Research Article

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Electro-magneto-hydrodynamic lubrication

https://doi.org/10.1515/phys-2018-0040
Received Nov 02, 2017; accepted Nov 12, 2017

Abstract: The topic of the presented paper aims to demonstrate a new principle of hydrodynamic lubrication in mechanical, thermal and electro-magnetic fields. Up till now, when dealing with the hydrodynamic theory lubrication, many authors of scientific papers have assumed the constant oil dynamic viscosity value without variations caused by temperature crosswise the film thickness. Simultaneously, due to the numerous AFM measurements, it appears that oil temperature gradients and oil viscosity changes in the bearing gap height directions cannot be omitted. Therefore, in this paper, the problem of the viscosity changes across the lubricant thin layer was resolved as the main novelty in principles of mechanical thermal lubrication. The method of solving the mentioned problem was manifested by a general model of semi-analytical solutions of isothermal electro-magneto-elastohydro-dynamic and non-Newtonian, lubrication problem formulated for two deformable rotational surfaces in curvilinear, co-ordinates.

Keywords: 3D-lubrication; viscosity variations crosswise film thickness; semi-analytical solutions; curvilinear coordinates

PACS: 47.50.-d, 47.65.-d, 47.10.ab, 47.10.ad, 47.15.gm

1 Introduction and governing equations

This paper presents a semi analytical method of solution of the asymmetrical, laminar, steady, non-Newtonian lubrication flow problem between two rotational and deformable, curvilinear orthogonal movable surfaces in conjugated elasto-hydro-electro-magnetic fields. The fluid flow between the two above mentioned solid surfaces in the electromagnetic field will be described by the 3 momentum equations of equilibrium in a vector form, a fluid continuity equation, and by equation of a conservation of energy equation in a scalar form. Hence, we obtain the following system [1, 2]:

$$Div\mathbf{S} + \mu_o(\mathbf{N}\nabla)\mathbf{H} + \frac{1}{2}\mu_o rot\,(\mathbf{N}\times\mathbf{H}) + \mathbf{J}\times\mathbf{B} = \rho\frac{d\mathbf{v}}{dt} \quad (1)$$

$$\frac{\partial\rho}{\partial t} + div\,(\rho v) = 0, \quad (2)$$

$$div\,(\kappa\; grad\; T) + \varphi_F = \rho\frac{d}{dt}(c_v T) + \mu_o T\Xi\,(\mathbf{v}\nabla)\,\mathbf{H} + \mathbf{J}^2/\sigma. \quad (3)$$

The above-mentioned system of equations is completed by magneto-thermo-elasticity equations describing problem in stresses for two solid surfaces restricting the thin fluid layer. This system consists of the three partial differential equations (4) in a vector form. Heat conductivity equation in a solid body (5) is added to this set of equations and we obtain a system in the following form [2–4]:

$$Div\,\mathbf{S}^* + \mathbf{J}^*\times\mathbf{B}^* + \mu_o\left(\mathbf{N}^*\nabla\right)\mathbf{H}^* = \rho\star\frac{\partial^2\mathbf{u}}{\partial t^2} \quad (4)$$

$$div\left(\kappa^* gradT^*\right) = \rho\star c_v^\star\frac{\partial T^\star}{\partial t} \quad (5)$$

Moreover, we add the Maxwell and Ohm equations as well for the two surfaces as for the thin boundary liquid layer between two surfaces. Thus we get such equations [5]:

$$\nabla\cdot\mathbf{B}^* = 0,\quad \nabla\cdot\mathbf{B} = 0, \quad (6)$$
$$\nabla\times\mathbf{H}^* = \mathbf{J}^* + \frac{\partial\mathbf{D}^*}{\partial t},\; \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\mathbf{E}^* = -\frac{\partial\mathbf{B}^*}{\partial t},\; \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}, \quad (7)$$
$$\mathbf{J}\star = \sigma\star\mathbf{E}^*,\; \mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

We assume the following notations: μ_o – magnetic permeability in vacuum H/m, T – fluid temperature in K, T^* – solid body temperature in K, **B** – magnetic induction vector in T, **N** - magnetization vector A/m, **E** – electric intensity vector V/m, **H** – magnetic intensity vector A/m, (– first

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derivative of magnetization vector with respect to temperature A/mK, σ - electrical conductivity coefficient S/m, **J** - electric current density in A/m^2, **D** - electric induction vector As/m, ρ - fluid density kg/m^3, κ - thermal conductivity coefficient W/mK, **v** – fluid velocity vector in m/s, φ_F – dissipation of energy in W/m^3, **S** - stress tensor in the fluid in Pa, **u** - displacement vector of the solid body in m, t - time in s, c_v - specific heat in J/kgK. The symbols with an asterisk are related to the solid body.

