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Variable Viscosity Effects on Time Dependent Magnetic Nanofluid Flow past a Stretchable Rotating Plate

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Abstract: An attempt has been made to describe the effects of geothermal viscosity with viscous dissipation on the three dimensional time dependent boundary layer flow of magnetic nanofluids due to a stretchable rotating plate in the presence of a porous medium. The modelled governing time dependent equations are transformed a from boundary value problem to an initial value problem, and thereafter solved by a fourth order Runge-Kutta method in MATLAB with a shooting technique for the initial guess. The influences of mixed temperature, depth dependent viscosity, and the rotation strength parameter on the flow field and temperature field generated on the plate surface are investigated. The derived results show direct impact in the problems of heat transfer in high speed computer disks (Herrero *et al.* [1]) and turbine rotor systems (Owen and Rogers [2]).

Keywords: Temperature and depth dependent viscosity; Magnetic Nanofluid; Porous medium; Stretchable rotating plate; Ferrohydrodynamic (FHD) interaction parameter

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1 Introduction

In the last few years, deep interest has been shown in investigating and understanding the boundary layer flow and convective heat transfer in magnetic nanofluid flow, which is a topic of major contemporary interest in both science and engineering. Magnetic nanofluids, popularly known as Ferrofluids, are fluids in which nano-sized particles of Fe_3O_4, γ-Fe_2O_3 or $CoFe_2O_4$, having an average size of range about 3-15 nm, are dispersed in a carrier fluid (*e.g.* water, kerosene, ethylene glycol, toluene, biofluids, or lubricants). These particles are coated with a stabilizing dispersing agent called surfactant (antimony, oleic acid, glycerol, fatty acids, etc.) to prevent agglomeration even when a strong magnetic field gradient is applied to the fluid. For the past few decades, ferrofluids have been widely research for their numerous heat exchanger applications in engineering, industry, the physical and medical sciences, etc. The pioneer study of ordinary viscous fluid flow over a rotating disk was described by Karman [3]. After his work, many scientists focused on the study of boundary layer flow due to a rotating disk (Cochran [4], Benton [5], El-Mistikawy and Attia [6], and El-Mistikawy *et al.* [7]). Turkyilmazoglu [8] focused on a class of exact solutions to the steady Navier-Stokes equations for incompressible Newtonian viscous fluid flow motion due to a rotating porous disk. By considering the field dependent viscosity, Ram and Sharma [9] described the behaviour of ferrofluid flow over a rotating disk. Soundalgekar [10] studied the effect ofviscous dissipation on unsteady free convection flow past an infinite, vertical porous plate with uniform suction.

The effects of viscosity variation due to the geophysical position and temperature, taken one at a time, have been carried out by many researchers in MHD and few in FHD. However, the effects of mixed depth and temperature dependent viscosity have not been popularly analyzed in the rotating disk problem yet. Geothermal (depth and temperature dependent) viscosity has a number of fruitful applications in geophysics and the geosciences. Important applications of flow induced by a stretching boundary are found in extrusion processes in the plastic and metal industries (Altan *et al.* [11]). Recently, Turkyilmazoglu [12] examined the steady magnetohydrodynamic(MHD) boundary layer flow over a radially stretchable rotating disk in the pres-

