transfer in ferromagnetic fluids. The Nusselt number for ferromagnetic fluids is defined as

$$Nu(\tau) = \frac{\text{Heat transfer by conduction+convection}}{\text{Heat transfer by conduction}} \quad (56) \\ = 1 + \frac{Pr\pi^2(Ra^\alpha)^{\frac{1}{2}}}{4\gamma^2(2\pi^2 + a^2)} A^2(\tau).$$

3.2 The general Lorentz type equations

We restrict the analysis to the case of two-dimensional disturbances so that all physical quantities are independent

of y . Using the stream function defined by

$$u = \frac{\partial \psi}{\partial z} \quad \text{and} \quad w = -\frac{\partial \psi}{\partial x}$$

equations (1)–(4) reduce to

$$\frac{\partial}{\partial t} \nabla_L^2 \psi - |\mathcal{J}(\psi, \nabla_L^2 \psi)| = \nabla_L^4 \psi - M_1 Ra^{\alpha \frac{1}{2}} \frac{\partial^2 \phi}{\partial x \partial z} - Ra^{\alpha \frac{1}{2}} (1 + M_1) \frac{\partial \theta}{\partial x} + Ta^{\frac{1}{2}} \frac{\partial \zeta}{\partial z}, \quad (57)$$

$$\frac{\partial \zeta}{\partial t} + |\mathcal{J}(\psi, \zeta)| = \nabla_L^2 \zeta - Ta^{\frac{1}{2}} \frac{\partial \psi}{\partial z}, \quad (58)$$

$$Pr \left(\frac{\partial \theta}{\partial t} + |\mathcal{J}(\psi, \theta)| \right) = \nabla_L^2 \theta - Ra^{\alpha \frac{1}{2}} \frac{\partial \psi}{\partial x}, \quad (59)$$

$$\frac{\partial^2 \phi}{\partial z^2} + M_3 \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial \theta}{\partial z}, \quad (60)$$

where \mathcal{J} is the Jacobian matrix. The solution of Eqs. (57)–(60) represented as a minimal double Fourier series of modes (1,1) for the stream function and magnetic potential and modes (0,2) and (1,1) for temperature and vorticity of the finite amplitude convection of the ferromagnetic fluid flows as

$$\psi =_{11} \sin ax \sin \pi z, \quad (61)$$

$$\theta = B_{11} \cos ax \sin \pi z + B_{02} \sin 2\pi z, \quad (62)$$

$$\zeta = C_{11} \sin ax \cos \pi z + C_{02} \sin 2\pi z, \quad (63)$$

$$\phi = D_{11} \cos ax \cos \pi z, \quad (64)$$

where $A_{11}, B_{11}, B_{02}, C_{11}, C_{02}$ and D_{11} are time t dependent amplitudes. This is equivalent to a truncated Galerkin method. Substituting and integrating over the domain, we obtain a set of four ordinary differential equations for the time evolution of the amplitudes of convection of a ferromagnetic fluid in the form

$$\frac{dA_{11}}{dt} = -\gamma^2 A_{11} - \frac{a\pi Ra^{\alpha \frac{1}{2}}}{\gamma^2(\pi^2 + a^2 M_3)} B_{11} - \frac{aRa^{\alpha \frac{1}{2}}(1 + M_1)}{\gamma^2} B_{11} + \frac{\pi Ta^{\frac{1}{2}}}{\gamma^2} C_{11}, \quad (65)$$

$$\frac{dC_{11}}{dt} = -\gamma^2 - \pi Ta^{\frac{1}{2}} A_{11}, \quad (66)$$

$$\frac{dB_{11}}{dt} = \frac{aRa^{\alpha \frac{1}{2}}}{Pr} A_{11} - \frac{\gamma^2}{Pr} B_{11} - a\pi A_{11} B_{02}, \quad (67)$$

$$\frac{dB_{02}}{dt} = \frac{a\pi}{2}A_{11}B_{11} - \frac{4\pi^2}{Pr}B_{02}. \quad (68)$$

To simplify the equations we introduce new variables

$$X_1 = \frac{a\pi}{\gamma^2}A_{11}, \quad X_2 = \frac{-a^2\pi Ra^{\frac{1}{2}}}{\gamma^6}B_{11}, \quad \tau = \gamma^2 t, \quad (69)$$

$$X_3 = \frac{-a^2\pi Ra^{\frac{1}{2}}}{\gamma^6}B_{02} \text{ and } X_4 = \frac{a\pi^2 Ta^{\frac{1}{2}}}{\gamma^6}C_{11}. \quad (70)$$

This reduces Eqs. (65)–(68) to general Lorentz type equations

$$\frac{dX_1}{d\tau} = -X_1 + KX_2 + X_4, \quad (71)$$

$$\frac{dX_2}{d\tau} = RX_1 - Pr^{-1}X_2 - X_1X_3, \quad (72)$$

$$\frac{dX_3}{d\tau} = \frac{1}{2}X_1X_2 - bPr^{-1}X_3, \quad (73)$$

$$\frac{dX_4}{d\tau} = -TaX_1 - X_4, \quad (74)$$

where $R = \frac{a^2}{\gamma^6}Ra^\alpha$, $b = \frac{4\pi^2}{\gamma^2}$,
and $K = \frac{(2\pi^2 + a^2 M_3) + M_1(\pi^2 + a^2 M_3)}{\pi^2 + a^2 M_3}$.

3.3 Stability of Lorentz equations

In this section we discuss the stability of the nonlinear systems of differential equations that describe the evolution of the convection amplitudes for a ferromagnetic fluid flow. Firstly, we note that the nonlinear Eqs. (71)–(74) are invariant under the transformation

$$S(X_1, X_2, X_3, X_4) = (-X_1, -X_2, -X_3, -X_4). \quad (75)$$

These equations are also uniformly bounded and dissipative in the phase space

$$\sum_{i=1}^4 \frac{\partial \dot{X}_i}{X_i} = - \left[1 + Pr^{-1} + bPr^{-1} \right] < 0 \quad (76)$$

Thus the volume of the phase space moving with the flow for time $\tau > 0$ is given by

$$V(t) = V(0) \exp \left(- \left[1 + Pr^{-1} + bPr^{-1} \right] \tau \right). \quad (77)$$

We find that the stationary points of the system of nonlinear Eqs. (71)–(74) are:

- The motionless conduction solutions $(0, 0, 0, 0)$.

– The steady solution represented by the point

$$(x_1^*, x_2^*, x_3^*, x_4^*) = \left(\pm \sqrt{\frac{2b(PrKR - (Ta + 1))}{Pr^2(1 + Ta)}}, \right. \\ \left. \pm \frac{Ta + 1}{K} \sqrt{\frac{2b(PrKR - (Ta + 1))}{Pr^2(Ta + 1)}}, \frac{kPrR - (Ta + 1)}{kPr}, \right. \\ \left. \mp \sqrt{\frac{2b(PrKR - (Ta + 1))}{Ta^2 Pr^2(1 + Ta)}} \right).$$

The stability of the stationary point associated with the motionless solution $X^* = (0, 0, 0, 0)$ is determined by roots of the following characteristic polynomial equation

$$P(\xi) = \xi^3 + d_1\xi^2 + d_2\xi + d_3 = 0, \quad (78)$$

where

$$d_1 = Pr^{-1}, \quad d_2 = Ta - 1 - KR$$

and

$$d_3 = \frac{Ta - PrKR - 1}{Pr}$$

for the eigenvalues ξ_i , ($i = 2, 3, 4$) and $\xi_1 = -\frac{b}{Pr}$. It is clear that ξ_1 is always negative as $Pr > 0$. The remaining eigenvalues are obtained from Eq. (78), and using the Routh-Hurwitz criteria [21], the polynomial Eq. (78) has negative real roots if and only if

$$R < \frac{Ta - 1}{KPr}$$

and $Pr > 1$. This implies that the stationary solution is a stable node. Hence the critical value of R where the stationary solution of ferromagnetic fluid flow loses stability and steady convective flow takes over is

$$R = \frac{Ta - 1}{KPr}.$$

The stability of the stationary point corresponding to the steady convective flow is determined by the roots of the characteristic equation

$$p(\xi) = \xi^4 + c_1\xi^3 + c_2\xi^2 + c_3\xi + c_4 = 0, \quad (79)$$

where

$$c_1 = 2Pr + 2bPr, \\ c_2 = 2b - 2Pr^2 - 2KPr^2R + 2Pr^2Ta \\ + Pr^2x_1^{*2} + 2KPr^2x_3^*, \\ c_3 = -2Pr - 2bPr - 2bKPrR - 2KPr^2R + 2PrTa \\ + 2bPrTa + KPr^3x_1^*x_2^* + 2bKPrx_3^* + 2KPr^2x_3^*, \\ c_4 = -2b - 2bKPrR + 2bTa - Pr^2x_1^{*2} + Pr^2Tax_1^{*2} \\ + KPr^2x_1^*x_2^* + 2bKPrx_3^*.$$

