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Spacetime Exterior to a Star: Against Asymptotic Flatness

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Abstract: In many circumstances the perfect fluid conservation equations can be directly integrated to give a geometric-thermodynamic equation: typically that the lapse N is the reciprocal of the enthalphy h, ($N = 1/h$). This result is aesthetically appealing as it depends only on the fluid conservation equations and does not depend on specific field equations such as Einstein's. Here the form of the geometric-thermodynamic equation is derived subject to spherical symmetry and also for the shift-free ADM formalism. There at least three applications of the geometric-thermodynamic equation, the most important being to the notion of asymptotic flatness and hence to spacetime exterior to a star. For asymptotic flatness one wants $h \to 0$ and $N \to 1$ simultaneously, but this is incompatible with the geometric-thermodynamic equation. Observational data and asymptotic flatness are discussed. It is argued that a version of Mach's principle does not allow asymptotic flatness.

Keywords: Aysmptotic flatness; length scale; exterior spaceimes; mach's principle; adm formalism

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1 Introduction.

1.1 A Relation Between Fluids and Geometry.

The perfect fluid stress can be covariantly differentiated to give the perfect fluid conservation equations. In many cases these differential equations can be directly integrated to give a *geometric-thermodynamic* equation, which typically equates the Eisenhart (1924) [1] Synge (1937) [2] fluid index w to the reciprocal of the lapse N. The Eisenhart-Synge index is essentially the fluids zero temperature enthalpy. In section 2 the index is calculated for several equations of state. The α–equation of state is a 2–parameter equation of state which describes polytropes. The β–equation of state is a 1–parameter equation of state obtained from the α–equation of state by assuming the second law of thermodynamics for an adiabatic process, Tooper (1965) [3], Zeldovich and Novikov (1971) [4]. It gives the γ–equation of state in all cases except for $\gamma = 0$ where the pressure free ($p = 0$) case is not recovered; but rather $\mu = p \ln(\frac{p}{K})$. For equations of state see also Eligier *et al* (1986) [5], and Ehlers [6]. The index is not defined in the pressure free case, thus solutions such as the Tolman (1934) [7]-Bondi (1947) [8] solution are not covered by description in terms of the fluid index.

1.2 Discussion of the preceddings role in asymptotics

The main application of the geometric-thermodynamic equation is to the description of spacetime exterior to a star. It is shown in many cases that asymptotic flat solutions do not exist. This is taken to imply that the notion of asymptotic flatness as usually understood is physically simplistic. In the literature diagrams are constructed which are supposed to represent the causal spacetime of a collapsing star. These diagrams usually require that the spacetime is asymptotically flat, but the inclusion of a non-vacuum stress is often sufficient for this requirement no longer to hold. An example of how these diagrams can be qualitatively altered by infinitesimal matter is given by solutions to the scalar-Einstein equations which often have no event horizons, Roberts (1985) [9]. To the lowest approximation the spacetime exterior to a star has no stress: the star exists in a vacuum. In order to take account of the matter that surrounds a star it is necessary to find an approximate stress which has contributions from planets, dust etc…There seems to be no systematic way of producing a stress which approximates such diverse forms of matter. When relativity is applied to macroscopic matter the stress is usually taken to be a perfect fluid, so that this is taken to be the form of the first order correction to the vacuum. Specifically the stress is taken to be a spherically symmet-

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ric perfect fluid with γ–equation of state and the result generalized where possible. The nature of the surface of the star is left open as boundary conditions to interior solutions are not discussed. The assumed equations of state are essentially of one variable so that the pressure p does not depend on the entropy s, i.e. $p = p(\mu)$ not $p = p(\mu, s)$ or in other words they are isentropic. Stars radiate and the radiation possess entropy whether there are equations of state that can describe this, and if so whether they are susceptible to a similar analysis as to that given here is left open: however it would be unusual for at the simplest level there to be asymptotically flat spacetimes, at the next level of complexity for there to be none, and at the full level of complexity for asymptotically flat spacetime to reappear.

1.3 Asymptotic Flatness.

It will be shown that many spacetimes with a perfect fluid stress do not have asymptotically flat solutions. Throughout it is assumed that the fluid permeates the whole spacetime and that the spacetime is of infinite extent. Also throughout it is assumed that a simple limiting process is appropriate: specifically as the luminosity radial coordinate tends to infinity the spacetime becomes Minkowskian. This does not always happen, two examples are: Krasiński's (1983) [10] analysis of the Stephani universe where $r = \infty$ labels a second center of symmetry, and type B Bekenstein conformal scalar extensions of ordinary scalar field solutions, Agnese and LaCamera (1985) [11], and Roberts (1996) [12], which have bizarre asymptotic properties. More rigorous definitionsof asymptotic flatness are give in Ludwig (1975) [13] and Reiris (2014) [14]. The result follows from the conservation equations so that it is **explicitly** independent of the gravitational field equations used. The conservation equations use a Christoffel symbol or a generalization of this. The connection depends on the metric which in turn can be thought of as a solution to gravitational field equations. In this **implicit** sense the result can be thought of as depending on field equations. It might not hold if there are other fields coupled to the fluid. Similarly asymptotically flat solutions are rare for theories with quadratic Lagrangians, Buchdahl (1973) [15]. The absence of asymptotically flat solutions might have application to the "missing mass" problem, see Roberts (1991,2004,2011) [16, 18, 19] and Capazziello and De Laurentis (2012) [17]. In Bradley *et al* (1999) [20] it is shown that the Wahlquist perfect fluid spacetime cannot be smoothly joined to an exterior asymptotically flat vacuum region. Boundary conditions for isolated horizons are described in Ashtekar *et al* (1999) [21]. Models of stars in general relativity have been discussed by Herrera *et al* (1984) [22], Stergioulas (1998) [23], and Nilsson and Uggla (2000) [24].

