Research Article **Open Access**

Gökhan Gökdere*, Mehmet Gürcan, and Muhammet Burak Kılıç

A New Method for Computing the Reliability of Consecutive *k*-out-of-*n*:F Systems

DOI 10.1515/phys-2016-0015
Received December 30, 2015; accepted March 2, 2016

Abstract: In many physical systems, reliability evaluation, such as ones encountered in telecommunications, the design of integrated circuits, microwave relay stations, oil pipeline systems, vacuum systems in accelerators, computer ring networks, and spacecraft relay stations, have had applied consecutive k-out-of-n system models. These systems are characterized as logical connections among the components of the systems placed in lines or circles. In literature, a great deal of attention has been paid to the study of the reliability evaluation of consecutive k-out-of-n systems. In this paper, we propose a new method to compute the reliability of consecutive k-out-of-n:F systems, with n linearly and circularly arranged components. The proposed method provides a simple way for determining the system failure probability. Also, we write R-Project codes based on our proposed method to compute the reliability of the linear and circular systems which have a great number of components.

Keywords: k-out-of-n system, Reliability evaluation, R programming.

PACS: 02.50.-r , 02.50.-CW , 02.70.-C

Notations

p : Reliability of components in the system
n : Number of components in the system
k : Minimum number of consecutive components whose failures cause system failure
i.i.d : Independent identically distributed

***Corresponding Author: Gökhan Gökdere:** Department of Statistics, Fırat University, 23119 Elazığ, Turkey, E-mail: g.g.gokdere@gmail.com
Mehmet Gürcan: Department of Statistics, Fırat University, 23119 Elazığ, Turkey, E-mail: mehmetgurcan2000@yahoo.com
Muhammet Burak Kılıç: Department of Statistics, Mehmet Akif Ersoy University, 15030 Burdur, Turkey, E-mail: muhammet.b.kilic@gmail.com

Lin/Con/k/n:F : Linear consecutive k-out-of-n:F
Cir/Con/k/n:F : Circular consecutive k-out-of-n:F
$R_L(n, k, p)$: Reliability of the linear consecutive k-out-of-n:F system with p
$R_C(n, k, p)$: Reliability of the circular consecutive k-out-of-n:F system with p
ξ : All possible position in the system by the number of components in the system
$\xi = \sum_{j=0}^{n-1} \alpha_{j(\xi)} 2^j, \quad \xi = 0, 1, 2, \ldots, 2^n - 1.$
$\alpha_{j(\xi)}$: The states of the each component at the position ξ in the system and can only take two possible states: failed (0) or good (1)
M_ξ: $\{j : \alpha_{j(\xi)} = 0\}, \quad j = 0, 1, 2, \ldots, n-1$
m_ξ : The maximum number of consecutive j in M_ξ
G_ξ : The system failure probability of the position in ξ if $m_\xi \geq k$

1 Introduction

A consecutive k-out-of-n:F system consists of n components such that the system breakdowns if and only if at least k consecutive components fail. The first report for the consecutive k-out-of-n:F system was presented by Kontoleon [7]. Then, to pay attention to the reliability of this system, much work has been done by Chiang and Niu [3], Bollinger and Salvia [13], Derman et al. [2], Bollinger [11, 12], Goulden [6], Hwang [4], Sarje and Prasad [1], Hwang and Yao [5], Zuo and Kuo [9], and Eryılmaz [14]. Another special type of system related to the consecutive k-out-of-n:F system, is the consecutive k-out-of-n:G system. A consecutive k-out-of-n:G system is an ordered sequence of n components such that the system works if and only if at least k consecutive components work. Similarly, for the reliability of this system, extensive work has been done by Tong [17], Kuo et al. [16], Zuo [10], and Zuo and Kuo [9]. Consecutive k-out-of-n:F and G systems are divided into linear and circular systems correspond to the components arranged along a line or a circle.