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ence of a transverse magnetic field together with viscous dissipation and Joule heating. Ram and Kumar [13] obtained the heat flow pattern of the boundary layer flow of a ferrofluid over a stretchable rotating disk. Nawaz *et al.* [14] investigated the laminar boundary layer flow of nanofluid induced by a radially stretching sheet. Turkyilmazoglu [15] examined the behaviour of stagnation point flow over a stretchable rotating disk subjected to a uniform vertical magnetic field. Viscous dissipation effects on nonlinear MHD flow in a porous medium over a stretching porous surface have been studied by Devi and Ganga [16]. Chen [17] studied the effects of viscous dissipation on radiative heat transfer in MHD flow past a stretching surface. Sidheshwar *et al.* [18] studied temperature dependent flow behaviour and heat transfer over an exponential stretching sheet in a Newtonian liquid. Ram and Kumar [19] described the temperature dependent viscosity on steady revolving axi-symmetric laminar boundary layer flow of an incompressible ferrofluid over a stationary disk. Maleque [20] investigated the unsteady flow of an electrically conducting fluid over a rotating disk by considering the viscosity of the fluids dependent on depth and temperature together. Al-Hadhrami *et al.* [21] proposed a model for viscous dissipation in a porous medium which is probably adequate for most of the practical purposes. Attia [22] studied the effect of a porous medium and temperature dependent viscosity on unsteady flow and heat transfer for a viscous laminar incompressible fluid due to an impulsively-started rotating, infinite disc. Mukhopadhyay [23] presented similarity solutions for unsteady flow and heat transfer over a stretching sheet. Turkyilmazoglu [24] obtained an exact numerical solution of steady fluid flow and heat transfer induced between two stretchable co-axial disks rotating in same or opposite direction. The case of an unsteady viscous fluid flow past a horizontal stretching sheet through a porous medium in the presence of a viscous dissipation effect and a heat source was investigated by Sharma [25].
The modelled rotating disk problem of our manuscript has a number of real world engineering and industrial applications. Disk-shaped bodies are often encountered in many real world engineering applications, and heat transfer problems of free convection boundary layer flow over a rotating disk, which occur in rotating heat exchangers, rotating disk reactors for bio-fuel production, and gas or marine turbines, are extensively used by the energy, chemical, and automobile industries [26]. Many application areas, such as rotating machinery, viscometry, computer storage devices, and crystal growth processes, require the study of rotating flows [27]. In this problem, the effects of geothermal viscosity on the time dependent boundary layer flow of an electrically non-conducting, magnetic nanofluid on a stretchable rotating plate in the presence of a porous medium and viscous dissipation have been investigated. Here, the viscosity of the fluid is considered as depth and temperature dependent. The plate is maintained at a temperature T_w and subjected to a magnetic field H $(H_r, H_\phi, 0)$. After transforming the governing equations into non-dimensionalized ordinary differential equations (ODEs) using the Von-Karman transformations, the resultant modelled set of equations is solved in Matlab to obtain the flow pattern. To study the effect of various entities of physical interest such as viscosity variation parameters, rotation parameter, porosity etc., on a hydrocarbon based magnetic nanofluid, EFH1, having the properties: fluid density $\rho = 1.21 \times 10^3\ kg/m^3$, dynamic viscosity $\mu = 0.006 kg/(ms)$, Prandtl number $Pr = 101.2$ at an average temperature of $25^0 C$, a magnetic particle concentration of 7.9%, is used. The nano particles contain a mixture of magnetite (Fe_3O_4 about 80%) with maghemite (Fe_2O_3 about 20%).

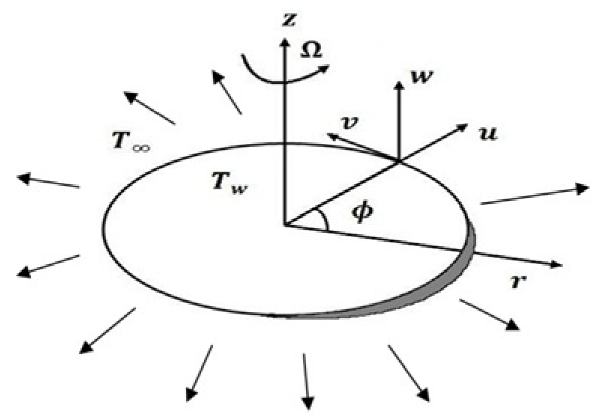

Figure 1: Schematic Diagram of the Physical Model

2 Mathematical Formulation of the Problem

Cylindrical polar coordinates (r,ϕ,z) are used to represent the problem mathematically, with an electrically non-conducting infinite rotating plate fixed at z = 0. The plate, subjected to a magnetic field H, is considered to revolve about z-axis with a constant angular velocity Ω at a uniform temperature T_w (at the plate) and temperature T_∞ (far away from it).
The viscosity of the magnetic nanofluid, considered to be

both depth and temperature dependent [28], is given as:

$$\mu(z,T)=\frac{\mu_\infty(1-\alpha z)}{1+\alpha(T-T_\infty)} \tag{1}$$

where μ_∞ is the uniform viscosity of the fluid and $\alpha \geq 0$ is a constant [29].

