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Study of Grodzins product $(E(2_1^+) \star B(E2)\uparrow)$ in the framework of the Asymmetric Rotor Model

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Abstract: A systematic dependence of Grodzins product $(E(2_1^+) \star B(E2)\uparrow)$ on the asymmetry parameter γ_0 is studied in the $Z = 50-82$, $N = 82-126$ major shell space. The Grodzins product provides contributions of $E(2_1^+)$ and $B(E2)\uparrow$ simultaneously, which further reflects the shape phase transitions with asymmetry parameter γ_0. In the region of deformed nuclei, Grodzins product $(E(2_1^+) \star B(E2)\uparrow)$ shows direct dependence on the asymmetry parameter γ_0. We discuss here for the first time the correlation between Grodzins product $(E(2_1^+) \star B(E2)\uparrow)$ and the asymmetry parameter γ_0.

Keywords: Asymmetric Rotor Model; Grodzins product rule

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1 Introduction

The collective model of Bohr and Mottelson [1] has been used extensively to study the vibrational and rotational states of nuclei. Later, many nuclear models have been introduced to study transitional nuclei [2–5] by expanding the $I(I+1)$ expression. Mariscotti *et al.* [2] proposed the variable moment of inertia (VMI) model to study the ground state band of even-even nuclei by using two limiting conditions: (1) the moment of inertia was considered to be proportional to the deformation parameter and (2) the potential energy was taken to be harmonic. The relation between $B(E2, 2_2^+ \rightarrow 2_1^+)/B(E2, 2_1^+ \rightarrow 0_1^+)$ and $R_{4/2} = E(4_1^+)/E(2_1^+)$ was also discussed. The shape fluctuation model of Satpathy and Satpathy [3] also yielded good results for the ground state band. The nuclear softness model of Gupta [4] obtained better fits of energy levels than the VMI model by varying the moment of inertia with angular momentum J. The study of collective nuclear structure with neutron number N, proton number Z or mass number A gives a deeper understanding of the nucleon-nucleon interaction involved. The nuclear structure of medium mass nuclei changes from anharmonic vibrator to deformed rotor as we move away from the closed shells of N or Z. In nuclear theory, it is desirable to reproduce these changes of nuclear structure with N or Z. In nuclear models, it is useful to find one, two or more variable parameters that depend sensitively on nucleon-nucleon interaction. For example, in the Interacting Boson Model 1 (IBM-1) [6], the total boson number $N_B = N_\pi + N_\nu$ (where N_π = proton boson number and N_ν = neutron boson number) plays an important role. The coefficient χ in the quadrupole operator of the IBM-1 Hamiltonian

$$Q = \left(d^+ s + s^+ d\right)^{(2)} + \chi\left(d^+ \tilde{d}\right)^{(2)}, \tag{1}$$

was used as a variable parameter to determine changes in structure of nuclei [7]. Casten [8] studied the variation of the first excited state energy, $E(2_1^+)$ and energy ratio, $R_{4/2}$ with N_B and $N_p N_n$ in the $A = 100-200$ mass region.
Davydov and Filippov [9] proposed an asymmetric rotor model (ARM) to determine the energy levels and transitional properties of inter- and intra-band excited states in even-even nuclei. They explain the variation in structure of nuclei in terms of the asymmetry parameter γ_0. The Hamiltonian of ARM is related to the energy of the nucleus and can be expressed as

$$H = \sum_{\lambda=1}^{3} \frac{A J_\lambda^2}{2 sin^2\left(\gamma - \frac{2\pi}{3}\lambda\right)}, \tag{2}$$

where $A = \frac{\hbar^2}{4B\beta^2}$ has the dimensions of energy and the J_λ^2 are the projections of the angular momentum operator on the axes of a coordinate system related to the nucleus. The rotational energy levels for the spins $2, 3, 5$ and the transitional probabilities between these energy levels in even-even nuclei were calculated by treating the nucleus as a triaxial ellipsoid. Davydov and Rostovsky [10] computed the rotational energy levels for spins $4, 6, 8$ for even-even nuclei. Gupta and Sharma [11], from a careful analysis of

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the $\gamma-g$ interband B(E2) ratios and the asymmetry parameter γ_0, showed that the $B(E2, 2_\gamma \rightarrow 0/2)$ and $B(E2, 3_\gamma \rightarrow 2/4)$ ratios have some relationship with the asymmetry parameter γ_0. Mittal *et al.* [12] extended the search to study neutron-deficient $Te-Sm$ nuclei for $N < 82$ and concluded the same. A systematic study of $B(E2) \uparrow$ values with the asymmetry parameter γ_0 has been presented in Ref. [13].