Applying the Routh-Hurwitz criteria to Eq. (79), it is clear that $c_1 > 0$, and $c_3 > 0$ if and only if

$$Ta + 1 < PrKR$$

and

$$R < -\left(\frac{b+1}{bK+Pr}\right).$$

Also, $c_4 > 0$ if and only if

$$x_1^{*2} < -\left(\frac{2b(1+PrKR)}{Pr^2}\right)$$

with

$$Ta + 1 < PrKR.$$

Hence the fixed point is stable if the condition

$$\begin{aligned} Pr^3 c_3^* + Pr^2 c_2^* + Pr c_1^* + c_0^* \\ > Pr^3 d_3^* + Pr^2 d_2^* + Pr d_1^* + d_0^*, \end{aligned} \quad (80)$$

is satisfied where

$$\begin{aligned} c_3^* &= \left(8KR + 8K^2R^2 + 4KTax_1^*x_2^* \right. \\ &\quad + 2Kx_1^{*3}x_2^* + 4Kx_1^{*2}x_3^* + 4K^2x_1^*x_2^*x_3^* \\ &\quad \left. + 8K^2x_3^{*2} + 8KTax_3^*\right)(1+b), \\ c_2^* &= 8(KR+1) + 16b + 8b^2 + 8KR(3+2b) \\ &\quad + 8K^2R^2(b+1) + 8Ta^2(b^2+2b+1) \\ &\quad + 8KTax_3^*(2b^2+3b+1) + (4Kx_1^{*2}x_3^* \\ &\quad + 8K^2x_3^{*2})(b+b^2) + 4K^2R(x_1^{*2}x_2^* + 2x_3^*), \\ c_1^* &= 8KR(b^3+b^2) + 4Kx_1^{*2}x_2^*(2b+b^2) \\ &\quad + (8Kx_3^* + 8KRTa + 4K^2Rx_1^{*2}x_2^*)(1+b) \\ &\quad + 16bK^2Rx_3^*, \quad c_0^* = 8b + 4b^2 + 8Ta(b^2 \\ &\quad + 2b+1) + 8Kx_3^*(b^3+2b^2+b) \\ &\quad + 8KRTa(b^2+b) + 8b^2K^2Rx_3^*, \end{aligned}$$

and

$$\begin{aligned} d_3^* &= \left(8KRTa + 4KRx_1^{*2} + 4(K+K^2)x_1^{*2}x_2^* \right. \\ &\quad \left. + 8Kx_3^* + 16K^2Rx_3^*\right)(1+b), \\ d_2^* &= \left(16Ta + 4Kx_1^*x_2^*\right)(b^2+2b+1) \\ &\quad + \left(8KRTa + 8Kx_3^*\right)(2b^2+3b+1) \\ &\quad + 16K^2Rx_3^*(b^2+b) + K^2(x_1^{*2}x_2^* + 4x_1^*x_2^*x_3^* + 4x_3^*), \\ d_1^* &= 8Kx_3^*(b^3+b^2) + (KTax_3^* + 4KTax_1^*x_2^* \\ &\quad + 8K^2R^2)(b+1) + 4bK^2(x_1^*x_2^*x_3^* + 2x_3^*), \\ d_0^* &= 8KR(b^3+2b^2+b) + 4Ta(b^2+2b+1) \\ &\quad + 8KTax_3^*(b^2+b) + 4b^2K^2x_3^{*2}. \end{aligned}$$

3.4 The method of solution

In this section, we describe the multi-domain spectral collocation method [15–18] used to obtain the solutions to Eqs. (71)–(74). The multi-domain technique assumes that the interval $\Lambda = [0, T]$ can be decomposed into p non overlapping sub-intervals. The sub-intervals are defined as

$$\Lambda_i = [\tau_{i-1}, \tau_i], \quad i = 1, 2, \dots, p. \quad (81)$$

In each sub-interval, the system of Eqs. (71)–(74) is written in the form

$$\begin{aligned} \dot{X}_{1,s+1}^i + \alpha_{1,1}X_{1,s+1}^i + \alpha_{1,2}X_{2,s}^i + \alpha_{1,3}X_{3,s}^i \\ + \alpha_{1,4}X_{4,s}^i + f_1(X_{2,s}^i, X_{3,s}^i, X_{4,s}^i) = g_1 \\ \dot{X}_{2,s+1}^i + \alpha_{2,1}X_{1,s+1}^i + \alpha_{2,2}X_{2,s+1}^i + \alpha_{2,3}X_{3,s}^i \\ + \alpha_{2,4}X_{4,s}^i + f_2(X_{1,s+1}^i, X_{3,s}^i, X_{4,s}^i) = g_2 \\ \dot{X}_{3,s+1}^i + \alpha_{3,1}X_{1,s+1}^i + \alpha_{3,2}X_{2,s+1}^i + \alpha_{3,3}X_{3,s+1}^i \\ + \alpha_{3,4}X_{4,s}^i + f_3(X_{1,s+1}^i, X_{2,s+1}^i, X_{4,s}^i) = g_3 \\ \dot{X}_{4,s+1}^i + \alpha_{4,1}X_{1,s+1}^i + \alpha_{4,2}X_{2,s+1}^i + \alpha_{4,3}X_{3,s+1}^i \\ + \alpha_{4,4}X_{4,s+1}^i + f_4(X_{1,s+1}^i, X_{2,s+1}^i, X_{3,s+1}^i) = g_4, \end{aligned} \quad (82)$$

with initial conditions

$$X_{n,s+1}^i(\tau_{i-1}) = X_n^{i-1}(\tau_{i-1}), \quad n = 1, 2, 3, 4. \quad (83)$$

Here $\alpha_{n,k}$ and g_n ($n, k = 1, 2, 3, 4$) are constants while f_n is the nonlinear component of each equation. Each sub-interval Λ_i is transformed to $[-1, 1]$ using the transformation

$$\tau = \frac{\tau_{i-1} - \tau_i}{2}t^* + \frac{\tau_{i-1} + \tau_i}{2}, \quad t^* \in [-1, 1]. \quad (84)$$

The Chebyshev-Gauss-Lobatto collocation points are used to discretize the unknown functions

$$t_j^{*i} = \cos\left(\frac{\pi j}{N}\right), \quad j = 0, 1, \dots, N. \quad (85)$$

The derivative of the unknown function at the collocation point is given by

$$\frac{dX_{n,s+1}^i}{d\tau}(\tau^i) = \sum_{k=0}^N \mathbf{D}_{jk} X_{n,s+1}^i(t_k^{*i}) = \mathbf{D}X_{n,s+1}^i \quad (86)$$

where

$$\mathbf{D} = 2D/\delta\tau_i$$

with $\delta\tau_i = \tau_{i-1} - \tau_i$ and D is Chebyshev differentiation matrix. The vector functions at the collocation points are

$$\mathbf{X}_{n,s+1}^i = \left(X_{n,s+1}^i(t_0^{*i}), \dots, X_{n,s+1}^i(t_N^{*i})\right)^T.$$

Substituting Eq. (86) into Eq. (82) and reducing to matrix form we obtain the system

$$\mathbf{A}_n \mathbf{X}_{n,s+1}^i = \mathbf{R}_n^i \quad (87)$$

with $\mathbf{A}_n = \mathbf{D} + \alpha_{n,n} \mathbf{I}$ and

$$\mathbf{R}_1^i = \mathbf{g}_1 - \left[\alpha_{1,2} \mathbf{X}_{2,s}^i + \alpha_{1,3} \mathbf{X}_{3,s}^i + \alpha_{1,4} \mathbf{X}_{4,s}^i + f_1(\mathbf{X}_{2,s}^i, \mathbf{X}_{3,s}^i, \mathbf{X}_{4,s}^i) \right] \quad (88)$$

$$\mathbf{R}_2^i = \mathbf{g}_2 - \left[\alpha_{2,1} \mathbf{X}_{1,s+1}^i + \alpha_{2,3} \mathbf{X}_{3,s}^i + \alpha_{2,4} \mathbf{X}_{4,s}^i + f_2(\mathbf{X}_{1,s+1}^i, \mathbf{X}_{3,s}^i, \mathbf{X}_{4,s}^i) \right] \quad (89)$$

$$\mathbf{R}_3^i = \mathbf{g}_3 - \left[\alpha_{3,1} \mathbf{X}_{1,s+1}^i + \alpha_{3,2} \mathbf{X}_{2,s+1}^i + \alpha_{3,4} \mathbf{X}_{4,s}^i + f_3(\mathbf{X}_{1,s+1}^i, \mathbf{X}_{2,s+1}^i, \mathbf{X}_{4,s}^i) \right] \quad (90)$$

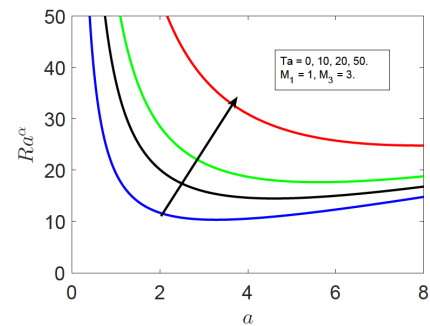
$$\mathbf{R}_4^i = \mathbf{g}_4 - \left[\alpha_{4,1} \mathbf{X}_{1,s+1}^i + \alpha_{4,2} \mathbf{X}_{2,s+1}^i + \alpha_{4,3} \mathbf{X}_{3,s+1}^i + f_4(\mathbf{X}_{1,s+1}^i, \mathbf{X}_{2,s+1}^i, \mathbf{X}_{3,s+1}^i) \right] \quad (91)$$

where \mathbf{g}_n is \mathbf{g}_n multiplied by a vector of ones of size $(N + 1) \times 1$ and \mathbf{I} is an identity matrix of size $(N + 1) \times (N + 1)$.

