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Unification of all hyperbolic tangent function methods

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Abstract: Over the last twenty years, several "different" hyperbolic tangent function methods have been proposed to search solutions for nonlinear partial differential equations(NPDEs). The most common of these methods were the tanh-function method, the extended tanh-function method, the modified extended tanh-function method, and the complex tanh-function method. Besides the excellent sides of these methods, weaknesses and deficiencies of each method were encountered. The authors realized that they did not actually give "very different and comprehensive results", and some of them are even unnecessary. Therefore, these methods were analysed and significant findings obtained. Firstly, they compared all of these methods with each other and gave the connections between them; and secondly, they proposed a more general method to obtain many more solutions for NPDEs, some of which having never been obtained before, and thus to overcome weaknesses and deficiencies of existing hyperbolic tangent function methods in the literature. This new method, named as the unified method, provides many more solutions in a straightforward, concise and elegant manner without reproducing a lot of different forms of the same solution. Lastly, they demonstrate the effectiveness of the unifed tanh method by seeking more exact solutions of the Rabinovich wave equation which were not obtained before.

Keywords: The unified method; The tanh-function method; The extended tanh-function method; The modified extended tanh-function method; The complex tanh-function method; The Rabinovich wave equation

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1 Introduction

Over the past two decades, several expansion methods for finding solutions of nonlinear differential equations(NPDEs) have been proposed, developed, and extended owing to the fact that there are a lot of applications of them describing different processes in many scientific areas. In the recent years, direct searching for exact solutions of NPDEs has become more and more attractive partly due to symbolic computation. One of the most effective direct methods to construct wave solutions of NPDEs is the family of the tanh-function methods firstly introduced by Malfliet [7], and developed and used in [8–20] among many others. The most known members of this family are the tanh-function method, the extended tanh-function method, the modified extended tanh-function method, and the complex tanh-function method. In the literature, there are a lot of names for these tanh methods. For instance, one can encounter in some papers the further extended tanh method or new extended tanh method instead of the modified extended tanh function method, and the tanh-coth method instead of the extended tanh function method.

The tanh method is introduced by Huibin and Kelin to solve a higher-order KdV equation in a straightforward but not practical manner [1]. After Huibin and Kelin, various extensions of the method have been developed. Malfliet and Hereman have a systemized version of the tanh method and used it to solve particular evolution and wave equations. They have obtained closed-form solutions of KdV-Burgers, MKdV-Burgers, as well as coupled equations in an elegant and straightfoward way by using this method. To avoid algebraic complexity, they have customized this technique by introducing tanh as a new variable, since all derivatives of the tanh function are represented by itself. Also, Malfliet and Hereman have refined and systemized this technique through the incorporation of boundary conditions and given a priori determination of the velocity of the travelling wave [7–10]. Fan has proposed "the extended tanh-function method " and used it to solve some (1+1)- and (2 +1)- dimensional nonlinear PDEs. He has shown this method to be readily applicable to a large vari-

ety of nonlinear PDEs [2]. Wazwaz has improved the tanh method, primarily named as the "tanh method" firstly, then renamed it as the "tanh-coth method". Wazwaz has applied the tanh-coth method to solve numerous PDEs [11–19]. Based on an extended tanh-function method, El-Wakil et al. have suggested a "modified extended tanh-function method (METF)" to obtain multiple travelling wave solutions for nonlinear PDEs and obtained some new exact solutions [3]. Soliman has extended the METF method to solve four different types of nonlinear differential such as the Burgers, KdV–Burgers, coupled Burgers, and 2D Burgers' equations [4]. Lü and Zhang have presented a "further extended tanh method" and applied it to the (3 +1)- dimensional Jumbo–Miwa equation. Some new soliton-like and periodic-form solutions of the equation have been obtained. Then, they made a little adaption of the method to obtain rational solutions to nonlinear evolution equations. Next, Lü and Zhang showed that a further extended tanh method can also be used to construct multi-soliton and multi-soliton like solutions to nonlinear evolution equations [5]. Khuri has introduced a "complex tanh-function method" for constructing exact travelling wave solutions of nonlinear partial differential equations with complex phases and solutions. He has obtained multiple soliton solutions to the nonlinear cubic Schrödinger equation and a generalized Schrödinger-like equation [20]. Wang et al. have proposed a new method, which is called the "(G'/G)-expansion method", to look for travelling wave solutions of nonlinear evolution equations [6].

The family of the tanh-function methods have some deficiencies and express the same solution in different forms as mentioned in papers [21–23] by Kudryashov. He has pointed out in [21–23], that some authors do not take arbitrary constants into consideration in the exact solutions of nonlinear differential equations resulting in many different forms of the same solution. We have compared the modified extended tanh method with the (G'/G)-expansion method and proved that all the solutions obtained by the (G'/G)-expansion method can be obtained by the modified extended tanh method [24]. However, using hyperbolic and trigonometric identities, it can be proved that these obtained solutions are merely disguised versions of previously known results. In fact, each method is only a developed variant of the former variation, if it produces the repeated

solutions. In overcoming the deficiencies of other tangent function methods, the unified method provides many more solutions in a straightforward, concise, and elegant way without reproducing a lot of different forms of the same solution, and generalizes the family of the tanh-function methods. Proposing the unified method, the main aim of this paper is to not only give a unification for the family of the tanh-function methods, but also obtain many more new solutions for NPDEs without producing the same solutions in different forms.