Very recently, Gupta [14] pointed out the relationship between reduced electric quadrupole transition probability $B(E2; 0_1^+ \rightarrow 2_1^+)$ values (also written as $B(E2) \uparrow$ values) and the first excited state energy, $E(2_1^+)$, presented by the following formula (given by Grodzins [15]):

$$(E(2_1^+) * B(E2) \uparrow) \sim constant(Z^2/A), \tag{3}$$

and concluded that the constancy of Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ breaks down in the combined effect of the $Z = 64$ subshell effect and the shape phase transition at $N = 88 - 90$.

The above study of Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ was related to proton number Z in quadrant-I ($N > 82$). In light of the above developments, it becomes desirable to study Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ in relation to the asymmetry parameter γ_0. This study may provide a useful insight into nuclear structure because the asymmetry parameter γ_0 determines change in shape of nuclei.

2 Theory

The asymmetry parameter γ_0 of ARM determines the deviation in shape of the nucleus from axial symmetry and it varies between 0 and $\pi/3$. When the asymmetry parameter γ_0 is equal to zero, the energy spectrum is found to be identical to an axially symmetric nucleus and the ARM model produces the symmetric rotor "γ-band" (non-staggered) energies. The deviation of axial symmetry of even-even nuclei (*i*.e., as the asymmetry parameter γ_0 increases) only slightly affects the rotational spectrum of the axial nucleus; however, a few new rotational energy states with spin 2, 3, 4, etc. may be visible (see Figure 1 of Ref. [9]). This effect becomes large when the asymmetry parameter $\gamma_0 = 20°$. The nucleus gets more deformed with increase in γ_0 and finally the nucleus becomes triaxial near $\gamma_0 = 30°$. There are many ways to calculate the asymmetry parameter γ_0. Varshni and Bose [16] used $R_{4/2}$ to calculate γ_0 and exclude the nuclei with $R_{4/2} < 8/3$. However, in Ref. [17], $E(2_1^+)$ and $E(4_1^+)$ were used to determine γ_0. Gupta *et al.* [18] first found the value of the quadrupole moment Q by using sum of B(E2) values of $(2_1^+ \rightarrow 0_1^+)$ and $(2_2^+ \rightarrow 0_1^+)$ and then calculated γ_0. As the value of $B(E2, 2_2^+ \rightarrow 0_1^+)$ was found to be small and of low accuracy, this method was not so fruitful. Davydov *et al.* [9] used $R_\gamma = E(2_2^+)/E(2_1^+)$ to determine γ_0, which was found to be valid as discussed by Gupta and Sharma [11]. We calculate the value of the asymmetry parameter γ_0 by using R_γ in the equation

$$\gamma_0 = \frac{1}{3} sin^{-1}\left[\frac{9}{8}\left(1 - \left(\frac{R_\gamma - 1}{R_\gamma + 1}\right)^2\right)\right]^{1/2}. \tag{4}$$

The values of $E(2_2^+)$ and $E(2_1^+)$ are taken from the National Nuclear Data Center (NNDC) website[1]. The reduced electric quadrupole transition probability $B(E2) \uparrow$ values are taken from Raman *et al.* [19]. They complied the $B(E2) \uparrow$ values for even-even nuclei for the $0 \leq A \leq 260$ mass region. The adopted $B(E2) \uparrow$ values have been compared with many theoretical nuclear models.

