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Vladimir Šauli*

Analytical solution for the correlator with Gribov propagators

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Abstract: Propagators approximated by meromorphic functions with complex conjugated poles are widely used to model the infrared behavior of QCD Green's functions. In this paper, analytical solutions for two point correlators made out of functions with complex conjugated poles or branch points have been obtained in the Minkowski space for the first time. As a special case the Gribov propagator has been considered as well. The result is different from the naive analytical continuation of the correlator primarily defined in the Euclidean space. It is free of ultraviolet divergences and instead of Lehmann it rather satisfies Gribov integral representation.

Keywords: confinement; Gribov propagator; Quantum Chromodynamics; dispersion relations; quantum field theory; Green's functions

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1 Introduction

The explanation of hadron properties in terms of QCD degrees of freedom represents a hard non-perturbative task, especially when the energy of a process does not comply with the asymptotic freedom and particle like description of hadron constituents. Chiral symmetry breaking and confinement are the main phenomena beyond the applicability of perturbation theory.

That confinement can be naturally encoded in analytical properties of QCD Green's functions is an old-fashioned conjecture [1–9]. Quark and gluon propagators with complex conjugated singularities are long-standing outcomes of many Bethe-Salpeter and Schwinger-Dyson equation (SDEs) studies [10–16], noting here that considered propagators, either calculated or phenomenologically used, usually exhibit not one but several complex conjugated poles (infinite number of poles with zero measure *i.e.* the cuts are possible as well). For instance, the quark propagator considered as a series

$$S_q = \sum_i \not{/} \frac{r_i}{p - m_i} + \not{/} \frac{r_i}{p - m_i^*}\,, \tag{1}$$

where m_i are complex numbers and r_i real residua, actually provides a good ingredient for calculations of pion observables. Note that in practice, most non-perturbative studies (and all cited above) are based on the use of Euclidean metric from the beginning and the calculations performed in Minkowski space that are due to the known obstacles very rare [17, 18]. Guiding by simple assumption that physics in Minkowski space can be read from the analytical continuation of the solutions via Euclidean theory, the lattice data has been checked against the form of Stjieltjes representation [19, 20]. Also the solutions of SDEs [21–23] have been discussed in the context of the usual dispersion relation.

The assumed structure of propagators describing the confinement of quarks and gluons, *i.e.* the form represented by (1), implies the loss of perturbative analyticity. In this paper we readdress some important issues of analytical properties of the 2-point Green's function. For this purpose we consider a 2-point correlation function of the following form

$$\Pi(p) = i \int \frac{d^4 l}{(2\pi)^4} \Gamma G(l) \Gamma G(l - p)\,. \tag{2}$$

Using the Gribov propagator:

$$G(p^2) = \frac{1}{p^2 + \frac{\mu^4}{p^2}}\,, \tag{3}$$

Here we ignore the spin structure as considering a tensorial structure is straightforward. We will explicitly calculate the correlator (2) in Minkowski space and show that the correlator does not satisfy spectral representation but it reflects and in fact it reproduces the Gribov form in its own continuous integral representation. Recall that the loss of reflection positivity is expected for theory with confinement. This is the specific form of analyticity, which ensures the non existence of spectral (Lehmann) representation at all.

***Corresponding Author: Vladimir Šauli:** Department of Theoretical Physics, Institute of Nuclear Physics Rez near Prague, CAS, Czech Republic; Email: sauli@ujf.cas.cz

The choice of the function (3) is well motivated due to the fact that the propagator with Gribov mass μ appears as the solution of Gribov copy problem [1, 3]. Actually, it was shown in [24–26] that if QCD is properly quantized then the gluon propagator in Landau gauge receives the Gribov form in a non-perturbative manner. Notice, it has the simplest non-trivial complex conjugated poles, which are located on the imaginary axis of the p^2 complex plane.

The purpose of this presented paper is twofold: the first point is to show that the evaluation of the loop with the propagators (3,1) in Minkowski space is feasible and its analytical evaluation is certainly not too demanding. For the second point, we mention the possible related physics.

The knowledge of correct analytical behaviour in the whole complex momentum plane is highly useful. However, let us stress that we start with Minkowskian definition and thus the final result is not given by analytical continuation of the expression primarily defined within the use of a real Euclidean theory. As a consequence, we will show that the correlator with Gribov propagators produces a purely imaginary result, contradicting thus the usual naive Wick rotation. Furthermore, the obtained result is ultraviolet finite which is another striking outcome of the presented calculation.