1.4 Sectional Contents.

Section 2 introduces the stress, conservation equations, and the relationship between the enthalpy h and the Eisenhart-Synge fluid index ω. In section 3 it is shown that there are no asymptotically flat static fluid spheres unless the fluid index $\omega \to 1$ at infinity, and using Einstein's field equations there are no asymptotically flat static fluid spheres with γ–equation of state. In the non-static case for γ–equation of state there are no asymptotically flat solutions provided that $\gamma \neq 0, 1$ and certain conditions hold on the metric. For both static and non-static cases there might be asymptotically flat solutions for α–polytropes. In section 4 it is shown for static spacetimes admitting non-rotating vector $U_\alpha = (N, 0)$ and having γ–equation of state that the lapse N is inversely proportional to the fluid index $N = 1/\omega$. For the non-static case, subject to $\dot{N} = 0$ and $\gamma(\ln(\sqrt{g^{(3)}}))_{,i} = 0$, the equation relating the lapse to the fluid index is $\omega = N^{-1} g^{(3)\frac{1}{2}(1-\gamma)} = \mu^{\frac{\gamma-1}{\gamma}}$. These results can be used to show that there are no asymptotically flat fluid filling spacetimes admitting the vector $U_a = (N, 0)$ with γ–equation of state, again also subject to certain conditions on the metric. The introduction of the vector $U_a = (N, 0)$ assumes that the fluid is non-rotating and that the spacetime admits a global time coordinate, unlike the vacuum Einstein equations, see for example Cantor et al (1976) [25], and Witt (1986) [26]. In section 5 a case against asymptotic flatness is presented. Outer solar system observations of orbital irregularities are discussed. Non-asymptoticness on length scales greater than the solar system, such as galaxies, is mentioned. The time-like geodesics for an arbitrary Newtonian potential are calculated. Modeling hypothetical galactic halos of "dark matter" with spherically symmetric fluid solutions so as to produce constant galactic rotation curves is attempted. The the rate of decay of various fields are discussed. It is argued that most perfect fluid spheres and some conformal scalar spheres rate of decay is in fact an increase prohibiting asymptotic flatness. There is the possibility of experimentally testing gravitational theory by measuring the deviation of the Yukawa potential from what would be expected in the absence of gravitation; how this might be done is briefly discussed, the possibility of an actual test seems remote. Various onion models of spacetime surrounding the Sun are discussed. It is argued that non-asymptoticness implies that a system cannot be gravitationally isolated and that this suggests a new formulation of Mach's princi-

ple: THERE ARE NO FLAT REGIONS OF PHYSICAL SPACETIME. The philosophy of what an "isolated system" entails is briefly discussed. In section 6 the Tolman-Ehrefest (1930) [27] relation is derived. Section 7 speculates on the relevance of the geometric-thermodynamic relation to cosmic censorship.

2 The Enthalpy and the Eisenhart-Synge fluid Index.

2.1 Perferct Fluids.

The stress of a perfect fluid is given by

$$T_{\alpha\beta} = (\mu + p)\, U_\alpha U_\beta + p\, g_{\alpha\beta} = nh\, U_\alpha U_\beta + p\, g_{\alpha\beta}, \quad (1)$$
$$U_\alpha U^\alpha_{\cdot} = -1,$$

where μ is the fluid density, p is the pressure, n is the particle number, h is the enthalpy, and $p + \mu = nh$. The unit timelike vector U_a defines the geometric objects

$$h_{\alpha\beta} = g_{\alpha\beta} + U_\alpha U_\beta, \quad \dot{U}_\alpha = U_{\alpha;\beta} U^\beta_{\cdot}, \quad (2)$$
$$\theta = U^\alpha_{\cdot;\alpha}, \quad K_{\alpha\beta} = U_{\chi;\delta} h^\chi_{\alpha\cdot} h^\delta_{\beta\cdot},$$
$$\omega_{\alpha\beta} = h^\chi_{\alpha\cdot} h^\delta_{\beta\cdot} U_{[\chi;\delta]}, \quad \sigma_{\alpha\beta} = U_{(\alpha;\beta)} + \dot{U}_{(\alpha} U_{\beta)} - \frac{1}{3}\theta h_{\alpha\beta},$$

called the projection tensor, the acceleration, the expansion, the second fundamental form, the rotation, and the shear, see for example page 83 of Hawking and Ellis [28]. The projection obeys $U_\alpha h^\alpha_{\cdot\beta} = 0$ and $\dot{U}_\alpha h^\alpha_{\cdot\beta} = \dot{U}_\beta$, also the acceleration obeys $U^\alpha_{\cdot}\dot{U}_\alpha = 0$. Formally the second fundamental form and its associated hypersurface only exist when the rotation vanishes. Transvecting the stress conservation equation $T^{\ \beta}_{\alpha\cdot;\beta}$ with $U^\alpha_{\cdot}$ and $h^\alpha_{\cdot\gamma}$ gives the first conservation equation

$$-U^\alpha_{\cdot} T^\beta_{\alpha\cdot;\beta} = \mu_\alpha U^\alpha_{\cdot} + (\mu + p) U^\alpha_{\cdot;\alpha} = \dot{\mu} + (\mu + p)\theta = 0 \quad (3)$$

and the second conservation equation

$$h^\alpha_{\cdot\gamma} T^\beta_{\alpha\cdot;\beta} = (\mu + p)\dot{U}_\alpha + h^\beta_{\alpha\cdot} p_\beta = 0, \quad (4)$$

respectively. These equations equate the derivatives of the vector field to the pressure and density. From a technical point of view, here we are investigating when these equations can be integrated. It turns out that assuming a specific form of vector field - say hypersurface orthogonal $U_\alpha = \lambda\phi_{,\alpha}$ is not directly of much use, but rather assumptions about the form of the metric have to be made. The first law of thermodynamics can be taken in the infinitesimal form

$$dp = n\, dh + nT\, ds, \quad (5)$$

where T is the temperature and s is the entropy. The Eisenhart[1]-Synge[2] fluid index is defined by

$$\ln(\omega) \equiv \int \frac{dp}{(\mu + p)} \quad (6)$$

after setting $T = 0$ in 5 and integrating it is apparent that up to a constant factor at zero temperature $\omega = h$. The index is also discussed on page 84 of Hawking and Ellis [28].

2.2 Polytropes

The α–polytrope has equation of state

$$p = \alpha\mu^\beta \quad (7)$$

and has

$$dp = \alpha\beta\mu^{\beta-1} d\mu, \quad (8)$$

or

$$\frac{\partial p}{\partial \alpha} = \frac{\partial p}{\partial \beta} = 0, \quad (9)$$

because of this the pressure is not an explicit function of two variables α and β, but only one. The index and particle number corresponding to 7 are

$$\omega = (1 + \alpha\mu^{\beta-1})^{\frac{\beta}{\beta-1}}, \quad n = \mu(1 + \alpha\mu^{\beta-1})^{\frac{1}{1-\beta}}, \quad (10)$$

The β–polytrope [4] has equation of state

$$p = Kn^\gamma, \quad (11)$$

where K is a constant and $V = 1/n$ is the volume occupied by one bayron. For an adiabatic process (no exchange of heat) the second law of thermodynamics is

$$p = -\frac{\partial E}{\partial V}, \quad (12)$$

where E is the total energy density per unit mass $E = \mu/n$. Then 12 becomes

$$p = n^2 \frac{\partial \mu/n}{\partial n}, \quad (13)$$

11 and 13 give

$$pn^{-2} = K\mu_o^{\gamma-2} = \frac{\partial \mu/n}{\partial n}, \quad (14)$$

which in the case $\gamma \neq 1$ can be integrated to give

$$\mu = \frac{K}{\gamma - 1} n^\gamma, \quad (15)$$

where the constant of integration is taken to be zero. Using 11, 15 becomes the equation of state of γ–polytrope

$$p = (\gamma - 1)\mu, \quad \gamma \neq 1, \quad (16)$$

which has index and particle number

$$\omega = \mu^{\frac{\gamma-1}{\gamma}}, \quad n = \gamma\mu^{\frac{1}{\gamma}}, \quad \gamma \neq 0, 1, \tag{17}$$