3 Results and Discussion

Gupta *et al.* [20] suggested splitting the major shell space ($Z = 50 - 82$, $N = 82 - 126$) into four quadrants and grouping them on the basis of valence-particles and valence-holes. Quadrant-I and Quadrant-III contain particle-particle and hole-hole bosons, respectively, whereas Quadrant-II and Quadrant-IV have hole-particle and particle-hole bosons, respectively. However, no nuclei lie in Quadrant-IV.

3.1 The $Ba - Gd$ nuclei, $N > 82$ region

This is a particle-particle bosons subregion of the major shell region $Z = 50–82$, $N = 82–126$, known as Quadrant-I. Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ is plotted against the asymmetry parameter γ_0 for $N > 82$ region (see Figure 1).

Table 1: The values of the asymmetry parameter γ_0 for Quadrant-I.

N	Ba	Ce	Nd	Sm	Gd
84	24.40	25.05	26.13		
86	19.42	19.94	21.39	23.71	
88	15.26	16.85	19.02	21.74	21.45
90	15.76	15.66	13.80	13.22	13.84
92				9.53	11.04
94					10.31
96					10.97

1 Chart of Nuclides, http://www.nndc.bnl.gov/chart/

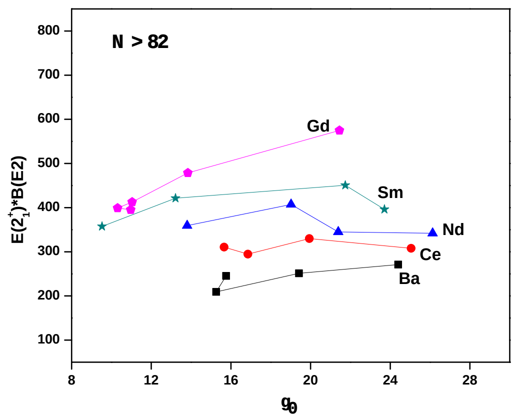

Figure 1: Plot of $(E(2_1^+) * B(E2) \uparrow)$ in $keVe^2b^2$ vs. asymmetry parameter γ_0 for the $N > 82$ region in $Ba - Gd$ nuclei.

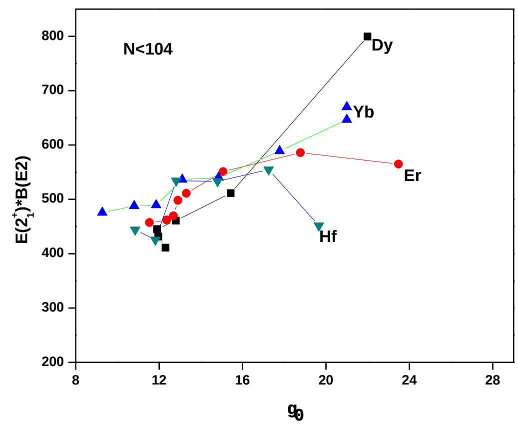

Figure 2: Plot of $(E(2_1^+) * B(E2) \uparrow)$ in $keVe^2b^2$ vs. asymmetry parameter γ_0 for $N < 104$ region in $Dy - Hf$ nuclei.

In Ba and Ce nuclei (with $\gamma_0 = 15.26°$ and $16.85°$ respectively), the Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ values are small as compared to other nuclei at $N = 88$. The high values of Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ occur especially for the transitional nuclei ^{148}Nd ($\gamma_0 = 19.02°$), ^{150}Sm ($\gamma_0 = 21.74°$) and ^{152}Gd ($\gamma_0 = 21.45°$). This shows the effect of the $Z = 64$ subshell at $N = 88$ isotones. When the same Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ is plotted against proton number Z, an increasing behaviour is observed (see Figure 3 of Ref. [14]). The role of the $Z = 64$ subshell effect with the N-dependence and the N_p-dependence of $B(E2) \uparrow$ values in the breakdown of Grodzins product at $N = 88$ is also discussed in Ref. [14]. Gupta *et al.* [20] concluded that the rotational spectra of nuclei are more constant in isotonic multiplets. The shape fluctuation energy for $Xe - Gd$ nuclei ($N = 84 - 104$) is independent of atomic number Z and also constant for isotones (see Ref. [21]). Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ shows smooth dependence on the asymmetry parameter γ_0. Values of γ_0 for different values of $N > 82$ in $Ba - Gd$ nuclei are shown in Table 1.