Lemma 1

If the reliability of component i in the consecutive k-out-of-n:F system is equal to the unreliability of component i in the consecutive k-out-of-n:G system for all i, and if both types of systems have the same k and n as well, then the reliability of one type of system is equal to the unreliability of the other type of system. And the consecutive k-out-of-n:F and G systems are mirror images of each other (Kuo et al. [16]).

From Lemma 1 we can easily provide that consecutive k-out-of-n:F and G systems are the duals of each other. To give a clear picture about these systems, we list the following examples (see Kuo and Zuo [15]).

Microwave Stations of a Telecom Network: The data transmission between the base stations uses microwave links. A system consists of n microwave station transmit signals from a particular location to another particular location. Microwave stations are located at equal intervals between the specific places and each station is able to relay signals to a distance including k other stations. It is clear that such a system fails if and only if at least k consecutive microwave stations fail.

Oil Pipeline System: Consider oil pipeline system with n pump stations. Each pump station is able to transport oil as far as the next k stations. If less than k consecutive stations fail, the flow of oil will not be interrupted and the system will still function properly. However, when at least k consecutive pump stations fail, the oil flow will be interrupted and the system fail.

Vacuum System in an Electron Accelerator: An accelerator comes either in the form of a ring, where a beam of particles travels repeatedly round a loop, or in a straight line, where the particle beam travels from one end to the other. In the vacuum system of an electronic accelerator, the core consists of a large number of identical vacuum bulbs. The vacuum system fails if at least a certain number of bulbs that are adjacent to one another fail.

Photographing of a Nuclear Accelerator: In analysis of the acceleration activities that occur in a nuclear accelerator, high-speed cameras are used to take pictures of the activities. A set of n cameras are installed around the accelerator. The photographing system operates if and only if at least k consecutive cameras function properly.

Most of the researchers have used the combinatorial approach for reliability evaluation of such systems. With the combinatorial approach, one is able to find an explicit expression of system reliability as a function of component reliability, p. However, this approach provides a disadvantage in terms of ease of application. In this paper, we present a simple computation method for the reliability of $Lin(Cir)/Con/k/n$:F systems based on a logical approach. This method leads to an advantage for computing the reliability of $Lin(Cir)/Con/k/n$:F systems

The rest of this paper is organized as follows. In Section 2, we explain the proposed method to compute the reliability of $Con/k/n$:F systems with i.i.d components. Section 3 gives some examples to show the usefulness of the proposed method. Section 4 contains some concluding remarks.

2 Proposed Method

In this section, we introduce a new method to compute the reliability of Lin and $Cir/Con/k/n$:F systems with i.i.d components.

2.1 $Lin/Con/k/n$:F system

For $Lin/Con/k/n$:F system, we will write the reliability of the system using a logical approach as shown below

$$R_L(n, k, p) = 1 - \sum_{m_\xi \geq k} G_\xi$$

where $\sum_{m_\xi \geq k}$ denotes the summation over all ξ under the condition $m_\xi \geq k$ and G_ξ is the system failure probability at the position ξ if $m_\xi \geq k$ and defines as

$$G_\xi = \prod_{j=0}^{n-1} g_{j(\xi)},$$

where

$$g_{j(\xi)} = \begin{cases} p & \alpha_{j(\xi)} = 1 \\ q & \alpha_{j(\xi)} = 0 \end{cases}$$

Here, we can write the $\alpha_{j(\xi)}$ values for n. These values can be shown from Table 1, whose columns are indexed by $\xi = 0, 1, 2, \ldots, 2^n - 1$ and rows indexed by 2^j, $j = 0, 1, 2, \ldots, n-1$.

2.2 $Cir/Con/k/n$:F system

In the circular system, the components are placed on a circle such that the first and the nth component become adjacent to each other. Similarly, the proposed method can be adopted $Cir/Con/k/n$:F system by adding $2^0, 2^1, \ldots, 2^{k-2}$ columns end of the 2^j column in Table 1.

Table 1: $\alpha_{j(\xi)}$ values for $Lin/Con/k/n$:F system.