To demonstrate the effectiveness of this proposed method, the Rabinovich wave equation with nonlinear damping has been solved as an illustrative example by using the unified method. Rabinovich has considered how the establishment of self-oscillations takes place for explosion instability [25]. He has investigated such a mechanism using the example of medium described by the equation

$$-\beta u_{xxtt} - u_{tt} + \left(-\gamma u + u^2 - \alpha u^3\right)_t + \left(V + \delta u^2\right) u_{xx} = 0. \tag{1.1}$$

This equation describes electric signals in telegraph lines on the basis of the tunnel diode. In [26], setting $\beta = -1$, $\gamma = \alpha = \delta = 0$ and $V = 1$ in Eq.(1.1) , Korpusov has considered the equation

$$u_{xxtt} - u_{tt} + u_t - \left(u^2\right)_t + u_{xx} = 0 \tag{1.2}$$

and named as "Rabinovich wave equation with nonlinear damping". Also he has obtained sufficient conditions of the blow-up for the Eq.(1.2).

This paper is organized as follows: In section 2, the authors have presented decriptions of the tanh method (the standard tanh method), the extended tanh method, the modified extended tanh method, and the complex tanh-function method. In section 3, they have proposed a unification for all the tanh methods named the unified method. In section 4, they have showed the connections of these methods and novelty of the unified method. In section 5, they have implemented the unified method to solve the Rabinovich wave equation and to obtain new solutions which could not be attained before. Lastly, a brief conclusion has been given in section 5.

2 Description of the tanh-function methods

A PDE

$$P(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots) = 0 \tag{2.1}$$

can be converted to an ODE

$$P(U, U', U'', U''', \ldots) = 0 \tag{2.2}$$

by using a wave variable $u(x, t) = U(\xi)$, $\xi = x - ct$. Eq.(2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

2.1 The standard tanh method

The tanh method developed by Malfliet in [7], was used in [8 – 19] and many others. In order to seek the travelling wave solutions of Eq. (2.1), assume that the solution of Eq. (2.2) can be expressed by a polynomial in Y as follows:

$$u(\xi) = S(Y) = \sum_{i=0}^{M} a_i Y^i, \tag{2.3}$$

where $a_i, 0 \le i \le M$ are constants to be determined later and $Y(\xi)$ satisfies the following Riccati differential equation

$$Y' = k\left(1 - Y^2\right), \tag{2.4}$$

where k is an arbitrary constant and $Y = Y(\xi), Y' = \frac{dY}{d\xi}$. The parameter M can be found by balancing the linear term of the highest order with the nonlinear term of highest degree. Inserting (2.3) and (2.4) into the ordinary differential equation (2.2) will yield a system of algebraic equations with respect to a_i, k , and c (where $0 \le i \le M$) because all the coefficients of Y^i have to vanish. Solving the resulting system of coefficients of Y, one can then determine a_i, k , and c. Considering the general solution of the Riccati differential equation in (2.4) as follows:

$$Y = \tanh\left(k\left(\xi + \xi_0\right)\right), \xi = x - ct, \tag{2.5}$$

and substituting a_i, k , and c into (2.3), the solutions of Eq. (2.1) can be obtained.

2.2 The extended tanh method

The extended tanh method [18, 19] follows the assumptions made in (2.2) and (2.4), and then admits the use of expansion

$$u(\xi) = S(Y) = a_0 + \sum_{i=1}^{M}\left(a_i Y^i + b_i Y^{-i}\right), \tag{2.6}$$

where M is a positive integer. Eq.(2.6) is an extension of Eq.(2.3) and giving for $b_i = 0, 1 \le i \le M$ can convert the extended tanh method to the standard tanh method[5]. The parameter M is usually obtained, as stated before, by balancing the linear term of highest order in the resulting equation with the nonlinear term of highest degree. If M is not an integer, then a transformation formula should be used to overcome this difficulty. Substituting (2.6) and (2.4) into the ODE (2.2) results in a system of algebraic equations in powers of Y which will lead to the determination of the parameters a_i ,b_i, k, and c.

2.3 The modified extended tanh method

In order to seek the travelling wave solutions of Eq. (2.1), the following ansatz is introduced

$$u(\xi) = S(\phi) = A_0 + \sum_{i=1}^{M}\left(A_i \phi^i + B_i \phi^{-i}\right), \tag{2.7}$$

$$\phi' = b + \phi^2, \tag{2.8}$$

where $A_0, A_i, B_i, 1 \le i \le M$ are constants to be determined later, b is a parameter, and $\phi = \phi(\xi), \phi' = \frac{d\phi}{d\xi}$. The parameter M can be found by balancing the linear term of the highest order with the nonlinear term of highest degree. Inserting (2.7) and (2.8) into the ordinary differential equation (2.2) will yield a system of algebraic equations with respect to A_0, A_i, B_i, b , and c (where $1 \le i \le M$) because all the coefficients of power ϕ have to vanish. Solving the resulting system of coefficients of power ϕ we can determine A_0, A_i, B_i, b , and c. Using the general solutions of the Riccati differential equation (2.8) as follows :

(i) If $b < 0$

$$\begin{aligned}\phi &= -\sqrt{-b}\tanh\left(\sqrt{-b}\left(\xi + \xi_0\right)\right) \text{ or } \phi \\ &= -\sqrt{-b}\coth\left(\sqrt{-b}\left(\xi + \xi_0\right)\right),\end{aligned} \tag{2.9}$$

(ii) If $b > 0$

$$\phi = \sqrt{b}\tan\left(\sqrt{b}\left(\xi + \xi_0\right)\right) \text{ or } \phi = -\sqrt{b}\cot\left(\sqrt{b}\left(\xi + \xi_0\right)\right), \tag{2.10}$$

(iii) If $b = 0$

$$\phi = -\frac{1}{\xi + \xi_0}, \tag{2.11}$$

and substituting A_0, A_i, B_i, b , and c into (2.7), we obtain the solutions of Eq. (2.1) .