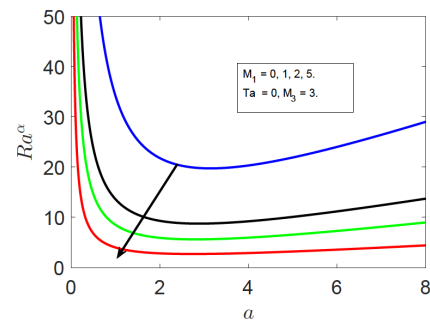
4 Results and discussion

We have presented a weakly nonlinear stability analysis of a rotating layer of a ferromagnetic fluid with temperature modulation at the boundary. We have obtained mathematical expressions for the stationary Rayleigh number Ra^a and the magnetic Rayleigh number N^a . Our results agree qualitatively with the results in [1, 9]. To provide a measure of validation of our results we give a comparison with [9] in Tables 1 and 2 of the influence of the Taylor number on the critical wave number and the corresponding Rayleigh numbers. Although the results in the two studies are not directly comparable, of interest is the general trend observed, namely that in both cases, increasing Ta increases the critical wave number and the Rayleigh numbers suggesting that the influence of Taylor number is to stabilize the system.

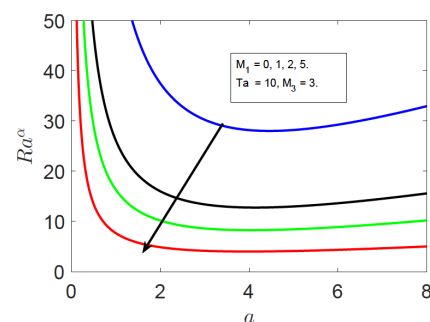
The instability curves are given in Figures 1–3. Figure 1(a)–1(d) shows the influence of various parameters



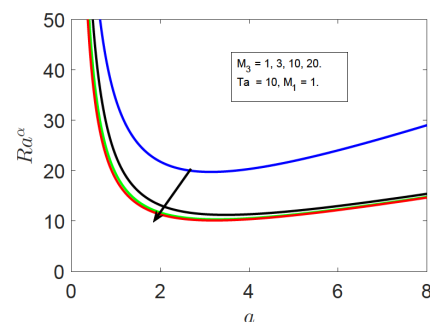
(a)



(b)



(c)



(d)

Figure 1: The effect of stationary Rayleigh number Ra^a versus wave number a for various values of (a) the Taylor number Ta , (b) the ratio of magnetic force to the buoyancy force parameter M_1 with $Ta = 0$ (c) the ratio of magnetic force to the buoyancy force parameter M_1 with $Ta = 10$ (d) the nonlinearity of magnetization parameter M_3

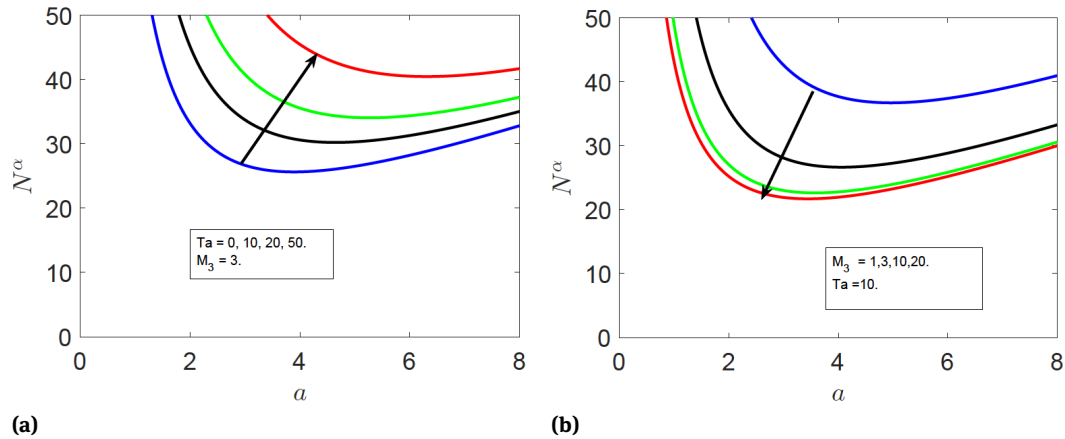


Figure 2: The effect of magnetic Rayleigh number N^a versus wave number a for various values of (a) the Taylor number Ta (b) the nonlinearity of magnetization parameter M_3

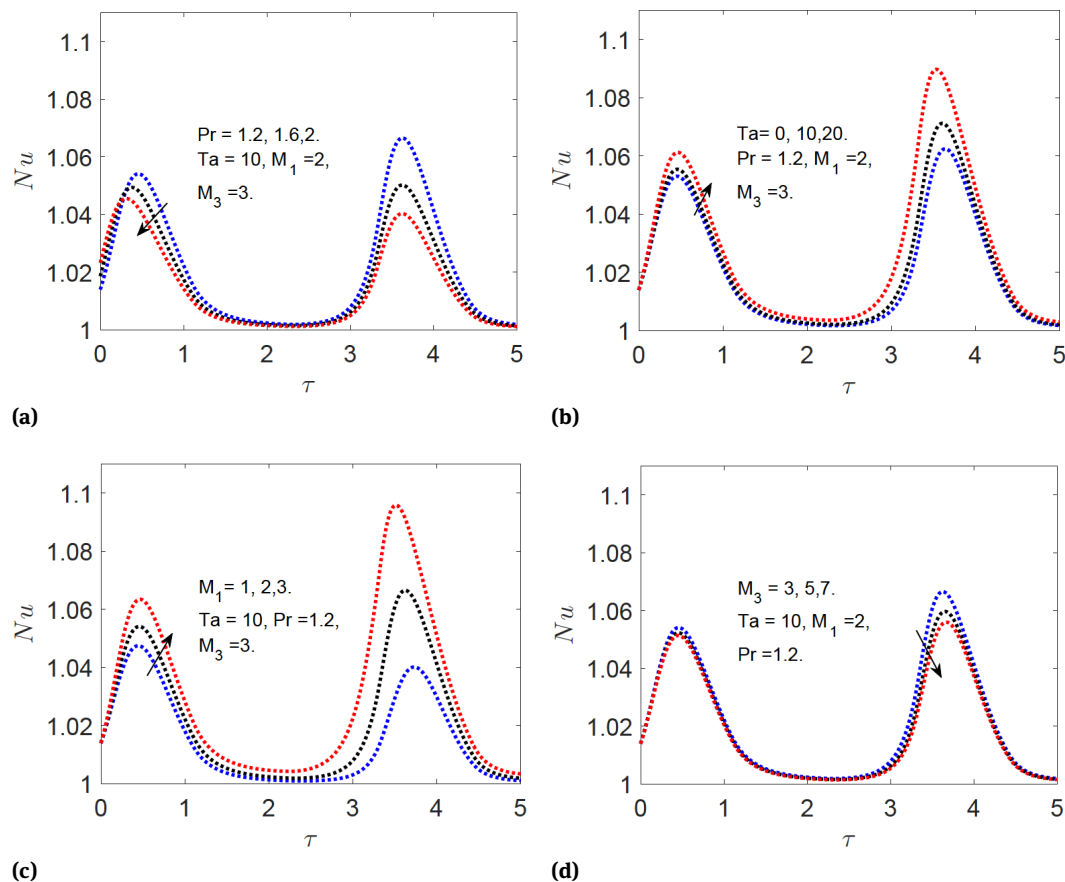


Figure 3: The variation of Nusselt number Nu with time τ in in-phase modulation ($\phi = 0$) for different values of : (a) the Prandtl number Pr , (b) the Taylor number Ta , (c) the ratio of magnetic force to buoyancy force M_1 and (d) the nonlinearity of magnetization M_3

on the stationary Rayleigh number. It can be seen in Figures 1(a) and 2(a) that as Ta increases from 0 to 50 the values of the stationary Rayleigh and the magnetic Rayleigh numbers both increase. This shows that rotation has a

stabilizing effect on the system. This result is similar to that of an ordinary viscous fluid. Rotation has a stabilizing influence on ferromagnetic fluid flow. Figures 1(b) and 1(c) show the relative influence of the size of the magnetic