ξ	2^0	2^1	2^2	...	2^j
0	0	0	0	...	0
1	1	0	0	...	0
2	0	1	0	...	0
3	1	1	0	...	0
4	0	0	1	...	0
5	1	0	1	...	0
6	0	1	1	...	0
7	1	1	1	...	0
...	...	...	...	...	...
$2^n - 1$	1	1	1	...	1

3 Examples

In this section, we apply the proposed method to find the reliability of system. Firstly, we use two numerical examples to illustrate the proposed method. First example consists of $Lin/Con/k/n$:F system and the second example consists of $Cir/Con/k/n$:F system. Secondly, for computational examples, we provide R codes with different n sizes for $Lin/Con/k/n$:F and $Cir/Con/k/n$:F systems

3.1 Numerical examples

Example 1. For the reliability of a $Lin/Con/k/n$:F system with $n = 3$, $k = 2$ and $p = 0.9$ at first we must know the $\alpha_{j(\xi)}$ values for $n = 3$. These values can be written by using Table 2 for $j = 0, 1, 2$ and ξ=0, 1, . . . , $2^3 - 1$. Using the approach mentioned in the previous section we can get

For $\xi = 0$,

$\alpha_{0(0)} = 0, \alpha_{1(0)} = 0, \alpha_{2(0)} = 0$
$M_0 = \{0, 1, 2\}, m_0 = 3$
$G_0 = g_{0(0)}g_{1(0)}g_{2(0)} = q^3$

For $\xi = 1$,

$\alpha_{0(1)} = 1, \alpha_{1(1)} = 0, \alpha_{2(1)} = 0$
$M_1 = \{1, 2\}, m_1 = 2$
$G_1 = g_{0(1)}g_{1(1)}g_{2(1)} = pq^2$

For $\xi = 2$,

$\alpha_{0(2)} = 0, \alpha_{1(2)} = 1, \alpha_{2(2)} = 0$
$M_2 = \{0, 2\}, m_2 = 0$

For $\xi = 3$,

$\alpha_{0(3)} = 1, \alpha_{1(3)} = 1, \alpha_{2(3)} = 0$
$M_3 = \{2\}, m_3 = 0$

For $\xi = 4$,

$\alpha_{0(4)} = 0, \alpha_{1(4)} = 0, \alpha_{2(4)} = 1$
$M_4 = \{0, 1\}, m_4 = 2$
$G_4 = g_{0(4)}g_{1(4)}g_{2(4)} = pq^2$

For $\xi = 5$,

$\alpha_{0(5)} = 1, \alpha_{1(5)} = 0, \alpha_{2(5)} = 1$
$M_5 = \{1\}, m_5 = 0$

For $\xi = 6$,

$\alpha_{0(6)} = 0, \alpha_{1(6)} = 1, \alpha_{2(6)} = 1$
$M_6 = \{0\}, m_6 = 0$

For $\xi = 7$,

$\alpha_{0(7)} = 1, \alpha_{1(7)} = 1, \alpha_{2(7)} = 1$
$M_7 = \{\}, m_7 = 0$

Table 2: $\alpha_{j(\xi)}$ values for $Lin/Con/2/3$:F system.

ξ	2^0	2^1	2^2
0	0	0	0
1	1	0	0
2	0	1	0
3	1	1	0
4	0	0	1
5	1	0	1
6	0	1	1
7	1	1	1

Finally, the reliability of the system is computed based on the sum of existing G_ξ as follows:

$$\begin{aligned} R_L(3, 2, 0.9) &= 1 - (G_0 + G_1 + G_4) \\ &= 1 - \left(q^3 + pq^2 + pq^2\right) \\ &= 0.981 \end{aligned}$$

Example 2. For the reliability of a $Cir/Con/k/n$:F system with $n = 3$, $k = 2$ and $p = 0.9$ at first we must know the $\alpha_{j(\xi)}$ values for $n = 3$. Note that, in circular system, the n components are placed on a circle so that the first and the nth components become adjacent to each other. So these values can be written by using Table 3 for $j = 0, 1, 2$ and $\xi = 0, 1, \ldots, 2^3 - 1$.