2.4 The complex tanh-function method

The complex tanh-function method can be summarized as follows [20]. Following the assumptions made in (2.2) and (2.4), a solution of the form

$$u(\xi) = S(Y) = \sum_{n=0}^{M} a_n \tanh^n\left(ik\left(\xi + \xi_0\right)\right), \tag{2.12}$$

is proposed, where $\xi = x - ct$ and $i = \sqrt{-1}$. Also, M is a positive integer that can be determined by balancing the

linear term of the highest order with the nonlinear term of highest degree, and $a_0, a_1, \ldots, a_M$ are parameters to be determined as stated before. Substituting a_i, k , and c into (2.12), we obtain the solutions of Eq. (2.1).

3 The unified method

The authors describe the unified method for finding solutions of nonlinear partial differential equations in the following steps. Suppose that a nonlinear partial differential equation(NPDE), say in two independent variables x and t, is given by

$$P(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \ldots) = 0 \tag{3.1}$$

where $u(x, t)$ is an unknown function, P is a polynomial in $u = u(x, t)$ and its various partial derivatives, in which highest order derivative and nonlinear terms are involved.

The summary of the unified method can be presented in the following six steps:

Step 1: To find the travelling wave solutions of Eq.(3.1) , using the wave variable

$$u(x, t) = U(\xi), \xi = x - ct, \tag{3.2}$$

where the constant c is generally termed the wave velocity. Substituting Eq.(3.2) into Eq.(3.1), the following ordinary differential equation(ODE) in ξ is obtained (which illustrates a principal advantage of a travelling wave solution, i.e., a partial differential equation(PDE) is reduced to an ordinary differential equation(ODE)).

$$P(U, cU', U', cU'', c^2U'', U'', \ldots) = 0 \tag{3.3}$$

Step 2: If necessary, one integrates Eq.(3.3) as many times as possible and sets the constants of integration to be zero for simplicity.

Step 3: Suppose the solution of nonlinear partial differential equation can be expressed by an ansatz as follows:

$$u(\xi) = a_0 + \sum_{i=1}^{M} \left[a_i \phi^i + b_i \phi^{-i} \right] \tag{3.4}$$

where $\phi = \phi(\xi)$ satisfies the Riccati differential equation

$$\phi'(\xi) = \phi^2(\xi) + b, \tag{3.5}$$

where $\phi' = \frac{d\phi}{d\xi}$,and a_i, b_i , and b are constants. The general solutions of Eq.(3.5) are as follows:

Family 1. When $b < 0$, the solutions of Eq.(3.5) are

$$\phi(\xi) = \begin{cases} \frac{\sqrt{-(A^2+B^2)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)+B} \\ \frac{-\sqrt{-(A^2+B^2)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)+B} \\ \sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)-\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)} \\ -\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)} \end{cases} \tag{3.6}$$

where A and B are two real arbitrary constants, and ξ_0 an arbitrary constant.

Family 2. When $b > 0$, the solutions of Eq.(3.5) are

$$\phi(\xi) = \begin{cases} \frac{\sqrt{(A^2-B^2)b} - A\sqrt{b}\cos\left(2\sqrt{b}(\xi+\xi_0)\right)}{A\sin\left(2\sqrt{b}(\xi+\xi_0)\right)+B} \\ \frac{-\sqrt{(A^2-B^2)b} - A\sqrt{b}\cos\left(2\sqrt{b}(\xi+\xi_0)\right)}{A\sin\left(2\sqrt{b}(\xi+\xi_0)\right)+B} \\ i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)-i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)} \\ -i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)} \end{cases} \tag{3.7}$$

where A and B are two real arbitrary constants, and ξ_0 an arbitrary constant.

Family 3. When $b = 0$, the solution of Eq.(3.5) is

$$\phi(\xi) = -\frac{1}{\xi + \xi_0} \tag{3.8}$$

where ξ_0 is an arbitrary constant.

Step 4: The positive integer M can be accomplished by considering the homogeneous balance between the linear term of the highest order with the nonlinear term of highest degree appearing in Eq.(3.3) as follows:

If it is defined that the degree of $u(\xi)$ as $D[u(\xi)] = M$, then the degree of the other expressions are defined by

$$\begin{aligned} D\left[\frac{d^q u}{d\xi^q}\right] &= M + q, \\ D\left[u^r \left(\frac{d^q u}{d\xi^q}\right)^s\right] &= Mr + s(q + M). \end{aligned}$$

Thus, the value of M in eq.(2.4) is found.

Step 5: Substituting Eq.(3.4) and (3.5) into Eq.(3.3) and collecting all terms with the same degrees of ϕ together, then setting each coefficient of terms with $\phi^i (-M \le i \le M)$ to zero, yield a set of algebraic equations for a_i, b_i, c, and b.