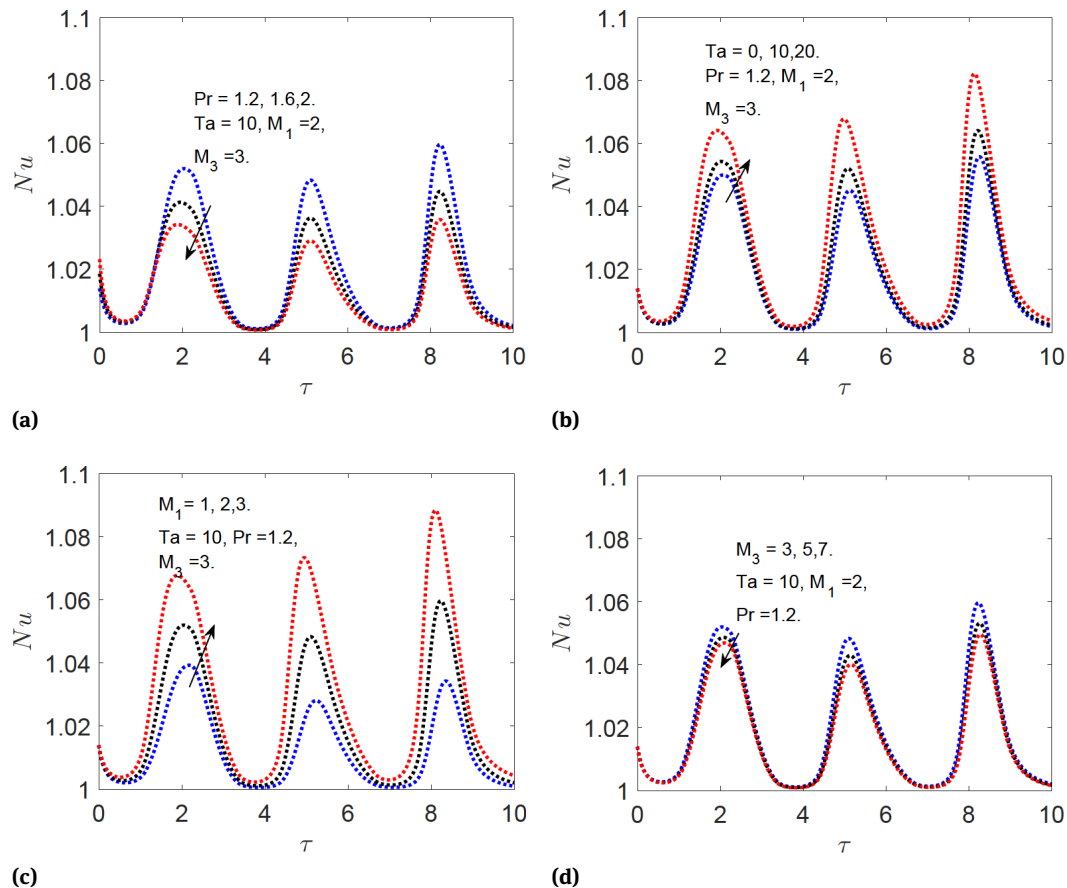


Figure 4: The variation of Nusselt number Nu with time τ in out-phase modulation ($\phi = \pi$) for different values of : (a) the Prandtl number Pr , (b) the Taylor number Ta , (c) the ratio of magnetic force to buoyancy force M_1 and (d) the nonlinearity of magnetization M_3

force to the buoyancy force parameter M_1 . As M_1 increases from 0 to 5 the stationary Rayleigh number is reduced. This suggests the magnetic and the buoyancy force are both destabilizing to the ferromagnetic fluid flow. Further, as M_1 increases with $Ta = 10$ fixed, the stationary Rayleigh number decreases, suggesting M_1 has a destabilizing effect for both low and high Taylor numbers. From Figures 1(d) and 2(b) it is observed that increasing M_3 from 1 to 20 reduces both the Rayleigh number and the magnetic Rayleigh number, this is destabilization to the system.

The Ginzburg-Landau equation is obtained using the nonlinear stability analysis at the third order of ϵ . The equation was solved using a multi-domain spectral method. The heat transfer coefficient, represented by the Nusselt number, is presented graphically for in-phase and out-phase modulation in Figures 3–4. Figure 3(a)–3(d) show the effect of Pr , Ta , M_1 and M_3 on the Nusselt number with time τ . It can be observed that on increasing the Pr and M_3 , the Nusselt number decreases. Hence increasing these parameters reduces the rate of heat transfer. In-

creasing Ta and M_1 increases the Nusselt number, thus the rate of heat transfer increases. Figures 4(a)–4(d) show changes in the Nusselt number with respect to time τ due to the influence of various parameters in the case of out of phase modulation. It can be observed that the Nusselt number for in-phase modulation is less than for out of phase modulation.

A multi-domain spectral collocation method was used to find the nonlinear amplitudes in ferromagnetic fluid convection equations for various values of R . The solution sets were obtained using initial conditions selected in the neighborhood of the stationary points corresponding to the motionless solutions. The simulations were done to a maximum time $\tau_{max} = 20$. For a sense of the accuracy of the method, the solutions were compared with solutions obtained using the Runge-Kutta based ode45 routine. Figures 5(a)–5(d) show the time series solution of $X_1(\tau)$ for different supercritical values of R . As R increases, periodic solutions are obtained. Here a comparison between the multi-domain spectral collo-

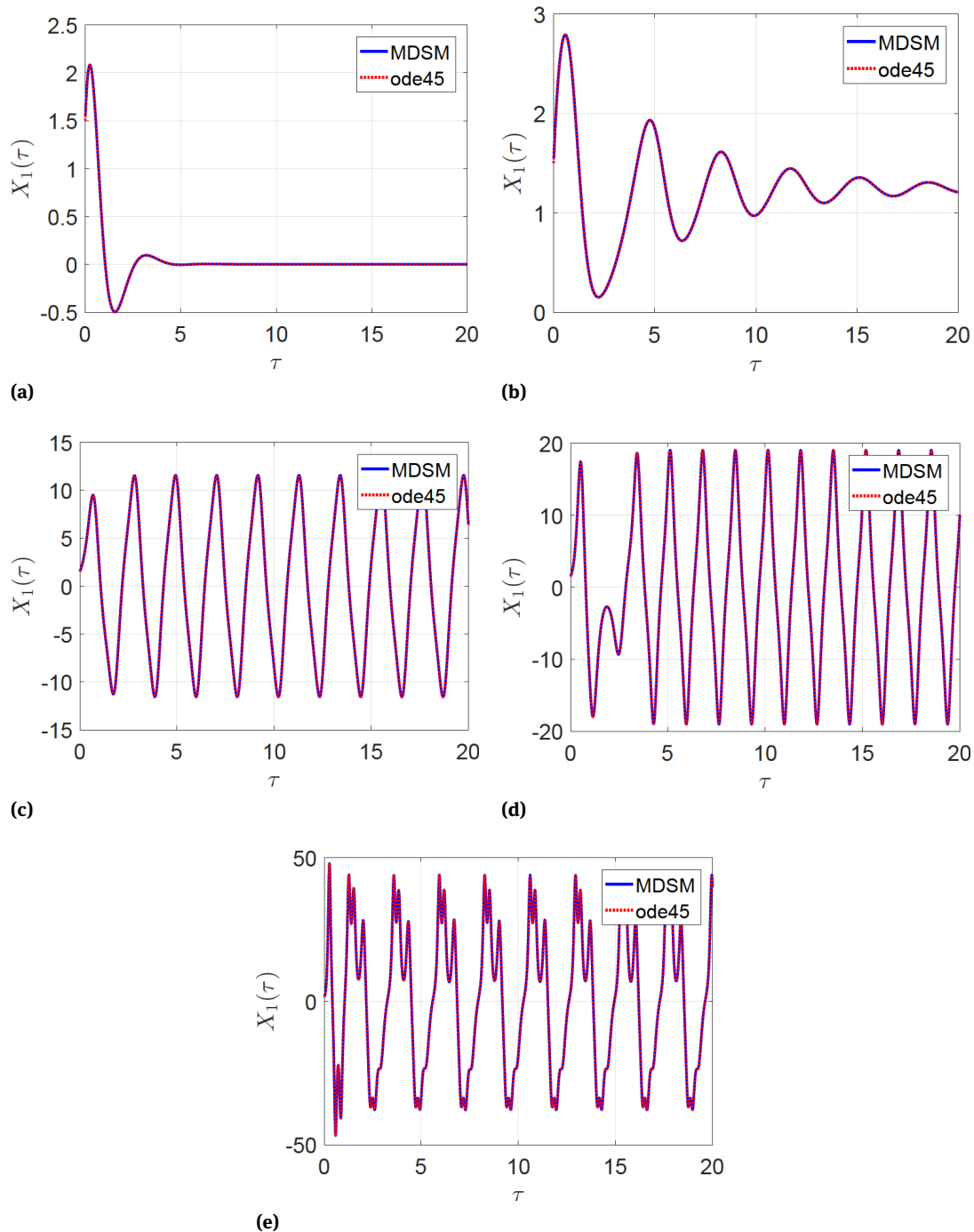


Figure 5: Comparison between a Multi-domain spectral collocation methods and ode45 methods for the solution of $X_1(\tau)$ for different values of R (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$

cation method and the ode45 is given. In Figures 6–11 we present a projection of the trajectories onto the (X_1, X_2) , (X_1, X_3) , (X_1, X_4) , (X_2, X_3) , (X_2, X_4) and (X_3, X_4) phase planes, respectively. The initial supercritical convective solution $R = 2$ is presented in part a in each figure. We observe that the trajectories attracted to equilib-

rium points that correspond to the motionless solution are stable spirals. The solutions are presented in part c and d of each figure when $R = 20$ and $R = 25$, respectively. For these Rayleigh numbers, chaotic solutions are obtained. These changes in solutions are further presented in Figures 6(c)–6(d), 7(c)–7(d) and 9(c)–9(d). The results pre-