In the pressure free case ($\gamma = 1$ in 16) the index 6 is not defined, an option is to replace p with $(\gamma-1)\mu$ in the definition 6 and then take $\gamma = 1$ to obtain $\ln(\omega) = 0$ or $\omega = 1$, then the condition $n\omega = \mu + p$ gives $n = \mu$. For the γ–equation of state the first 3 and second 4 conservation laws can be written in terms of μ, where $\mu = \omega^{\frac{\gamma}{\gamma-1}}$, and are

$$\dot{\mu} + \gamma\mu\theta = 0, \tag{18}$$

and

$$\gamma\mu\dot{U}_\alpha + (\gamma - 1)h^\beta_\alpha\mu_\beta = 0, \tag{19}$$

respectively. The γ–equation of state has been derived under the assumption that $\gamma \neq 1$. Perhaps the correct $\gamma = 1$ equation of state for a β–polytrope is found by putting $\gamma = 1$ in 14 and integrating to give

$$\mu = p \ln\left(\frac{p}{K}\right); \tag{20}$$

however the speed of sound

$$v_s \equiv \frac{\partial p}{\partial \mu} = \left(\ln\left(\frac{p}{K}\right) + 1\right)^{-1}, \tag{21}$$

is 1 or the speed of light when $p/K = 1$, it is less than the speed of light for $p/K > 1$, and it diverges as $p/K \to \exp(-1)$. That the speed of sound can take these values suggests that this equation of state is essentially non-relativistic. Some writers refer to 20 as dust, others call the pressure free case $p = 0$ dust. 20 has index and particle number

$$\omega = \left(1 + \ln(\frac{p}{K})\right)^{\frac{1}{K}}, \quad n = p\left(1 + \ln(\frac{p}{K})\right)^{\frac{K-1}{K}}. \tag{22}$$

3 Asymptotically Flat Fluid Spheres

3.1 Spherical Symmetry.

The line element of a spherically symmetric spacetime can be put in the form

$$ds^2 = -C\,dt^2 + A\,dr^2 + B\,d\Sigma^2. \tag{23}$$

Choosing the timelike vector field

$$U_a = (\sqrt{C}, 0, 0, 0), \tag{24}$$

the rotation vanishes and the projection tensor, acceleration, expansion, shear, and second fundamental form are

$$h^r_{r.} = h^\theta_{\theta.} = h^\phi_{\phi.} = 1, \tag{25}$$

$$\dot{U}_\alpha = (0, \frac{C'}{2C}, 0, 0),$$
$$\theta = -\frac{1}{\sqrt{C}}(\frac{\dot{A}}{2A} + \frac{\dot{B}}{B}),$$
$$\sigma_{rr} = -\frac{2A}{B}\sigma_{\theta\theta} = -\frac{2A}{Bsin^2\theta}\sigma_{\phi\phi} = \frac{1}{3}\frac{1}{\sqrt{C}}(-\dot{A} + \frac{A}{B}\dot{B}),$$
$$K_{rr} = -\frac{1}{2}\frac{1}{\sqrt{C}}\dot{A},$$
$$K_{\theta\theta} = \frac{1}{sin\theta}K_{\phi\phi} = -\frac{1}{2}\frac{1}{\sqrt{C}}\dot{B},$$

where the overdot denotes absolute derivative with respect to τ as in $\dot{U}_\alpha = \frac{DU_\alpha}{d\tau}$, but otherwise the overdot denote partial derivative with respect to time. Noting that $d\mu/d\tau = dt/d\tau\ \mu_{,t} = \dot{\mu}/\sqrt{C}$, the first conservation equation 3 becomes

$$\dot{\mu} - (\mu + p)(\frac{\dot{A}}{2A} + \frac{\dot{B}}{B}) = 0, \tag{26}$$

and the second conservation equation 4 becomes

$$p' + (\mu + p)\frac{C'}{2C} = 0, \tag{27}$$

only the r component is non-vanishing in the second equation.

3.2 The Static Case.

In the static case the first conservation equation 26 vanishes identically and the second conservation equation 27 integrates to give

$$\omega = \frac{1}{\sqrt{C}}, \tag{28}$$

the constant of integration is taken to be independent of θ and ϕ and is absorbed into C, for example by redefining t. For the line element 23 to be asymptotically flat it is necessary that as $r \to \infty$, the line element 23 becomes Minkowski spacetime in other words as r increases $C \to 1$, $A \to 1$ and $B \to r^2$. Now from 28, $C \to 1$ implies that $\omega \to 1$. Thus any static spherical fluid sphere with a well defined index not equal to 0 or 1 cannot be asymptotically flat. To see this result in particular cases first consider the γ–equation of state. From 17 and 28

$$\mu = C^{\frac{\gamma}{2(1-\gamma)}}, \tag{29}$$

and as $C \to 1$, μ tends to a constant and thus the spacetime cannot be asymptotically flat; also the spacetime cannot be asymptotically DeSitter as this would necessitate μ tending to a constant time r^2. In the pressure free case, the index is not defined and there are the asymptotically flat solutions given by Tolman [7] and Bondi [8]. Next consider the β–equation of state, from 22 and 28

$$C = \left(1 + \ln(\frac{p}{K})\right)^{-\frac{2}{K}}, \tag{30}$$

now asymptotically as $C \to 1$, $p \to K$; however a constant value of p asymptotically is not consistent with asymptotic flatness, therefore there are no asymptotically flat solutions. Finally consider the α–equation of state, from 10 and 28

$$C = (1 + \alpha\mu^{\beta-1})^{\frac{2\beta}{1-\beta}}, \tag{31}$$

in the case $\mu \to 0$, $C \to 1$ and there might be asymptotically flat α–polytropic spheres. The same results are obtained using the more general vector

$$U_\alpha = (a\sqrt{C}, \sqrt{(a^2-1)A}, 0, 0), \tag{32}$$

where a is a constant.