Step 6: Substituting a_i, b_i, c, and b into (3.4) , which is obtained in step 5, and using the general solutions of

Eq.(3.5) in (3.6) , (3.7) and (3.8), the explicit solutions of Eq.(3.1) immediately depending on the value of b, can be obtained.

4 Comparison of all tanh methods

In this section, after comparing all of the tanh methods given in section 2, respectively, the advantages of the unified method are given.

Extension in (2.6) can be reduced to the standard tanh method (2.3) for $b_i = 0, 1 \le i \le M$. Because we can not obtain the solutions for that combination of tanh and coth functions, the extended tanh method is more powerful than the standard tanh method.

For comparision of the extended tanh method and the modified extended tanh function method, a transformation

$$Y(\xi) = -\frac{\phi(\xi)}{k}, \tag{4.1}$$

can be used in (2.4) . Thus, this transformation converts (2.4) and (2.6) to the form, respectively:

$$\phi' = -k^2 + \phi^2, \tag{4.2}$$

$$u(\xi) = a_0 + \sum_{i=1}^{m} \left[\frac{a_i}{(-k)^i} \phi(\xi)^i + b_i (-k)^i \phi(\xi)^{-i} \right] \tag{4.3}$$

Therefore, the relations of coefficients in (2.6) and (2.7) are determined from (4.3) as follows:

$$\begin{aligned} A_0 &= a_0, \\ A_i &= \frac{a_i}{(-k)^i}, \\ B_i &= b_i (-k)^i . \end{aligned} \tag{4.4}$$

where $k^2 = -b$ and $1 \le i \le M$. The deficiency of the extended tanh method is not to take into account more solutions for the Riccati differential equation. Using the transformation in (4.1) and considering not only hyperbolic function but also trigonometric and rational solutions for the Riccati differential equation, it has been indicated that the solutions given in these two methods are equal.

The complex tanh function method does not give us all types of solutions for NPDEs. Taking the expansion (2.7) in the modified extended tanh function method for $B_n = 0, 1 \le n \le M$ with the first solution in (2.10) , we obtain

$$u(\xi) = A_0 + \sum_{n=1}^{M} A_n \phi^n, \text{ where} \phi = \sqrt{b} \tan\left(\sqrt{b}(\xi + \xi_0)\right). \tag{4.5}$$

Considering the identity $\tanh(ix) = i\tan(x)$ in (2.12) , the connection between coefficients of (2.7) and (2.12) can be found as follows:

$$\begin{aligned} A_0 &= a_0, \\ A_n &= \left(\frac{i}{\sqrt{b}}\right)^n a_n, \\ B_n &= 0. \end{aligned} \tag{4.6}$$

where $k = \sqrt{b}$ and $1 \le n \le M$. Thus, the complex tanh function method gives us only some part of the trigonometric function solutions for NPDEs. Furthermore, the complex tanh function method failed to obtain not only the solutions of the combination of the tan-cot function, but also the hyperbolic and rational solutions.

To summarize, the modified extended tanh function method is the most powerful tanh method in section 2. As easily seen, the main differences of these methods are to extend the solution form and take into account more solutions of the Riccati differential equation. Therefore, considering these significant findings, the authors have proposed the unified method both to unify all of the tanh methods, and to obtain many more solutions for NPDEs.

The unified method is compared to the modified extended tanh method because it gives all the solutions of the family of the tanh function method in a simple manner. It was shown that the modified extended tanh method can give a maximum seven type solutions such as tanh, coth, tan, cot, tanh – coth, tan – cot , and rational functions so far. However, taking ξ_0 as an arbitrary constant and the identities $\tanh\left(x - \frac{\pi}{2}i\right) = \coth(x)$ and $\tan\left(x - \frac{\pi}{2}\right) = -\cot(x)$ into account, it can be noticed that the solutions of tanh, coth and tan, cot are exactly same. Therefore, the method can give a maximum five type solutions including tanh, tan, tanh – coth, tan – cot , and rational function, such that they can be obtained by using the unified method.

After putting $B = 0$ first in (3.6) and (3.7), tanh and tan functions can easily be obtained using some hyperbolic and trigonometric identities as follows:

$$\begin{aligned} &\frac{A\sqrt{-b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}(\xi + \xi_0)\right)}{A\sinh\left(2\sqrt{-b}(\xi + \xi_0)\right)} \\ &= \frac{\sqrt{-b}\left(1 - \cosh\left(2\sqrt{-b}(\xi + \xi_0)\right)\right)}{\sinh\left(2\sqrt{-b}(\xi + \xi_0)\right)} \\ &= -\sqrt{-b}\tanh\left(\sqrt{-b}(\xi + \xi_0)\right), \end{aligned}$$

$$\frac{A\sqrt{b}-A\sqrt{b}\cos\left(2-b\left(\xi+\xi_0\right)\right)}{A\sin\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)}$$
$$=\frac{\sqrt{b}\left(1-\cos\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)\right)}{\sin\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)}$$
$$=\sqrt{b}\tan\left(\sqrt{b}\left(\xi+\xi_0\right)\right).$$

On the other hand, the unified method gives many more solutions as it can be seen in (3.6) and (3.7), besides the solutions of the modified extended tanh method. Consequently, the unified method gives many more solutions than the family of the tanh function method. Consequently, the unifed method is the most powerful method to solve NPDEs if compared others.