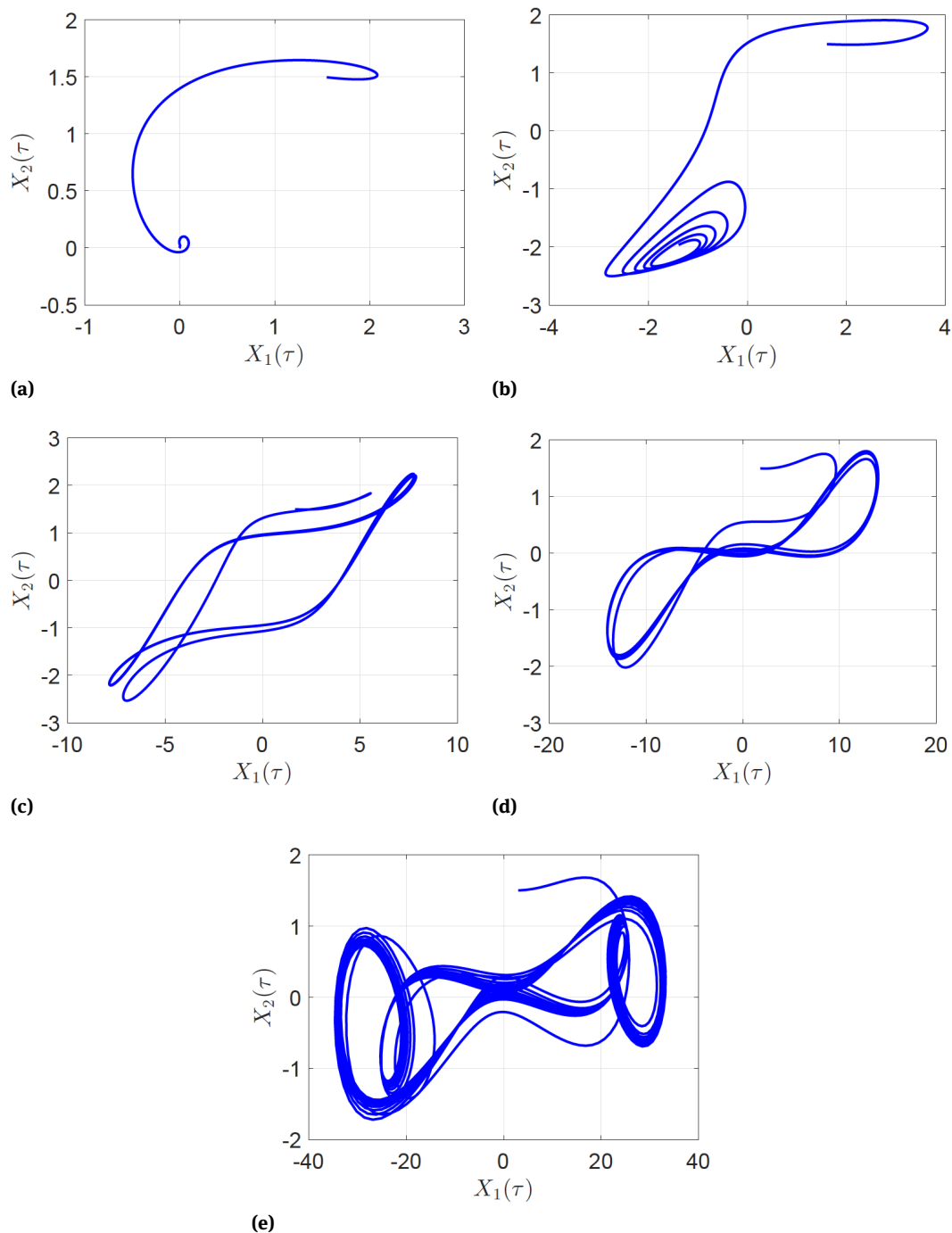


Figure 6: The evolution of trajectory over times in phase space for increasing values of Rayleigh number. (in term of R) corresponding to (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$. The graph represented the projection of the solution into X_1X_2 plane

sented in Figures 8(c)–8(d) and 10(c)–10(d) show a transition to a limit cycle. Increasing the values of R , for example, when $R = 100$ the results are complex with a significant level of unpredictability.

Figure 12 shows the streamlines patterns for the flow of a ferromagnetic fluid. Two different eddies are observed.

The clockwise and anti-clockwise flows are shown via negative and positive stream function values, respectively. With the Rayleigh number increasing from 2 to 200, the magnitude of the stream function values increase. The sense of motion in the subsequent cells is opposite that of an adjoining cell, indicating symmetry in the forma-

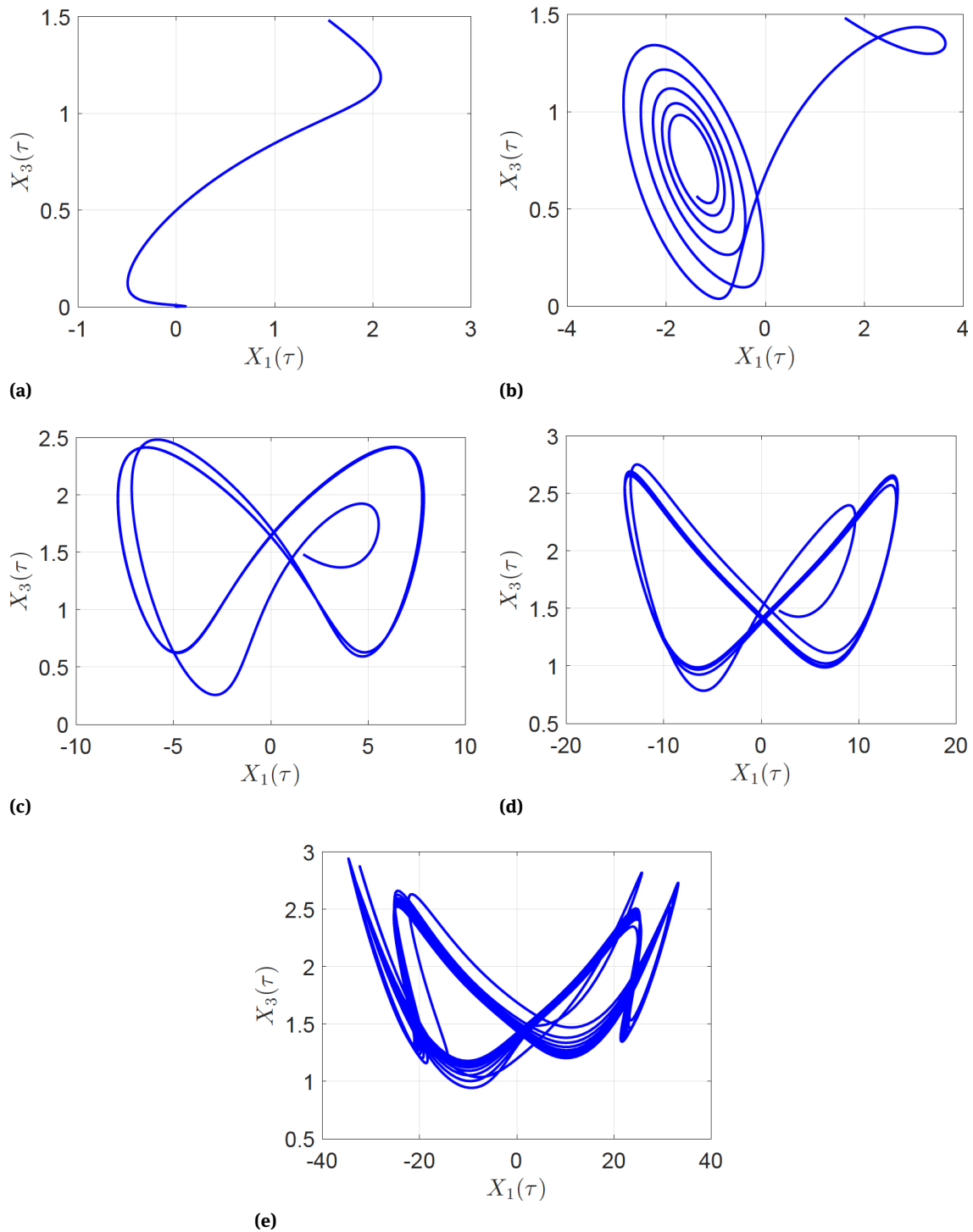


Figure 7: The evolution of trajectory over times in phase space for increasing values of Rayleigh number. (in term of R) corresponding to (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$. The graph represented the projection of the solution into X_1X_3 plane

tion of ferromagnetic convective cells. Figure 13 shows the isotherm patterns as the Rayleigh number changes from 2 to 200. Three different eddies are observed. The small eddy at the left corner diminishes as R increases from 2 to 200.

Also, increasing R reduces the density of the isotherms implying a delay of the onset of instability.