The governing equations of unsteady FHD boundary layer flow along with the boundary conditions in component form are given as (Ram and Kumar [19]):

Equation of Continuity:

$$u_r+\frac{u}{r}+w_z=0 \tag{2}$$

Equations of Momentum:

$$\rho(u_t+uu_r-\frac{v^2}{r}+wu_z)+p_r=(\mu u_r)_r+(\mu\frac{u}{r})_r + (\mu u_z)_z+\mu_0|\vec{M}||\vec{H}|_r-\frac{\mu}{k_0}u \tag{3}$$

$$\rho(v_t+uv_r+\frac{uv}{r}+wv_z)=(\mu v_r)_r+(\mu\frac{v}{r})_r + (\mu v_z)_z-\frac{\mu}{k_0}v \tag{4}$$

$$\rho(w_t+uw_r+ww_z)+p_z=(\mu w_r)_r+\frac{1}{r}(\mu w)_r + (\mu w_z)_z+\mu_0|\vec{M}||\vec{H}|_z-\frac{\mu}{k_0}w \tag{5}$$

Energy Equation:

$$\rho C_p(T_t+uT_r+wT_z) = k(T_{rr}+\frac{1}{r}T_r+T_{zz})+\mu[(u_z)^2+(v_z)^2] \tag{6}$$

where ρ is the fluid density, μ is the magnetic permeability in free space, k is the thermal conductivity, and C_p is the specific heat at constant pressure. The boundary conditions for the flow are:

$$\left.\begin{aligned} &u=sr, v=r\Omega, w=0, T=T_w\ at\ z=0\\ &u\to 0, v\to 0, T\to T_\infty, p\to p_0\ as\ z\to z_\infty\\ &w\to \text{some finite negative value as } z\to z_\infty \end{aligned}\right\} \tag{7}$$

2.1 Solution of the Problem

The magnetic scalar potential due to the magnetic dipole m is given by

$$\psi_m=\frac{m}{2\pi r}cos\phi.$$

The corresponding magnetic field H, considering negligible variation along z-axis, has the components,

$$H_r=-\frac{\partial\psi_m}{\partial r}=\frac{1}{2\pi}\frac{mcos\phi}{r^2},$$

$$H_\phi=-\frac{1}{r}\frac{\partial\psi_m}{\partial\phi}=\frac{1}{2\pi}\frac{msin\phi}{r^2}, H_z=0$$

$$\Rightarrow H=\sqrt{H_r^2+H_\phi^2+H_z^2}=\frac{m}{2\pi r^2} \tag{8}$$

The magnetic field H is assumed to be sufficiently strong to produce the state of maximum magnetization of the magnetic nanofluid and the magnetization M is approximated, as considered by Neuringer [30] in his work, by

$$M=K(T_c-T) \tag{9}$$

where T_c is he Curie temperature and K is the pyromagnetic coefficient.

The outward flow of the fluid particles due to the centrifugal force is balanced by the radial pressure force. So, the boundary layer approximation for equation (3) is

$$\frac{1}{\rho}p_r=r\Omega^2. \tag{10}$$

We introduce the following similarity transformations to non-dimensionalize the governing equations:

$$\left.\begin{aligned} &\eta=\frac{z}{\delta},\ u=r\Omega U(\eta),\ v=r\Omega V(\eta),\\ &w=\frac{\nu}{\delta}W(\eta)\text{ and } T-T_\infty=\Delta T\theta(\eta) \end{aligned}\right\} \tag{11}$$

with $\Delta T=T_w-T_\infty$ and $\delta(t)$ is a scalar factor.

