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Research Article

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Multivariable polynomial fitting of controlled single-phase nonlinear load of input current total harmonic distortion

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Abstract: The power systems usually include a number of nonlinear receivers. Nonlinear receivers are the source of disturbances generated to the power system in the form of higher harmonics. The level of these disturbances describes the total harmonic distortion coefficient THD. Its value depends on many factors. One of them are the deformation and change in RMS value of supply voltage. A modern LED luminaire is a nonlinear receiver as well. The paper presents the results of the analysis of the influence of change in RMS value of supply voltage and the level of dimming of the tested luminaire on the value of the current THD. The analysis was made using a mathematical model based on multivariable polynomial fitting.

Keywords: curve fitting, total harmonics distortion, single-phase nonlinear loads, LED luminaire

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1 Introduction

Nonlinear receivers are the source of many negative phenomena occurring in the power system. They receive the distorted currents from the power system which, when flowing by the power systems elements, result in voltage drops caused by higher harmonics of current. Consequently, this causes distortion of the supply voltage. The distorted current causes increased heating of wires and other power systems elements, as well as insulation ageing, increased power losses, increased probability of resonance, and many more. In three-phase systems, another problem concern the occurrence of current in neutral conductor.

Nowadays, more devices are equipped with SMPS power supplies. Such devices are receivers with nonlinear current-voltage characteristics *e.g.* LED luminaires that are equipped with SMPS power supplies. Modern luminaire solutions allow to control of the luminous flux. This is accomplished by controlling the current or output voltage of the power supply. Usually, most of power supplies are equipped with PFC systems which are used to improve the power factor. They are generally designed for increasing the rated power of the power supply. In the case of power supplies, the influence of the level of control on the generated disturbances is observed. With the decrease of the dimming level of control gear, *THD* coefficient of the current increases and the power factor decreases. In addition, the THD_I is influenced by a number of factors: the power supply design and algorithm, the level of the supply voltage distortion and its RMS value, the control level.

The voltage and current *THD* coefficients are described by (1) and (2).

$$THD_U = \frac{U_H}{U_1} = \sqrt{\left(\frac{U}{U_1}\right)^2 - 1} \quad (1)$$

$$THD_I = \frac{I_H}{I_1} = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1} \quad (2)$$

According to standard [23] for steady-state and non-sinusoidal currents and voltages, the RMS values of voltage U and current I have two components:

$$U^2 = U_0^2 + U_H^2 \quad (3)$$

$$I^2 = I_0^2 + I_H^2 \quad (4)$$

A component with a lower index 0 denotes a constant component. For other components, the index is the actual harmonic or voltage number.

$$U_H^2 = U_0^2 + \sum_{H\neq 1} U_H^2 = U^2 - U_1^2 \quad (5)$$

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$$I_H^2 = I_0^2 + \sum_{H \neq 1} I_H^2 = I^2 - I_1^2 \quad (6)$$

A new problem with the generation of harmonics is the fact that increasingly responsible for this phenomenon are the receivers with low power although present in large quantities. In particular, it concerns adjustable light sources and luminaires dimmed light sources and luminaires.

Many analytical methods are used to evaluate this effect. One method uses a simplified schematic diagram of a nonlinear load described by time differential equations based on the knowledge of basic electrical laws [1]. Another method include replacing the nonlinear receiver with a nonlinear resistance [4]. In the literature [12, 17, 19], a method of creating a mathematical model of a nonlinear equipment based on measurements is presented. It is based on accepting the input and output quantities which characterize the given nonlinear equipment in the studied phenomena. The method described was used to assess the impact of computer power supplies on the quality of electricity in the mains [12, 14, 17, 18, 20].

As a tested object the LED luminaire with controlled power and light flux was selected. For the analysis, the methods described in [6–8] using the measurement results of a real object were used. Using these methods a mathematical model was developed. Input values of the model are the RMS value of the supply voltage and the control level. The output value of the model is the value of the THD_I. Mathematical model is a n-th degree polynomial of two input variables. The model was developed using the *Curve Fitting Toolbox* available in MATLAB™.