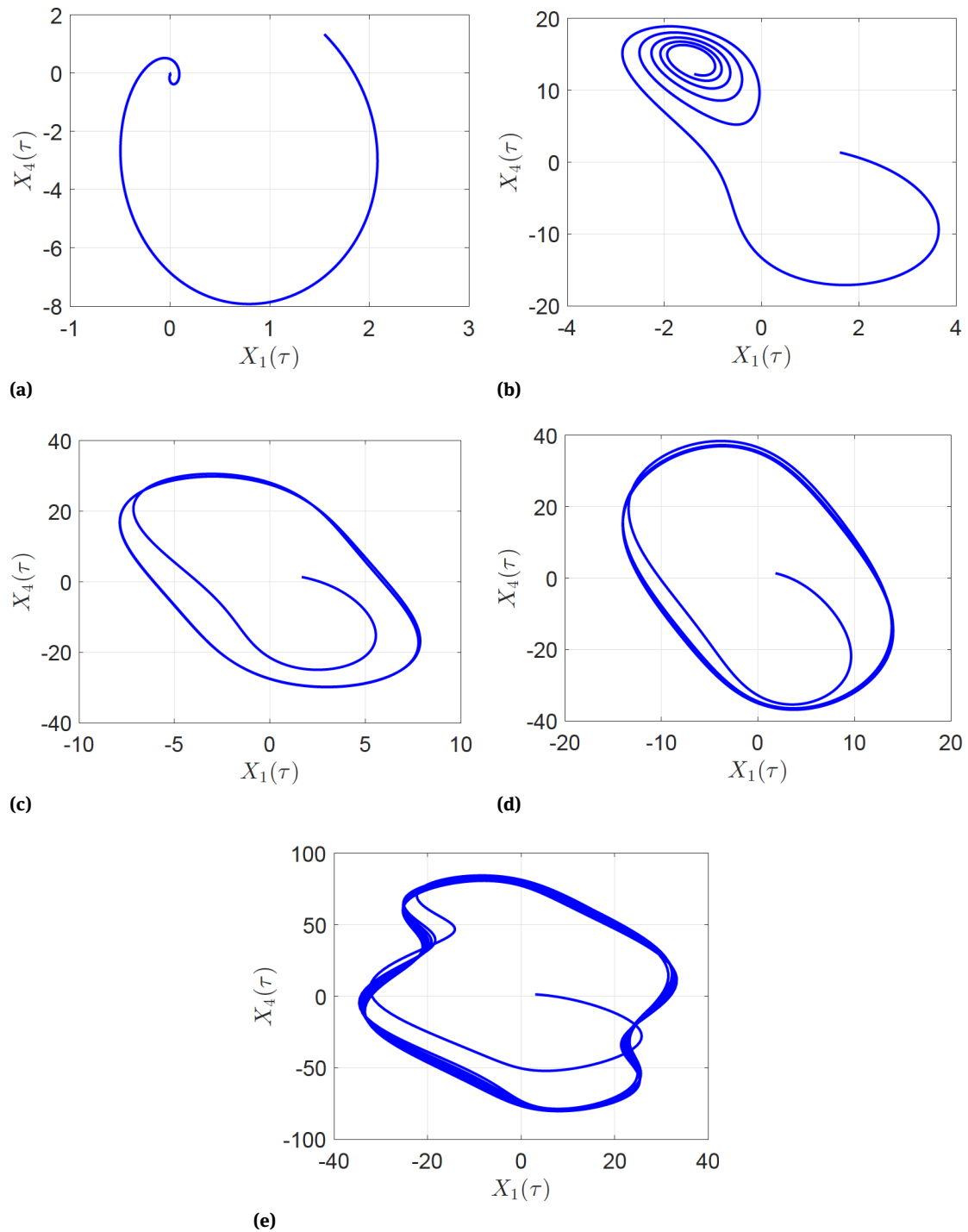


Figure 8: The evolution of trajectory over times in phase space for increasing values of Rayleigh number. (in term of R) corresponding to (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$. The graph represented the projection of the solution into X_1X_4 plane

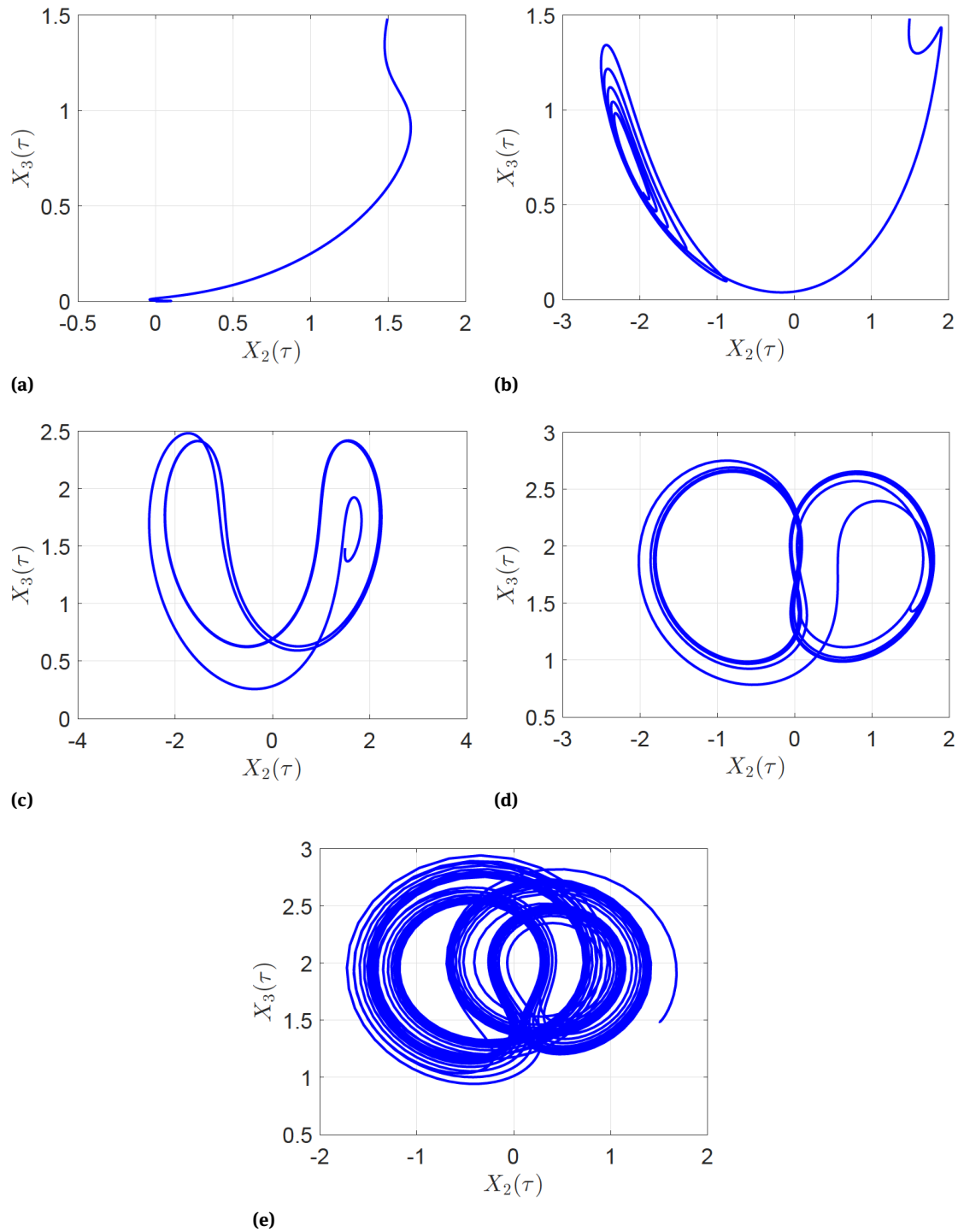


Figure 9: The evolution of trajectory over times in phase space for increasing values of Rayleigh number. (in term of R) corresponding to (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$. The graph represented the projection of the solution into X_2X_3 plane

