

Research Article

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The Greek parameters of a continuous arithmetic Asian option pricing model via Laplace Adomian decomposition method

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Abstract: The Greek parameters in option pricing are derivatives used in hedging against option risks. In this paper, the Greeks of the continuous arithmetic Asian option pricing model are derived. The derivation is based on the analytical solution of the continuous arithmetic Asian option model obtained via a proposed semi-analytical method referred to as Laplace-Adomian decomposition method (LADM). The LADM gives the solution in explicit form with few iterations. The computational work involved is less. Nonetheless, high level of accuracy is not neglected. The obtained analytical solutions are in good agreement with those of Rogers & Shi (J. of Applied Probability 32: 1995, 1077-1088), and Elshegmani & Ahmad (ScienceAsia, 39S: 2013, 67–69). The proposed method is highly recommended for analytical solution of other forms of Asian option pricing models such as the geometric put and call options, even in their time-fractional forms. The basic Greeks obtained are the Theta, Delta, Speed, and Gamma which will be of great help to financial practitioners and traders in terms of hedging and strategy.

Keywords: Option pricing, Black-Scholes model, Adomian decomposition method, Laplace transform

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1 Introduction

In financial mathematics, the Greeks also referred to as sensitivity parameters are partial derivatives of the option prices with respect to some fundamental parameters. These Greeks are of great interest for hedging and risk management [1, 2]. Different dimensions to the risk associated with an option position are measured accordingly by different and unique Greeks. The following basic Greeks: Delta, Speed, Theta, and Gamma are studied with respect to (w.r.t.) Asian option while their associated mathematical expressions follow in the later part of this paper. Asian option is a special form of option contract whose value is hinged on the average value of the associated underlying asset over the option life time. Asian options are path dependent in nature unlike other options such as the European, American, lookback options and so on [3–10]. Basically, Asian options are of two kinds viz: geometric Asian option (GAO) and a rithmetic Asian option (AAO). The GAO is noted to have a closed form solution. However, the AAO is difficult to price in terms of closed form solution [11, 12]. Hence, many researchers have developed solution techniques to that effect [13–20]. Other numerical methods that are of interest are [21–30]. Some vital research approaches involving neural networks in relation to stochastic differential equations (SDEs) and/or option pricing are captured in [31–36].

In this paper, we propose the Laplace-Adomian decomposition method (LADM) for the first time in literature as a semi-analytical method, for analytical solution of a continuous arithmetic Asian option pricing model. Thereafter, the basic Greeks of the AAO model are explicitly derived. The remaining parts of the paper are organized as follows: in section 2, a brief note on the Asian option pricing model is given. In section 3, the proposed solution method (LADM) is presented, section 4 contains the application and the Greek-terms while in section 5, concluding remark is made.

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2 Asian option pricing model

The stock price $S(t)$ at time t , is assumed to satisfy a geometric Brownian motion (GBM) governed by the stochastic dynamic:

$$dS(t) = S(t)(r dt + \sigma dW(t)), \quad t \in \mathbb{R}_+ \quad (1)$$

where σ is a volatility coefficient, r a drift term indicating average rate of growth, and $W(t)$, $t \in [0, T]$ a standard Brownian motion. The payoff for an Asian option with arithmetic average strike is given as:

$$\Xi(T) = \max \left(S(T) - \frac{1}{T} \int_0^T S(\zeta) d\zeta, 0 \right). \quad (2)$$

Note, the option price at $t \in [0, T]$ is a risk neutral pricing formula defined as [3]:

$$\Xi(t) = E \left(e^{-r(T-t)} \Xi(T) | \mathcal{F}_t \right) \quad (3)$$

where $E(\cdot)$ and \mathcal{F}_t denote mathematical expectation operator and filtration respectively.

The payoff, $\Xi(T)$ is path-dependent. Hence, the introduction of a stochastic process [4]:

$$\begin{cases} I(t) = \int_0^T S(\zeta) d\zeta \\ dI(t) = S(t) dt, \quad S(0) = S_0, \quad (\text{SDE form}) \end{cases} \quad (4)$$

where $I(t)$ is the running sum of the strike price. Therefore, the corresponding Asian call option price is characterized by the model:

$$\frac{\partial \Xi}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Xi}{\partial S^2} + rS \frac{\partial \Xi}{\partial S} + S \frac{\partial \Xi}{\partial I} - r\Xi = 0 \quad (5)$$

which is satisfied by $\Xi(S, I, t)$ for continuous arithmetic average strike such that $t \geq 0$, and $S > 0$. Equation (5) is similar to the classical time-fractional Black-Scholes model at $\alpha = 1$ in [21, 22] except for the averaging term $\left(S \frac{\partial \Xi}{\partial I} \right)$.