5 The Rabinovich wave equation with nonlinear damping as an illustrative example

The Rabinovich wave equation with nonlinear damping is given by

$$u_{xxtt}-u_{tt}+u_t-\left(u^2\right)_t+u_{xx}=0 \tag{5.1}$$

which describe electric signals in telegraph lines on the basis of the tunnel diode [27–29]. Using the wave variable $\xi=x-ct$ in Eq.(5.1), then integrating this equation and considering the integration constant to not be zero, we obtain

$$c^2U'''+\left(1-c^2\right)U'-cU+cU^2=0. \tag{5.2}$$

Balancing U^2 and U''' gives $M=3$. Therefore, the solutions of Eq.(5.2) can be written in the form

$$U(\xi)=b_3\phi^{-3}+b_2\phi^{-2}+b_1\phi^{-1}+a_0+a_1\phi+a_2\phi^2+a_3\phi^3, \tag{5.3}$$

where $b_3, b_2, b_1, a_0, a_1, a_2$ and a_3 are constants which are unknowns to be determined later.

Substituting Eq.(5.3) and its derivatives into Eq.(5.2) and equating each coefficients of ϕ^i $(-3\le i\le 3)$ to zero, we obtain a set of nonlinear algebraic equations for $b_3, b_2, b_1, a_0, a_1, a_2, a_3$, and c. Solving this system using Maple, we obtain

Set 1.

$$c=\mp\frac{1}{\sqrt{76b+1}},\ b_3=60cb^3,\ b_2=0,\ b_1=\frac{180b^2}{c\left(76b+1\right)},$$
$$a_3=a_2=a_1=0,\ a_0=\frac{1}{2};$$

Set 2.

$$c=\mp\frac{11}{\sqrt{121-836b}},\ b_3=60cb^3,\ b_2=0,\ b_1=\frac{-540b^2}{c\left(76b-11\right)},$$
$$a_3=a_2=a_1=0,\ a_0=\frac{1}{2};$$

Set 3.

$$c=\mp\frac{1}{\sqrt{76b+1}},\ b_3=b_2=b_1=0,\ a_3=-60c,\ a_2=0,$$
$$a_1=\frac{-180b}{c\left(76b+1\right)},\ a_0=\frac{1}{2};$$

Set 4.

$$c=\mp\frac{11}{\sqrt{121-836b}},\ b_3=b_2=b_1=0,\ a_3=-60c,\ a_2=0,$$
$$a_1=\frac{540b}{c\left(76b-11\right)},\ a_0=\frac{1}{2};$$

Set 5.

$$c=\mp\frac{1}{\sqrt{304b+1}},\ b_3=60cb^3,\ b_2=0,\ b_1=\frac{540b^2}{c\left(304b+1\right)},$$
$$a_3=-60c,\ a_2=0,\ a_1=\frac{-540b}{c\left(304b+1\right)},\ a_0=\frac{1}{2};$$

Set 6.

$$c=\mp\frac{11}{\sqrt{121-3344b}},\ b_3=60cb^3,\ b_2=0,\ b_1=\frac{-180b^2}{c\left(304b-11\right)},$$
$$a_3=-60c,\ a_2=0,\ a_1=\frac{180b}{c\left(304b-11\right)},\ a_0=\frac{1}{2};$$

Using these values and assuming $b\neq 0$, we obtain the following general solutions respectively:

$$u_1(x,t) = \frac{1}{2} + \frac{180b^2\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{c(76b+1)\left(\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)} + 60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.4}$$

$$u_2(x,t) = \frac{1}{2} - \frac{180b^2\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{c(76b+1)\left(\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)} - 60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.5}$$

$$u_3(x,t) = \frac{1}{2} + \frac{180b^2}{c(76b+1)\left(\sqrt{-b}+\frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(\sqrt{-b}+\frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)^3}, \tag{5.6}$$

$$u_4(x,t) = \frac{1}{2} + \frac{180b^2}{c(76b+1)\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)^3}, \tag{5.7}$$

where $c = \mp\frac{1}{\sqrt{76b+1}}$, $b < 0$, and A and B are two real constants;

$$u_5(x,t) = \frac{1}{2} + \frac{180b^2\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}{c(76b+1)\left(\sqrt{\left(A^2-B^2\right)b}-A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)\right)} + 60cb^3\left(\frac{\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}{\sqrt{\left(A^2-B^2\right)b}-A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.8}$$

$$u_6(x,t) = \frac{1}{2} - \frac{180b^2\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}{c(76b+1)\left(\sqrt{\left(A^2-B^2\right)b}+A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)\right)} - 60cb^3\left(\frac{\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}{\sqrt{\left(A^2-B^2\right)b}+A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.9}$$

$$u_7(x,t) = \frac{1}{2} + \frac{180b^2}{c(76b+1)\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3}, \tag{5.10}$$

$$u_8(x,t) = \frac{1}{2} + \frac{180b^2}{c(76b+1)\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)^3}, \tag{5.11}$$

where $c = \mp\frac{1}{\sqrt{76b+1}}b > 0$, and A and B are two real constants;

$$u_9(x,t) = \frac{1}{2} - \frac{540b^2\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{c(76b-11)\left(\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)} + 60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.12}$$

$$u_{10}(x,t) = \frac{1}{2} + \frac{540b^2\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{c(76b-11)\left(\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)} - 60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.13}$$