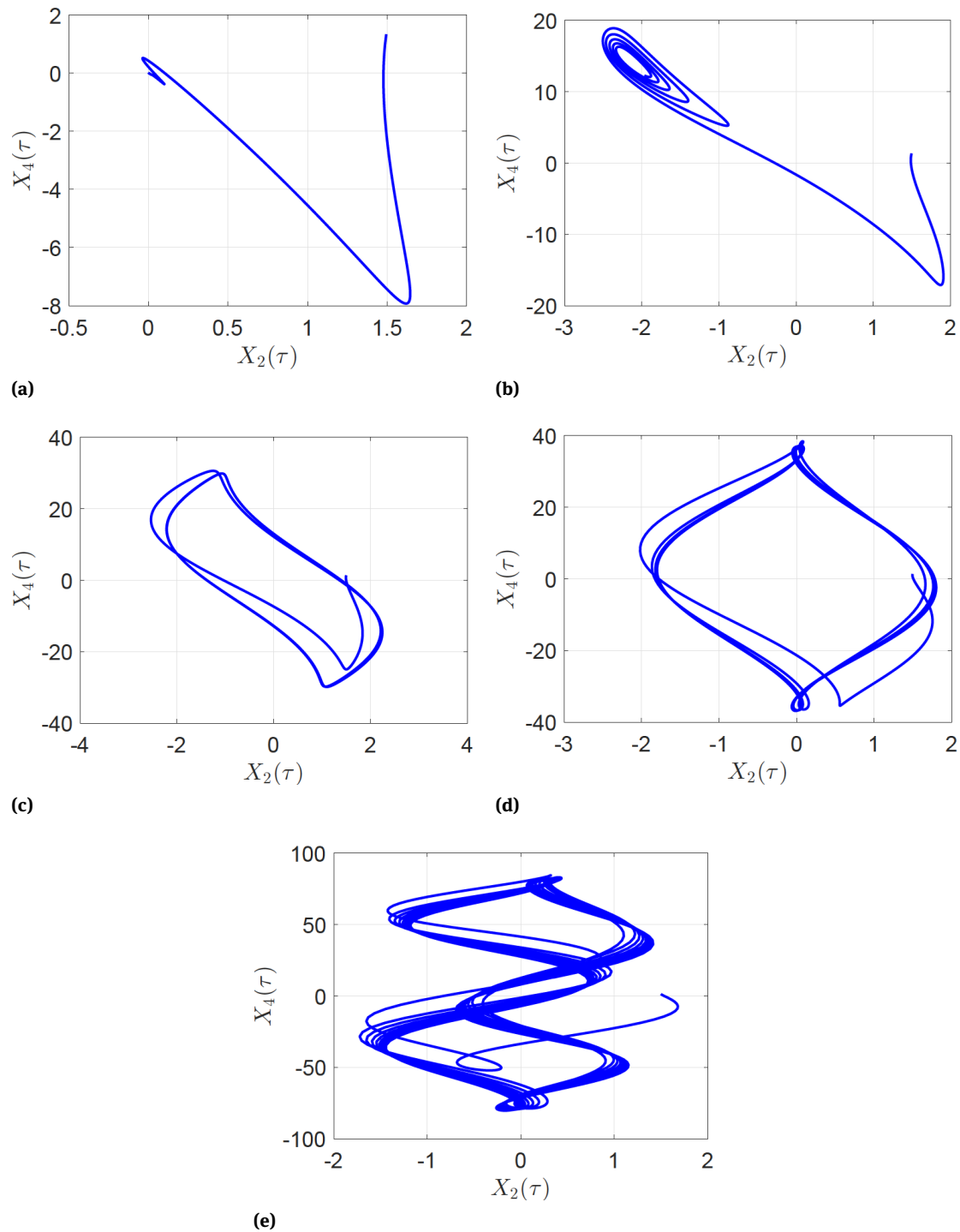


Figure 10: The evolution of trajectory over times in phase space for increasing values of Rayleigh number. (in term of R) corresponding to (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$. The graph represented the projection of the solution into X_2X_4 plane

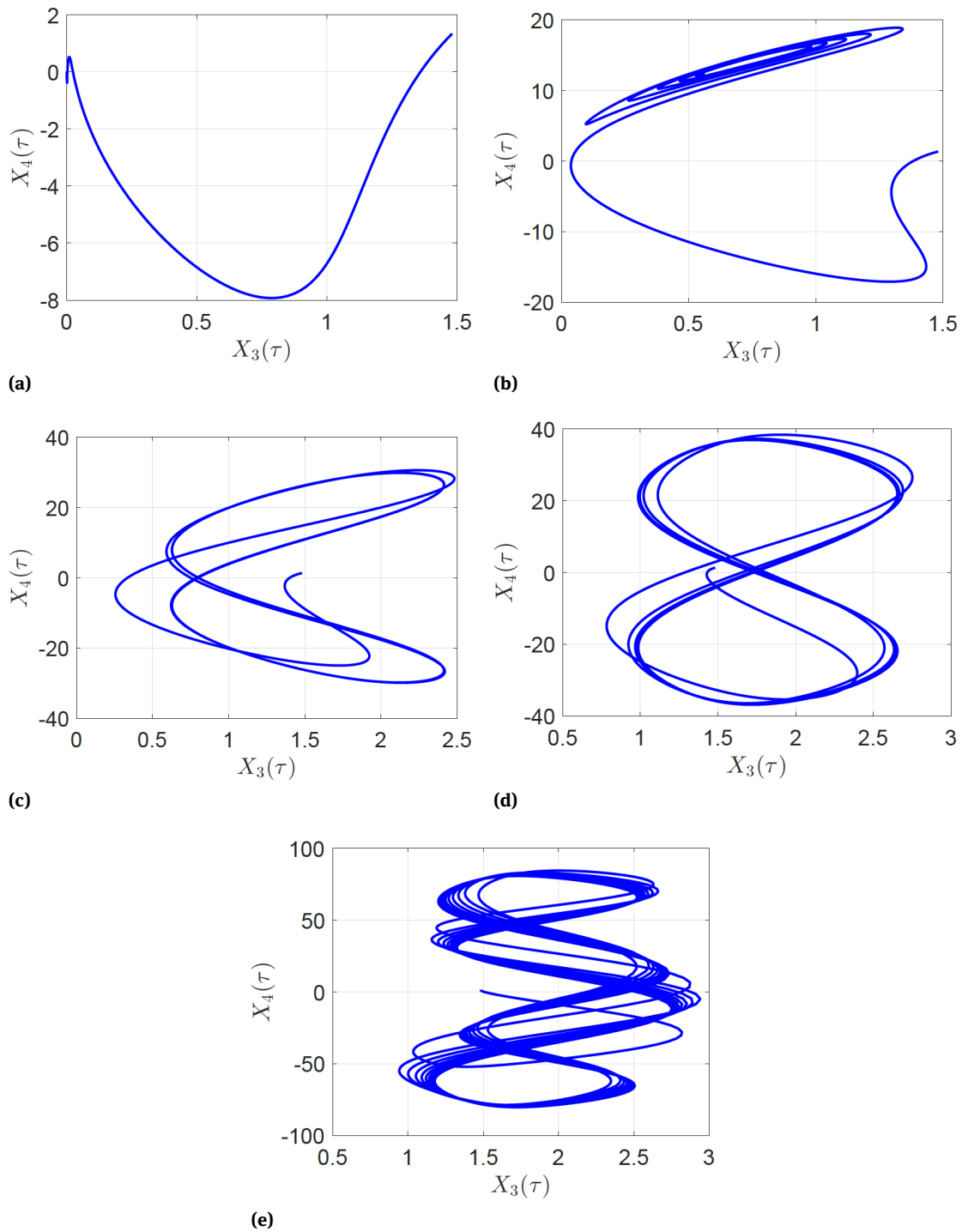


Figure 11: The evolution of trajectory over times in phase space for increasing values of Rayleigh number. (in term of R) corresponding to (a) $R = 2$, (b) $R = 4$, (c) $R = 10$, (d) $R = 20$ and (e) $R = 100$. The graph represented the projection of the solution into X_3X_4 plane

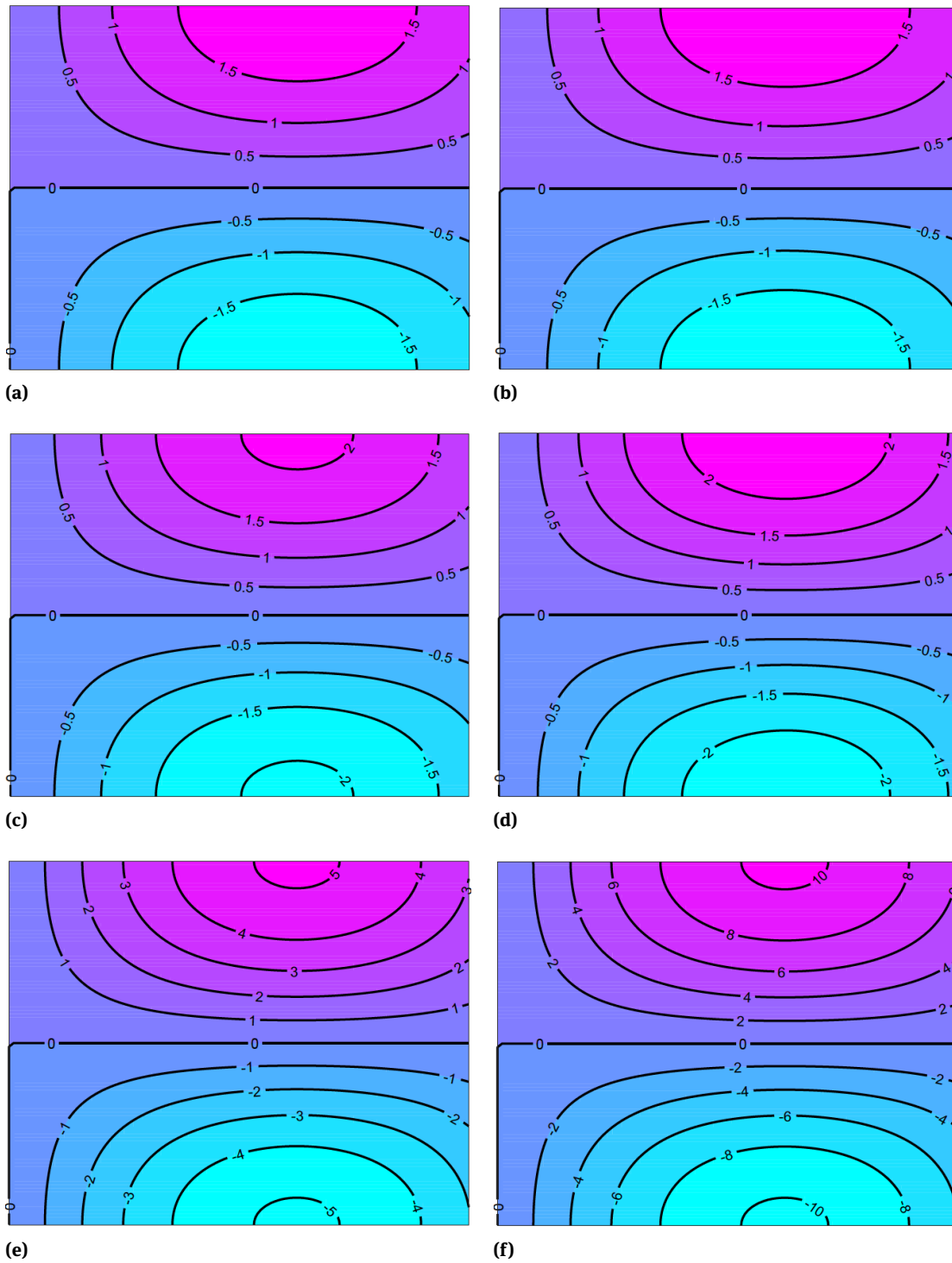


Figure 12: The streamlines for different values of the Rayleigh number R e.g. $R = 2, 4, 10, 20, 100, 200$