2 Laboratory results

LED luminaire equipped with power supply with analogue control input made of standard 1 - 10V has been subjected to the laboratory tests. This luminaire has rated power equal to 32W. The RMS value of the supply voltage was varied in the range 230V ± 10% (from 207 to 253 V). The limits of voltage variations aredescribed in EN 50160 [21] for acceptable voltage changes in low voltage networks. The second input values was the input control level of the luminaire, determined by the value of the control voltage from 1V to 10V, with a step equal to 1 volt. The output value was the value of the THD_I coefficient. Measurements were made using the FLUKE 1760 which is anelectrical power quality analyzer. The laboratory power supply as a source of DC control voltage was used.

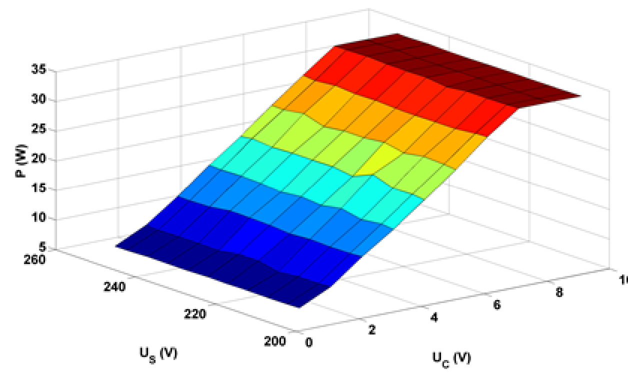

Figure 1: Relation of the power of the lighting fixture P as a function of the supply voltage U_S and control voltage U_C

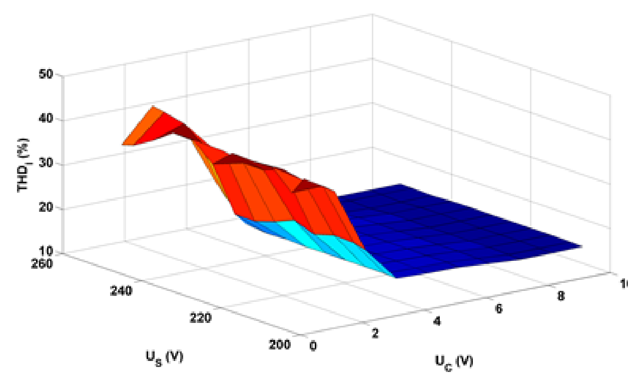

Figure 2: Relation of THD_I coefficient as a function of supply voltage U_S and control voltage U_C

Figure 1 shows the relation of the power of the lighting fitting as a function of the control voltage U_C and of the supply voltage U_S. When analyzing its waveform, it can be seen that in the range of U_C voltage from 1V to 8V the dependency is linear. In the range of 8V to 10V it can be assumed that the luminaire is working at full power. Table 1 summarizes the results of the measurement of the power of the luminaire P as a function of the control voltage U_C. For the tested luminaire, 1V of control voltage ofthe luminaire is achievedat 19.32% of the rated power. This corresponds to the lower limit of regulation, at which the luminaire achieves a minimum power of 6.28W.

In Figure 2 the relationship between the THD_I of the U_S supply voltage and the U_C control voltage is presented. Figure 2 shows two different areas of variation. For control voltages greater than approx. 5V, the THD_I value of the supply current varies in a small range. In the control voltage range of 1V to 5V, the THD_I coefficient shows a strong upward trend.

The presented dependence is characteristic for the tested luminaire. Other types of luminaires with control

Table 1: Relation between the luminaire power *P* and control voltage U_c

U_C (V)	P (W)	P (%)
1	6.28	19.32
2	9.05	27.85
3	12.84	39.54
4	16.62	51.18
5	20.68	63.65
6	24.48	75.36
7	28.36	87.29
8	32.48	100.00
9	32.48	100.00
10	32.48	100.00

systems may had different dependencies whilethe general nature of the dependency changes are similar.