Regarding the two point correlator matrix (2), it has played a long-standing and historically unprecedented role in physics. Such two point functions can stand for QED like $V-V$ or $A-A$ current correlators considered originally in QCD Sum Rules [27, 28] or it can represent colored gluon polarization functions, quark self-energy etc. depending on what we mean by the vertex Γ and the propagator(s) G. While the results do not support too much the old conjecture of quark-hadron duality, the integral representation derived thorough this paper can actually be used in practice for bound states and form factor calculations. Remembering here its weak coupling prerequisite: the Perturbation Theory Integral Representation [29], which has found its own application in bound state calculations [30, 31] in non-confining theory.

We will consider not only a Gribov propagator, but a wider class of the functions which have the complex conjugated singularities including the poles as well as branch points in general. Before going ahead we simplify and consider the analytical structure alone. Note however, that using suitable "trace projectors" and after some trivial algebra, any correlator matrix (2) can be cast into the sum of the product of matrices,tensors and the following scalar form factors

$$\Pi(p^2) = iI(p^2) = i\int \frac{d^4l}{(2\pi)^4}\Gamma(l,p)G(l)G(l-p)\,, \qquad (4)$$

where all the functions in Rel. (4) are Lorentz scalars. Throughout this paper we also neglect the momentum dependence of the vertex function and simply take $\Gamma = 1$, implying thus that all the analytical behaviour is solely due to the Minkowski space measure and the form of functions G. Obviously a more complete solution of the SDEs with nontrivial momentum dependence of the vertex would require analysis beyond the scope of the presented study. We present Minkowski space calculations for the correlator (4) made out of Gribov propagators in the next Section (2).

In order to see the effect of changing the pole position we also consider the correlator made out of propagators with shifted poles. For this purpose we consider the following super-convergent toy model propagator function

$$G(l) = \frac{1}{(l^2-a)^2+b^2}\,, \qquad (5)$$

where a represents a real part of the pole location. As in the last case we consider also the convolution of "propagators" which have complex conjugated branch points. The later example with the function G defined as

$$G(l) = \frac{1}{\sqrt{(l^2-a)^2+b^2}}\,, \qquad (6)$$

is considered in the Section 4.

2 Correlator with Gribov propagators

The Gribov propagator (3) represents a simple rational function and whilst it has a usual perturbative ultraviolet asymptotic, its infrared structure is drastically different from the free particle propagator. Instead of the real pole associated with free particle modes, it has two simple imaginary poles associated with the confinement scale b. Its reality and the absence of the real axis singularity implies that the function $I = -i\Pi$ should be real again. Also the direct integration in the Minkowski space is well established without the need of (sometimes unavoidable) analytical continuation to the auxiliary Euclidean space. However, as the momentum integration is particularly easy within the use of the Euclidean metric, the method of analytical continuation still remains a powerful technical tool and we will use it carefully in our case as well. Before starting doing so, it is convenient to make a little algebra and we rewrite the correlator $\Pi(p^2)$ in the following way:

$$\Pi(p^2) = \frac{i}{4}\int \frac{d^4l}{(2\pi)^4}\left[\frac{1}{(l^2+ib)(q^2+ib)} + \frac{1}{(l^2-ib)(q^2-ib)} + \frac{2}{(l^2+ib)(q^2-ib)}\right], \qquad (7)$$

where each line in bracket is hermitean and we label momentum $q = l - p$ and the square of Gribov mass $b = \mu^2$. Feynman parametrization represents a useful trick, which allows evaluation of perturbative loops in closed form. Using this the first line in (7) can be written as

$$i\int\frac{d^4l}{(2\pi)^4}\int_0^1 dx\left[\frac{1}{[l^2+p^2x(1-x)+ib]^2}+\frac{1}{[l^2+p^2x(1-x)-ib]^2}\right], \tag{8}$$

where we assume the shift of variable l as well as the interchange of the integration order granted by the symmetry (we will add the omitted factor $1/4$ in (7) at the end). Considering l_o integration we will use the usual contour, which is literally known as a Wick rotation (WR), for the first term, while we will use the contour which is mirror symmetric to the Wick rotation (MWR) to integrate the second term (for Wick rotation see the standard textbook ([32]) . Cauchy lemma then allows us to write

$$\int\frac{d^4l_E}{(2\pi)^4}\int_0^1 dx\left[\frac{-1}{[-l_E^2-p_E^2x(1-x)+ib]^2}+\frac{1}{[-l_E^2-p_E^2x(1-x)-ib]^2}\right], \tag{9}$$

where $-p^2\to p_E^2=p_4^2+p_1^2+p_2^2+p_3^2$ in our metric convention.