The relationship between stress tensor **S** and strain rate tensor $2\mathbf{T}_d = \mathbf{A}_1$ *i.e.* constitutive equations are assumed for the lubricant in the following form [3]:

$$\mathbf{S} = -p\boldsymbol{\delta} + \eta_p \mathbf{A_1} \tag{8}$$

whereas unit tensor $\boldsymbol{\delta}$, strain rate tensor $\mathbf{A}_1$ have the following components: δ_{ij}, Θ_{ij} in s^{-1}. For power law of compressible fluid the apparent viscosity η_p has the form [2–4]:

$$\eta_p \equiv \eta_{pr} = 2^{n-1} m(n) \left| \frac{1}{2}\mathbf{I}_1^2 - \mathbf{I}_2 \right|^{\frac{n-1}{2}}, \tag{9a}$$

$$\varphi_F = 4\eta_p \left(\frac{1}{2}\mathbf{I}_1^2 - \mathbf{I}_2 \right) - p\mathbf{I}_1,$$

$$\mathbf{I}_1 = div v, \tag{9b}$$

$$\mathbf{I}_2 = \Theta_{11}\Theta_{22} - \Theta_{12}^2 + \Theta_{11}\Theta_{33} - \Theta_{13}^2 + \Theta_{22}\Theta_{33} - \Theta_{23}^2,$$

We assume: n – dimensionless flow index (0.5, 1.2), *m=m(n)* fluid consistency coefficient in Pas^n.

The Duhamel Neumann relations between the components τ_{ij} of the stress tensor S* of the elastic body on the sleeve and the strain tensor components ϵ_{ij} take the following form:

$$\tau_{ij} = 2G\varepsilon_{ij} + \left(\Lambda \varepsilon_{kk} - 3\alpha_T K \Delta T^\star \right) \delta_{ij} \tag{10}$$

for *i,j*=1,2,3, where: δ_{ij} – Kronecker unit tensor component (δ_{ij} =1 for *i=j* and δ_{ij} =0 for *i≠j*), $K = \Lambda + 2G/3$, $G = E_Y/[2(1+\vartheta)]$, $\Lambda = E_Y\vartheta/(1+\vartheta)(1-2\vartheta)$, whereas E_Y - Young‘s Modulus in Pa, α_T – thermal coefficient of linear expansionin K^{-1}, ϑ – Poisson ratio.

And by virtue of known linear geometrical relations, the strain tensor components $\Theta_{ij}(v_i)$ in the oil, and strain components $\epsilon_{ij}(u_i)$ in the sleeve are mutually connected with the oil velocity components v_i and displacement vector of solid body components u_i, respectively:

$$\Theta_{ij} = 0.5(v_{i|j} + v_{j|i}), \quad \varepsilon_{ij} = 0.5(u_{i|j} + u_{j|i}). \tag{11}$$

2 Electro-magneto-hydrodynamic equations

We assume: unsymmetrical, incompressible (for invariant $\mathbf{I}_1 = 0$), steady, pseudo-plastic lubrication in electromagnetic field for power law model without viscous-elastic properties. Oil apparent viscosity η_p varies in length, width and gap-height directions and depends on pressure, temperature flow shear ratio, and electromagnetic field [1–5]. Inertia forces and terms of energy convections are neglected, pressure is constant in gap height direction. The total distance ϵ_T between the two surfaces is signifucantly smaller than other dimensions of the considered surfaces. Taking into account the layer boundary simplification *i.e.* neglecting in (1)-(7) the negligibly small terms of order $\Psi = 0.001$ (radial clearance) presenting the quotient of characteristic gap height ε_0 to the radius R of journal, the problem is made in local curvilinear and orthogonal coordinates (α_1, α_2, α_3) connected with one of the movable surfaces, where α_2 denotes the direction of gap height. The parallel and longitudinal intersections of the cooperating deformed, rotational surfaces have curvilinear and non-monotone generating lines. Hence, Lame coefficients have the form: $h_1=h_1(\alpha_3)$, $h_2=1$, $h_3=h_3(\alpha_3)$.

We put physical and geometrical dependencies (9), (10), (11) into expanded boundary simplified equations (1)-(7). Hence, we obtain the system of equations of conservation of momentum, continuity, energy, thermo-elasticity, heat transfer, Maxwell simplified results in the following form of curvilinear orthogonal co-ordinates (α_1, α_2, α_3):

$$0 = -\frac{1}{h_i}\frac{\partial p}{\partial \alpha_i} + \frac{\partial}{\partial \alpha_2}\left[\eta_p(\alpha_1, \alpha_2, \alpha_3) \frac{\partial v_i}{\partial \alpha_2} \right] + M_i, \tag{12}$$

$$\frac{\partial p}{\partial \alpha_2} = 0,$$

$$0 = \frac{1}{h_1}\frac{\partial v_1}{\partial \alpha_1} + \frac{\partial v_2}{\partial \alpha_2} + \frac{1}{h_1 h_3}\frac{\partial}{\partial \alpha_3}(h_1 v_3) \tag{13}$$

$$\frac{\partial}{\partial \alpha_2}\left(\kappa \frac{\partial T}{\partial \alpha_2} \right) + [\eta_p(\alpha_1, \alpha_2, \alpha_3)] \; V(v_1, v_3) = M_T. \tag{14}$$