Using eq. (11) in the Equation of Continuity (2), we get

$$W=-\frac{2\omega}{R_1Re^2}U. \tag{12}$$

Using the above transformations (8)–(12) in the system of governing equations (3)–(6), we have the transformed ODEs as:

$$\begin{aligned}&\left(1-\frac{\varepsilon_1\eta}{Re}\right)U_{\eta\eta\eta}-\frac{\varepsilon_1}{Re}U_{\eta\eta}\\ &-\left(1-\frac{\varepsilon_1\eta}{Re}\right)(1+\varepsilon\theta)^{-1}\varepsilon\theta_\eta U_{\eta\eta}\\ &+(1+\varepsilon\theta)\left[\frac{\delta}{\nu}\delta_t\frac{\eta}{Re^2}U_{\eta\eta}+\frac{\omega}{R_1Re^2}(V^2-{U_\eta}^2)\right.\\ &\left.+\frac{2\omega}{R_1Re^2}UU_{\eta\eta}-\frac{\omega R_1}{Re^2}-\frac{2B\omega}{R_1Re^3}-\frac{\omega}{Re^2}\beta U_\eta\right]=0\end{aligned} \tag{13}$$

$$\begin{aligned}&\left(1-\frac{\varepsilon_1\eta}{Re}\right)V_{\eta\eta}-\frac{\varepsilon_1}{Re}V_{\eta\eta}\\ &-\left(1-\frac{\varepsilon_1\eta}{Re}\right)(1+\varepsilon\theta)^{-1}\varepsilon\theta_\eta V_{\eta\eta}+(1+\varepsilon\theta)\\ &\left[\frac{\delta}{\nu}\delta_t\frac{\eta}{Re^2}V_\eta+\frac{2\omega}{R_1Re^2}(UV_\eta-VU_\eta)-\frac{\omega}{Re^2}\beta V_\eta\right]=0\end{aligned} \tag{14}$$

Energy Equation:

$$\theta_{\eta\eta}+Pr\left\{\frac{\delta}{\nu}\delta_t\frac{\eta}{Re^2}+\frac{2\omega}{R_1Re^2}U\right\}\theta_\eta \tag{15}$$

$$+ PrEc(U_{\eta\eta}^2 + V_\eta^2) = 0,$$

where $\varepsilon = \alpha\Delta T$ and $\varepsilon_1 = \alpha\delta$ are the viscosity variation parameter and modified viscosity variation parameter, respectively, $\omega = \frac{\Omega\delta^2}{\nu}$ is the rotation parameter, $R_1 = \frac{\Omega}{s}$ is the rotation strength parameter measuring the ration of swirl to stretch, $Re = \sqrt{\frac{s}{\nu}}$ is the Reynolds number, $B = \frac{m}{2\pi}\mu_0 K(T_c - T)\frac{\rho}{\mu^2}$ is the FHD interaction parameter, $\beta = \frac{\mu}{k_0\rho\Omega}$ is the permeability parameter, $Ec = \frac{r^2 s^2}{\Delta T C_p}$ is the Eckert number, and $Pr = \frac{\mu C_p}{k}$ is the Prandtl number.
For unsteady flow, the term $\frac{\delta}{\nu}\delta_t$ in equations (13)–(15) should not be dropped. We consider the usual scaling factor for various unsteady boundary layer flows [31]

$$\delta = 2\sqrt{\nu t} + L \tag{16}$$

where L represents the length scale of steady flow.
Introducing (16) in equations 13)–(15, we get the following set of modelled equations:

$$\begin{aligned} &\left(1 - \frac{\varepsilon_1\eta}{Re}\right) U_{\eta\eta\eta} - \frac{\varepsilon_1}{Re} U_{\eta\eta} \\ &- \left(1 - \frac{\varepsilon_1\eta}{Re}\right)(1+\varepsilon\theta)^{-1}\varepsilon\theta_\eta U_{\eta\eta} \\ &+ (1+\varepsilon\theta)\left[\frac{2\eta}{Re^2}U_{\eta\eta} + R(V^2 - U_\eta{}^2) + 2RUU_{\eta\eta}\right. \\ &\left. - RR_1^2 - \frac{2BR}{Re} - RR_1\beta U_\eta\right] = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} &\left(1 - \frac{\varepsilon_1\eta}{Re}\right) V_{\eta\eta} - \frac{\varepsilon_1}{Re} V_{\eta\eta} \\ &- \left(1 - \frac{\varepsilon_1\eta}{Re}\right)(1+\varepsilon\theta)^{-1}\varepsilon\theta_\eta V_{\eta\eta} \\ &+ (1+\varepsilon\theta)\left[\frac{2\eta}{Re^2}V_\eta + 2R(UV_\eta - VU_\eta) - RR_1\beta V\right] = 0 \end{aligned} \tag{18}$$

$$\theta_{\eta\eta} + Pr\left\{\frac{2\eta}{Re^2} + RU\right\}\theta_\eta + PrEc(U_{\eta\eta}^2 + V_\eta^2) = 0, \tag{19}$$

The boundary conditions (7) reduce to

$$\left.\begin{aligned} U_\eta(0) = 1,\ V(0) = R_1,\ W(0) = 0,\ \theta(0) = 1 \\ U_\eta(\infty) = V(\infty) = \theta(\infty) = 0 \\ W(\infty) \text{ tends to some negative values} \end{aligned}\right\}, \tag{20}$$

where $R = \frac{\omega}{R_1 Re^2}$ is the modified rotation parameter. The solutions of the above modelled equations provide us with the variation within the velocity profiles and temperature profile. Besides these results, the radial and tangential skin friction and the Nusselt number at the surface of the plate are also calculated. These are given as, respectively:

$$\begin{cases} (1-\varepsilon)^{-1} Re^{\frac{1}{2}} C_{f_r} = U_{\eta\eta}(0), \\ (1-\varepsilon)^{-1} Re^{\frac{1}{2}} C_{f_\phi} = V_\eta(0), \\ Re^{\frac{1}{2}} Nu = -\theta_\eta(0), \end{cases}$$

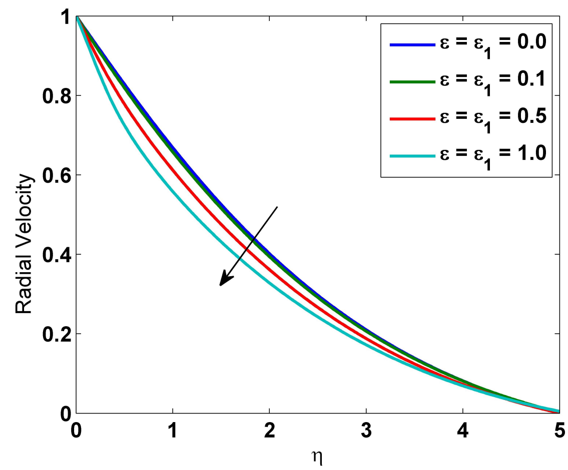

Figure 2: Effect of the viscosity variation on the radial velocity profile

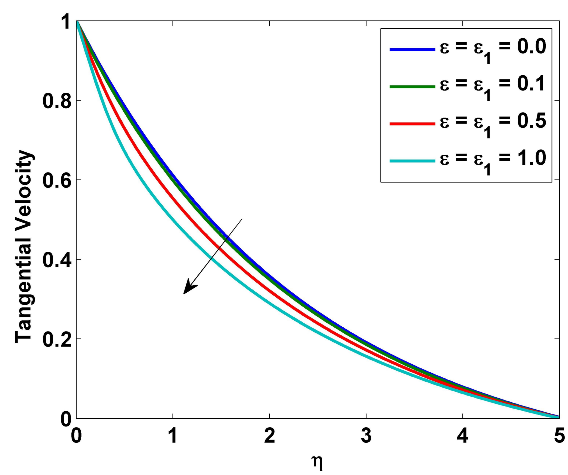

Figure 3: Effect of the viscosity variation on the tangential velocity profile

where C_{f_r} and C_{f_ϕ} are the radial and tangential skin frictions coefficients, respectively, Nu is the Nusselt number, and Re is the local rotational Reynolds number.