Matching two terms in (9) together in the following manner

$$\int\frac{d^4l_E}{(2\pi)^4}\int_0^1 dx\,\frac{4ib(-l_E^2-p_E^2x(1-x))}{[-l_E^2-p_E^2x(1-x)-ib]^2[-l_E^2-p_E^2x(1-x)+ib]^2} = \int\frac{d^4l_E}{(2\pi)^4}\int_0^1 dx\int_0^1 dy\frac{\Gamma(4)4iby(1-y)(-l_E^2-p_E^2x(1-x))}{[l_E^2+p_E^2x(1-x)+ib(1-2y)]^4}. \tag{10}$$

The result is obviously finite and one can integrate over the momentum l without use of any regulator. However it requires the integration over two auxiliary variables x, y and there is a slightly easy way, which is to integrate each term individually. This step requires some usual regularization (translation and other symmetries keeping), however the infinite pieces cancel each other. Independently on the procedure, it leads to the result:

$$\frac{1}{(4\pi)^2}\int_0^1 dx\ln\left(\frac{p_E^2x(1-x)+ib}{p_E^2x(1-x)-ib}\right). \tag{11}$$

In order to get the analytical structure more explicit, it is advantageous to have all logs linearly dependent on the integral variable. An easy way is to exploit the substitution $u = x - 1/2$ as a first. Then , realizing the integrand is the even function of u one can change the u-integral boundaries such that $\int_{-1/2}^{1/2}du\to 2\int_0^{1/2}du$. Then changing variable $u\to\omega$ such that $u=\sqrt{(1/4-b/\omega)}$ one gets

$$\frac{b}{(4\pi)^2}\int_{4b}^{\infty}\frac{d\omega}{\omega^2\sqrt{\frac{1}{4}-\frac{b}{\omega}}}\ln\left(\frac{p_E^2+i\omega}{p_E^2-i\omega}\right) = \frac{-4ib}{(4\pi)^2}\int_{4b}^{\infty}\frac{d\omega}{\omega^2\sqrt{1-\frac{4b}{\omega}}}\tan^{-1}\left(-\frac{\omega}{p^2}\right). \tag{12}$$

The expression (12) defines function of p^2 which has two symmetric cuts along the imaginary axis going from $i4b$ to $i\infty$ and from $-i4b$ to $-i\infty$.

Note that there is no real cut associated with the particle threshold and the usual of dispersion relation between absorptive and real part does not apply there. In other words: there is no Lehmann representation for the correlator made out of Gribov propagators in Minkowski space. The correlator has no real part anywhere for the real Minkowski space argument p^2. In formal analogy with perturbation theory, which deals with the usual particle like propagators, an analogous integral representation to spectral ones can be written down, however here, they have Gribov form again (*i.e.* denominator of such representation has complex conjugated zeros). To write down such representation explicitly one can use per-partes integration in ω variable getting the following:

$$\frac{i}{(4\pi)^2}\left[\pi\,sgn(p^2)-2\int_{4b}^{\infty}d\omega\frac{p^2\sqrt{1-\frac{4b}{\omega}}}{{p^2}^2+\omega^2}\right], \tag{13}$$

which shows us how the spectral representation for particles turns to the continuous sum of Gribov propagators of confined objects. The same arguments apply for the remaining term in Eq. (7) for which we are going to derive the appropriate result now. Matching together its denominators by using Feynman variable x we can write for the second line of Rel. (7)

$$i\int\frac{d^4l}{(2\pi)^4}\int_0^1\frac{2}{\left[l^2+p^2x(1-x)+ib(2x-1)\right]^2}. \tag{14}$$

To arrive to the known Euclidean integral we use x–parameter dependent contours in the complex l_o plane. For $b(2x-1)$ positive (negative) we can use WR (MWR) and employ Cauchy lemma (assuming the same for the exter-

nal momentum). This directly gives the following result:

$$2\int\frac{d^4l_E}{(2\pi)^4}\int_0^1 dx\frac{\left[-\Theta(2x-1)+\Theta(1-2x)\right]}{\left[l_E^2+p_E^2x(1-x)-ib(2x-1)\right]^2}. \quad (15)$$