$$\frac{\partial}{\partial \alpha_2}\left(\kappa^\star \frac{\partial T^\star}{\partial \alpha_2} \right) = 0, \quad \frac{\partial}{\partial \alpha_2}\left[(2G + \Lambda\delta_{2i}) \frac{\partial u_i}{\partial \alpha_2} \right] = \tag{15}$$

$$= \delta_{2i}\left[\frac{\partial}{\partial \alpha_2}\left(3K\alpha_T^\star T^\star \right) + \left(\mathbf{J}^\star \times \mathbf{B}^\star \right)_i - \mu_0 \left(\mathbf{N}^\star \nabla \right) H_i^\star \right]$$

$$\eta_p(\alpha_1, \alpha_2, \alpha_3) \equiv [m(n, T, B, E)] \cdot V^g(v_1, v_3), \tag{16}$$

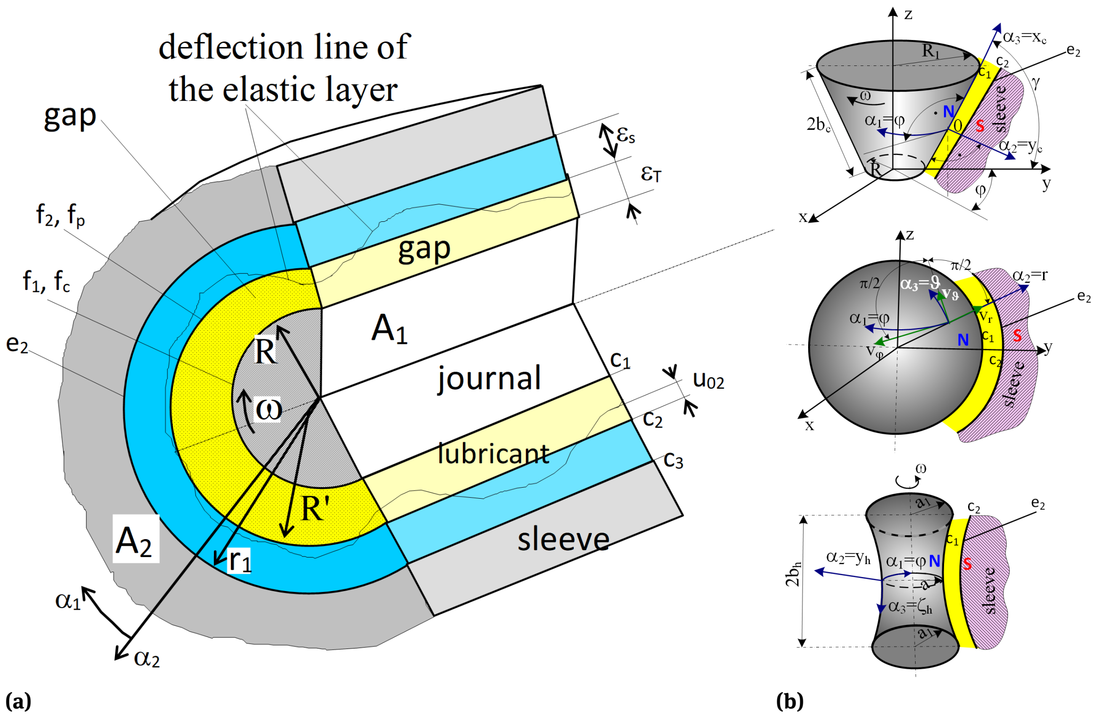

Figure 1: Motionless solid A_2 and moving body A_1 loaded by means of pressure and temperature and magnetic field; N, S - magnetic poles; a) cylindrical bearing, b) other bearing

$$V \equiv \left(\frac{\partial v_1}{\partial \alpha_2}\right)^2 + \left(\frac{\partial v_3}{\partial \alpha_2}\right)^2,$$

$$g(n) \equiv \frac{n-1}{2}, \tag{17}$$

$$M_T(\alpha_1, \alpha_3) \equiv \mu_0 T \Xi (v\nabla) H + J^2/\sigma,$$

$$M_i(\alpha_1, \alpha_3) \equiv \mu_0 (\mathbf{N}\nabla) H_i + \frac{1}{2}\mu_o rot(\mathbf{N} \times \mathbf{H})_i + (\mathbf{J} \times \mathbf{B})_i,$$

where the length, width and gap- height directions, are limited respectively: $0 < \alpha_1 \leq 2\pi$, $-b_m \leq \alpha_3 b_s$, $0 \leq \alpha_2 \leq \epsilon_T$ and i=1,2,3. The system of equations (12)((17) contains the following unknown dimensional values: pressure $p(\alpha_1, \alpha_3)$, temperature in oil $T(\alpha_1, \alpha_2, \alpha_3)$ and sleeve T^*, three oil velocity components $v_i(\alpha_1, \alpha_2, \alpha_3)$ and three layer displacement components $u_i(\alpha_1, \alpha_2, \alpha_3)$, for i=1,2,3 in three curvilinear, orthogonal dimensional directions: $\alpha_1, \alpha_2, \alpha_3$.