3.2 The $Dy - Hf$ nuclei, $N < 104$ region

These nuclei lie in Quadrant-II of the $Z = 50-82$, $N = 82-126$ shell space which contains proton-hole and neutron-particle bosons. The Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ shows a very different pattern as compared to the $N < 82$ mass region. Plotting Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ versus asymmetry parameter γ_0 yields a smooth increasing curve for $\gamma_0 \le 19°$. This region is described as the deformed SU(3) region according to IBM. It can be concluded that Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ increases with increasing asymmetry parameter γ_0 in the $N < 104$ region. Our results agree with Gupta [14] that the Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ shows more constancy for deformed nuclei.

Table 2: The values of asymmetry parameter γ_0 for Quadrant-II.

N	Dy	Er	Yb	Hf
88	21.96	23.44	20.97	
90	15.41	18.75	17.76	19.65
92	12.78	15.05	14.83	17.22
94	11.88	13.28	13.09	14.77
96	11.95	12.88	11.84	12.64
98	12.29	12.66	10.79	11.80
100		12.34	9.26	10.83
102		11.52		

The curve however gets scattered after $\gamma_0 = 19°$ and shows a shape transition from SU(3) to the O(6) limit (γ–soft) or SU(5) (vibrator) in these neutron deficient $Dy - Hf$ nuclei. This is similar to the conclusion obtained by Gupta *et al.* [22] about the shape fluctuation energy plot for $Dy - Pt$ nuclei. The large increase in Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ is clearly seen in Figure 2 at $\gamma_0 = 21°$; hence ^{154}Dy is a shape phase transitional nucleus. Values of γ_0 for different values of $N < 104$ in $Dy - Hf$ nuclei are shown in Table 2.

3.3 The $Hf - Pt$ nuclei, $N > 104$ region

These nuclei lie in Quadrant-III of the $Z = 50-82$, $N = 82-126$ major shell space with hole-hole subspace. The plot of Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ versus asymmetry

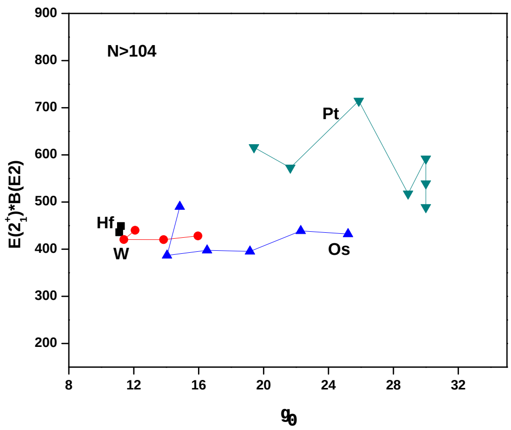

Figure 3: Plot of $(E(2_1^+) * B(E2) \uparrow)$ in $keVe^2b^2$ vs. asymmetry parameter γ_0 for the $N > 104$ region in $Hf - Pt$ nuclei.

Table 3: The values of asymmetry parameter γ_0 for Quadrant-III.

N	Hf	W	Os	Pt
106	11.19	12.06	14.82	19.38
108	11.09	11.37	14.03	21.62
110		13.82	16.50	25.83
112		15.92	19.14	28.86
114			22.22	30.00
116			25.17	30.00
118				30.00

parameter γ_0 is almost constant for $Hf - W$ as these are well deformed nuclei, but in the case of $^{188-192}Os$ nuclei, the Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ increases slightly with increase in asymmetry parameter at $\gamma_0 = 19.14°$ (see Figure 3).