Integrating over the Euclidean momentum we arrive into the finite expression

$$\frac{2}{4\pi^2}\left(-\int_{1/2}^1 dx+\int_0^{1/2}dx\right)\ln\left(-p_E^2x(1-x)+ib(2x-1)\right), \quad (16)$$

noting that separate integration over the first or over the second step function in Eq. (15) would require the introduction of some regulator method (like in the previous case, the regulator can be avoided by summing these terms before the integration, for which purpose one can use Feynman trick once again. We are not showing these trivial details).

In addition we offer several integral representations, which in principle can be useful in the future. Making the substitution $x=(1+z)/2(x=(1-z)/2)$ in the first (the second) term one immediately gets

$$\frac{1}{(4\pi)^2}\int_0^1 dz\ln\frac{-p_E^2(1-z^2)/4-ibz}{-p_E^2(1-z^2)/4+ibz} \quad (17)$$
$$=\frac{2i}{(4\pi)^2}\int_0^1 dz\tan^{-1}\frac{4bz}{p_E^2(1-z^2)}.$$

As aforementioned it cannot be cast into the form of spectral representation. The reason for this is obvious as the correlator has a branch cut along the imaginary axis of p^2 instead of the real one. Actually using the substitution $z\to\omega$ one gets for (17) the following representation

$$\frac{i}{(4\pi)^2}\left[-\pi\,sgn(p^2)+\int_0^\infty\frac{d\omega}{(p^{2\,2}+\omega^2)}\frac{p^2\omega}{(b+\sqrt{b^2+\omega^2/4})}\right] \quad (18)$$

or alternatively

$$\frac{1}{(4\pi)^2}\left[-i\pi\,sgn(p^2)-\frac{1}{2}\int_{-\infty}^\infty\frac{d\omega}{(ip^2-\omega)}\frac{|\omega|}{(b+\sqrt{b^2+\omega^2/4})}\right]. \quad (19)$$

To conclude, the correlator Π is given by (quarter of) the sum of two contributions (13) and (18) and satisfies continuous "Gribov" integral representation

$$\Pi(p^2)=i\int_0^\infty\frac{d\omega\,\rho_G(\omega)\,p^2}{p^{2\,2}+\omega^2}\rho_G(\omega)(4\pi)^2 \quad (20)$$
$$=\frac{1}{4}\frac{\omega}{b+\sqrt{b^2+\omega^2/4}}-\frac{1}{2}\sqrt{1-\frac{4b}{\omega}}\Theta(\omega-4b)$$

Likewise the Lehmann representation reflects the analyticity of Feynman propagator $G^{-1}=p^2-m^2+i\epsilon$, the correlator made out two Gribov propagators copiously reproduces the analytical structure of Green's functions involved.

The last undone integration can be performed as well but being a lengthy expression it is not presented here. Numerical values for the function Π are shown against the Minkowski variable p^2 in Fig. 1. There is infra-red discontinuity in the origin of the complex p^2 plane, which can be adjusted by the change of position pole of the Gribov propagator. This analytical behaviour is worthwhile to study and we do this in the next Section.

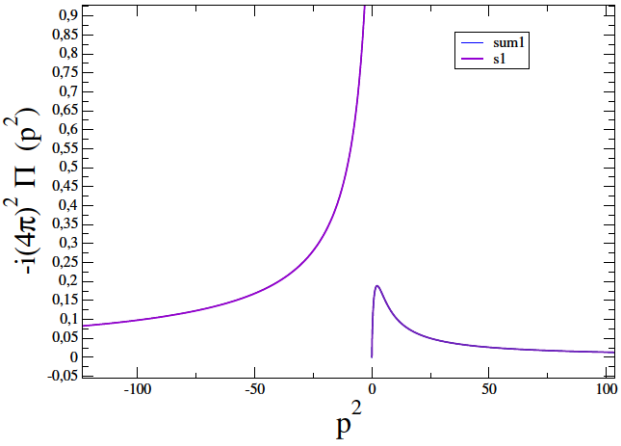

Figure 1: Correlator with Gribov propagators, the momentum is in units where $b=1$.