3 Boundary conditions

The lubricant flow in the bearing gap is generated by the rotation of a rotational, curvilinear journal. Hence the boundary conditions for lubricant velocity components have the form:

$$v_1 = \omega h_1 \text{ for } \alpha_2 = 0,\ v_1 = 0 \tag{18}$$
$$\text{for } \alpha_2 = \epsilon_T,\ v_2 = 0 \text{ for } \alpha_2 = 0,$$
$$\text{and for } \alpha_2 = \epsilon_T,\ v_3 = 0 \text{ for } \alpha_2 = 0,\ v_3 = 0 \text{ for } \alpha_2 = \epsilon_T,$$

where ω denotes the angular velocity of rotational journal, in the circumferential direction.

Decrements or increments that are above the characteristic environmental temperature T_0 have constant value f_c on the journal surface and variable unknown values $f_p(\alpha_1, \alpha_3)$ on the sleeve surface. Heat flux is transferred from the rotational surface of bearing journal into lubricant. Hence the boundary temperature values are as follows:

$$T(\alpha_1, \alpha_2, \alpha_3) = T_0 + f_c \text{ for } \alpha_2 = 0, \tag{19a}$$

$$T(\alpha_1, \alpha_2, \alpha_3) = T_0 + f_p(\alpha_1, \alpha_3)\ for\ \alpha_2 = \epsilon_T \tag{19b}$$

$$\kappa \frac{\partial T}{\partial \alpha_2} = -q_c \equiv -\upsilon \Delta f \text{ for } \alpha_2 = 0, \tag{19c}$$

where Δf – temperature difference across the film thickness, υ – heat transfer coefficient in W/m^{2}K, q_c - heat flux density in oil on journal surface in W/m^2.

The elastic layer of bearing alloy is laying on the rigid ring in place $\alpha_2 = c_3$, and therefore the contact surface of these bodies is not deformed by the pressure. The contact surface $\alpha_2 = c_3$ is yet deformed by variable temperature, which implies the shape and the total volume change of the rigid ring. These deformations are not considered now.

The temperatures and surfaces localization is presented in Figure 1.

The boundary conditions in the elastic layer have the following form:

$$\tau^{\star}_{ij}(\alpha_1, \alpha_2 = c_2, \alpha_3) = -p(\alpha_1, \alpha_3)\delta_{i2}\delta_{ij} \quad (20)$$

$$T^{\star}(\alpha_1, \alpha_2 = c_3, \alpha_3) = T_0 + e_2(\alpha_1, \alpha_3) \quad (21)$$

$$T^{\star}(\alpha_1, \alpha_2 = c_2, \alpha_3) = T_0 + f_2(\alpha_1, \alpha_3) \quad (22)$$

$$\kappa^{\star}\frac{\partial T^{\star}}{\partial \alpha_2} = -q_p, \quad (23)$$

where i=1,2,3. Moreover: T_0 - average ambient temperature, q_p - heat flux density across the elastic layer located on the sleeve surface, $f_2(\alpha_1, \alpha_3)$ - temperature increases above the temperature T_0 on the external surface of the elastic layer for $\alpha_2 = c_2$, additionally $e_2(\alpha_1, \alpha_3)$ temperature increases in the excess of temperature T_0 on the bottom surface of the elastic layer in the place of $\alpha_2 = c_3$.

4 Oil consistency and viscosity variations

Temperature decreases and magnetic induction, electric intensity increases the oil consistency m and dynamic viscosity coefficient η. This phenomenon is described in the following form:

$$\begin{aligned} m(\alpha_1, \alpha_2, \alpha_3) &= m(n, T, B, E) \quad (24)\\ &= \eta_0\eta_1(T, B, E)\cdot(v_0/\varepsilon_0)^{1-n},\\ \eta_1(T, B, E) &= \exp\,[\delta_B B + \delta_E E - \delta_T T], \end{aligned}$$

with coefficient δ_B, δ_E of magnetic, electric influence on the oil viscosity and coefficient δ_T of temperature influence on the oil viscosity Mentioned phenomena are confirmed by virtue of authors own measurements.

5 Sketch of semi-analytical solutions

After double integration of two equations (15) in α_2 direction we apply the low heat flux continuity ($q_p = q_c$) in the contact region between the elastic alloy layer and lubricant, and finally we obtain the external displacement of the sleeve surface in the following dimensional form:

$$\begin{aligned} u_{02} &= u_2(\alpha_1, \alpha_2 = c_2, \alpha_3) = \quad (25)\\ &= \varepsilon_s\frac{1+\vartheta}{1-\vartheta}\cdot\frac{1-2\vartheta}{E}p - \left[f_p - \frac{1}{2}\frac{\varepsilon_s q_p}{\kappa^{\star}}\right]\varepsilon_s\alpha^{\star}_T\frac{1+\vartheta}{1-\vartheta} +\\ &+ \frac{1}{2}\cdot\frac{1-2\vartheta}{E}\cdot\frac{1+\vartheta}{1-\vartheta}\varepsilon_s^2\left[\mathbf{N}^{\star}\nabla B^{\star}_2 + \left(\mathbf{J}^{\star}\times\mathbf{B}^{\star}\right)_2\right] \end{aligned}$$