This may later call for modification while numerical or semi-analytical methods are adopted [23, 24]. It is obvious that (5) will eventually lead to a greater computational problem because of its three-dimensional form. Hence, the need for a reduction to a lower level dimensional form using the following transformation variables [4, 26]:

$$\begin{cases} \Xi(S, I, t) = Sm(t, \omega), \\ \omega S = k - \frac{I}{T}. \end{cases} \quad (6)$$

Hence, (5) becomes:

$$\begin{cases} \frac{\partial m}{\partial t} + \frac{1}{2} \sigma^2 \omega^2 \frac{\partial^2 m}{\partial \omega^2} - \left(\frac{1}{T} + r\omega \right) \frac{\partial m}{\partial \omega} = 0, \\ m(T, \omega) = \varphi(\omega). \end{cases} \quad (7)$$

Obviously, (7) is now reformed to be in two dimensional whose solution will be used to obtain the Asian option price via the link in (6).

3 The Analysis of the Laplace Adomian decomposition method

In this sequel, we consider w.r.t. LADM [37–44] the following differential equation (partial or ordinary) of the form:

$$g\zeta(x, t) = h(x, t) \quad (8)$$

where g signifies a first order differential operator in t , which may be nonlinear, thereby including linear and nonlinear terms. Hence, (8) is decomposed as:

$$\underbrace{L_t \zeta(x, t) + R\zeta(x, t) + N\zeta(x, t)}_{g\zeta(x, t)} = h(x, t) \quad (9)$$

where $L_t(\cdot) = \frac{\partial}{\partial t}(\cdot)$, R is a linear differential operator, N represents the nonlinear differential operator equivalent to an analytical nonlinear term, while $h(x, t)$ is the associated source term. Hence, (9) becomes:

$$L_t \zeta(x, t) = h(x, t) - (R\zeta(x, t) + N\zeta(x, t)). \quad (10)$$

We proceed by introducing the Laplace operator to differentiate the solution technique from the classical ADM. This is done as follows via the definitions:

Definition 1: Let $f(t)$ be defined on $t \in [0, \infty)$, then the Laplace transform of $f(t)$ is $F(s)$ defined as:

$$F(s) = \hat{L}\{f(t)\} = \int_0^\infty f(t) e^{-st} dt. \quad (11)$$

Definition 2: For a continuous function $f(t)$ such that $F(s) = \hat{L}\{f(t)\}$, $f(t)$ called the inverse Laplace transform (ILT) is defined as:

$$\hat{L}^{-1}\{F(s)\} = f(t). \quad (12)$$

Definition 3: For an n -th order differential equation, the associated Laplace transform is:

$$\begin{cases} \hat{L}\{f^{(n)}(t)\} = s^n \hat{L}\{f(t)\} - \sum_{i=0}^{n-1} s^{n-1-i} f^{(i)}(0), \\ \hat{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s), \end{cases} \quad (13)$$

where (n) denotes the n -th derivative with respect to t and with respect to s associated with $f^{(n)}(t)$ and $F^{(n)}(s)$ respectively.

The Laplace Transform (LT) is incorporated in the ADM [37–44] by taking the LT of both sides of (10) as follow:

$$\hat{L}\{L_t\zeta(x, t)\} = \hat{L}\{h(x, t) - (R\zeta(x, t) + N\zeta(x, t))\}. \quad (14)$$

By using the derivative properties as noted in (13), therefore (14) becomes:

$$\begin{aligned} s\zeta(x, s) - \zeta(x, 0) \\ = \hat{L}\{h(x, t) - (R\zeta(x, t) + N\zeta(x, t))\}. \end{aligned} \quad (15)$$

It thus implies that:

$$\begin{aligned} \zeta(x, s) \\ = \frac{1}{s} \left\{ \zeta(x, 0) + \hat{L} \left(\begin{array}{c} h(x, t) \\ -R\zeta(x, t) - N\zeta(x, t) \end{array} \right) \right\}. \end{aligned} \quad (16)$$

Hence, for non-homogeneous cases (NHC) and homogeneous cases (HC), we have:

$$\begin{aligned} \zeta(x, s) \\ = \left\{ \begin{array}{l} \left(\frac{1}{s} \left(\zeta(x, 0) + \hat{L}\{h(x, t)\} \right) \right. \\ \left. - \frac{1}{s} \hat{L}\{(R\zeta(x, t) + N\zeta(x, t))\}, \text{ NHC}, \right. \\ \left. \frac{1}{s} \zeta(x, 0) - \frac{1}{s} \hat{L}\{(R\zeta(x, t) + N\zeta(x, t))\}, \text{ HC}. \right. \end{array} \right\} \end{aligned} \quad (17)$$