At last but not at least we comment the ultraviolet finiteness of the result. The result we obtained here is finite, however, we should stress that ultraviolet finiteness is not a general property of the correlator made out of propagators with complex conjugated poles. Usual "log" divergence arises for the convolution of two different Gribov propagators. This divergence is proportional to the difference of the pole positions $\simeq b_1-b_2$ and if necessary it can removed by some sort of symmetry keeping regularization. Recall here, the Gribov form of QCD propagators is assumed to be a good approximation in the infrared, while it is likely less useful for the study of ultraviolet properties. However, as the serious Minkowski space study of QCD with Gribov propagators is lacking, an outcome could be challenging.

3 Correlators with generalized Gribov propagators

3.1 Shifting the branch point

In the previous section we have derived Gribov integral representation, which arises when two Gribov propagators convolute in 3+1 momentum space. An obvious question arises: what would happen to the correlator Π when one considers a more general structure for the propagator, e.g. shifted pole, branch points, etc. As an example we will consider the correlator $\Pi(p^2)$ with propagators of the following form:

$$G(l) = \frac{1}{(l^2 - a)^2 + b^2}. \tag{21}$$

Due to the same reasoning, the function $\Pi(p^2)$ should be a purely imaginary function for all p^2. Contrary to the Gribov case the finiteness is obvious from the beginning since its Euclidean counterpart is finite (note the UV behaviour has no dramatic effect here). Thus the adjective "generalized" in the title is solely due to the shift of the pole position by the real amount a.

To arrive at the analytical expression here we slightly change the calculation procedure and start with the Feynman parametrization from the beginning. For this purpose let's write

$$\begin{aligned}\frac{1}{(p^2 - a)^2 + b^2} &= \frac{1}{p^2 - a + ib}\frac{1}{p^2 - a - ib} \\ &= \int_0^1 dx \frac{1}{[p^2 - a + ib - 2ibx]^2}.\end{aligned} \tag{22}$$

The product of two such propagators in (4) can be written as

$$\begin{aligned}&\int_0^1 dx_1 dx_2 \\ &\frac{1}{[l^2 - a + ib - 2ibx_1]^2}\frac{1}{[(l-p)^2 - a + ib - 2ibx_2]^2} \\ &= \int_0^1 dy dx_1 dx_2 \\ &\frac{y(1-y)\Gamma(4)}{[l^2 y + (l-p)^2(1-y) - a + ib - 2ib(x_1 y + x_2(1-y))]^4}\end{aligned} \tag{23}$$

Lorentz invariance of the measure as well as the kernel here both imply that the finite shift of the integral variable leaves the result invariant. This symmetry dictated property is valid for reasonably theory, *e.g.* QCD, and we cannot discuss details here (we assume that all re-normalizable models belong to this class but interesting questions arise if for instance about what would happen to more complicated diagrams, e.g. anomaly triangle). Shifting momentum l one gets for the correlator:

$$\begin{aligned}\Pi(p^2) &= i\int \frac{d^4 l}{(2\pi)^4}\int_0^1 dy dx_1 dx_2 \frac{y(1-y)\Gamma(4)}{[l^2 + p^2(1-y)y - \Omega]^4}, \\ \Omega &= a - ib + 2ib(x_1 y + x_2(1-y)).\end{aligned} \tag{24}$$

The position of the pole now depends on the parameters and we interchange the ordering of integrations and integrate over the four-momentum as a first. Like in the previous case we use MWR for $\Im\Omega < 0$, while for positive Ω, when the pole is located in the lower half plane of complex l^2, we will use the usual WR. In both cases the inner part of the intended curve is thus free of singularities and the use of Cauchy lemma switches to the Euclidean metric (performing similar for the external momentum). Doing this explicitly, one gets the following prescription for the integral

$$\begin{aligned}i\int \frac{d^4 l}{(2\pi)^4} f(l,p) \to &-\int \frac{d^4 l_E}{(2\pi)^4}\Theta(\Im\Omega) f(l_E, p_E) \\ &+\int \frac{d^4 l_E}{(2\pi)^4}\Theta(-\Im\Omega) f(l_E, p_E),\end{aligned} \tag{25}$$

wherein the subscript E implies the arguments of f_E uses an Euclidean metric $l_E^2 = l_1^2 + l_2^2 + l_3^2 + l_4^2$. In accordance with causality, we assume $a > 0$ in order to avoid poles at the space like region of momenta. Integrating over the momentum l one gets

$$\Pi(p^2) = \int_0^1 dy dx_1 dx_2 \frac{y(1-y)\left[-\Theta(\Im\Omega) + \Theta(-\Im\Omega)\right]}{(4\pi)^2[p^2 y(1-y) - \Omega]^2}, \tag{26}$$

where p is Minkowski momentum again, and the function $\Pi(p^2 > 0)$ is an analytical continuation of $\Pi(p^2 < 0)$.