3.3 The Non-static Case.

In the non-static case it is necessary to assume an equation of state in order to calculate a geometric-thermodynamic relation. The γ–equation of state is assumed. Then either from 18 and 19 and 25, or from 16 and 26 and 27 the first and second conservation laws are

$$\dot{\mu} - \gamma\mu\left(\frac{\dot{A}}{2A} + \frac{\dot{B}}{B}\right) = 0, \tag{33}$$

and

$$(\gamma - 1)\mu' + \gamma\mu\frac{C'}{2C} = 0, \tag{34}$$

respectively. The equation

$$\begin{aligned} d\mu &= \dot{\mu}dt + \mu' dr \\ &= -\gamma\mu\left(\frac{\dot{A}}{2A} + \frac{\dot{B}}{B}\right)dt + \frac{\gamma\mu}{\gamma-1}\frac{C'}{2C}dr, \end{aligned} \tag{35}$$

can be integrated when

$$\dot{C} = 0, \tag{36}$$

and

$$(AB^2)' = 0, \tag{37}$$

to give

$$\mu = \omega^{\frac{\gamma}{\gamma-1}} = A^{+\frac{\gamma}{2}}B^{+\gamma}C^{\frac{\gamma}{2(1-\gamma)}}, \quad \gamma \neq 0, 1, \tag{38}$$

where the constant of integration have been taken to be independent of θ and ϕ and is absorbed into the line element. The assumption $(AB^2)' = 0$ is coordinate dependent and holds rarely as for example $(AB^2)' \approx 4r^3$ for Minkowski spacetime in spherical coordinates whereas $(AB^2)' = 0$ for Minkowski spacetime in rectilinear spacetime. Taking the limits $A, C \to 1$, $B \to r^2$, for $\gamma > 0$, $\mu \to$ a constant, and for $\gamma < 0$, μ diverges; thus there are no asymptotically flat solutions. The α–equation of state 7 cannot be investigated without further information. Discussion of non-existence of time dependent fluid spheres can also be found in Mansouri (1977) [30].

4 The Geometric-Thermodynamic equation in the ADM formalism.

4.1 Vanishing Shift ADM Formalism.

In the ADM (-1,+3) [31] formalism with vanishing shift the metric is given by

$$\begin{aligned} g_{\alpha\beta} &= (-N^2, g_{ij}), \quad g^{\alpha\beta} = (-N^{-2}, g^{ij}), \\ \sqrt{-g^{(4)}} &= N\sqrt{g^{(3)}}. \end{aligned} \tag{39}$$

where $g^{(3)}$ is the determinant of the 3–dimensional metric. The reason the shift is taken to vanish will become apparent later. The timelike unit vector field used here

$$\begin{aligned} U_\alpha &= (N, 0), \quad U^\alpha = (-\frac{1}{N}, 0), \\ U_{i;t} &= -N_{,i}, \quad U_{i;j} = -\frac{1}{2N}g^{(3)}_{ij,t}, \\ U_{t;t} &= U_{t;i} = 0, \end{aligned} \tag{40}$$

there are other choices such as $U_\alpha = (-N, 0)$, and also $U_\alpha = (aN, bN_i)$ for which the unit size condition $U_\alpha U^\alpha_{.} = -1$ implies $g^{ij}N_iN_j = \frac{a^2-1}{b^2}$. For 40 the rotation vanishes and the remaining geometric objects 2 are

$$\begin{aligned} h_{ij} &= g_{ij}, \quad \dot{U}_\alpha = (0, \frac{N_i}{N}), \quad \theta = -\frac{1}{N}\left(\ln(g^{(3)})\right)_{,t}, \\ \sigma_{ij} &= -g^{(3)}_{ij,t} + g^{(3)}_{ij}\left(\ln(g^{(3)})\right)_{,t}, \\ K_{ij} &= K_{ji} = -g_{ij,t}. \end{aligned} \tag{41}$$

The first conservation equation 3 becomes

$$\mu_{,t} - (\mu + p)\left(\ln\sqrt{g^{(3)}}\right)_{,t} = 0, \tag{42}$$

and the second conservation equation 4 becomes

$$p_i + (\mu + p)\frac{N_{,i}}{N} = 0, \tag{43}$$

the t component of the second conservation equation 43 vanishes identically. If the shift is included in the above vector 40 one finds

$$2N^2U^0_{.i} = 2NN_{,i} + (N_kN^k)_{,i} + N^j(2N_{j,i} - N^k g_{\{ik,j\}}), \tag{44}$$

and further calculation proves intractable.

4.2 Static Case.

In the static case the first conservation equation vanishes identically and the second conservation equation integrates immediately and independently of the equation of state to give

$$\omega = \frac{1}{N}, \tag{45}$$

where the constant of integration has been absorbed into N.

4.3 Non-static Case.

In the non-static case assume the γ–equation of state has to be assumed to accommodate the first conservation law 3. With γ–equation of state 16 the conservation equations 42 and 43 integrate to give

$$\omega = \frac{1}{N} g^{(3)\frac{1}{2}(\gamma-1)}, \qquad \gamma \neq 0, 1, \tag{46}$$

where in place of 36 and 37

$$\dot{N} = 0, \tag{47}$$

and

$$\gamma \left(\ln \left(\sqrt{g^{(3)}} \right) \right)_{,i} = 0, \tag{48}$$

respectively. Constants of integration have been absorbed into the line element. Substituting the spherically symmetric values of the previous section into 46 gives 38 times a function of $\sin\theta$ which has been taken to be asorbable there. The equations 45 and 46 depend on the choice of velocity vector 40, for example if a geodesic velocity vector is chosen then the acceleration vanishes and 45 and 46 do not hold. The conditions 47 and 48 do not appear to have an invariant formulation. There are three things to note. The *first* is that these derivatives do not occur in the covariant derivatives of the vector field 40 and hence do not occur in the geometric objects 41. The *second* is that 36 and 48 are satisfied if

$$\{^{t}_{tt}\} = 0, \tag{49}$$

and

$$\{^{i}_{jk}\} = 0, \tag{50}$$

respectively, as they only occur in these Christoffel symbols. The *third* is that 47 and 48 might **solely** be a gauge condition; but 47 puts on one constraint and 48 puts on three constraints totaling four, the usual number of differential gauge constraints. The Plebanski-Ryten (1961) [29] gauge condition is

$$[(-g)^{w} g^{ab}_{..}]_{,b} = 0, \tag{51}$$

for $w = \frac{1}{2}$ this is the harmonic gauge condition. For $a = t$, 51 is

$$-\frac{1}{N^2}(\ln(g^{(3)w}))_{,t} + \frac{\dot{N}}{N^3} = 0. \tag{52}$$

For $a = x^i$, 51 is

$$-\frac{N_{,i}}{N} + (\ln(g^{(3)w}) g^{ij}_{..})_{,j} = 0. \tag{53}$$

For $w \neq 0$, 47 and 48 cannot be recovered, except for Minkowski spacetime in rectilinear coordinates. Thus the conditions 47 and 48 on the metric appear not to be an example of Plebanski-Ryten gauge conditions. It can be asked, is there a non-static geometric-thermodynamic relation which involves familiar gauge conditions instead of metric constraints such as 47 and 48. Inspection of 18 and 19 with arbitrary vector field instead of 40 does not immediately give a choice of vector field for which application of the Plebanski-Ryten gauge 51 simplifies matters enough for the problem to be tractable.

4.4 γ-equation of state and the ADM.