Finally, in addition to the calculations made in Example 1, only G_2 is calculated and the following result obtained.

$$\begin{aligned} R_C(3,2,0.9) &= 1-(G_0+G_1+G_2+G_4) \\ &= 1-\left(q^3+pq^2+pq^2+pq^2\right) \\ &= 0.972 \end{aligned}$$

As it can be seen from the examples we have given above,

Table 3: $\alpha_{j(\xi)}$ values for Ci $r/Con/2/3$:F system.

ξ	2^0	2^1	2^2	
0	0	0	0	0
1	1	0	0	1
2	0	1	0	0
3	1	1	0	1
4	0	0	1	0
5	1	0	1	1
6	0	1	1	0
7	1	1	1	1

in a very large consecutive k-out-of-n system, we require a lot of complex calculations. Therefore, we provide our method's R-project implementation to compute the reliability of the linear and circular systems, respectively.

3.2 Computational examples

Here, we write R codes based on the proposed method. For $Lin/Con/k/n$:F system, R codes are given by

```
#Linear
RL <- function(n,k,p){
q<-1-p
A=matrix(0,2^n,n)
for (i in 1:n){
A[,i]<-rep(0:1,each=2^(i-1),len=2^n)
}
colnames(A)<-paste("A",0:(n-1),sep="")
A<-data.frame(A)
AA <- c()
for(j in 1:(n-k+1)) {
for(m in j:(j+k-1)){ AA = c(AA,colnames(A)[m])
}
}
subb <- t(matrix(AA,k,(n-k+1)))
aa <- c()
for(j in 1:dim(subb)[1]){
  yy <- c()
  for(i in 1:dim(A)[1]){
  yy[i]=sum(A[i,match(subb[j,],names(A))])
 }
 aa <- c(aa,which(yy==0))
}
aa
l <- as.matrix(sort(unique(aa)))
A.sub<-A[l,]
A.sub<-as.matrix(A.sub)
A.sub[A.sub == 1]<-p
A.sub[A.sub == 0]<-q
G<-apply(A.sub[,1:n],1,prod)
Prob<-sum(G)
print(1-Prob)
}
>RL(n,k,p)
```

For $Cir/Con/k/n$:F system, R codes are given by

```
#Circular
RC <- function(n,k,p){
q<-1-p
A=matrix(0,2^n,n)
for (i in 1:n){
A[,i]<-rep(0:1,each=2^(i-1),len=2^n)
}
colnames(A)<-paste("A",0:(n-1),sep="")
A<-cbind(A,A[,1:(k-1),drop=FALSE])
AA <- c()
for(j in 1:n) {
for(m in j:(j+k-1)){ AA = c(AA,colnames(A)[m])
}
}
AA
subb <- t(matrix(AA,k,n))
subb
A<-data.frame(A)
aa <- c()
for(j in 1:dim(subb)[1]){
  yy <- c()
  for(i in 1:dim(A)[1]){
  yy[i]=sum(A[i,match(subb[j,],names(A))])
 }
 aa <- c(aa,which(yy==0))
}
aa
l <- as.matrix(sort(unique(aa)))
A.sub<-A[l,]
A.sub<-as.matrix(A.sub)
A.sub[A.sub == 1]<-p
A.sub[A.sub == 0]<-q
G<-apply(A.sub[,1:n],1,prod)
```

```
Prob<-sum(G)
print(1-Prob)
}
>RC(n,k,p)
```

We compute the reliability of the $Lin(Cir)/Con/k/n$:F system by using R codes when $k = 3$, $p = 0.7$ and $n = 5, 10, 15$ and 20. These results are shown in Tables 4 and 5.