3 Numerical Results and Discussion

Using the finite difference scheme in the MATLAB tool ODE45, the effects of various parameters on the flow behaviour near the plate for the considered flow equations (2)–(6) are discussed. From the present simulation (Table 1), it is observed that the skin friction coefficients increase with increasing viscosity variation parameters ε, ε_1, and modified rotational parameter R. The rate of heat transfer is represented by the value of $-\theta_\eta(0)$. The present

Table 1: Various values of the skin friction coefficients and the Nusselt number for $R_1 = 1.0$; Re = 40.0; B = 2.0; $\beta = 1.0$ and Ec = 0.25.

ε	ε_1	R	Pr	$U''(0)$	$V'(0)$	$\theta'(0)$
0.0	0.0	0.1	101.2	0.349533145015	0.477964239598	0.228767975032
0.1	0.1	0.1	101.2	0.375095331450	0.511642395981	0.512479202287
0.5	0.5	0.1	101.2	0.465096593315	0.623501942396	1.798767975032
1.0	1.0	0.1	101.2	0.549465965933	0.725019423959	3.769876797503
0.5	0.5	0.1	101.2	0.465096593315	0.623501942396	1.798767975031
0.5	0.5	0.2	101.2	0.639596593315	0.870896250194	1.908984279977
0.5	0.5	0.3	101.2	0.781797996593	1.059601942396	2.253298984279
0.5	0.5	0.4	101.2	0.901797996593	1.217205960194	2.470789842799
0.5	0.5	0.1	78.4	0.465046965933	0.623219423959	1.283167975031
0.5	0.5	0.1	79.3	0.464696593315	0.623219423959	1.316797503213
0.5	0.5	0.1	101.2	0.465096593315	0.623501942396	1.798767975030
0.5	0.5	0.1	176.4	0.466024596593	0.624059670194	3.342973423798

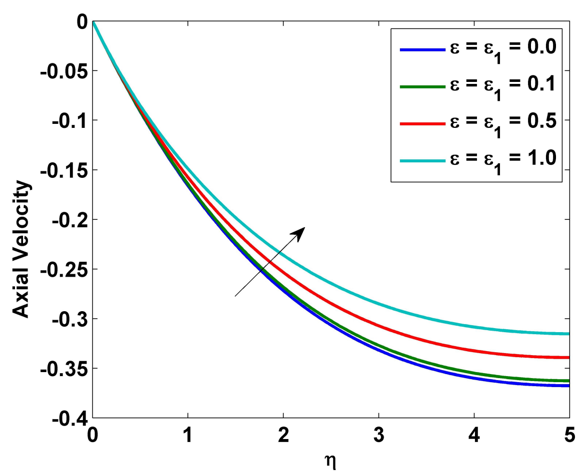

Figure 4: Effect of the viscosity variation on the axial velocity profile

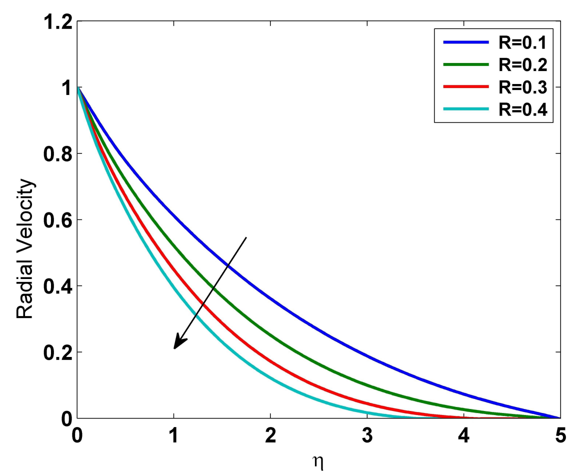

Figure 6: Effect of the modified rotation parameter on the radial velocity profile