For the γ–equation of state 46 becomes

$$\mu = N^{\frac{\gamma}{1-\gamma}}, \qquad \gamma \neq 0, 1, \tag{54}$$

for the spacetime to be asymptotically flat the density μ must vanish asymptotically implying that the lapse N must vanish, contradicting the assumption that the spacetime is asymptotically flat. For the γ–equation of state 16, 45 becomes

$$\mu = N^{\frac{\gamma}{1-\gamma}} g^{(3)\gamma/2}, \quad \gamma \neq 0, 1, \tag{55}$$

asymptotically $\mu \to r^2$ and the spacetime cannot be asymptotically flat. For α–polytropes the static case 45 gives

$$N = \left(1 + \alpha\mu^{\beta-1}\right)^{\frac{\beta}{1-\beta}}, \tag{56}$$

and in this case it is possible for $N \to 1$ and $\mu \to 0$ simultaneously as $r \to \infty$ Thus for spacetimes where the rotation free vector 40 can be introduced, and subject to the caveats mentioned above for the non-static case: i)there are no asymptotically flat γ–polytropes except possibly for $\gamma = 0$ or 1, ii)there are no asymptotically flat fluid spacetimes unless the fluid index tends to a finite non-vanishing constant.

5 Against Asymptotic Flatness.

5.1 Length Scales.

On length scales from the outer solar system to cosmology there are observations indicating that asymptotic flatness of the systems under consideration are not correct. It is known that the dynamics of the outer solar system have unexplained irregularities. For example from the figures of Seidelmann *et al* (1980) [32] it appears that the irregularity in Pluto's orbit is that the RA increases by about 2 arcsec

more than expected in 50 years, similarly the declination decreases by about 1 arcsec in 50 years. The irregularities are not neatly expressible by a single quantity, as for example the orbit of Mercury was prior to general relativity; but roughly this means that the orbit is boosted by about 2 arcsec in 50 years. This makes the construction of theories to explain the irregularities difficult. In Roberts (1987) [33] the effect of a non-zero cosmological constant was investigated in order to explain the irregularities of Pluto's orbit and it was found that the cosmological constant would have to be about 12 orders of magnitude bigger than the upper bound Zel'dovich (1968) [37] finds from cosmological considerations. Axenides *et al* (2000) [34] also discuss dynamical effects of the cosmological constant.

Scheffer (2001) [35] and Anderson *et al* (2001) discuss dynamical irregularities in the paths of spacecraft. The orbit of comets, Marsden (1985) [38], Rickman (1988) [39], and Sitarski (1994) [40] have unexplained irregularities, for example at least 6 comets have unexplained forces acting toward the elliptic. Qualitatively this is exactly what would be expected from Kerr geodesics [41] page 363,

> In summary then, the bound and the marginally bound orbits must necessarily cross the equatorial plane and oscillate about it.

but qualitatively the effect is many orders of magnitude out: on solar system length scales the Kerr modification of Schwarzschild geometry is intrinsically short ranged. These solar system orbital problems might originate from the oblateness of the sun, Landgraf (1992) [42]. There are theories which have gravitational potential with an exponential term and mass scale $m_p(m_H/m_P)^n$, where m_H is a typical hadron mass, m_P is the Planck mass, and n = 0, 1, and sometimes 2. Satellite and geophysical data for $n = 2$ theories show that they are not viable unless $m_H > 10^3 GeV$, Gibbons and Whiting (1981) [43]. Other searches for an adjusted potential have been undertaken by Jarvis (1990) [44].

5.2 The Exterior Schwarzschild Solution as a Model.

The exterior Schwarzschild solution is a reasonable model of the solar system outside the sun. A fluid solution can be argued to be a better approximation to the matter distribution as it takes some account of interplanetary space not being a vacuum. Any exterior fluid spacetime would have different geodesics than the vacuum Schwarzschild solution, consequently the orbits of the planets would be different from that suggested by the Schwarzschild solution: how to calculate these geodesics for spherically symmetric spacetimes is shown below. The magnitude of the upper limit of the effective cosmological constant is about $\rho_\Lambda = 10^{-16}$g. cm.$^{-3}$, that this is too small to explain Pluto's irregular orbit was shown in Roberts (1987) [33]. Thus to explain Pluto's irregular orbit using a fluid the critical density must be larger than ρ_Λ. ρ_Λ is much larger than the mean density of interplanetary space which is of the order of 10^{-29}g. cm.$^{-3}$ (or 10^{-5} protons cm.$^{-3}$). The density of interplanetary matter is insignificant compared to the density contribution from the planets, for example for Jupiter $\rho_{\text{Jupiter}} = \frac{3}{4\pi} M_{\text{Jupiter}} r^{-3}_{\text{Jupiter}} \approx 2.10^{-4}$g. cm.$^{-3}$, where the radius r_{Jupiter} is the semi-major axis of the planets orbit. This density is above ρ_Λ and might be above ρ_C. Taking a fluid to model the planets is an unusual step, but the alternative of seeking an $n-$body solution to the field equations is not viable because even the 2−body solution is not known. Looking at constant galactic rotation curves one might try an approximation. As noted in the last paragraph of section 5 of [16]:

> For constant circular velocities over a large distance it is necessary to have an approximately logarithmic potential. Thus the metric will have an approximately logarithmic term. The Riemann tensor is constructed from the second derivatives of the metric and the square of the first derivatives of the metric. For a logarithmic potential these will both be of the order r^{-2} and thus a linear analysis might not be appropriate.

This suggests that only an approach using an exact solution will work. One can assume that the system under consideration can be modelled by a static spherically symmetric spacetime with line element 23. Constructing the geodesics using Chandrasekhar's (1983) [41] method, the geodesic Lagrangian is given by

$$2\mathcal{L} = -C\dot{t}^2 + A\dot{r}^2 + B\dot{\theta}^2 + B\sin^2\theta\dot{\phi}^2. \tag{57}$$

The momenta are given by

$$p_a = \frac{\partial \mathcal{L}}{\partial \dot{x}_a} \tag{58}$$

and are

$$p_t = -C\dot{t}, \quad p_r = A\dot{r}, \quad p_\theta = B\dot{\theta}, \quad p_\phi = B\sin^2\theta\dot{\phi}. \tag{59}$$

Euler's equations are

$$\dot{p}_a = \partial_a \mathcal{L} \tag{60}$$

For static spacetimes with $\partial_t A = \partial_t B = \partial_t C = 0$, giving $\frac{\partial \mathcal{L}}{\partial t} = 0$ so that the time component of the Euler equation 60 gives $\frac{dp_t}{d\tau} = 0$, integrating

$$-p_t = C\frac{dt}{d\tau} = E \text{ a constant along each geodesic.} \tag{61}$$