where $\epsilon_s = c_3 - c_2$, $0 < \alpha_1 < 2\pi$, $-b_m < \alpha_3 < +b_m$. Solution (25) valid if and only if E_Y - Young modulus and solid body coefficients α_T, $\kappa^{\star}$, ϑ - Poisson ratio, are independent of α_2, The hydrodynamic pressure p, the load carrying capacity and temperature T distributions with oil velocity components v_1, v_2, v_3 in machine slide journal bearing are solved by the means of the quasi half numerical Frobenius small parameter method implemented by the finite difference method using Mathcad 15 Professional Program. A system of partial differential equations (12)-(14) defined in thin space $(\alpha_1, \alpha_2, \alpha_3, t)$ between two movable rotational surfaces and for continuous $m(n)$, $\eta == m(n = 1)$, has an analytical solution in the form of the following infinite uniform convergent power function series in following form:

$$v_i(\alpha_1, \alpha_2, \alpha_3) = v_i^{(0)} + gv_i^{(1)} + \ldots + g^k v_i^{(k)} + \ldots \quad (26a)$$
$$T(\alpha_1, \alpha_2, \alpha_3) = T^{(0)} + gT^{(1)} + \ldots + g^k T^{(k)} + \ldots$$

$$p(\alpha_1, \alpha_3) = p^{(0)} + gp^{(1)} + \ldots + g^k p^{(k)} + \ldots \quad (26b)$$
$$q_c = q_c^{(0)} + gq_c^{(1)} + \ldots + g^k q_c^{(k)} + \ldots$$

where $q_c^{(0)} = q_c^{(1)} = \ldots = q_c^{(k)} \equiv q$, with $q_c = q/(1-g)$; for i=1,2,3; k=0,1,2,..., g=$g(n)$≡ (n-1)/2 where for $0 < n \leq 3/2$ the small parameter g is less than +1/4 and greater than −1/2. In order to achieve the linearization of apparent viscosity (16), we put functions (26a) into Eq. (16) and expand function in closed interval [1, n] or [n, 1] for $0 < n \leq 3/2$ using Taylor series in the neighborhood of point n=1 with respect to the small parameter in the following form:

$$\begin{aligned} \eta_p(\alpha_1, \alpha_2, \alpha_3) &= \eta_0\cdot[\eta_1(T, B, E)] \quad (27)\\ &\cdot\left(1 + g\eta_{pr1} + \ldots + g^k\eta_{prk} + \ldots\right) \end{aligned}$$

where η_1(T,B,E) = $\eta_1(\alpha_1, \alpha_2, \alpha_3)$ and functions $\eta_{prk} \equiv \eta_{prk}(v_1, v_3)$ for $k = 1, 2, \ldots$ – are the dimensionless expansion coefficients of dynamic viscosity dependent on $\alpha_1, \alpha_2, \alpha_3$, whereas for k=0 $\eta_{pr0} = 1$. Such functions after involved calculations are determined in analytical form. Putting the infinite series (26), (27) into the system (12)-(14), multiplying the series by Cauchy methods and equating the terms by the same powers k=0,1,2,... of the small parameter g, and then using the L. Kronecker symbol *i.e.* $\delta_{k0} = 1$ for k=0, $\delta_{k0} = 0$ for k=1,2,..., we obtain the linear differential systems of equations for k=0,1,2,... i=1,3 in following form:

$$\frac{1}{h_i}\frac{\partial p^{(k)}}{\partial \alpha_1} = \frac{\partial}{\partial \alpha_2}\left[\eta_0\eta_1\left(B, E, T^{(k)}\right)\frac{\partial v_i^{(k)}}{\partial \alpha_2}\right] + \frac{\partial}{\partial \alpha_2}\left[\eta_0\eta_1\left(B, E, T^{(k)}\right)S_i^{(k)}\right] + M_i\delta_{k0}, \quad \frac{\partial p^{(k)}}{\partial \alpha_2} = 0, \tag{28}$$

$$\frac{1}{h_1}\frac{\partial}{\partial \alpha_1}\left(v_1^{(k)}\right) + \frac{\partial}{\partial \alpha_2}\left(v_2^{(k)}\right) + \frac{1}{h_1 h_3}\frac{\partial}{\partial \alpha_3}\left(h_1 v_3^{(k)}\right) = 0, \tag{29}$$

$$\frac{\partial}{\partial \alpha_2}\left(\kappa\frac{\partial T^{(k)}}{\partial \alpha_2}\right) = -\eta_0\eta_1\left(B, E, T^{(k)}\right)V_T^{(k)} + M_T\delta_{k0}. \tag{30}$$