Performing the substitution $x_1 \to z$ such that $z = 1 - 2(x_1 y + x_2(1-y))$, one gets

$$\begin{aligned}&\Pi(p^2) = \\ &\int_0^1 dy dx_2 \int_{1-2(y+x_2(1-y))}^{1-2x_2(1-y)} dz \frac{(1-y)\left[-\Theta(z) + \Theta(-z)\right]}{(4\pi)^2[p^2 y(1-y) - a + ibz]^2}.\end{aligned} \tag{27}$$

For the purpose of the integration over the variable z let's distinguish three cases. The first case we define is such that upper and down z–integral boundaries are negative. This allows to consider only the mirror Wick rotation. Integrating over the variable z leads to the following formula:

$$\frac{-1}{2ib}\int_0^1 dy dx_2 \frac{(1-y)}{(4\pi)^2}\Sigma_{i=1,2}\frac{(-1)^{1+i}\left[\Theta(2x_2(1-y) - 1)\right]}{p^2 y(1-y) - a + ibz_i}, \tag{28}$$

where $z_1 = 1 - 2x_2(1-y)$ and $z_2 = 1 - 2(y + x_2(1-y))$.

The second case is defined by inequalities $z_1 > 0$ and $z_2 < 1$. Both step functions are relevant and they just split the z integration domain to two integrals with boundaries z_2, 0 and 0, z_1 respectively. Integrating over the variable z is straightforward and the result for the second case reads

$$\frac{-1}{2ib}\int_0^1 dy \int_0^1 dx_2 \frac{(1-y)\Theta(z_1)\Theta(-z_2)}{(4\pi)^2} \left[\frac{2}{p^2y(1-y)-a} - \Sigma_{i=1,2}\frac{1}{p^2y(1-y)-a+ibz_i}\right], \tag{29}$$

with two variables z_i defined previously.

The third and the ultimate case corresponds with the condition $z_1 > z_2 > 0$, for which one gets

$$\frac{-1}{2ib}\int_0^1 dy \int_0^1 dx_2 \frac{(1-y)\Theta(z_2)}{(4\pi)^2}\Sigma_{i=1,2}(-1)^i \frac{1}{p^2y(1-y)-a+ibz_i}. \tag{30}$$

The scalar polarization correlator Π is then equal to the sum of expressions (28), (29) and (30).

Further analytical integration is possible as well, for instance the integration over the variable x_2 gives for (29)

$$\frac{1}{(2ib)^2}\frac{1}{(4\pi)^2}\left[\int_0^{1/2} dy \ln\frac{R-2iby}{R+2iby} + \int_{1/2}^1 \ln\frac{R+ib(2y-1)}{R+ib(1-2y)} + \int_{1/2}^1 \ln\frac{R+ib}{R-ib}\right] + \frac{i}{2b}\frac{1}{(4\pi)^2}\left[\int_0^{1/2} dy\frac{-y}{R} + \int_{1/2}^1 dy\frac{1}{R}\right], \tag{31}$$

where we have defined $R = p^2y(1-y) - a$ for purpose of brevity.

Using the identity

$$\ln\frac{R-2iby}{R+2iby} = 2i\tan^{-1}\frac{2by}{p^2y(1-y)-a} \tag{32}$$

for the first and similarly for other terms in (31), one immediately sees that the result for (29) is purely imaginary.

Further, summing (28) and (30) together and integrating over x_2 gives, after some trivial algebra, the following formula:

$$\frac{1}{(-2ib)^2}\frac{1}{(4\pi^2)}\int_0^{1/2} dy \left[-\ln(R+2ib) + \ln(R+ib) - \ln(R+ib(1-2y)) + \ln(R) + c.c.\right], \tag{33}$$

where c.c. stands for complex conjugated term. Recall, as was discussed in the beginning, the total result must be completely imaginary. Thus since Rel. (29) already is, while the Rel. (33) is purely real, the later must be exactly zero for all p and the total contribution is given solely by the expression (31).