For the illuminated luminaire, the characteristics of the THD_I coefficient are described by two polynomials according to (7):

$$THD_I = \begin{cases} THD_{I-15}\,(U_C, U_S) & \forall U_C \in \langle 1; 5) \wedge \forall U_S \in \langle 207; 253\rangle \\ THD_{I-510}\,(U_C, U_S) & \forall U_C \in \langle 5; 10\rangle \wedge \forall U_S \in \langle 207; 253\rangle \end{cases} \tag{7}$$

Polynomials THD_{I-15} and THD_{I-510} are presented as the sum of the product of the supply voltage U_S, the control voltage U_C and the coefficients a_{ij} and b_{ij} (8) (9), respectively:

$$THD_{I-15}\,(U_C, U_S) = \sum_{i=0}^{m}\sum_{j=0}^{m} a_{ij} {U_C}^i {U_S}^j \tag{8}$$

$$THD_{I-510}\,(U_C, U_S) = \sum_{i=0}^{m}\sum_{j=0}^{m} b_{ij} {U_C}^i {U_S}^j \tag{9}$$

The coefficients a_{ij} and b_{ij} can be written in matrix form as A and B (4):

$$\mathbf{A} = \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} & a_{04} & a_{05} \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & 0 \\ a_{20} & a_{21} & a_{22} & a_{23} & 0 & 0 \\ a_{30} & a_{31} & a_{32} & 0 & 0 & 0 \\ a_{40} & a_{41} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{m\times m} \tag{10}$$

$$\mathbf{B} = \begin{bmatrix} b_{00} & b_{01} & b_{02} & b_{03} & b_{04} & b_{05} \\ b_{10} & b_{11} & b_{12} & b_{13} & b_{14} & 0 \\ b_{20} & b_{21} & b_{22} & b_{23} & 0 & 0 \\ b_{30} & b_{31} & b_{32} & 0 & 0 & 0 \\ b_{40} & b_{41} & 0 & 0 & 0 & 0 \\ b_{50} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{m\times m}$$

For the study, the luminaire coefficients a_{ij} and b_{ij} take the following values.

$$\mathbf{A} = \begin{bmatrix} 5.909\cdot 10^5 & -1.306\cdot 10^4 & 115.40 & -0.5098 & 0.001127 & -9.97\cdot 10^{-7} \\ 1792 & -63.24 & 0.6836 & -0.3671 & 4.222\cdot 10^{-6} & 0 \\ 1235 & -20.34 & 0.09958 & -0.0001527 & 0 & 0 \\ 83.91 & -0.3671 & 0.0001371 & 0 & 0 & 0 \\ -6.322 & 0.02564 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} -1.639\cdot 10^5 & 36.36 & -32.17 & 0.142 & -0.0003127 & 2.749\cdot 10^{-7} \\ -1361 & 20.32 & -0.1216 & 0.0003184 & -3.284\cdot 10^{-6} & 0 \\ 56.74 & -0.3298 & 0.001436 & -7.035\cdot 10^{-7} & 0 & 0 \\ -4.782 & 0.001031 & -4.391\cdot 10^{-5} & 0 & 0 & 0 \\ 0.3367 & 0.0006142 & 0 & 0 & 0 & 0 \\ -0.01304 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The polynomials THD_{I-15} and THD_{I-510} from the dependence (1) were obtained using the two-variable polynomial fitting method to the measurement results using the *Curve Fitting Toolbox* [7]. The proposed tool for data vectors U_S, U_C and THD_I have found the vector of polynomial coefficients of the assumed degrees. Polynomial coefficients were selected to obtain a minimum value of the mean square deviation between the given function $THD_I = f(U_S, U_C)$, and approximating polynomial $THD_{I-15}(U_S, U_C)$ for $U_S \in \langle 1; 5)$ and $THD_{I-510}(U_S, U_C)$ for $U_S \in \langle 5; 10\rangle$.

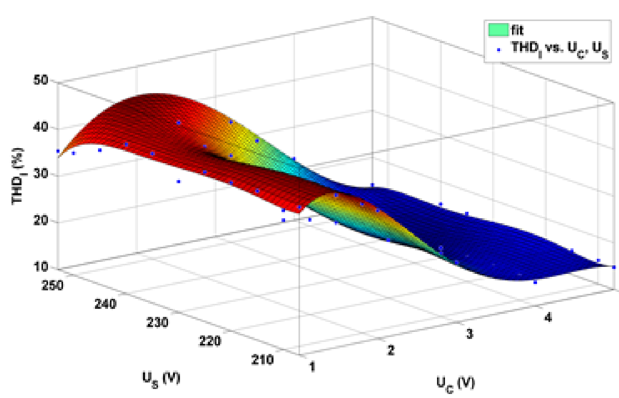

Figure 3: Graph presenting the approximating polynomial $THD_{I-15}(U_S,U_C)$

For the obtained approximating polynomials (Figures 3 and 4) the mean square errors were respectively 0.95 and 0.99.