Scholten [23] referred to these nuclei as shape phase transition nuclei and predicted them to have O(6) symmetry. The shape fluctuation energy is independent of the neutron number N for $Hf - Os$ along the isotopic lines (see Ref. [20, 21]). The increase in Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ at $\gamma_0 = 25.8°$ shows a shape phase transition in the ^{188}Pt nucleus. This was also observed by Gupta *et al.* [21]. A neutron shell gap is also present at $N = 114$ which may be the main reason of this different behaviour in Pt nuclei. The microscopic view of this neutron shell gap at $N = 114$ was discussed in Ref. [24]. This observation supports the presence of the $N = 114$ neutron shell gap which is effective at $Z = 78$. Values of the asymmetry parameter γ_0 for the $N > 104$ region in $Hf - Pt$ nuclei are presented in Table 3.

4 Conclusion

In all the quadrants studied above, Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ shows a direct dependence on the asymmetry parameter γ_0. In Quadrant-I ($N > 82$), Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ shows variation in curve due to the presence of a $Z = 64$ subshell effect at $N = 88$ isotones. However, in the case of Quadrant-II, Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ increases with an increase in the asymmetry parameter γ_0. For $Hf - Pt$ nuclei, ($N > 104$) Grodzins product $(E(2_1^+) * B(E2) \uparrow)$ shows sharp increase for transitional nuclei, as in the case of Pt at $\gamma_0 = 25.8°$, whereas the curve tends to become smooth for $Hf - Os$ nuclei. A small neutron shell gap at $N = 114$ is observed in Pt nuclei. Grodzins product provides contributions of $E(2_1^+)$ and $B(E2) \uparrow$ simultaneously, which further reflects shape phase transitions which depend sharply or gradually on the asymmetry parameter γ_0.

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References

[1] A. Bohr, B.R. Mottelson, Nuclear Structure Vol. II (Benjamin, New York, 1975)
[2] M.A.J. Mariscotti, G. Scharff-Goldhaber, B. Buck, Phys. Rev. 178, 1864 (1969)
[3] M. Satpathy, L. Satpathy, Phys. Lett. B 34, 377 (1971)
[4] R.K. Gupta, Phys. Lett. B 36, 173 (1971)
[5] G. Scharff-Goldhaber, C.B. Dover, A.L. Goodman, Ann. Rev. Nucl. Science 26, 239 (1976)
[6] F. Iachello, A. Arima, The Interacting Boson Model (Cambridge University Press, Cambridge, 1987)
[7] D.D. Warner, R.F. Casten, Phys. Rev. C 25, 2019 (1982)
[8] R.F. Casten, Nucl. Phys. A 443, 1 (1985)
[9] A.S. Davydov, G.F. Filippov, Nucl. Phys. 8, 237 (1958)
[10] A.S. Davydov, V.S. Rostovsky, Nucl. Phys. 12, 58 (1959)
[11] J.B. Gupta, S. Sharma, Phys. Scr. 39, 50 (1989)
[12] H.M. Mittal, S. Sharma, J.B. Gupta, Phys. Scr. 43, 558 (1991)
[13] R. Kumar, R. Kaushik, S. Sharma, DAE Symp. Nucl. Phys. 58, 236 (2013)
[14] J.B. Gupta, Phys. Rev. C 89, 034321 (2014)
[15] L. Grodzins, Phys. Lett. 2, 88 (1962)
[16] Y.P. Varshni, S. Bose, Nucl. Phys. A 144, 645 (1970)
[17] J. Meyer-Ter-Vehn, Nucl. Phys. A 249, 111 (1975)
[18] K.K. Gupta, V.P. Varshney, D.K. Gupta, Phys. Rev. C 26, 685 (1982)

[19] S. Raman, C.W. Nestor, Jr., P. Tikkanen, At. Data Nucl. Data Tables 78, 1 (2001)
[20] J.B. Gupta, J.H. Hamilton, A.V. Ramayya, Int. J. Mod. Phys. 5, 1155 (1990)
[21] J.B. Gupta, H.M. Mittal, S. Sharma, Phys. Scr. 41, 660 (1990)
[22] J.B. Gupta, H.M. Mittal, J.H. Hamilton, A.V. Ramayya, Phys. Rev. C 42, 1373 (1990)
[23] O. Scholten, Interacting bosons in Nuclear Physics, 17 (Plenum Press, New York, 1979)
[24] R.F. Casten, Phys. Rev. Lett. 54, 1991 (1985)