$$u_{11}(x,t) = \frac{1}{2} - \frac{540b^2}{c(76b-11)\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)^3}, \tag{5.14}$$

$$u_{12}(x,t) = \frac{1}{2} - \frac{540b^2}{c(76b-11)\left(-\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(-\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)^3}, \tag{5.15}$$

where $c = \mp\frac{11}{\sqrt{121-836b}}$, $b < 0$, and A and B are two real constants;

$$u_{13}(x,t) = \frac{1}{2} - \frac{540b^2\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}{c\,(76b-11)\left(\sqrt{\left(A^2-B^2\right)b}-A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)\right)} + 60cb^3\left(\frac{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}{\sqrt{\left(A^2-B^2\right)b}-A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.16}$$

$$u_{14}(x,t) = \frac{1}{2} + \frac{540b^2\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}{c\,(76b-11)\left(\sqrt{\left(A^2-B^2\right)b}+A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)\right)} - 60cb^3\left(\frac{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}{\sqrt{\left(A^2-B^2\right)b}+A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3, \tag{5.17}$$

$$u_{15}(x,t) = \frac{1}{2} - \frac{540b^2}{c\,(76b-11)\left(i\sqrt{b}+\frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(i\sqrt{b}+\frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3}, \tag{5.18}$$

$$u_{16}(x,t) = \frac{1}{2} - \frac{540b^2}{c\,(76b-11)\left(-i\sqrt{b}+\frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)} + \frac{60cb^3}{\left(-i\sqrt{b}+\frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)^3}, \tag{5.19}$$

where $c = \mp\frac{11}{\sqrt{121-836b}}$, $b > 0$, and A and B are two real constants;

$$u_{17}(x,t) = \frac{1}{2} - \frac{180b}{c\,(76b+1)}\frac{\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{\left(\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)} - 60c\left(\frac{\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{\left(\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)}\right)^3, \tag{5.20}$$

$$u_{18}(x,t) = \frac{1}{2} + \frac{180b}{c\,(76b+1)}\frac{\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{\left(\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)} + 60c\left(\frac{\left(A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B\right)}{\left(\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)\right)}\right)^3, \tag{5.21}$$

$$u_{19}(x,t) = \frac{1}{2} - \frac{180b}{c(76b+1)}\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right) \quad (5.22)$$
$$-60c\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3,$$

$$u_{20}(x,t) = \frac{1}{2} - \frac{180b}{c(76b+1)}\left(-\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)}\right) \quad (5.23)$$
$$-60c\left(-\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)}\right)^3,$$

where $c = \mp\frac{1}{\sqrt{76b+1}}$, $b < 0$, and A and B are tworeal constants;

$$u_{21}(x,t) = \frac{1}{2} - \frac{180b}{c(76b+1)}\left(\frac{\sqrt{\left(A^2-B^2\right)b} - A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}\right) \quad (5.24)$$
$$-60c\left(\frac{\sqrt{\left(A^2-B^2\right)b} - A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}\right)^3,$$

$$u_{22}(x,t) = \frac{1}{2} + \frac{180b}{c(76b+1)}\left(\frac{\sqrt{\left(A^2-B^2\right)b} + A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}\right) \quad (5.25)$$
$$+60c\left(\frac{\sqrt{\left(A^2-B^2\right)b} + A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{\left(A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B\right)}\right)^3,$$

$$u_{23}(x,t) = \frac{1}{2} - \frac{180b}{c(76b+1)}\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right) \quad (5.26)$$
$$-60c\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3,$$

$$u_{24}(x,t) = \frac{1}{2} - \frac{180b}{c(76b+1)}\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\,(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}\,(\xi+\xi_0)\right)}\right) \quad (5.27)$$
$$-60c\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\,(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}\,(\xi+\xi_0)\right)}\right)^3,$$

where $c = \mp\frac{1}{\sqrt{76b+1}}$, $b > 0$, and A and B are two real constants;

$$u_{25}(x,t) = \frac{1}{2} + \frac{540b}{c(76b-11)}\left(\frac{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right) \quad (5.28)$$
$$-60c\left(\frac{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right)^3,$$

$$u_{26}(x,t) = \frac{1}{2} - \frac{540b}{c(76b-11)}\left(\frac{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)}{A\sinh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)+B}\right) \tag{5.29}$$
$$+60c\left(\frac{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)}{A\sinh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)+B}\right)^3,$$

$$u_{27}(x,t) = \frac{1}{2} + \frac{540b}{c(76b-11)}\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)-\sinh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)}\right) \tag{5.30}$$
$$-60c\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)-\sinh\left(2\sqrt{-b}\left(x-ct+\xi_0\right)\right)}\right)^3,$$

$$u_{28}(x,t) = \frac{1}{2} + \frac{540b}{c(76b-11)}\left(-\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\left(\xi+\xi_0\right)\right)+\sinh\left(2\sqrt{-b}\left(\xi+\xi_0\right)\right)}\right) \tag{5.31}$$
$$-60c\left(-\sqrt{-b} + \frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\left(\xi+\xi_0\right)\right)+\sinh\left(2\sqrt{-b}\left(\xi+\xi_0\right)\right)}\right)^3,$$

where $c = \mp\frac{11}{\sqrt{121-836b}}$, $b < 0$, and A and B are two real constants;