with

$$\eta_1\left(B, E, T^{(k)}\right) = [\eta_{1BE}(B, E)]\, e^{-\delta_T T^{(k)}(\alpha_1,\alpha_2,\alpha_3)}, \quad \eta_{1BE}(B, E) \equiv \eta_{1BE}(\alpha_1, \alpha_3), \quad S_i^{(0)} = 0, \quad V_T^{(0)} \equiv V\left(v_1^{(0)}, v_3^{(0)}\right), \quad S_i^{(1)} \equiv \frac{\partial v_i^{(0)}}{\partial \alpha_2}\left[\left(\frac{\varepsilon_0}{v_0}\right)^2 V_T^{(0)}\right], \tag{31a}$$

$$V_T^{(1)} \equiv V_T^{(0)}\cdot \ln\left[\left(\frac{\varepsilon_0}{v_0}\right)^2 V_T^{(0)}\right] + 2\left(\frac{\partial v_1^{(0)}}{\partial \alpha_2}\right)\left(\frac{\partial v_1^{(1)}}{\partial \alpha_2}\right) + 2\left(\frac{\partial v_3^{(0)}}{\partial \alpha_2}\right)\left(\frac{\partial v_3^{(1)}}{\partial \alpha_2}\right), \tag{31b}$$

For $k = 0$ the abovementioned system of equations (28)-(31) determines functions: $v_i^{(0)}$, $T_i^{(0)}$, $p^{(0)}$ for Newtonian oil taking into account the boundary conditions in steady motion (18), (19). For k=1,2,... and i=1,2,3 the system of equations (28)-(31) determines the first, second,... corrections: $v_i^{(k)}$, $T^{(k)}$, $p^{(k)}$ caused by the non-Newtonian oil. The boundary conditions for k=0 are as follows:

$$v_1^{(0)} = \omega\, h_1,\ v_2^{(0)} = 0, v_3^{(0)} = 0,\ T^{(0)} = T_0 + f_c \quad \text{for } \alpha_2 = 0, \qquad v_1^{(0)} = 0,\ v_2^{(0)} = 0,\ v_3^{(0)} = 0,\ T^{(0)} = T_0 + f_p^{(0)} \quad \text{for } \alpha_2 = \varepsilon_T; \tag{32a}$$

and for k=1,2,... we have:

$$v_i^{(k)} = 0,\ T^{(k)} = sf_p^k, \quad \text{for} \quad s = \alpha_2/\varepsilon_T, \alpha_2 = 0, \alpha_2 = \varepsilon_T; \tag{32b}$$

The total gap height ϵ_T includes surface displacement (25). Integrating twice (28) and (29) once with respect to the variable α_2, then the oil velocity components for k=0,1,2,... by virtue of boundary conditions (32a,b) have the following form:

$$v_i^{(k)} = \frac{1}{\eta_0\eta_{1BE}}\cdot\frac{1}{h_i}\frac{\partial p^{(k)}}{\partial \alpha_i}K_i^{(k)}(\alpha_1, \alpha_2, \alpha_3) + \left[K_2^{(k)}(\alpha_1, \alpha_2, \alpha_3)\right]\cdot\int_0^{\varepsilon_T} S_i^{(k)}d\alpha_2 - \int_0^{\alpha_2} S_i^{(k)}d\alpha_2 + \omega h_1[1 - K_2^{(k)}(\alpha_1, \alpha_2, \alpha_3)]\delta_{i1}\delta_{k0} + \delta_{k0}\Delta M_i, \tag{33}$$

$$v_2^{(k)} = -\frac{1}{h_1}\int_0^{\alpha_2}\frac{\partial v_1^{(k)}}{\partial \alpha_1}d\alpha_2 - \frac{1}{h_1 h_3}\int_0^{\alpha_2}\frac{\partial}{\partial \alpha_3}\left(h_1 v_3^{(k)}\right)d\alpha_2, \tag{34}$$

for $i = 1, 3$; $0 < \alpha_{21} < \alpha_2 < \epsilon_T$, $0 < \alpha_1 < 2\pi$, $-b_m < \alpha_3 < +b_m$, whereas we denote:

$$\Delta M_i = -\frac{1}{\eta_0\eta_{1BE}}\int_0^{\alpha_2}\int_0^{\alpha_{21}} M_i d\alpha_{21}d\alpha_2 + \frac{1}{\eta_0\eta_{1BE}}s\int_0^{\varepsilon_T}\int_0^{\alpha_{21}} M_i d\alpha_{21}d\alpha_2 = \frac{\varepsilon_T^2}{2\eta_0\eta_{1BE}}M_i(s - s^2), \tag{35a}$$

$$K_1^{(k)}(\alpha_1, \alpha_2, \alpha_3) \equiv \frac{\det\left\|\Omega^{(k)}\right\|}{\Omega_1^{(k)}(\alpha_1, \alpha_2 = \varepsilon_T, \alpha_3)}, \quad K_2^{(k)}(\alpha_1, \alpha_2, \alpha_3) \equiv \frac{\Omega_1^{(k)}(\alpha_1, \alpha_2, \alpha_3)}{\Omega_1^{(k)}(\alpha_1, \alpha_2 = \varepsilon_T, \alpha_3)}, \tag{35b}$$