Similarly by spherical symmetry one can take $\partial_\phi A = \partial_\phi B = \partial_\phi C = 0$, giving $\frac{\partial \mathcal{L}}{\partial \phi} = 0$ so that the ϕ component of the Euler equation 60 gives $\frac{dp_\phi}{d\tau} = 0$, integrating

$$p_\phi = B \sin^2\theta \frac{d\phi}{d\tau} = \text{a constant}. \tag{62}$$

For the θ component

$$\frac{\partial \mathcal{L}}{\partial \theta} = B \sin\theta \cos\theta \frac{d\phi}{d\tau}, \tag{63}$$

the Euler equation 60 is

$$\frac{d}{d\tau} p_\theta = \frac{d}{d\tau} B\dot{\theta} = \frac{\partial \mathcal{L}}{\partial \theta} = B \sin\theta \cos\theta \frac{d\phi}{d\tau}, \tag{64}$$

choosing to assign the value $\pi/2$ to θ when $\dot{\theta}$ is zero, then $\ddot{\theta}$ will also be zero; and θ will remain constant at the assigned value. The geodesic is described in an invariant plane which can be taken to be $\theta = \pi/2$. Equation 62 now gives

$$p_\phi = B\dot{\phi} = L \text{ a constant along each geodesic} \tag{65}$$

where L is the angular momentum about an axis normal to the invariant plane. Substituting into the Lagrangian

$$-\frac{E^2}{C} + A\dot{r}^2 + \frac{L^2}{B} = 2\mathcal{L} = -1 \text{ or } 0, \tag{66}$$

where $2\mathcal{L} = -1$ or 0 depending on whether time-like or null geodesics are being considered. Rearranging

$$A\dot{r}^2 = -\frac{L^2}{B} + \frac{E^2}{C} + 2\mathcal{L}. \tag{67}$$

Taking r to be a function of ϕ instead of τ and using 65 gives

$$\left(\frac{dr}{d\phi}\right)^2 = -\frac{B}{A} + \frac{B^2}{AL^2}\left(\frac{E^2}{C} + 2\mathcal{L}\right), \tag{68}$$

now letting

$$u \equiv \frac{1}{r} \tag{69}$$

as in the usual Newtonian analysis

$$\left(\frac{du}{d\phi}\right)^2 = -\frac{u^4 B}{A} + \frac{u^4 B^2}{AL^2}\left(\frac{E^2}{C} + 2\mathcal{L}\right), \tag{70}$$

seeking a thermodynamic interpretation one can substitute the enthalpy h for the lapse $C = h^2$, but A and B are still arbitrary so that this is not pursued. Inserting the Köttler (Schwarzschild solution with cosmological constant) values of the metric

$$B = r^2, \quad C = \frac{1}{A} = 1 - \frac{2m}{r} + \frac{\Lambda}{3}R^2, \tag{71}$$

and taking $2\mathcal{L} = -1$ for time-like geodesics equation 70 becomes

$$\left(\frac{du}{d\phi}\right)^2 = -u^2 + 2mu^3 + \frac{2mu}{L^2} - \frac{1-E^2}{L^2} - \frac{\Lambda}{3u^2L^2} - \frac{\Lambda}{3} \tag{72}$$

which is equation (4) of Reference [33], the last term suggesting the possibility of constant rotation curves. One can if investigate if there is any adjustment of the Newtonian potential which will give constant geodesics, as required for galactic rotation. Taking (c.f. Will (1993)[45] eq.4.6)

$$g_{tt} \approx -1 + 2U, \tag{73}$$

where U is the Newtonian gravitational potential. Now in assume additionally the particular form for a spherically symmetric spacetime

$$B = r^2, \quad A = \frac{1}{C} \approx \frac{1}{1-2U} \approx 1 + 2U \tag{74}$$

inserting in 70 and expanding for small U everywhere

$$\left(\frac{du}{d\phi}\right)^2 = -\frac{1-E^2}{L^2} - u^2(1+2U) + \frac{2U}{L^2} \tag{75}$$

In particular one might expect that constant rotation is given by the middle term so that

$$-u^2(1+2U) = \alpha \text{ a constant}, \tag{76}$$

rearranging for U we find

$$U = -\frac{1}{2} - \frac{\alpha^2}{2}r^2 \tag{77}$$

this suggests that the correct addition to U to produce constant curves is a function in r^2, this is given by the addition of a cosmological constant and such a spacetime is given by Köttler's solution 71. One might ask what is the next simplest space-time after on with a cosmological constant which has an r^2 increasing potential and perhaps this is the interior Schwarzschild solution. This can be thought of as modelling the halo of a galaxy with the interior Schwarzschild solution and calculating the geodesics to see if they give constant motion. Newtonian modelling has been done by Binney and Tremaine (1987) [46]. For the interior Schwarzschild solution Adler, Bazin, and Schiffer, (1975) [48] equation number 14.47 one has

$$A = \frac{1}{1-\frac{r^2}{\hat{R}^2}}, \quad B = r^2, \tag{78}$$
$$C = \left[\frac{3}{2}\sqrt{1-\frac{r_0^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{\hat{R}^2}}\right]^2,$$
$$\text{for} \quad r \le r_0, \quad \hat{R}^2 = \frac{3c^2}{8\pi\kappa\rho},$$

inserting into 70 one gets

$$\left(\frac{du}{d\phi}\right)^2 = u^2 - \frac{1}{\hat{R}^2} \tag{79}$$

$$+\frac{1}{L^2}\left[-1+\frac{1}{u^2\hat{R}^2}+\left(\frac{2E}{3\sqrt{\frac{u^2\hat{R}^2-u^2r_0^2}{u^2\hat{R}^2-1}}-1}\right)^2\right].$$

The $\frac{1}{\hat{R}^2}=\frac{8\pi\kappa\rho}{3c^2}$ term can be thought of as giving constant rotation curves proportional to the halo density $v_c\propto\rho$. What one expects from the Tully-Fisher (1977) [47] relationship is that $v_c^4\propto L\propto M$.

5.3 Rates of Decay.

In general one can ask what exact solutions to gravitational field equations give what rate of decay. This problem could also be studied numerically or in terms of Weyl and Ricci scalars Ludwidg (1975) [13]. The rate of decay of scalar fields has been discussed in the last paragraph of the introduction of [12]. Including other fields one roughly gets the rates of decay: type B conformal scalars > perfect fluids > the gravitational field > type O scalars > electromagnetic fields > type A conformal scalars > coupled and interacting fields and fluids. The type B conformal scalars and perfect fluids are not usually asymptotically flat. Of course, for example, one would expect there to be conformal scalar solutions which are neither type A or B and these have unpredictable rates of decay, so this ordering is not absolute.