Applying the ILT $(\hat{L}^{-1}(\cdot))$ to (17) gives:

$$\begin{aligned} \zeta(x, t) \\ = \left\{ \begin{array}{l} \left(\zeta(x, 0) + \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L}\{h(x, t)\} \right\} \right) \\ \left(-\hat{L}^{-1} \left\{ \frac{1}{s} \hat{L}\{(R\zeta(x, t) + N\zeta(x, t))\} \right\} \right), \\ \zeta(x, 0) - \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L}\{(R\zeta(x, t) + N\zeta(x, t))\} \right\}, \end{array} \right\} \end{aligned} \quad (18)$$

Next, the LADM proposes representing the solution as an infinite series given as:

$$\zeta(x, t) = \sum_{n=0}^{\infty} \zeta_n(x, t) \quad (19)$$

with $\zeta_n(x, t)$ to be computed recursively. Also, the nonlinear term $N\zeta(x, t)$ is defined as:

$$N(\zeta(x, t)) = \sum_{n=0}^{\infty} A_n(\zeta_0, \zeta_1, \zeta_2, \dots, \zeta_n) \quad (20)$$

and A_n referred to as Adomian polynomials is given as:

$$A_n = \frac{1}{n!} \frac{\partial^n}{\partial \lambda^n} \left[N \left(\sum_{i=0}^n \lambda^i \zeta_i \right) \right]_{\lambda=0}. \quad (21)$$

Therefore, substituting (19) and (20) in (18) gives:

$$\sum_{n=0}^{\infty} \zeta_n(x, t) = \left(\zeta(x, 0) + \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L}\{h(x, t)\} \right\} - Q \right). \quad (22)$$

where:

$$Q = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(R \sum_{n=0}^{\infty} \zeta_n(x, t) + \sum_{n=0}^{\infty} A_n \right) \right\} \right\}.$$

From (22), the solution $\zeta(x, t)$ is therefore determined via the recursive relation:

$$\begin{cases} \zeta_0 = \zeta(x, 0) + \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L}\{h(x, t)\} \right\} \\ \zeta_{n+1} = -\hat{L}^{-1} \left\{ \frac{1}{s} \hat{L}\{(R(\zeta_n) + NA_n)\} \right\}, \quad n \geq 0 \end{cases} \quad (23)$$

while $\zeta(x, t)$ is finalized as:

$$\zeta(x, t) = \lim_{j \rightarrow \infty} \sum_{n=0}^j \zeta_n(x, t). \quad (24)$$

4 Illustrative examples and applications

Here, the consideration of the analytical solution is made based on the proposed LADM with two illustrative cases.

Case I: Consider (5) via (6-7) in an operator form as follows:

$$\begin{cases} m_t = \left(\frac{1}{T} + r\omega \right) m_\omega - \frac{1}{2} \sigma^2 \omega^2 m_{\omega\omega} \\ m(0, \omega) = \varphi(\omega), \end{cases} \quad (25)$$

where the subscripts denote partial derivatives w.r.t. the subscripted variables.

Hence, taking the Laplace transform of (25) gives:

$$m(\omega, s) = \frac{1}{s} \left[m(\omega, 0) + \hat{L} \left\{ \begin{array}{c} \left(\frac{1}{T} + r\omega \right) m_\omega \\ -\frac{1}{2} \sigma^2 \omega^2 m_{\omega\omega} \end{array} \right\} \right]. \quad (26)$$

Thus, applying the ILT $(\hat{L}^{-1}(\cdot))$ to (26) gives:

$$m(\omega, t) = m(\omega, 0) + \hat{L}^{-1} \left[\frac{1}{s} \hat{L} \left\{ \begin{array}{c} \left(\frac{1}{T} + r\omega \right) m_\omega \\ -\frac{1}{2} \sigma^2 \omega^2 m_{\omega\omega} \end{array} \right\} \right]. \quad (27)$$

Therefore, with the initial condition, and the infinite series solution form: $m(\omega, t) = \sum_{n=0}^{\infty} m_n(\omega, t)$, the following recursive relation is obtained via the proposed LADM:

$$\begin{cases} m_0 = m(\omega, 0) \\ m_{n+1} = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left(\begin{array}{c} \left(\frac{1}{T} + r\omega \right) (m_n)' \\ -\frac{1}{2} \sigma^2 \omega^2 (m_n)'' \end{array} \right) \right\}, \quad n \geq 0. \end{cases} \quad (28)$$

Note: the prime notations in (27) denote derivatives w.r.t. ω .