$$\left\|\Omega^{(k)}\right\| \equiv \left\|\begin{matrix}\Omega_2^{(k)}(\alpha_1, \alpha_2, \alpha_3) & \Omega_2^{(k)}(\alpha_1, \alpha_2 = \varepsilon_T, \alpha_3)\\ \Omega_1^{(k)}(\alpha_1, \alpha_2, \alpha_3) & \Omega_1^{(k)}(\alpha_1, \alpha_2 = \varepsilon_T, \alpha_3)\end{matrix}\right\|, \tag{35c}$$

$$\Omega_1^{(k)}(\alpha_1, \alpha_2, \alpha_3) \equiv \int_0^{\alpha_2} e^{-\delta_T\cdot T^{(k)}}d\alpha_2, \quad \Omega_2^{(k)}(\alpha_1, \alpha_2, \alpha_3) \equiv \int_0^{\alpha_2}\alpha_2\cdot e^{-\delta_T\cdot T^{(k)}}d\alpha_2.$$

It is easy to see, that:

$$K_1^{(k)}(\alpha_1, \alpha_2 = 0, \alpha_3) = 0, \tag{35d}$$

$$K_1^{(k)}(\alpha_1, \alpha_2 = \varepsilon_T, \alpha_3) = 0,$$
$$K_2^{(k)}(\alpha_1, \alpha_2 = 0, \alpha_3) = 0, \quad K_2^{(k)}(\alpha_1, \alpha_2 = \varepsilon_T, \alpha_3) = 1.$$

Imposing the boundary condition (32ab) in point $\alpha_2 = \epsilon_T$ on the solution (34) and taking into account the law of integral differentiation with variable limits of integration, we obtain the following modified Reynolds equation, which determines the unknown function $p^{(k)}(\alpha_1, \alpha_3)$:

$$\frac{1}{h_1}\frac{\partial}{\partial\alpha_1}\left\{\frac{1}{\eta_0\eta_{1BE}(\alpha_1,\alpha_3)}\frac{\partial p^{(k)}(\alpha_1,\alpha_3)}{\partial\alpha_1}\int_0^{\varepsilon_T} K_1^{(k)}(\alpha_1,\alpha_2,\alpha_3)d\alpha_2\right\} + \frac{1}{h_3}\frac{\partial}{\partial\alpha_3}\left\{\frac{1}{\eta_0\eta_{1BE}(\alpha_1,\alpha_3)}\frac{h_1}{h_3}\frac{\partial p^{(k)}(\alpha_1,\alpha_3)}{\partial\alpha_3}\int_0^{\varepsilon_T} K_1^{(k)}(\alpha_1,\alpha_2,\alpha_3)d\alpha_2\right\} + \tag{36}$$
$$+\frac{\partial}{\partial\alpha_1}\left[\Pi_{NNi}^{(k)} - \frac{\delta_{k0}\varepsilon_T^3 M_1}{\eta_0\eta_{1BE}(\alpha_1,\alpha_3)}\right] + \frac{\partial h_1}{h_3\partial\alpha_3}\left[\Pi_{NN3}^{(k)} - \frac{\delta_{k0}\varepsilon_T^3 M_3}{\eta_0\eta_{1BE}(\alpha_1,\alpha_3)}\right] =$$
$$= \omega h_1\frac{\partial}{\partial\alpha_1}\left[\int_0^{\varepsilon_T} K_2^{(k)}(\alpha_1,\alpha_2,\alpha_3)d\alpha_2 - \varepsilon_T\right],$$

with

$$\Pi_{NNi}^{(k)}(\alpha_1,\alpha_3) \equiv \left[\int_0^{\varepsilon_T} S_i^{(k)} d\alpha_2\right]\left[\int_0^{\varepsilon_T} K_2^{(k)}(\alpha_1,\alpha_2,\alpha_3)d\alpha_2\right] - \left[\int_0^{\varepsilon_T}\left(\int_0^{\alpha_2} S_i^{(k)} d\alpha_2\right)d\alpha_2\right], \tag{37}$$

for i=1,3; k=1,2,3.

To obtain the temperature functions $T^{(k)}(\alpha_1\alpha_2, \alpha_3)$ for k=0,1,2,..., we put solutions of oil velocity components (33), (34) for i=1,3; into the temperature equation (30). After double integration with respect to the variable α_2 *i.e.* in film thickness direction, using boundary conditions (19ab) in the form: $T^{(k)} = f_c\delta_{k0}$ for $\alpha_2 = 0$, $T^{(k)} = f_p\delta_{k0}$ for ($\alpha_2 = \epsilon_T$, and after the term ordering, we finally obtain:

$$T^{(k)} = \eta_0\eta_{1BE}\left\{s\int_0^{\varepsilon_T}\left[\frac{1}{\kappa}\int_0^{\alpha_2} V_T^{(k)} e^{-\delta_T T^{(k)}} d\alpha_2\right]d\alpha_2 - \int_0^{\alpha_2}\left[\frac{1}{\kappa}\int_0^{\alpha_2} V_T^{(k)} e^{-\delta_T T^{(k)}} d\alpha_2\right]d\alpha_2\right\} + \left\{\int_0^{\alpha_2}\left[\frac{1}{\kappa}\int_0^{\alpha_2} M_T\, d\alpha_2\right]d\alpha_2 - s\int_0^{\varepsilon_T}\left[\frac{1}{\kappa}\int_0^{\alpha_2} M_T\, d\alpha_2 d\alpha_2\right]d\alpha_2\right\}\delta_{k0} \tag{38}$$
$$+ \delta_{k0}\left[f_c(1-s) + f_p s\right],$$

where $s \equiv \alpha_2/\epsilon_T$, $0 \le s \le 1$. Non linear differential equation (38) determines temperature functions $T^{(k)}$ for k=0,1,2,... Imposing the condition (19c) on the temperature $T^{(k)}$ determined by the (38), we obtain the unknown temperature $f_p(\alpha_1, \alpha_3)$ on the sleeve.

To obtain the oil velocity components (33), hydrodynamic pressure (36) and temperature (38) for Newtonian oil, and taking into account oil viscosity variations crosswise the gap height, we put k=0 and hence: $S_i^{(0)} = 0$, $\Pi_{NNi}^{(0)} = 0$ for i=1.2.3. In this case we obtain oil velocity components $v_i^{(0)}$, pressure $p^{(0)}$, temperature $T^{(0)}$. Assuming cylindrical coordinates: $\alpha_1 = \phi$, $\alpha_2 = r$, $\alpha_3 = z$, $h_1 = R$, $h_2 = h_3 = 1$, where R is the radius of the cylindrical journal, we put the temperature $T^{(0)}$, determined from the reduced Eq. (38), into the reduced pressure Equation (36). Finally, the Eq. (36) determines the pressure $p(\phi, z)$, $0 \le \phi \le 2\pi$, $-b \le z \le b$, and tends to the following form:

$$\frac{\partial}{R\partial\phi}\left(\frac{\partial p^{(0)}}{R\partial\phi}\mathbf{K}_{J1}\right) + \frac{\partial}{\partial z}\left(\frac{\partial p^{(0)}}{\partial z}\mathbf{K}_{J1}\right) = \omega\frac{\partial\mathbf{K}_{J2}}{\partial\phi}, \tag{39}$$
$$\mathbf{K}_{J1} \equiv \frac{\varepsilon_T^3}{\eta_o\eta_{1BE}} + \delta_T\frac{\varepsilon_T^3}{\eta_o\eta_{1BE}}\left\{f_c - \frac{q}{2\kappa}\varepsilon_T - \frac{1}{5}\frac{(\omega R)^2\eta_o\eta_{1BE}}{\kappa} - \frac{1}{60\eta_o\eta_{1BE}\kappa}\varepsilon_T^4\left[\left(\frac{\partial p^{(0)}}{R\partial\phi}\right)^2 + \left(\frac{\partial\, p^{(0)}}{\partial\, z}\right)^2\right]\right\},$$
$$\mathbf{K}_{J2} = 6\varepsilon_T - 6\delta_T\varepsilon_T\left\{\frac{1}{6}\frac{q}{\kappa}\varepsilon_T + \frac{1}{24}\frac{(\omega R)^2\eta_o\eta_{1BE}}{\kappa} + \frac{1}{144\eta_o\eta_{1BE}\kappa}\varepsilon_T^4\left[\left(\frac{\partial p^{(0)}}{R\partial\phi}\right)^2 + \left(\frac{\partial p^{(0)}}{\partial z}\right)^2\right]\right\},$$

$0 \le \phi \le 2\pi$, $-b \le z \le b$, $p = p(\phi, z)$, $\eta_{1BE} = \exp(\delta_B B + \delta_E E)$.

For $\delta_T = 0$, viscosity is constant in gap height direction and (39) tends to the well known classical form of Reynolds equation.

6 Conclusions and results

1. After semi-analytical solutions and initial numerical calculations it appears that the pressure obtained directly for the oil dynamic viscosity variations crosswise the film thickness caused by the temperature gradients in gap height directions are about 5 to 7 percent different in comparison with the pressure values calculated for constant oil viscosity across the film thickness *i.e.* in the case when the temperature is constant in gap height direction and

varies only in bearing length and circumference direction.

2. The mutually direct interactions of influences between hydrodynamic pressure on temperature as well as temperature on pressure, are valid only if the temperature, hence oil dynamic viscosity, varies in the gap height direction.
3. It is worse to notice that constant temperature in the gap height direction does not correspond with 3D temperature field obtained directly from energy equation. The assumptions of constant temperature and viscosity crosswise the bearing gap are in contradiction with the contemporary achievements connected with new devices such as micro-bearing, nano-bearing, magnetic bearings, artificial joints in humanoid robots, micro-motors. Unfortunately, numerous authors in the field of hydrodynamic lubrication, avoid to assume the oil viscosity variations in the gap height direction.
4. The performed numerical calculations show that oil, presented in the calculation, increases the pressure and load capacity in the slide journal bearing for Newtonian and non-Newtonian oil, in the presence of magnetic induction field.

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