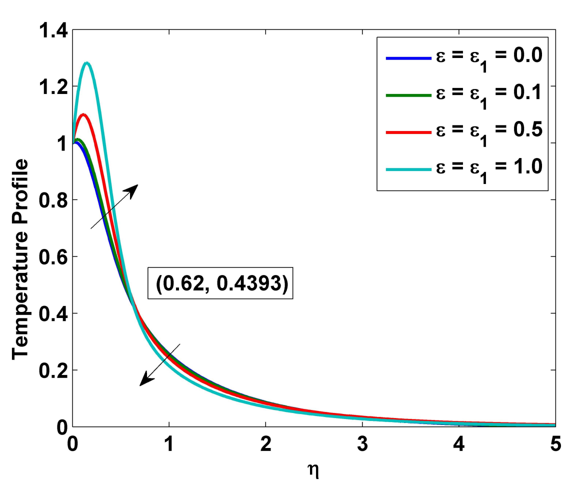

Figure 5: Effect of the viscosity variation on the temperature profile

computation shows that the rate of heat transfer increases with increasing modified rotation parameter R and Prandtl number Pr, while it decreases with increasing viscosity variation parameters.

Figures (2)–(5) exhibit the trends of the velocity and temperature profiles vs. the non-dimensional parameter η for viscosity variation parameters (ε & ε_1) with modified rotational parameter R = 0.1, stretching parameter $R_1 = 1.0$, Reynolds number Re = 40.0, FHD interaction parameter B = 2.0, permeability parameter $\beta = 1.0$, Prandtl number Pr = 101.2, and Eckert number Ec = 0.25. The parameter of viscosity variation ε depends on the temperature of the heated surface. The positive values of ε taken in the problem are for a heated surface, while for $\varepsilon = \varepsilon_1 = 0$ the problem reduces to the case of uniform viscosity flow. An increase in the viscosity variations causes a decrease in

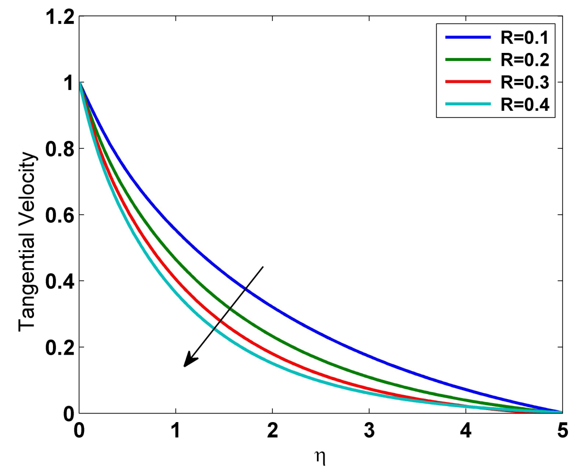

Figure 7: Effect of the modified rotation parameter on the tangential velocity profile