Therefore, the recursive relation in (28) yields:

$$\begin{cases} m_0 = m(\omega, 0) \\ m_1 = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(\frac{1}{T} + r\omega \right) (m_0)' - \frac{1}{2} \sigma^2 \omega^2 (m_0)'' \right\} \right\}, \\ m_2 = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(\frac{1}{T} + r\omega \right) (m_1)' - \frac{1}{2} \sigma^2 \omega^2 (m_1)'' \right\} \right\}, \\ m_3 = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(\frac{1}{T} + r\omega \right) (m_2)' - \frac{1}{2} \sigma^2 \omega^2 (m_2)'' \right\} \right\}, \\ m_4 = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(\frac{1}{T} + r\omega \right) (m_3)' - \frac{1}{2} \sigma^2 \omega^2 (m_3)'' \right\} \right\}, \\ m_5 = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(\frac{1}{T} + r\omega \right) (m_4)' - \frac{1}{2} \sigma^2 \omega^2 (m_4)'' \right\} \right\}, \\ \vdots \\ m_k = \hat{L}^{-1} \left\{ \frac{1}{s} \hat{L} \left\{ \left(\frac{1}{T} + r\omega \right) (m_{k-1})' - \frac{1}{2} \sigma^2 \omega^2 (m_{k-1})'' \right\} \right\}. \end{cases} \quad (29)$$

Thus, we obtain the following by subjecting (29) to the initial condition:

$$m_0 = m(\omega, 0) = \frac{1}{rT} (1 - e^{-rT}) - \omega e^{-rT}. \quad (30)$$

$$\begin{cases} m_1 = -\left(\frac{1}{T} + r\omega\right) t e^{-rT}, \\ m_2 = -\frac{1}{2!} \left(\frac{1}{T} + r\omega\right) r t^2 e^{-rT}, \\ m_3 = -\frac{1}{3!} \left(\frac{1}{T} + r\omega\right) r^2 t^3 e^{-rT}, \\ m_4 = -\frac{1}{4!} \left(\frac{1}{T} + r\omega\right) r^3 t^4 e^{-rT}, \\ m_5 = -\frac{1}{5!} \left(\frac{1}{T} + r\omega\right) r^4 t^5 e^{-rT}, \\ \vdots, \\ m_k = -\frac{1}{k!} \left(\frac{1}{T} + r\omega\right) r^{k-1} t^k e^{-rT}, \quad k \geq 1 \end{cases}$$

Hence,

$$\begin{aligned} m(\omega, t) &= \sum_{j=0}^{\infty} m_j \quad (31) \\ &= \left(\frac{1}{rT} (1 - e^{-rT}) - \omega e^{-rT} \right) \\ &\quad - \left(\frac{1}{T} + r\omega \right) \left(t + \frac{r t^2}{2!} + \dots \right) e^{-rT} \\ &= \left(\frac{1}{rT} (1 - e^{-rT}) - \omega e^{-rT} \right) \\ &\quad - \frac{1}{r} \left(\frac{1}{T} + r\omega \right) \left(r t + \frac{(r t)^2}{2!} + \dots \right) e^{-rT} \\ &= \left(\frac{1}{rT} (1 - e^{-rT}) - \omega e^{-rT} \right) \\ &\quad - \frac{1}{r} \sum_{i=1}^{\infty} \frac{(r t)^i}{i!} \left(\frac{1}{T} + r\omega \right) e^{-rT} \\ &= \left(\frac{1}{rT} (1 - e^{-rT}) - \omega e^{-rT} \right) \\ &\quad - \frac{1}{r} \left(-1 + \sum_{i=0}^{\infty} \frac{(r t)^i}{i!} \right) \left(\frac{1}{T} + r\omega \right) e^{-rT} \\ &= \left(\frac{1}{rT} (1 - e^{-rT}) - \omega e^{-rT} \right) \\ &\quad - \frac{1}{r} (-1 + e^{rt}) \left(\frac{1}{T} + r\omega \right) e^{-rT} \\ &= \left(\frac{1}{rT} (1 - e^{-r(T-t)}) - \omega e^{-r(T-t)} \right), \quad \omega \leq 0. \end{aligned}$$

$$\text{But from (6), } \begin{cases} \Xi(S, I, t) = S m(t, \omega), \\ \omega S = k - \frac{I}{T}. \end{cases}$$

Therefore,

$$\begin{aligned} \Xi(S, I, t) & \quad (32) \\ &= \left(\frac{S}{rT} (1 - e^{-r(T-t)}) - \left(k - \frac{I}{T} \right) e^{-r(T-t)} \right). \end{aligned}$$

Equation (32) is the analytical solution of (5) corresponding to the continuous arithmetic Asian option pricing model.

Case II: Suppose (25) via (29) is considered based on a different initial condition:

$$m_0 = m(\omega, 0) = \left(\omega r - \frac{1}{rT} \right) S e^{-rT}. \quad (33)$$

Then, by the same approach, we have:

$$\begin{cases} m_1 = (T^{-1} + r\omega) r t S e^{-rT}, \\ m_2 = \frac{1}{2!} (T^{-1} + r\omega) (r t)^2 S e^{-rT}, \\ m_3 = \frac{1}{3!} (T^{-1} + r\omega) (r t)^3 S e^{-rT}, \\ m_4 = \frac{1}{4!} (T^{-1} + r\omega) (r t)^4 S e^{-rT}, \\ m_5 = \frac{1}{5!} (T^{-1} + r\omega) (r t)^5 S e^{-rT}, \\ m_6 = \frac{1}{6!} (T^{-1} + r\omega) (r t)^6 S e^{-rT}, \\ \vdots, \\ m_p = \frac{1}{p!} (T^{-1} + r\omega) (r t)^p S e^{-rT}, \quad p \geq 1. \end{cases}$$

So,

$$\begin{aligned} m(\omega, t) &= \sum_{j=0}^{\infty} m_j \quad (34) \\ &= \left\{ \left(\omega r - \frac{1}{rT} \right) + (-1 + e^{rt}) \left(\frac{1}{T} + r\omega \right) \right\} S e^{-rT}. \end{aligned}$$

$$\therefore \Xi(S, I, t) = \left\{ \left(\omega r - \frac{1}{rT} \right) + (-1 + e^{rt}) \left(\frac{1}{T} + r\omega \right) \right\} S^2 e^{-rT}. \quad (35)$$

4.1 The Greeks of the Asian Option Model

Here, the Greeks (*G1-G4*) of the continuous AOPM are briefly introduced as their mathematical expressions are given. This is considered for a certain option value, $\Xi_*(S, I, t) = \Xi_*$.

G1: The Theta-Greek of an option measures the rate of change of the option price w.r.t. the passage of time. Mathematically, the Delta is obtained by differentiating once the option value w.r.t the time variable say:

$$\Xi_{*t} = \frac{\partial \Xi_*}{\partial t}.$$

G2: The Delta-Greek of an option measures the rate of change of the option price w.r.t. the underlying asset price. It is the sensitivity of the option to the price of the asset. Mathematically, the Delta is obtained by differentiating once the option value w.r.t the spatial variable say:

$$\Xi_{*S} = \frac{\partial \Xi_{*}}{\partial S}.$$

G3: The Gamma-Greek of an option defines the rate of change of the Delta-Greek w.r.t. the spatial variable. Mathematically, the Gamma is obtained by differentiating the Delta-Greek w.r.t the spatial variable say:

$$\Xi_{*SS} = \frac{\partial^2 \Xi_{*}}{\partial S^2}.$$

G4: The Speed-Greek of an option defines the rate of change of the Gamma-Greek w.r.t. the spatial variable. Mathematically, the Speed is obtained by differentiating the Gamma-Greek w.r.t the spatial variable say:

$$\Xi_{*SSS} = \frac{\partial^3 \Xi_{*}}{\partial S^3}.$$

5 Conclusions

This paper considered the Greek parameters: Theta, Delta, Speed, and Gamma associated with a continuous arithmetic Asian option pricing model. The derivation is based on the analytical solution of the continuous arithmetic Asian option model obtained via a proposed semi-analytical method: Laplace-Adomian decomposition method (LADM). To the best of the Authors' knowledge, the LADM is applied, for the first time, to the continuous arithmetic Asian option model for analytical solution. The solutions are provided in explicit form with few iterations, and less computational work is involved with high level of accuracy being maintained. For conformity, references are made to the analytical solutions obtained by Rogers & Shi (J. of Applied Probability 32: 1995, 1077-1088) [3], and Elshegmani & Ahmad (ScienceAsia, 39S: 2013, 67–69) [4]. The proposed method is highly recommended for analytical solution of other forms of Asian option pricing models such as the geometric put and call options, even in their time-fractional forms. The basic Greeks parameters obtained will be of great help to financial practitioners and traders in terms of hedging and portfolio management. Future research can include the application of the modified ADM, and the restarted ADM for speed and accuracy comparison.

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