5.4 The Spacetime of Elementary Particles.

Rates of decay are not only important on long distance scales. As pointed out in the second paragraph of the introduction of [12] the exact solution of the spherically symmetric spacetime of the Klein-Gordon-Einstein equations is not known, except in the massless case where the static spherically symmetric field equations $R_{ab}=2\phi_a\phi_b$ have the solution

$$\begin{aligned}ds^2=&\exp\left(-\frac{2m}{r}\right)dt^2\\&-\frac{\eta^4}{r^4}\exp\left(\frac{2m}{r}\right)\mathrm{cosech}^4\left(\frac{\eta}{r}\right)dr^2\\&-\eta^2\exp\left(\frac{2m}{r}\right)\mathrm{cosech}^2\left(\frac{\eta}{r}\right)d^2\Sigma,\quad \phi=\sigma/r\end{aligned}\tag{80}$$

where $\eta^2=m^2+\sigma^2$ and m is interpreted as the mass and σ the scalar charge, see for example, Roberts (1985) [9]. Only the massless exact solution is known so that the exact modification of the shape of the Yukawa potential for the meson is not known: it proves difficult to approximate. The spacetime of mesons has been discussed by Fisher (1948) [49], Ross (1972) [50], Nagy (1979) [51], and Ho (1995) [52]. The Yukawa potential was invented, Yukawa (1935) [53], Landau (1990) [54], to account for the π-meson as the exchange quantum in the force between two nucleons; this is by analogy with electromagnetism where it is the exchange of a photon that is the origin of the electric and magnetic forces between electrons. The exact form of the potential is $V=-(1/r)\exp(-r/m_\pi)$ times a function which involves the relative spin orientations. The Yukawa potential is only an approximation as Quantum Chromodynamics is really the theory of strong interaction; the π-meson or pion successfully describes the residual force, and is thought to work up to momentum transfer of about 1 GeV/c. In strong interaction there is also evidence of a linear confining potential. The Yukawa potential for mesons can be measured using form factors, several mesons are needed, see for example Gross, Van Orden and Holinde (1992) [55]. The hypothetical Higgs particle also has a Yukawa potential. In order to measure the Higg's Yukawa potential one needs to measure the coefficients of the H^3 and H^4 terms in

$$V=M_{H^2/2}H^2+\left(M_{H^2/2v}\right)H^3+\left(M_{H^2/2v^2}\right)H^4,\tag{81}$$

and verify that the coefficients are as expected. At present there are no measurements of these terms. To measure them one would need to observe multi-Higgs production: this is further discussed in Djouadi *et al* (1999) [56]. In the standard model the Higgs coupling enters by a quartic coupling: however in supersymmetric theories the quartic couplings are connected to gauge couplings which are known, so that in supersymmetric models it is easier to calculate the coefficients.

5.5 What Happens on Long Distance Scales.

If asymptotic flatness is incorrect then what does happen on long scales? Non-asymptotic flatness introduces the problem of what happens as the potential goes to infinity - does it increase for ever? What could happen is that the one body problem becomes inappropriate: one needs a solution which takes into account two or more bodies. In particular for the solar system if there is a growing term in the potential one might take that it has stopped growing well before the next star. Torbett and Smoluchowski (1984) [57] argue that there are bodies orbiting the Sun at $10^5\ AU\approx 5\times10^{-3}pc.$, which might be a maximum orbiting distance. Puyoo and Jaffel (1998) [58] study the interface between the heliopause and the interstellar medium, this is at about $10^3\ AU$ and they find a high interstellar hydrogen density of 0.24 ± 0.05 g.cm.$^{-3}$, a proton density of 0.043 ± 0.005 g.cm.$^{-3}$, a helium density of ($2.7\pm$

$0.5)10^{-2}$ g.cm.$^{-3}$, and so forth. One consequence of these non-vanishing densities is that in gravitation, as in quantum field theory, it becomes difficult to say what a vacuum is and whether it has energy, see Roberts (2000) [59]. One can ask what sort of metric describes spacetime at various distances from the Sun, and it seems that some sort of onion model is called for. The standard picture is that the interior Schwarzschild solution is matched to the exterior Schwarzschild solution as in Adler *et al* (1975) §14.2., and then match the exterior Schwarzschild solution to a Friedman model with a specific equation of state as in Stephani (1985) [60] §27.3. Perhaps there should be more than three regions. The sun has a mean density of $\rho_{Sun} = 1.409$g.cm.$^{-3}$ Allen (1962) [61]; however its density varies considerably depending on its distance from the centre from Allen (1962) [61] table on page 163; there is a big jump at about half its radius, which can be modelled by a dense core, so perhaps two interior solutions are needed to describe it. Dziembowski *et al* (1990) [62] and Basu *et al* (2000) [63], use inversion techniques to show that the sun has many layers with different speeds of sound and densities. The solar system splits up into three regions: the inner where the general relativistic corrections to Newtonian theory are needed, the middle where Newtonian theory works, and the outer where a term explaining the irregularity in Pluto's orbit is needed. Next one needs a metric to describe the effect of local stars, then of the galaxy, and then of groups of galaxies. the Robertson-Walker cosmological region comes next, and after this perhaps a chaotic region. One can ask if a particle, say at 1 parsec from the Sun is not in a flat region what is it that causes the most deviation from flatness. For simplicity assume that a Newtonian potential will give correct ratios between the contributions, so that the quantity $\phi/G = M/R$ is calculated in units of the Sun's mass over parsecs. A parsec from the Sun is about as isolated as a particle in the nearby galaxy could be expected to be. The deviation from flatness of the metric is approximately given by equation 73 with $U = \phi$. The quantities in Allen (1962) [61] §132,133,135,136, for the masses and distances associated with the local star system (Gould belt), the galaxy, the local group of galaxies, the Universe are used. Working to the nearest order of magnitude, the local star system has diameter 1,000 pc. and mass $1\times10^8\ M_{Sun}$, assuming the Sun is near the edge gives the potential $M/R \approx 10^5\ M_{Sun}$pc.$^{-1}$. The galaxy has diameter 25 kpc. and mass $1.1 \times 10^{11}\ M_{Sun}$, but the distance of the Sun from the centre is 8.2 ± 0.8 kpc., using this distance $M/R \approx 10^8\ M_{Sun}$pc.$^{-1}$. The local group of galaxies consists of 16 galaxies, suggesting an approximate mass of $10^{12}\ M_{Sun}$, whose centre is 0.4 Mpc. away giving $M/R \approx 10^7\ M_{Sun}$pc.$^{-1}$. Van den Berg (1999) [64] finds 35 local group members and mass $M_{LG} = (2.3 \pm 0.6) \times 10^{12} M_{Sun}$; and that the zero surface velocity, which separates the local group from the field that is expanding with the Hubble flow, has radius $R_0 = 1.18 \pm 0.15\ Mpc.$. The Universe has a characteristic length sale $R = c/H \approx 3,000$ Mpc. and the mass of the observable Universe is $10^{54}g.$, again one can form a ratio M/R, but it has no direct meaning because of homogeneity, one finds $M/R \approx 10^{11}$. To compare with the potential on the surface of the Earth note that the Earth's mean radius $R_{Earth} \approx 6 \times 10^3$Km. $= 2 \times 10^{-12}$pc. and has mass $M_{Earth} \approx 6 \times 10^{27}$g. $= 3 \times 10^{-6} M_{Sun}$, giving $M/R \approx 10^6\ M_{Sun}$pc.$^{-1}$ for the contribution from the Earth's mass, $M/R \approx 10^5\ M_{Sun}$pc.$^{-1}$ for the contribution from the Sun's mass. Collecting these results together gives the ratios