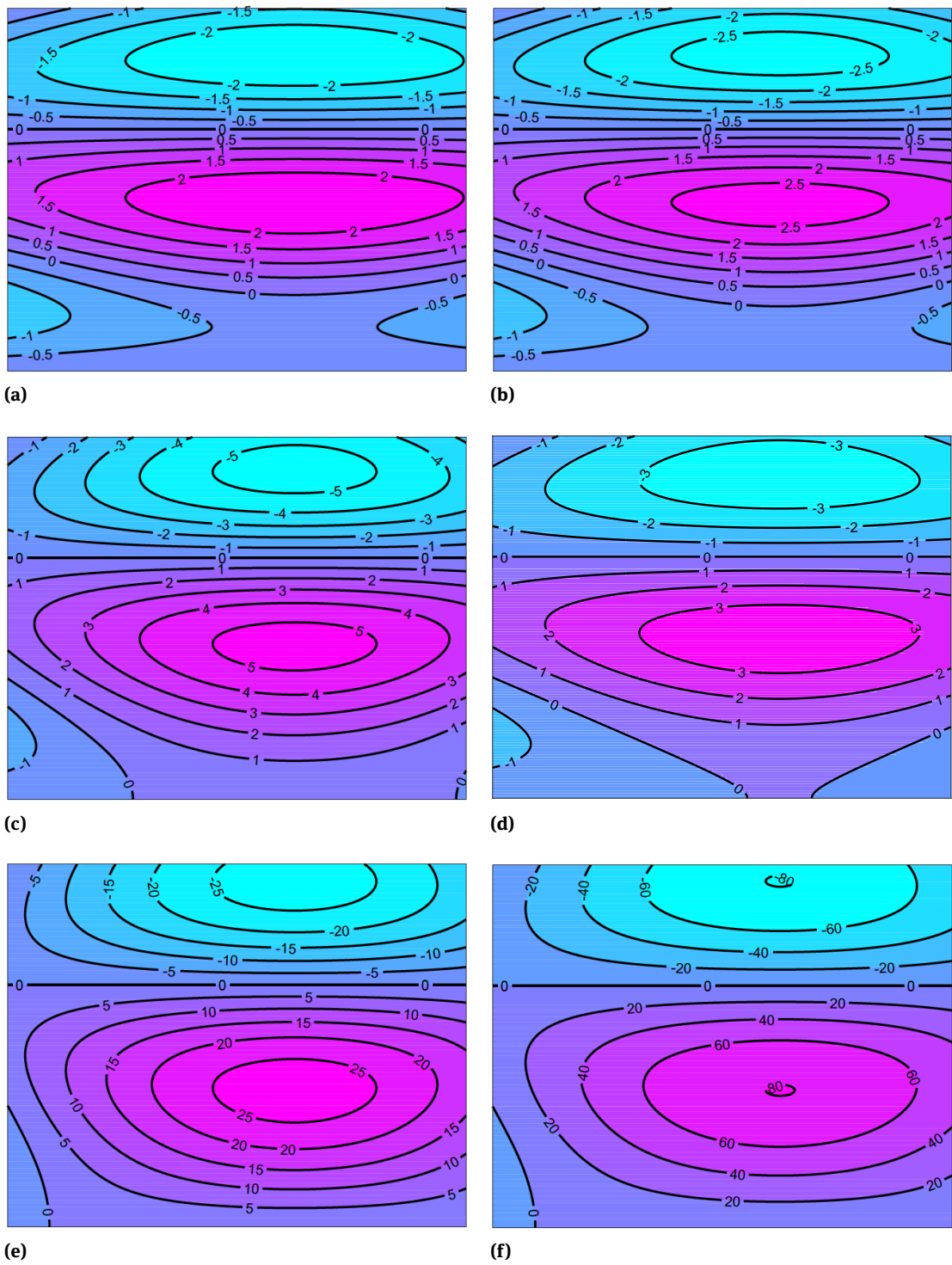


Figure 13: The isotherms for different values of the Rayleigh number R e.g. $R = 2, 4, 10, 20, 100, 200$

5 Conclusion

We investigated the thermoconvective instability in a rotating ferromagnetic fluid layer with time periodic temperature boundary conditions. The influence of flow parameters such as the Rayleigh number on the onset of instability was determined using a weakly nonlinear stability analysis. The results are broadly in line with the earlier findings in [1, 3, 8]. The heat transport has been analyzed for both the in-phase and out of phase temperature modulations. The influence of the parameters such as the Prandtl number, Taylor number and the magnetization parameter on the Nusselt number for the in-phase modulation was found to be less significant compared to the case of out of phase modulation.

The set of nonlinear differential equations for convection amplitude was solved using a multi-domain spectral collocation method. The accuracy of the solutions was determined by comparison with solutions using a different independent method, namely the Runge-Kutta based ode45 Matlab solver. The stability of the equilibrium solutions of the nonlinear differential equations has been analyzed. Transitions from different states have been demonstrated for different parameter values, for example from steady convection to chaotic solutions at high Rayleigh numbers.

References

- [1] Finlayson, B.A., Convective instability of ferromagnetic fluids, *J. Fluid Mech.*, 1970, 40(4), 755–767.
- [2] Chandrasekhar, S., Hydrodynamic and hydromagnetic stability, 1961, Oxford University Press, Oxford.
- [3] Rosensweig, R.E., Ferrohydrodynamics, 1985, Cambridge University Press.
- [4] Schwab L., Hildebrandt U., Stierstadt K., Magnetic Benard convection, *J. Magn. Magn. Mat.*, 1983, 39(1-2), 113–114.
- [5] Stiles P.J., Kagan M., Thermoconvective instability of a horizontal layer of ferrofluid in a strong magnetic field, *J. Magn. Magn. Mat.*, 1990, 85(1-3), 196–198.
- [6] Ruessli C.L., Blinnerhassett P.J., Stiles P.J., Large wave number convection in magnetized ferrofluids, *J. Magn. Magn. Mat.*, 1995, 149(1-2), 196–121.
- [7] Greenspan H.P., The theory of rotating fluids, 1968, Cambridge University press, London.
- [8] Gupta M.D., Gupta A.S., Convective instability of a layer of a ferromagnetic fluid rotating about a vertical axis, *Int. J. Eng. Sci.*, 1979, 17(3), 271–277.
- [9] Venkatasubramanian S., Kaloni P.N., Effects of rotation on the thermo-convective instability of a layer of a ferrofluids, *Int. J. Eng. Sci.*, 1994, 32(2), 237–256.
- [10] Gotoh K., Yamada M., Thermal convective in a horizontal layer of magnetic fluid, *J. Phys. Soc. Japan*, 1982, 51(9), 3042–3048.
- [11] Bhasauria B.S., Siddheshwar P.G., Kumar J., Suthar O.P., Weakly nonlinear stability analysis of temperature/gravity modulated stationary Rayleigh-Benard convection in a rotating porous medium, *Transp. Porous Med.*, 2012, 92(3), 633–647.
- [12] Rudraiah N., Sekhar N.G., Convective on magnetic fluids with internal heat generation, *ASME J. Heat Transfer*, 1991, 113(1), 122–127.
- [13] Aniss S., Belhaq M., Souhar M., Effects of amagnetic modulation on the stability of a magnetic liquid layer heated from above, *J. Heat Transfer*, 2001, 123(3) 428–432.
- [14] Singh J., Bajaj R., convective instability in a ferrofluids layer with temperature modulation rigid boundaries, *Fluid Dyn. Res.*, 2011, 43(2), 025502.
- [15] Trefethen L.N., Spectral methods in MATLAB, SIAM, Philadelphia, 2000, 26(2), 199.
- [16] Motsa S.S., Dlamini P., Khumalo M., A new multistage spectral relaxation method for solving chaotic initial value systems, *Nonlin. Dyn.*, 2013, 72(1-2) 265–83.
- [17] Motsa S.S., Magagula V.M., Goqo P.S., Oyelakin I.S., Sibanda P., A multi-domain spectral collocation approach for solving Lane-Emden type equations, Chapter 7, Numerical Simulation - From Brain Imaging to Turbulent flows, 2016, 10, 5772-63016.
- [18] Noreldin O.A.I., Sibanda P., Mondal S., Weakly Nonlinear Stability in a Horizontal Porous Layer Using a Multi-domain Spectral Collocation Method, chapter 9, Complexity in Biological and Physical Systems, 2018, 71066.
- [19] Bhadauria B.S., Fluid convection in a rotating porous layer under modulated temperature on the boundaries, *Transp. Porous Med.*, 2007, 67 297–315.
- [20] Mielke A., The Ginzburg-Landau Equation in Its Role as a Modulation Equation, Chapter 15, Handbook of Dynamical Systems, 2002, 2, 759–834.
- [21] Routh-Hurwitz criterion, Encyclopedia of Mathematics, http://www.encyclopediaofmath.org/index.php?title=Routh-Hurwitz_criterion&oldid=33371