All integrals in (31) can be further integrated analytically, providing the following final result:

$$\begin{aligned}\Pi = \frac{i}{2b(4\pi)^2}\Bigg\{ &\frac{1}{2s}\ln\frac{(a+s/4)^2+b^2}{a^2+b^2} \\ &- \frac{1}{4s}\ln\frac{(a-s/4)^2}{a^2} + \frac{1}{2b}\tan^{-1}\frac{b}{a} \\ &- \frac{1}{s\sqrt{\frac{4a-s}{s}}}\tan^{-1}\sqrt{\frac{s}{4a-s}}\Bigg\} \\ &- \frac{1}{4b^2(4\pi)^2}\Bigg\{\frac{\sqrt{D}}{s}\tan^{-1}\frac{-2ib+s}{\sqrt{D}} - \frac{\sqrt{D}}{s}\tan^{-1}\frac{-2ib}{\sqrt{D}} \\ &- c.c. + \frac{\sqrt{D_1}}{s}\tan^{-1}\frac{2ib}{\sqrt{D_1}} - \frac{\sqrt{D_1}}{s}\tan^{-1}\frac{2ib-s}{\sqrt{D_1}} - c.c. \\ &- \sqrt{\frac{D_2}{s}}\tan^{-1}\sqrt{\frac{s}{D_2}} - c.c.\Bigg\}.\end{aligned} \tag{34}$$

where we have used the following abbreviations

$$D = s(4a-s) + 4b^2 \;;\; D_1 = s(4a-s) + 4b^2 + 4ibs; \quad D_2 = 4a + 4ib - s\,; s = p^2. \tag{35}$$

The inverse tangent function of complex argument is defined through the multi-valuable complex logarithm. As one can see, apart of complex singularities, there is a real branch point presented as well. This branch point has coincided with the origin of the complex plane in the previous case, where $a = 0$ case was considered only.

3.2 Π composed from propagators with branch points

Quark and gluon propagators can be obtained via solutions of SDEs and the result can be in principle approximated as a series (1). It is well known that the interaction is reflected in the analytical properties of amplitudes. For instance, in a relatively simple theory like QED, the effect of dressing an electron propagator by photon self-exchange entails that a simple pole structure of the electron propagator turns to the branch point singularity. In QCD one can expect the similar, if not stronger effect and it is plausible that the other considerable expansions are much faster convergent, especially when their first terms catch the main properties of the exact solution. As a further reasonable candidate, we consider a propagator with

square root non-analyticity. Guiding by simplicity as well as its ultraviolet asymptotic of a free propagator, let us calculate the correlator with the propagator (6). In this special case the argument of the correlator reads

$$\frac{1}{\sqrt{((l-p)^2-a)^2+b^2}}\frac{1}{\sqrt{(l^2-a)^2+b^2}}. \tag{36}$$

As the first step we start with the root decomposition of square-root arguments, then using the Feynman trick it gives us

$$\frac{1}{(p^2-a+ib)^{1/2}}\frac{1}{(p^2-a-ib)^{1/2}} = \int_0^1 dx \frac{x^{1/2}(1-x)^{1/2}\frac{\Gamma(1)}{\Gamma^2(1/2)}}{p^2-a+ib-2ibx}. \tag{37}$$

Depending on the value of variable x, the denominator in the right hand side of Eq. (37) has a positive imaginary part for a small x and positive b. It turns to be negative for a larger value of x. Using the variable x_1 and x_2 for each propagator in (36) and further using the variable y to match propagators together one gets for Rel. (36) the following expression

$$\int_0^1 \frac{dx_1\,dx_2\,dy\,\sqrt{x_1(1-x_1)x_2(1-x_2)}\frac{\Gamma(2)}{\Gamma^4(1/2)}}{l^2y+(l-p)^2(1-y)-\Omega}, \tag{38}$$

where Ω is defined by (24) in the previous Section. After the standard shift one gets for the correlator

$$\Pi(p^2) = i\int\frac{d^{3+1}l}{(2\pi)^4}\int_0^1 \frac{dx_1\,dx_2\,dy\,\sqrt{x_1(1-x_1)x_2(1-x_2)}\frac{\Gamma(2)}{\Gamma^4(1/2)}}{[l^2-\Delta]^2} \tag{39}$$