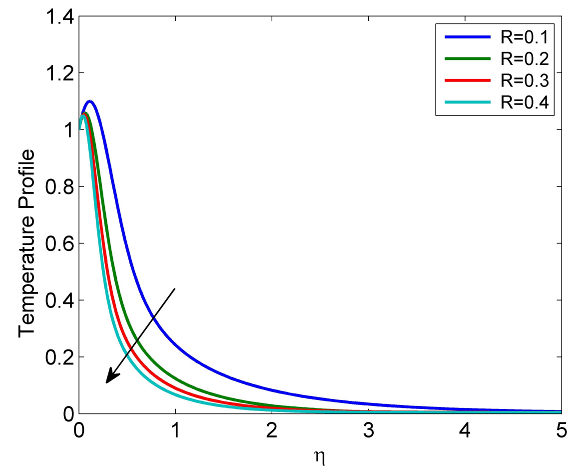

Figure 9: Effect of the modified rotation parameter on the temperature profile

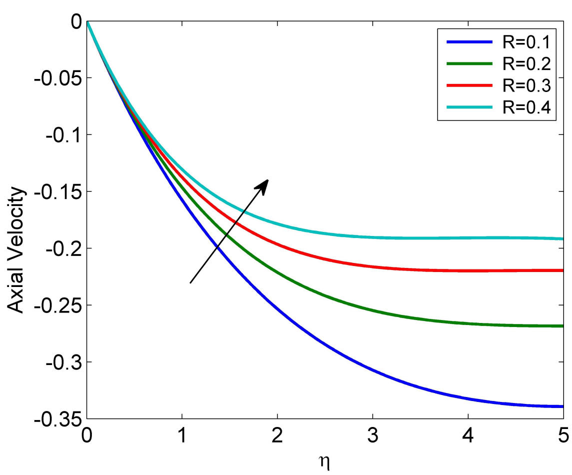

Figure 8: Effect of the modified rotation parameter on the axial velocity profile

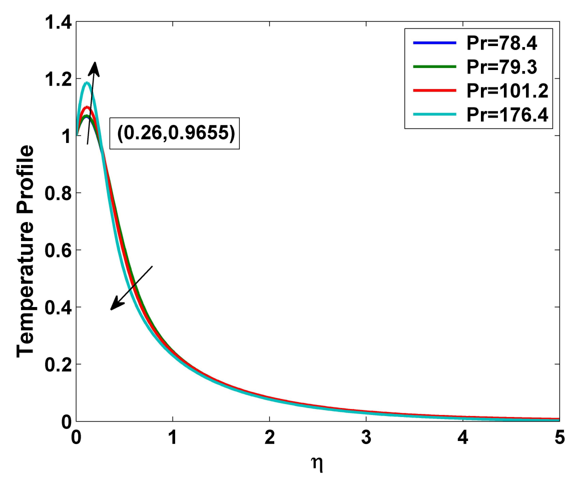

Figure 10: Effect of the Prandtl number on the temperature profile

the velocity profiles, while the dual nature of the temperature profile is noted along the critical point (0.62, 0.4393). The temperature profile changes its nature at the critical point from increasing to decreasing with increased viscosity variation. The fact is due to the increase in the skin friction which is caused by variation in the viscosity.

Figures 6–9 depict the influence of the modified rotational parameter on the boundary layer flow profiles. The trend shows that the effect of the rotation parameter is to decrease the velocity at all layers of fluid within the boundary layer. From Figures 6–8, it is observed that the velocity increases with decreasing values of the rotation parameter. Figure 9 shows that temperature profile decreases with increasing rotation of the plate. Also, the rate of reaching of the state of free steam velocity is faster in the velocity and temperature profiles as the value of modified rotation parameter increases.

To explicitly illustrate the influences of the Prandtl number Pr on the temperature profiles with a specified set of values of various physical parameters, such as $\varepsilon = \varepsilon_1 = 0.5$; R = 0.1; R_1 = 1.0; Re = 40.0; B = 2.0; β=1.0 & Ec = 0.25, Equations (17)–(19) have been solved and results are numerically presented in Figure 10. It is observed that the Prandtl number shows negligible effects on the velocity profile, however, an interesting behaviour is noticed in the temperature profile. A critical point (0.26, 0.9655) is found before which temperature the profile increases while after this point the temperature profile has a reverse trend. The numerical results reveal that an increase in Prandtl number results in an increase in the rate of heat transfer and a decrease in the thermal boundary layer thickness.

4 Conclusion

1. When we move from uniform viscosity to variable viscosity (temperature and depth dependent), the velocity profiles thins and approaches quickly to its steady state. The temperature profile increases with increasing viscosity variation parameter, causing thickening of the thermal boundary layer.
2. The temperature profile has a dual nature for viscosity variation and Prandtl number along the critical points (0.62, 0.4393) and (0.26, 0.9655) respectively.
3. In the present model, fast cooling of the device can be achieved by increasing the modified rotation parameter (R) and Prandtl number (Pr).

The present study considers the range of Prandtl numbers which have a direct application in controlling heat losses or in keeping an instrument cool and avoiding the damage caused due to the heat generation by the motion of its blades/shafts such as in the cases of thermal-power generating systems, high speed rotating machinery, and aerodynamic extrusion of plastic sheets.

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