Table 4: Computing the $R_L\,(n, k, p)$ for $k = 3$, $p = 0.7$ and selected values of n.

n	5	10	15	20
$R_L\,(n, 3, 0.7)$	0.93	0.84	0.76	0.69

Table 5: Computing the $R_C\,(n, k, p)$ for $k = 3$, $p = 0.7$ and selected values of n.

n	5	10	15	20
$R_C\,(n, 3, 0.7)$	0.90	0.81	0.73	0.67

4 Conclusion

The usefulness of a system depends on its performance. A common measure of the system is reliability. Reliability of a system has become more important in various designs,such as telecommunication networks and vacuum systems in an electron accelerator. Reliability evaluation of such systems deal with computing or approximating the probability of the system functions. Here, by considering Lemma 1 which provides a way of using the existing methods for the consecutive k-out-of-n:F system to find the reliability of the consecutive k-out-of-n:G system, we provide a new approach for obtaining the reliability of Lin and $Cir/Con/k/n$:F systems.

The proposed method described here is a simple and good alternative approach for computing the reliability of the systems and may be more efficient in very large consecutive k-out-of-n systems using a computer program.

One compared the calculated reliabilities in Examples 1 and 2, and in Tables 1 and 2 with previous methods with same n, k and p values, the same reliabilities can be obtained. Further, using the R codes based on the proposed method, we find the reliability of the linear system for n=11, k=3 and p=0.9 to be 0.9918. This value is equal to example 9.5 in [15].

As a future extension, we would like to emphasize that the proposed method can be examined for the performance of linear and circular consecutive k-out-of-n systems as a function of time.

References

[1] Sarje A.K., Prasad E.V., An efficient non-recursive algorithm for computing the reliability of k-out-of-n:F systems, IEEE Trans. Reliab., 1989, 38, 234-235.

[2] Derman C., Lieberman G.J., Ross S.M., et al., On the consecutive-k-out-of-n:F system, IEEE Trans. Reliab., 1982, 31, 57-63.

[3] Chiang D.T., Niu S.C., Reliability of consecutive-k-out-of-n:F system, IEEE Trans. Reliab., 1981, 30, 87-89.

[4] Hwang F.K., Simplified reliabilities for consecutive-k-out-of-n:F systems, SIAM Journal on Algebraic and Discrete Methods, 1986, 7, 258-264.

[5] Hwange F.K., Yao Y.C., A direct argument for Kaplansky's theorem on cyclic arrangement and its generalization, Oper. Res. Lett., 1991, 10, 241-243.

[6] Goulden I.P., Generating functions and reliabilities for consecutive k-out-of-n:F systems, Utilitas Mathematic, 1987, 32, 141-147.

[7] Kontoleon J.M., Reliability determination of a r-successive-out-of-n:F system, IEEE Trans. Reliab., 1980, 29, 437.

[8] Lambiris M., Papastavridis S.G., Exact reliability formulas for linear and circular consecutive-k-out-of-n:F systems, IEEE Trans. Reliab., 1985, 34, 124-126.

[9] Zuo M., Kuo W., Design and performance analysis of consecutive-k-out-of-n structure, Naval Research Logistics, 1990, 37, 203-230.

[10] Zuo M., Reliability and component importance of a consecutive-k-out-of-n systems, Microelectronics and Reliability, 1993, 33, 243–258.

[11] Bollinger R.C., Direct computation for consecutive-k-out-of-n:F systems, IEEE Trans. Reliab., 1982, 31, 444-446.

[12] Bollinger R.C., An algorithm for direct computation in consecutive-k-out-of-n:F systems, IEEE Trans. Reliab., 1986, 35, 611-612.

[13] Bollinger R.C., Salvia A.A., Consecutive-k-out-of-n:F networks, IEEE Trans. Reliab., 1982, 31, 53-55.

[14] Eryılmaz S., Parallel and consecutive k-out-of-n:F systems under stochastic deterioration, Appl. Math. Comput., 2014, 227, 19-26.

[15] Kuo W., Zuo M.J., Optimal Reliability Modeling: Principles and Applications, John Wiley & Sons, 2003.

[16] Kuo W., Zhang W., Zuo M., et al., A consecutive-k-out-of-n:G system: The mirror image of a consecutive-k-out-of-n:F system, IEEE Trans. Reliab., 1990, 39, 244–253.

[17] Tong Y.L., A rearrangement inequality for the longest run, with an application to network reliability, J. Appl. Probab., 1985, 22, 386–393.