$$\Delta = -p^2(1-y)y+\Omega.$$

Now we will integrate over the momenta as in the previous case. It means that for case when $\Im\Delta < 0$ WR is used, while when the denominator has singularities at the first and the third quadrant of complex l_o MWR is used. The conditions $\Im\Delta < 0$ and $\Im\Delta > 0$ limits the Feynman parameters integration domain. The integration over the momentum, if taken separately, includes UV divergence. Obviously, resulting infinite constants cancel against each other at the end. Using dimensional regularization for this purpose one can write

$$\Pi(p^2) = \int\frac{1}{(4\pi)^2}\int_0^1 dx_1\,dx_2\,dy \tag{40}$$
$$\sqrt{x_1(1-x_1)x_2(1-x_2)}\left(\frac{2}{\epsilon}-\ln(\Delta)-\gamma+O(\epsilon)\right)$$
$$\left[\Theta(-\Im\Omega)-\Theta(\Im\Omega)\right].$$

Note that only the $\Im$ parts of the expression in the large bracket in Eq. (40) can survive at the end. The result will not depend on the re-normalization scale at all and as mentioned it is finite. The remaining 3d integral over the Feynman variables can be performed numerically. If one wishes, the vanishment of the real part can be regarded as test of numerical precision. Actually, numerically we get $\Re\Pi/\Im\Pi < 10^{-15}$ (here we did not find the analytical expression for the integration over the Feynman parameters due to the presence of square-root function). The resulting correlator is plotted in Fig. (2). Obviously one can see the evidence for a real branch point at the momentum $p^2 = 4a^2$.

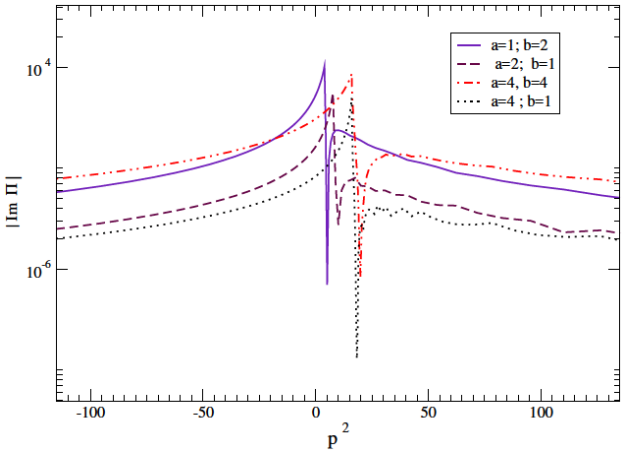

Figure 2: Scalar crrelator made out of the two propagators with branch point singularity. Momentum is scaled by the values of a, b, which are shown explicitly.

4 Conclusion

Correlators defined as a convolution of two Green's functions with the analytical structure, which admits confinement, have been studied in the Minkowski space. We have restricted to the choice of real propagators with complex conjugated singularities. The Gribov propagator, which plays important role in $SU(3)$ Yang-Mills theory was considered primarily. It was shown that Feynman parametrization allows the analytical integration over the momentum in all studied cases, providing unique and ultraviolet finite results in the Minkowski space. Contrary to calculations performed in the Euclidean space [7–9], the Minkowski space correlator with Gribov propagators

remains finite and does not require renormalization. Neither of correlators satisfies Khallen-Lehmann representation and in the case of the Gribov propagator the integral representation copiously reproduces the Gribov form in its continuous version. In other cases, the analytical structure is more complicated and there is no easy way to classify all branch points and related cuts. The correlator made out of generalized Gribov propagators exhibits the real quasithreshold as well. The later strikes itself as a sharp cusp in the graph of $\Pi(p^2)$ and is located at the usual point $p_T^2 = 4a^2$ ($p_T^2 = 0$ for Gribov).

The ramifications of the Minkowski space calculations can only be explored in conjunction with quantum Chromodynamics in strong coupling regions. It is very likely the Gribov form can appear as a not an ultimate approximation of QCD propagators and many steps remain to be finished in this respect. Meanwhile there are the first attempts [18, 33] to solve Schwinger-Dyson and Bethe-Salpeter equations directly in the momentum Minkowski space, the presented study shows the pertinent existence of discontinuities (cuts) when passing the real axis of momenta in Minkowski space. It obviously can make the direct Minkowskian momentum space integration of SDEs difficult, cumbersome, if not even impossible in some cases. In this respect, the integral representation derived here can be very useful when looking for hadronic observables in the framework of SDEs.

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