Verification of the correctness of the model has been made in order to determine the error values at the calculation points. The calculation results are shown in Figure 5 for the THD_{I-15} polynomial approximation and Figure 6 for the THD_{I-510} polynomial.

Due to the high nonlinearity according to $THD_I = f(U_S, U_C)$ in range of the control voltage from 1V to 5V the error values are within the range of 0.10% to 26.73%.

The maximum error value occurred for U_C = 3V and U_S = 245V. An error graph is shown in Figure 5. Within the control voltage range of 5 to 10V, the maximum error value is 2.52%, as illustrated in Figure 6.

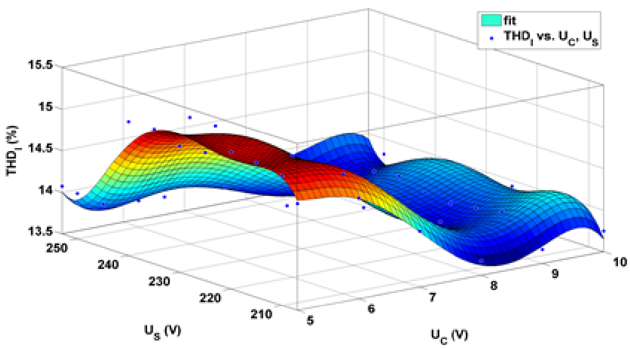

Figure 4: Graph presenting the approximating polynomial $THD_{I-510}(U_S,U_C)$

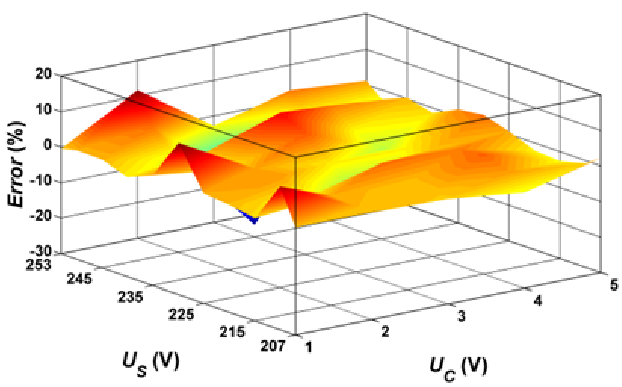

Figure 5: Graph presenting the polynomial approximation error of $THD_{I-15}(U_S,U_C)$

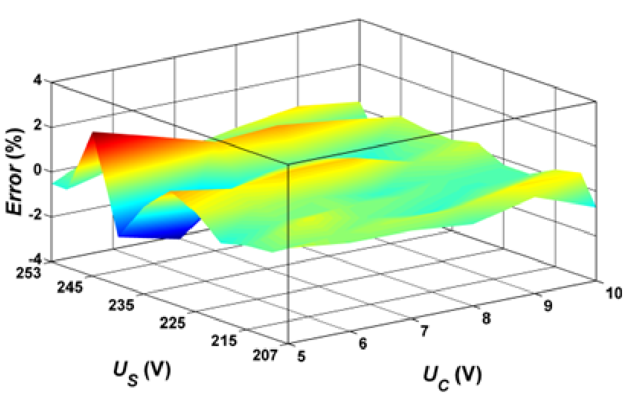

Figure 6: Graph presenting polynomial approximation error of $THD_{I-510}(U_S,U_C)$

3 Computational example designation of THD_I coefficient

The proposed method of estimating the value of THD_I of the power supply was used to calculate this factor for the tested luminaire and the assumed daily schedule of its operation. The luminaire works at full power from 16.00 to 23.00 and from 5.00 to 7.00. At night from 23.00 to 5.00 in the morning it is operated at 50% of the rated power. In order to implement the adopted work schedule of luminaire, it is necessary to know the power P depending on the input value U_C. This relationship is shown in Figure 7 and it is practically linear. For analytical purposes according to $U_C = f(P)$ it may therefore take advantage of the approximation as a linear function.