$$u_{29}(x,t) = \frac{1}{2} + \frac{540b}{c(76b-11)}\left(\frac{\sqrt{\left(A^2-B^2\right)b} - A\sqrt{b}\cos\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)}{A\sin\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)+B}\right) \tag{5.32}$$
$$-60c\left(\frac{\sqrt{\left(A^2-B^2\right)b} - A\sqrt{b}\cos\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)}{A\sin\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)+B}\right)^3,$$

$$u_{30}(x,t) = \frac{1}{2} - \frac{540b}{c(76b-11)}\left(\frac{\sqrt{\left(A^2-B^2\right)b} + A\sqrt{b}\cos\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)}{A\sin\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)+B}\right) \tag{5.33}$$
$$+60c\left(\frac{\sqrt{\left(A^2-B^2\right)b} + A\sqrt{b}\cos\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)}{A\sin\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)+B}\right)^3,$$

$$u_{31}(x,t) = \frac{1}{2} + \frac{540b}{c(76b-11)}\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)-i\sin\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)}\right) \tag{5.34}$$
$$-60c\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)-i\sin\left(2\sqrt{b}\left(x-ct+\xi_0\right)\right)}\right)^3$$

$$u_{32}(x,t) = \frac{1}{2} + \frac{540b}{c(76b-11)}\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)+i\sin\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)}\right) \tag{5.35}$$
$$-60c\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)+i\sin\left(2\sqrt{b}\left(\xi+\xi_0\right)\right)}\right)^3,$$

where $c = \mp\frac{11}{\sqrt{121-836b}}$, $b > 0$, and A and B are two real constant;

$$
\begin{aligned}
u_{33}(x,t) &= \frac{1}{2} - \frac{540b}{c(304b+1)}\left(\frac{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right) \\
&\quad -60c\left(\frac{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad +\frac{540b^2}{c(304b+1)}\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad +60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b} - A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.36}
$$

$$
\begin{aligned}
u_{34}(x,t) &= \frac{1}{2} + \frac{540b}{c(304b+1)}\left(\frac{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right) \\
&\quad +60c\left(\frac{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad -\frac{540b^2}{c(304b+1)}\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad -60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b} + A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.37}
$$

$$
\begin{aligned}
u_{35}(x,t) &= \frac{1}{2} - \frac{540b}{c(304b+1)}\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad -60c\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3 \\
&\quad +\frac{540b^2}{c(304b+1)\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)} \\
&\quad +\frac{60cb^3}{\left(\sqrt{-b} + \frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)^3},
\end{aligned}
\tag{5.38}
$$

$$
\begin{aligned}
u_{36}(x,t) &= \frac{1}{2}-\frac{540b}{c(304b+1)}\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right) \\
&\quad -60c\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)^3 \\
&\quad +\frac{540b^2}{c(304b+1)\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)} \\
&\quad +\frac{60cb^3}{\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)^3},
\end{aligned}
\tag{5.39}
$$

where $c=\mp\frac{1}{\sqrt{304b+1}}$, $b<0$, and A and B are two real constants;

$$
\begin{aligned}
u_{37}(x,t) &= \frac{1}{2}-\frac{540b}{c(304b+1)}\left(\frac{\sqrt{(A^2-B^2)b}-A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}\right) \\
&\quad -60c\left(\frac{\sqrt{(A^2-B^2)b}-A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad +\frac{540b^2}{c(304b+1)}\left(\frac{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)b}-A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right) \\
&\quad +60cb^3\left(\frac{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)b}-A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.40}
$$

$$
\begin{aligned}
u_{38}(x,t) &= \frac{1}{2}+\frac{540b}{c(304b+1)}\left(\frac{\sqrt{(A^2-B^2)b}+A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}\right) \\
&\quad +60c\left(\frac{\sqrt{(A^2-B^2)b}+A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad -\frac{540b^2}{c(304b+1)}\left(\frac{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)b}+A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right) \\
&\quad -60cb^3\left(\frac{A\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)b}+A\sqrt{b}\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.41}
$$

$$
\begin{aligned}
u_{39}(x,t) &= \frac{1}{2}-\frac{540b}{c(304b+1)}\left(i\sqrt{b}+\frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right) \\
&\quad -60c\left(i\sqrt{b}+\frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3 \\
&\quad +\frac{540b^2}{c(304b+1)\left(i\sqrt{b}+\frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)} \\
&\quad +60cb^3\left(i\sqrt{b}+\frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.42}
$$

$$
\begin{aligned}
u_{40}(x,t) &= \frac{1}{2}-\frac{540b}{c(304b+1)}\left(-i\sqrt{b}+\frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right) \\
&\quad -60c\left(-i\sqrt{b}+\frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)^3 \\
&\quad +\frac{540b^2}{c(304b+1)}\left(-i\sqrt{b}+\frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right) \\
&\quad +60cb^3\left(-i\sqrt{b}+\frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.43}
$$

where $c=\mp\frac{1}{\sqrt{304b+1}}b>0$, and A and B are two real constants;

$$
\begin{aligned}
u_{41}(x,t) &= \frac{1}{2}+\frac{180b}{c(304b-11)}\left(\frac{\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)+B}\right) \\
&\quad -60c\left(\frac{\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad -\frac{180b^2}{c(304b-11)}\left(\frac{A\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right) \\
&\quad +60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b}-A\sqrt{-b}\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.44}
$$

$$
\begin{aligned}
u_{42}(x,t) &= \frac{1}{2}-\frac{180b}{c(304b-11)}\left(\frac{\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right) \\
&\quad +60c\left(\frac{\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad +\frac{180b^2}{c(304b-11)}\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad -60cb^3\left(\frac{A\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)+B}{\sqrt{-\left(A^2+B^2\right)b}+A\sqrt{-b}\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3,
\end{aligned}
\tag{5.45}
$$

$$
\begin{aligned}
u_{43}(x,t) &= \frac{1}{2}+\frac{180b}{c(304b-11)}\left(\sqrt{-b}+\frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad -60c\left(\sqrt{-b}+\frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}\,(x-ct+\xi_0)\right)}\right)^3 \\
&\quad -\frac{180b^2}{c(304b-11)\left(\sqrt{-b}+\frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)} \\
&\quad +\frac{60cb^3}{\left(\sqrt{-b}+\frac{-2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(x-ct+\xi_0)\right)-\sinh\left(2\sqrt{-b}(x-ct+\xi_0)\right)}\right)^3},
\end{aligned}
\tag{5.46}
$$

$$
\begin{aligned}
u_{44}(x,t) &= \frac{1}{2}+\frac{180b}{c(304b-11)}\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)}\right) \\
&\quad -60c\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}\,(\xi+\xi_0)\right)}\right)^3 \\
&\quad -\frac{180b^2}{c(304b-11)\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)} \\
&\quad +\frac{60cb^3}{\left(-\sqrt{-b}+\frac{2A\sqrt{-b}}{A+\cosh\left(2\sqrt{-b}(\xi+\xi_0)\right)+\sinh\left(2\sqrt{-b}(\xi+\xi_0)\right)}\right)^3},
\end{aligned}
\tag{5.47}
$$

where $c = \mp\frac{11}{\sqrt{121-3344b}}b < 0$, and A and B are two real constants;

$$
\begin{aligned}
u_{45}(x,t) &= \frac{1}{2} + \frac{180b}{c(304b-11)}\left(\frac{\sqrt{(A^2-B^2)\,b} - A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}\right) \\
&\quad -60c\left(\frac{\sqrt{(A^2-B^2)\,b} - A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad -\frac{180b^2}{c(304b-11)}\left(\frac{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)\,b} - A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad +60cb^3\left(\frac{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)\,b} - A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3,
\end{aligned} \tag{5.48}
$$

$$
\begin{aligned}
u_{46}(x,t) &= \frac{1}{2} - \frac{180b}{c(304b-11)}\left(\frac{\sqrt{(A^2-B^2)\,b} + A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}\right) \\
&\quad +60c\left(\frac{\sqrt{(A^2-B^2)\,b} + A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}\right)^3 \\
&\quad +\frac{180b^2}{c(304b-11)}\left(\frac{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)\,b} + A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad -60cb^3\left(\frac{A\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)+B}{\sqrt{(A^2-B^2)\,b} + A\sqrt{b}\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3,
\end{aligned} \tag{5.49}
$$

$$
\begin{aligned}
u_{47}(x,t) &= \frac{1}{2} + \frac{180b}{c(304b-11)}\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right) - i\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right) \\
&\quad -60c\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}\,(x-ct+\xi_0)\right) - i\sin\left(2\sqrt{b}\,(x-ct+\xi_0)\right)}\right)^3 \\
&\quad -\frac{180b^2}{c(304b-11)\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)} \\
&\quad +\frac{60cb^3}{\left(i\sqrt{b} + \frac{-2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(x-ct+\xi_0)\right)-i\sin\left(2\sqrt{b}(x-ct+\xi_0)\right)}\right)^3},
\end{aligned} \tag{5.50}
$$

$$u_{48}(x,t) = \frac{1}{2} + \frac{180b}{c(304b-11)}\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right) \tag{5.51}$$
$$-60c\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)^3$$
$$-\frac{180b^2}{c(304b-11)\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)}$$
$$+\frac{60cb^3}{\left(-i\sqrt{b} + \frac{2Ai\sqrt{b}}{A+\cos\left(2\sqrt{b}(\xi+\xi_0)\right)+i\sin\left(2\sqrt{b}(\xi+\xi_0)\right)}\right)^3},$$

where $c = \mp\frac{11}{\sqrt{121-3344b}}$, $b > 0$, and A and B are two real constants;

$$u_{49}(x,t) = \frac{1}{2} \mp \frac{60}{(x \pm t + \xi_0)^3} \tag{5.52}$$

where $b = 0$.

6 Conclusion

In this paper, firstly have been presented and compared the family of the tanh-function methods. Afterwards, taking into account more solutions of the Riccati differential equation, it has been proposed that a new method called the unified method be used to solve for nonlinear partial differential equations(NPDEs). More precisely, as it can be seen in (3.6) and (3.7), the unified method defines the solution sets generally and extends them for NPDEs. Namely, the solutions of the family of the tanh-function methods can be obtained simply as in 4.7, while the first and second parts in (3.6) and (3.7) indicate a more general form. On the other hand, the third and fourth parts in (3.6) and (3.7) are new solution forms for PDEs when compared to the family of the tanh-function methods.

To sum up, the main contribution of the unified method is to give many more new solutions which were not obtained before in a straightforward, concise, and elegant manner without reproducing a lot of different forms of the same solutions.

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