$$10^6 : 10^5 : 1 : 10^5 : 10^8 : 10^7 : 10^{11} \quad (82)$$

This suggests that either the Newtonian approximation is not appropriate, that asymptotic flatness is not a physical notion, or both. For the onion model it suggests that the metric describing the effect of local stars, the galaxy, and the local group of galaxies might not be needed because of the Universe's higher ratio. Another approach to what sort of notions are useful in describing stellar systems is as follows. *A priori* one would not wish to exclude the possibility that near the centre of the galaxy there are stellar systems consisting of several stars, many planets, many asteroids and comets, lots of dust, and which are close say only a light year away from other stellar systems. Dynamics for such a stellar system perhaps could still be calculated in some regions, but there are no notions of a one-body system, vacuum field equations, or asymptotic flatness to use in an explicit manner. It is possible to produce characteristic length scales for the system under consideration Roberts (2016) [65].

5.6 Mach's Principle.

Mach's principle can be formulated in many ways: Barbour and Pfister (1995) [66] p.530 list 21, Bondi and Samuel (1997) [67] list 10. Different formulations can lead to contradictory conclusions: for example, Bondi and Samuel's (1997) [67] Mach3 and Mach10 give rise to diametrically opposite predictions when applied to the Lense-Thiring effect. A Newtonian formulation has equations which can be used to describe dynamics rather than recourse to dark matter, Roberts (1985) [68]. Lack of asymptotic flatness suggests that a system cannot be isolated. This is unlike thermodynamics where isolated heat baths are ubiquitous, and unlike electrostatics where the charge inside a

charged cavity can be zero. So why should a Minkowski cavity in a Robertson-Walker universe be excluded? Field equations and junction conditions allow this to be done, it has to be excluded by principle. The answer is that it is different from electrostatics as gravitation is monopolar in nature. Any departure from homogeneity in the exterior region to a charged cavity would mean a change in charge which would quickly attract the opposite charge and cancel out: however in the gravitational case this does not happen, a change in homogeneity exterior to a Minkowski cavity (I think) would quickly change the spacetime from being flat. The above suggests a new formulation of Mach's principle: THERE ARE NO FLAT REGIONS OF PHYSICAL SPACETIME. What happens for an initial value formulation of this is unclear: presumably it means that a well-defined initial surface does not develop into a surface part of which is flat. The above statement of Mach's principle is a particular case of the statement of Einstein (1953) [69], Ehlers (1995) [70], and Bondi and Samuel (1997) [67] Mach9 THERE ARE NO ABSOLUTE ELEMENTS: a flat metric is an absolute element.

5.7 Isolated Systems.

Another way of looking at asymptotic flatness is to note that it implies that the solar system is isolated. **Isolated systems** seem to be an ideal which is appealed to in order to make problems soluble. The necessity of addressing soluble problems is discussed in Medewar (1982) [71]. A formal approach to isolated systems is given in Ehlers (1980) [72]. In practice an isolated system is only an approximation, there is always some interaction with the external world and for the assumption of an isolated system to work this must be negligible. The assumption that systems can be isolated appears through out science, but there appears to be no discussion of what this involves in texts in the philosophy of science. Three examples of isolated systems are now given. The *first* is photosynthesis: one can think of each leaf on a tree as an isolated entity with various chemical reactions happening independent of the external world, but this is only an approximation as the leaf exchanges chemicals with the rest of the tree so perhaps the tree should be thought of as the isolated system, further one can think of the entire biosphere as an isolated entity which converts 3×10^{21} Joules per year into biomass from a total of 3×10^{24} Joules per year of solar energy falling on the Earth, see for example Borisov (1979) §1.2.1 [73]. The *second* is in thermodynamics and statistical mechanics, here the isolability of systems is taken as a primitive undefined concept, see for example Rosser (1982) [74] page 38. The *third* is of experiments where a single electron is taken to be isolated, Ekstrom and Wineland (1980) [75]: the single electron is confined for weeks at a time in a "trap" formed out of electric and magnetic fields.

6 The Tolman-Ehernfest Relation.

6.1 The Radiation Fluid.

For a radiation fluid $\gamma = \frac{4}{3}$, and by 17 the fluid index is

$$\omega = (3p)^{\frac{1}{4}}. \tag{83}$$

The Stefan-Boltzman law is

$$p = \frac{a}{3}T^4, \tag{84}$$

where T is the temperature and a is the Stefan-Boltzmann constant. Thus

$$\omega = a^{\frac{1}{4}}T. \tag{85}$$

Assuming the spacetime is static and admits the rotation free vector 40, equations 45 and 85 give

$$N = a^{-\frac{1}{4}}T^{-1}, \tag{86}$$

thus showing that the lapse N is inversely proportional to the temperature T. This is the Tolman-Ehrenfest (1930) [27] relation. Lapse only spacetimes have been studied by Roberts (1994) [76] and Schmidt (1996) [77]. For the non-static case 46 and 85 gives

$$Ng^{(3)\frac{1}{6}} = a^{-\frac{1}{4}}T^{-1}. \tag{87}$$

7 The Geometric-thermodynamic equation and Cosmic Censorship.

7.1 Scalar Field Solutions.

It is known that spherically symmetric asymptotically flat solutions to the Einstein massless scalar field equations do not posses event horizons, both in the static case Roberts (1985) [9] and in the non-static case Roberts (1996) [12]. Massless scalar field solutions are equivalent to perfect fluid solutions with $\gamma = 2$ and $U_a = \phi_a(-\phi_c\phi^c_{\cdot})^{-\frac{1}{2}}$; for the above scalar field solutions the vector field is not necessarily timelike so that the perfect fluid correspondence does not follow through. It can be argued that an asymptotically flat fluid would be a more realistic model of a collapsed object, because a fluid provides a better representation of

the stress outside the object. In the spherically symmetric case a global coordinate system of the form 23 can be chosen and a necessary condition for there to be an event horizon is that, at a finite non-zero value of r, $C \to \infty$. From 28, 29, 30, and 38 it is apparent that this only occurs from some exceptional equations of state and values for the fluid density. Relaxing the requirement of spherical symmetry equations 55 and 56 show that for there to be a null surface $N \to 0$, or $\omega \to \infty$; however the derivation of both 55 and 56 requires the vector 40 and components of this diverge as $N \to 0$, also to show that 55 and 56 hold globally it is necessary to show that the coordinate system 39 can be set up globally. The above suggests that it is unlikely that spacetimes with a perfect fluid present have event horizons except in contrived circumstances.

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