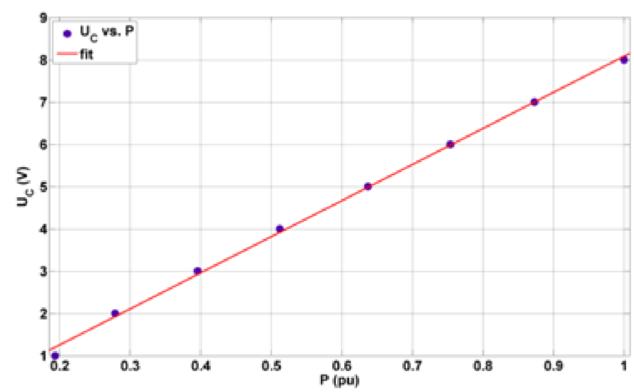

Figure 7: Graph presenting the approximating polynomial $U_C(P)$

The obtained approximation function $U_C = f(P)$ is represented by the expression (11) and obtained for the mean square error is equal to 0.9984.

$$U_C(P) = \begin{cases} 10 & \text{for} \quad P = 1 \ \text{(pu)} \\ 8.541 \cdot P - 0.4562 & \text{for} \quad 0.1932 \le P < 1 \ \text{(pu)} \end{cases} \tag{11}$$

For the further calculations it is assumed that in order to achieve the full power of the luminaire U_C = 10V and for the remaining control levels, the U_C value is calculated from the relationship (11). The dependence (11) allows to determine the value of the U_C control voltage to obtain the assumed power of the luminaire in any schedule of its operation.

The second input value (supply voltage U_S) was derived from measurements made in an existing lighting installation.

Figure 8 shows the time dependent waveforms of RMS supply voltage from measurements and the THD_I values obtained from the calculations. These results correspond to a full daylight cycle with a scheduled power reduction. For the full power of the luminaire, the predicted THD_I value 14%. The reduction of the power of the luminaire causes it to increase to a range of 15.5 to 17%. This corresponds to a 19% increase in THD_I.

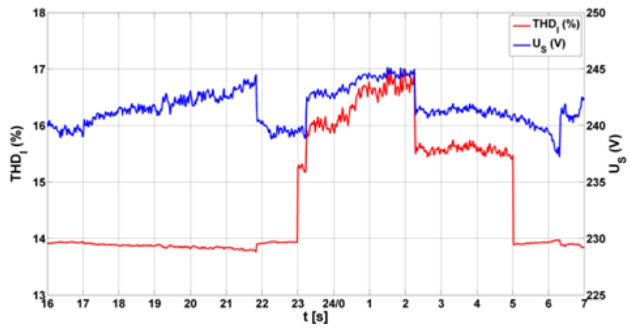

Figure 8: Results of calculations using the model

4 Conclusions

The problem of the quality of electricity is one of the most important aspects related to the operation of power devices. According to EN 61000-3-2 [21], class C receivers cannot generate harmonic currents exceeding their limit values. One of the indicators of the level of disturbances generated to the power supply in the form of higher harmonic current is the total harmonic distortion factor THD_I. On the basis of the measurements it was proved that for the LED luminaire with power regulation its value depends on the level of control and the value of the supply voltage. From an operational point of view, it is important to predict THD_I, values under given power conditions at a given level of control.

The applied mathematical model of the nonlinear receiver based on the results of the measurements allows for accurate estimation of the THD_I level of the given receiver. The knowledge of the THD_I characteristics as a function of changes in the input values allows to determine permissible changes of these values to maintain the current deformation at a given level. For the tested luminaire, this method determined the acceptable range of power regulation and luminous flux. The analysis can be carried out for the assumed range of voltage supply variations, ranging from 230V ± 10% (from 207V to 253V and full range of power P control).

References

[1] Mansoor A., Grady W.M., Chowdhury A.H., Samotyj M.J., An Investigation of Harmonics Attenuation and Diversity Among Distributed Single-Phase Power Electronic Loads, IEEE Transactions on Power Delivery, 1995, 10, 1.

[2] Nassif A.B., Yong J., Xu W., Chung C.Y., Indices for comparative assessment of the harmonic effect of different home appliances, International Transactions on Electrical Energy Systems, 2013, 23, 5,638-654.

[3] Emanuel A.E., Janezak J., Pileggi D.J., Gulachenski E.M., Root C.E., Breen M., Gentile T., Voltage distortion in distribution feeders with nonlinear loads, IEEE Transactionon Power Delivery, 1994, 9, 1, 79-87.

[4] Acarkan B., Zorlu S., Kilic O., Nonlinear Resistance Modeling using MATLAB and Simulink in Estimation of City Street Lighting Harmonic Activity, EUROCON 2005, Serbia&Montenegro, Belgrade, November 22-24, 2005, 1251.

[5] Rust B.W., Fitting nature's basic functions part I: polynomials and linear least squares,IEEE Computational Science and Engineering, 2001, 3, 4, 84-89.

[6] Rust B.W., Fitting nature's basic functions part II: estimating uncertainties and testinghypotheses, IEEE Computational Science and Engineering, 2001, 36, 60-64.

[7] Curve Fitting Toolbox™ User's Guide® COPYRIGHT 2001–2014 by The MathWorks, Inc.

[8] Koval D.O., Carter C., Power quality characteristics of computer loads, IEEE Transaction on Industry Application, 1997, 33, 3, 613-621.

[9] Taubin G., An improved algorithm for algebraic curve and surface fitting, Fourth International Conference onComputer Vision, 11-14 May 1993, Berlin, Germany, 658-665.

[10] Xu J., Zhang H., Zhu X., Li L., Ding P.,Curve Surface Fitting Based on an Improved GeneticAlgorithm, 2013 6th International Congress on Image and Signal Processing (CISP 2013), Hangzhou, China, 16-18 Dec. 2013, 747-752.

[11] Mesas J.J., Sainz L., Sala P., Statistical study of personal computer cluster harmonic currents from experimental measurements, Electric Power Components and Systems,2015,43, 1, 56-68.

[12] Aikio J.P., Rahkonen T., Polynomial fitting of nonlinear sources with correlating inputs, The International Journal for Coputation and Mathematics in Electrical and Electronic Engineering, 2012, 33, 4, 1097-1106.

[13] Moore P.J., Portugues I.E., The influence of personal computer processing modes online current harmonics, IEEE Transaction on Power Delivery, 2003, 18, 4, 1363-1368.

[14] Khan R.A.J., Akmal M., Mathematical modeling of current harmonics caused by personal computers, World Academy of Science, Engineering and Technology, 2008, 2, 3, 325-329.

[15] Herraiz S., Sainz L., Pedra J., Behavior of single-phase full-wave rectifier, EuropeanTransaction on Electrical Power, 2003,13, 3, 185-192.

[16] Herraiz S., Sainz L., Pedra J., Unified and simple model foruncontrolled rectifiers, Electric Power System Research, 2005, 74, 3, 331-340.

[17] Mujovic S., Djukanovic S., Radulovic V., Multi-parameter mathematical model for determination of PC cluster total harmonic distorsion of input curret, The International Journal for Coputation and Mathematics in Electrical and Electronic Engineering, 2016, 35, 1, 305-325.

[18] Mujović S., Katić V.A., Radović J., Improved analytical expression for calculating total harmonic distortion of PC clusters, Electric Power System Research, 2011, 81, 7, 1317-1324.

[19] Grasselli U., Lamedica R., Prudenzi A., Characteriation of fluctuating harmonics from single-phase power electronics-based equipment, The International Journal for Coputation and Mathematics in Electrical and Electronic Engineering, 2015, 23, 1, 133-147.

[20] Katić V.A., Mujović S., Radulović V., Radović J., The impact of the load side parameters on PC cluster's harmonics emission, Advances in Electrical and ComputerEngineering, 2011, 11, 1, 103-110.
[21] EN 50160:2010 Voltage characteristics of electricity supplied by public electricity
[22] EN 61000-3-2:2006 Electromagnetic compatibility (EMC) - Part 3-2: Limits - Limits for harmoniccurrent emissions (equipment inputcurrent < 16 A per phase) networks
[23] IEEE 1459 Std-2010, IEEE Standard Definitions for the Measurement of Electric Power Quanities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions