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Primacy analysis in the system of Bulgarian cities

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Abstract: The concept of "primacy" as introduced by Jefferson in 1939 in urban geography leads to the notion of "dominant city" also known as the primate city. Practically, the notion was extended by Sheppard in view of discussing some "hierarchy". The type of dominance is not universal nor any hierarchy reversal. Both can be time and sample dependent. Thus, as an example taking into consideration the existence of both pieces of the puzzle, we consider and discuss the Bulgarian urban system. It is also interesting to compare data on two groups of cities in different time intervals: (i) the whole Bulgaria city system which contains about 250 cities, - studied in the time interval between 2004 and 2011, and (ii) a system of 33 cities, - studied over the time interval 1887 till 2010. These latter cities are selected because the population was already over 10 000 inhabitants in 1946. It is shown that new additional indices are interestingly introduced in order to compensate defects in the Sheppard index. Numerical illustrations are illuminated through a "length ratio" measure, which allows to distinguish the (often) observed departures from the hyperbolic ranking seen by Jefferson.

Keywords: city sizes; Sheppard index; Zipf's law; primacy; primacy indices

1 Introduction

The concept of primacy was introduced by Jefferson [1] in urban geography. He observed that the largest city was more than twice the second ranked city in population size. Later a hyperbolic rank-size rule (or Zipf's law) was imagined to describe city size hierarchy [2]. Many recent studies have found deviations from Zipf's law. i.e. cities other than the largest one are far larger than should be expected from the basic rank-size rule. It might appear at first that this is an old problem; indeed, - but still with questions needing more data analysis and understanding. Two questions seem still recurrent in view of modern developments of mega-cities: (i) is there some understanding on the reversal of hierarchies? (ii) can one better describe hierarchies ?

We approach the puzzle in a different way than in previous work, relying on concepts derived from complexity theory. In fact, nonlinearity [3, 4] and complexity [5, 6] are common features for a large number of systems studied in modern social systems, as in [7, 8]. Such systems are much investigated by nonlinear dynamics methods, and time series analysis [9, 10]. In the last decade or so, these methods have been applied in various ways to many social, economic, and financial systems [11, 12]. In many cases, researchers have detected the existence of *power (= scaling)* laws, for different characteristic quantities of these complex systems. This gives some "universal character" to Jefferson's and Zipf's observations.

In general, power laws turn out to be useful features in studying complex systems because scaling relations may indicate that the system is controlled by a few rules that propagate across a wide range of scales [13, 14].

Do we have an interesting case to compare with, in "modern European history? The data must pertain to a country, contain markedly different city sizes, and if possible cover different time intervals. In this paper, we discuss the human population of Bulgaria along those lines. In Bulgaria, there exist about 250 cities and about 4000 villages. The human population of the country reached almost 9 millions in 1985 but later on has decreased steadily in the last 25 years down to 7.3 millions in 2011. Below, we examine two sets of urban population data. The first set is the yearly count of the population of whole Bulgarian cities from 2004 till 2011, as recorded by the National Statistical Institute of the Republic of Bulgaria (*http : www.nsi.bg*). The second data set is the yearly population count for the 33 Bulgarian cities which had a population over 10 000 citizens in 1946, in specific years: 1887, 1910, 1934, 1946, 2000 and 2011 The data for 1887, 1910, 1934,

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1946 is available from Mladenov and Dimitrov [15] while the data from 2000 and 2010 are from the National Statistical Institute of Republic of Bulgaria.

An interesting point will be observed the different growth (or rather decay in primacy) of the previous capital Veliko Tarnovo, due to the political vote when choosing a new capital, Sofia, in 1879, thus *in fine* inducing a hierarchy reversal. This can be related to another case, far away from Bulgaria: in India, Kolkata was the primate city, but later on Delhi became the primate one under the British Rule.

With respect to some pertinent literature on some hierarchy and of a city primacy origin, not considering reversal, recall that a primate city was often thought to occur in underdevelopped countries [16, 17], analysing 75 countries, in fact demonstrated some positive correlation between primacy and development. Lyman [18] has hypothesized that countries under colonial rule are more likely to exhibit primate urban systems than colonizing nations. Interestingly, Smith [19] has argued that a shift in the Guatemala city system hierarchy is related to a transition to capitalism. All such papers selected from the relevant literature points to some political cause, suggesting to look at countries having non-colonial and non-capitalist history, a few years ago, - like Bulgaria, not quite a colony, yet having moved on the capitalist side, but originally having modified the primacy hierarchy through an unexpected parliamentary voting result.

Note that another theoretical input to the puzzle will arise from pointing and resolving defects in the numerical notion of primacy first based on the global primacy index of Sheppard [20]. In order to compensate defects in the Sheppard index. Several (new) additional indices are next introduced starting from a scaling law distribution. This differs from Chase-Dunn [21] attempt to calculate a primary index to express the deviations of city size distribution from the log-normal (rank-size) rule. Numerical illustrations will further illuminate the discussion through a so called "length ratio" concept.

2 Analysis of primacy

In the course of time, the cities in a country develop a hierarchy. An expression of this hierarchy is the city population size distribution that can be easily constructed for any urban system. Zipf [2] suggested that a large number of observed city population size distributions could be approximated by a simple *scaling (= power) law* $N_r = N_1/r$, where N_r is the population of the r-th largest city, i.e. with $r = 1, \ldots$ and $N_r(t) \geq N_{r+1}(t)$, at some time t. A more flexible equation, with two parameters, reads $N_r = N_1/r^{\beta}$, is called the rank-size scaling law. Zipf suggested that the particular case $\beta = 1$ represents a desirable situation, in which forces of concentration balance those of decentralization. Such a case is called the *rank-size rule*.

It has been seen that, in a few cases, the city size distributions can be close to the rank-size rule of Zipf. However, in many cases these distributions are primate distributions [20], i.e. i) one or very few but very large cities (the capital and several other cities) dominate the distribution; (ii) there is a "large" number of equivalently large cities leading to convex distributions; or (iii) distributions with some mix of primacy and convexity, leading to a S-shape like or even a more complicated structure.

The urban population size distribution of several developed countries, like the USA, fits very well the rank-size rule over several decades [22, 23].

2.1 Sheppard index of primacy measure

Going beyond Zipf's simple law entices to introduce a measure of primacy through an index like

$$Pr^{(k)} = \frac{N_1}{\sum_{r=2}^{k+1} N_r}, \qquad k = 1, 2, 3, \ldots. \tag{1}$$

i.e. giving, as in Eq. (1), a numerical value for the "primacy" of the largest city with respect to the next $k-1$ cities, if the cities are ordered according to the decreasing number of their inhabitants. If one wishes to compare the primacy of other cities, with respect to the following ones, one can generalize Eq. (1), to read

$$Pr_j^{(k+j-1)} = \frac{N_j}{\sum_{r=j+1}^{k+j} N_r}, \quad k = 1, 2, 3, \ldots, \quad j = 1, \ldots, . \tag{2}$$

If a power law like $N_r = N_1/r^{\beta}$ is substituted into each of these measures, Eq. (1), for various cities, it is obvious that the corresponding index of primacy depends on β, whence rank-size relationships with different slopes (β, on a log-log plot) will have different (numerical) levels of primacy. Then, it will be not possible to discriminate between a country where a primate city dominates a city size distribution, which otherwise may have a low and fairly consistent negative slope, from a country exhibiting a rank-size relationship with steep slope β. Sheppard tried to avoid this puzzle by formulating a *primacy index* that is indepen-

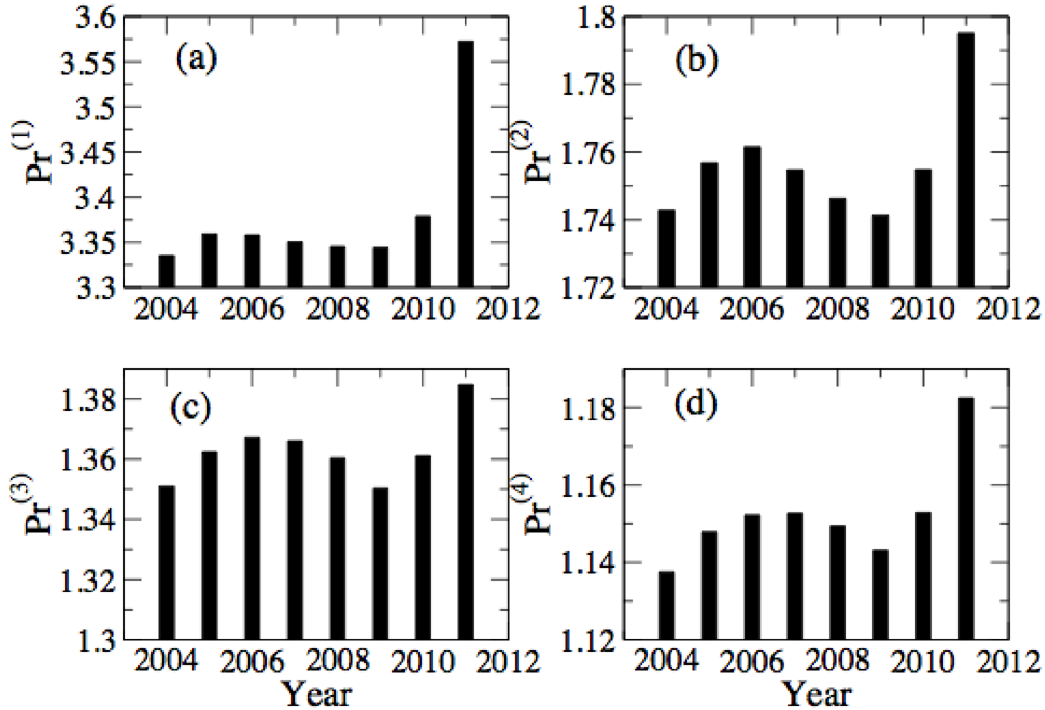

Figure 1: Evolution of the first 4 primacy indices for Sofia, the capital of Bulgaria, from 2004 till 2011. Remember that $Pr^{(k)} = N_1/(\sum_{r=2}^{k+1} N_r)$, for $k = 1, 2, 3, \ldots$, Eq. (1). Till 2006, the values of the primacy indices increase; later, till 2009, the values of the indices decrease. Next, a very sharp increase is observed in 2010 and 2011

dent of β, i.e., he defined

$$Pr_N = \frac{1}{N-2} \sum_{r=1}^{N-2} \left[\frac{\ln(N_r+1) - \ln(N_r)}{\ln(N_{r+2}) - \ln(N_{r+1}))} \right] \left[\frac{\ln(r+2) - \ln(r+1)}{\ln(r+1) - \ln(r)} \right] \tag{3}$$

The logics behind this index is as follows. Let us substitute here the power law rank-size relationship $N_r = N_1 r^{-\beta}$. The result is $Pr_N = (1/(N-2)) \sum_{r=1}^{N-2} 1 = 1$. Thus, for a perfect power law rank-size relationship, the index Pr_N has a value of 1, irrespective of the slope of the relationship.

We have applied the Sheppard index, Eq. (3), to study the primacy (or "hierarchy") of Bulgarian cities in the years between 2004 and 2011.

Figure 1 shows the changes in the first 4 primacy indices $Pr^{(1)}$, ..., $Pr^{(4)}$ for the largest city (and to-day capital) of Bulgaria, i.e. Sofia, i.e. $r = 1$; next, respectively, i.e., $r = 2, 3, 4$, are Plovdiv, Varna and Burgas. A decreasing of primacy is observed between 2006 and 2009. One reason for this is economic: the good economic development before the crisis (that appeared in Bulgaria in 2009). Because of favorable economic conditions, there was enough inflow of people to the second, third, and the fourth largest city, thereby *decreasing* the primacy of the capital, Sofia. However the subsequent economic crisis worsened the job perspectives in these large cities which led to an increased inflow of people back to Sofia. This led to re-*increasing* the primacy of the capital in the last few years.

Figs. 2-3 show $Pr^{(1)}$, $Pr^{(2)}$, and $Pr^{(3)}$ illustrating the evolution of the population of the capital Sofia within the class of 33 cities (with population exceeding 10 000 in 1946). It is seen that the primacy in 1887 was *below* 1 (Fig. 2). Observe the consistency, bearing a change of scale, between $Pr^{(2)}$ (Fig. 2) and $Pr^{(3)}$ (Fig. 3), indicating the stability of the different ratios of populations between these major cities.

Nevertheless, "on the advantage to be a capital" is interestingly shown through the ratio displayed in Fig. 4. Recall that Sofia was the capital but not the largest city in Bulgaria up to 1890. Fig.4 shows the ratio of the population of the two cities that were candidates for capital of Bulgaria in 1879. The choice of Sofia was markedly favorable for a population increase. One should remind the reader that in 1879, almost a year after the creation of the Third Bulgarian State, a new capital "had to be" selected. There were two candidate cities: Sofia and Veliko Tarnovo, the capital of the Second Bulgarian State. Sofia was selected to be the capital of Bulgaria: Sofia won by 1 vote over Veliko Tarnovo. At that time, the population of Sofia was about *twice* larger than the population of Veliko Tarnovo. The concentration process led to a situation in which the pop-

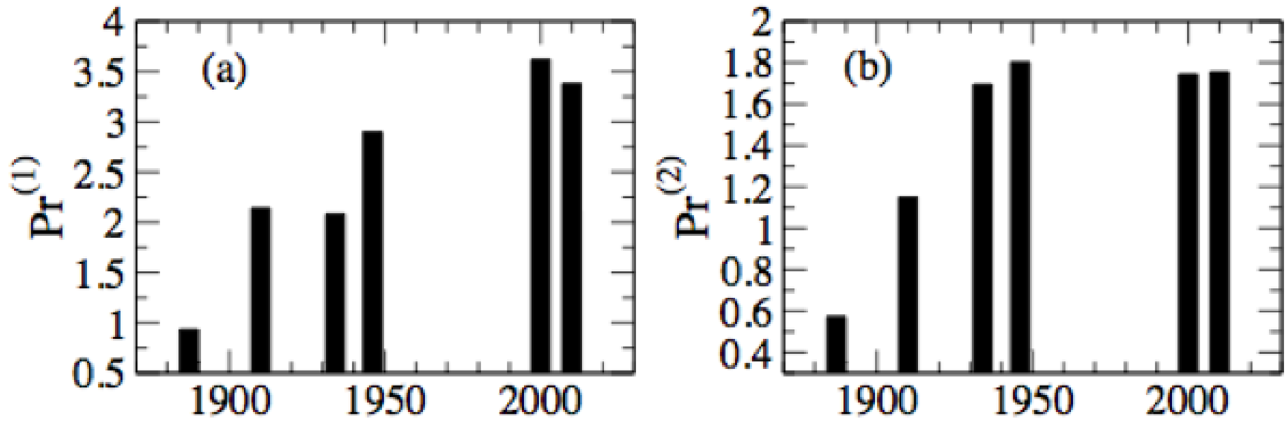

Figure 2: (a)-(b): The first two primacy indices of Sofia in the 33 city system for 1887, 1910, 1934, 1946, 2000, and 2010.

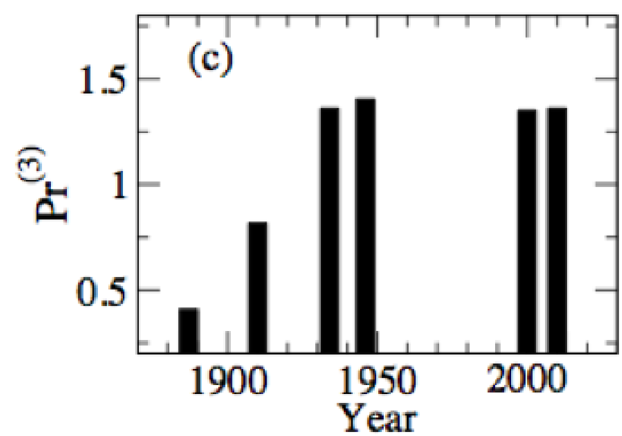

Figure 3: The third primacy index of Sofia in the system of 33 cities for 1887, 1910, 1934, 1946, 2000, and 2010.

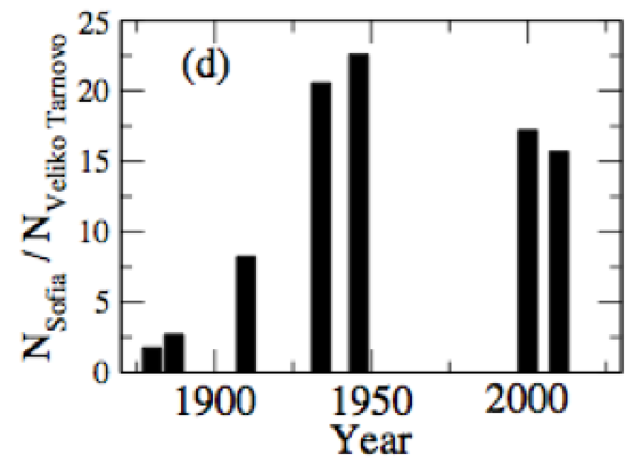

Figure 4: Ratio of the population of the two cities that were candidates for being the capital of Bulgaria in 1879 (Sofia and Veliko Tarnovo). The ratios are for 1880, 1887, 1910, 1934, 1946, 2000, and 2010.

ulation of Sofia became 25 *times larger* than the population of Veliko Tarnovo. In the last 25 years, the total country population as well as the urban population have decreased but the population of these two cities has further increased: the recent (*ca*. since 1950) rate of increase of the Veliko Tarnovo population is in fact *larger* that the rate of increase of the Sofia population. Thus, in 2010 the population of Sofia is only about 15 times larger than the population of Veliko Tarnovo (r = 16, in 2011); see Fig. 4. This is also a strong evidence for the fact that the population growth of the Bulgarian cities is city population size (and time) dependent [24].

2.2 Extended local primacy measures

However, the primacy index of Sheppard contains a difference of two logarithms in the denominator. When two cities have almost the same number of citizens, - which can be often the case for small cities and villages, this difference can be very small thus leading to large value of the Sheppard index. Actually, this happened: when we analyzed the primacy index values in the (large) system of about 250 Bulgarian cities, there were two cities for which the number of citizens differs by 1 only. In order to avoid such a kind of problem, we propose to consider two other local primacy measures where the difference of logarithms is present only in the numerator, as follows. Let the cities be ranked, in the each studied city system, according to the population, i.e. $N_r \geq N_{r+1}$. These measures are:

$$V_r = \frac{\ln(N_{r-1}) - \ln(N_r)}{\ln(r) - \ln(r-1)}, \tag{4}$$

and

$$W_r = \frac{\ln(N_r) - \ln(N_{r+1})}{\ln(r+1) - \ln(r)} - \frac{\ln(N_{r+1}) - \ln(N_{r+2})}{\ln(r+2) - \ln(r+1)} \equiv V_{r+1} - V_{r+2}. \quad (5)$$

- For the case of a power law relationship as $N_r = N_1 r^{-\beta}$ $(r = 1, \ldots, N)$, for each $\beta \geq 0$, the values of the measures V_r and W_r are: $V_r = \beta$ $(r = 2, \ldots, N-1)$ and $W_r = 0$ $(r = 2, \ldots, N-2)$.
- Let us now discuss cases when deviations from the power law occur. Let us consider the $(\ln(r), \ln(N_r))$-plane. Suppose first that $\ln(N_{r-1})$ is found to be above the straight line formed between the points $\ln(N_r)$ and $\ln(N_{r+1})$, a case of *local primacy*. In such a case, $V_r > V_{r+1}$ and $W_{r-1} > 0$. However, if $\ln(N_{r-1})$ is below the straight line formed by the points $\ln(N_r)$ and $\ln(N_{r+1})$, a case of *local convexity*, then $V_r < V_{r+1}$ and $W_{r-1} < 0$. Of course, if $\ln(N_{r-1})$ lies on the straight line formed by the points $\ln(N_r)$ and $\ln(N_{r+1})$, i.e. the strict power law case fulfilling, then $V_r = V_{r+1}$ and $W_{r-1} = 0$.

Thus, for the power law case, the V_r's will form a straight line as a function of r. In contrast, the deviation of the V_r distribution from a straight line will be a signal indicating a deviation of the city size distribution from a power law function.

Next, for the system of all Bulgarian cities in 2004 and 2011, consider the W-measures as reported in Fig. 5. If a single power law was present in the rank-size relationship then $W_r = 0$ and W_r would be a straight line as a function of r.

As easily observed, this is not the case for the system of Bulgarian cities, since what is observed, in Fig. 5, is a mix of regimes of local primacy and regimes of local convexity.

For completeness, the W-measures, Eq. (5), are compared for the system of the largest 170 USA cities in (a) 1990 and (b) 2010, according to the *US Census Bureau, Statistical Abstract of the United States: 2012*, in Fig.6. These plots can be compared to those pertinent to the BG case. In the USA case, the W_r values become more erratic with time. This is not the case for the BG cities, on the contrary. However, in both cases, when $r \leq 150$, the behaviors look very similar.

3 Length ratio

It has been observed here above that a hierarchy can be measured. However, the theoretical ground is incomplete if one does not obtain a measure of the deviations from the basic empirical laws. Thus, in order to characterize the deviation from a power law of a system of cities, from a system with the same number of cities, but the latter (theoretically) obeying a power law, one can e.g. measure the length L_β of the curves corresponding to the V-measures here above defined. Indeed, let us consider N cities. If the rank-size distribution of these cities is a single power law, then the W-measure of each 3 neighboring cities is equal to 0. For a system of N cities, there will be $N-2$ points in the (r, W_r) plane with coordinates $(j, 0)$ where $j = 1, \ldots, N-2$. These $N-2$ points connect $N-3$ segments of the W-curve and each segment has the same length 1. Then, the total length of the W_r line in the (r, W_r)-plane is $L_\beta = N-3$.

Let us now consider the other case, when the distribution of the population in cities does *not* behave according to a power law. Then, the W_r curve is not a straight line (see Fig. 3 for an example); the length of such a curve is bigger than L_β.

Thus, it seems of interest to define the *length ratio*

$$R_N = \frac{L_N}{L_\beta} \quad (6)$$

where L_N is the length of the line associated with the corresponding W_r-index:

$$L_N = \sum_{r=1}^{N-3} \sqrt{1 + (W_{r+1} - W_r)^2} \quad (7)$$

The results for the length ratio R_N for several classes of Bulgarian cities are shown in Table 1. For a given number of cities, the evolution of the deviation of the city size distribution from a power law[1] can be calculated. For example, for the 50 largest cities, R_{50} increases steadily since 2006. This means that the populations of cities change in such a manner that the corresponding rank-size distribution deviates *more and more* from a single power law as a function of time. The evolution with respect to R_N of the 100 largest cities is even more interesting, since between 2006 and 2008 the distribution appears to be more like a single power law than for 2005 (Figure not shown).

Finally, note that the length ratio R_N can be generalized in order to investigate the distribution deviation from a power law for any sub-class of cities, e.g. ranking between N_1 and N_2. One can define, e.g.,

$$R_{N_1,N_2} = \frac{1}{L_\beta} \sum_{r=N_1}^{N_2-3} \sqrt{1 + (W_{r+1} - W_r)^2}, \quad (8)$$

implying a set of new indices for presenting a hierarchy. The notion of "length" also suggests to represent the city

1 for a single power law $R_N = 1$

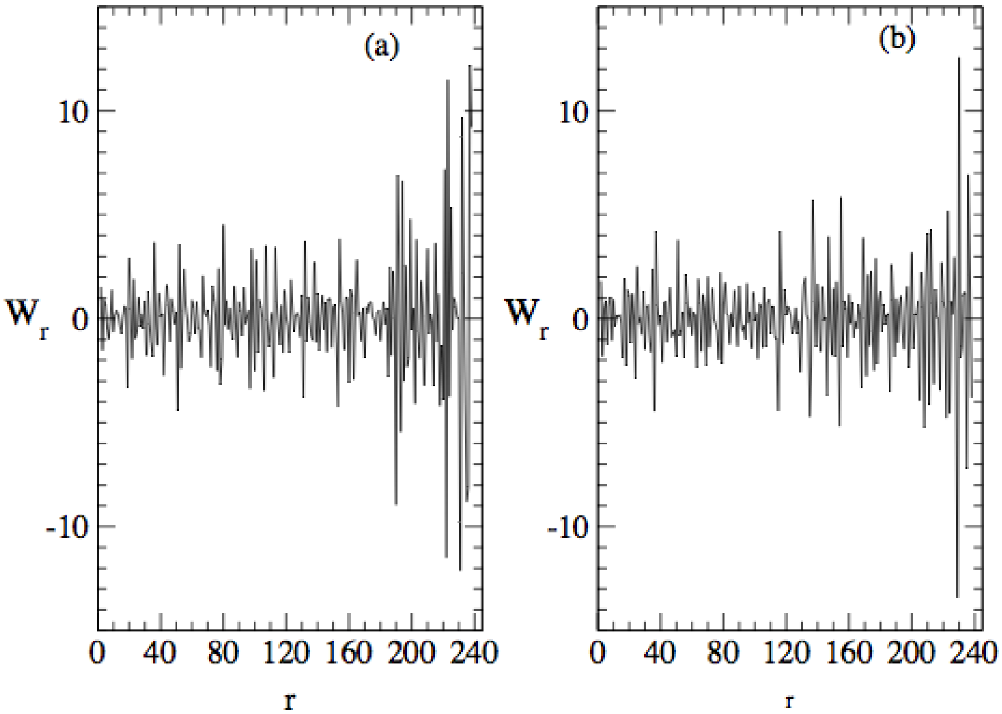

Figure 5: W-measures, Eq. (5), for the system of 250 Bulgarian cities: (a) 2004; (b) 2011

Table 1: Length ratio R_N of the V-measure line for different number N of Bulgarian cities, ranking from $r = 1$ till N, from 2004 till 2011. Observe the need to to consider four decimals.

year:	2004	2005	2006	2007	2008	2009	2010	2011
N:								
50	2.1171	2.2677	2.1652	2.2441	2.3363	2.3823	2.4169	2.4442
100	2.3458	2.3564	2.2130	2.1390	2.2488	2.3856	2.2360	2.2755
150	2.3987	2.5607	2.3736	2.3122	2.4431	2.7221	2.4188	2.3798
240	3.2635	3.5600	2.9865	2.7529	3.2125	3.4156	3.2255	3.0221

Table 2: Length ratio R_N of the V-measure line for different number N of 170 largest USA cities, ranking from $r = 1$ till N, in 1990 and 2010.

year:	1990	2010
N:		
50	1.9827	1.9899
100	2.1189	2.0661
150	2.2953	2.1976
170	2.2972	2.4290

network under a topological form rather than a geographical one. This can be worked out in further work, but this is much outside this section and paper purpose.

For further comparison, the Length Ratio in the 170 USA cities case is given in Table 2. Observe that there is almost no value change for the 50 largest agglomerations. The next 50 become closer to a power law in 2010, in comparison to the 1990 year. The same trend is seen for the 101 to 150 agglomerations. Finally, the last 20 deviate from a power law in 2010, more than the 1990 case. The Krugman [23] conclusions might have to be revised and monitored as a function of time. The Length Ratio measure thereby seems to be a new and simple measure of power law deviations.

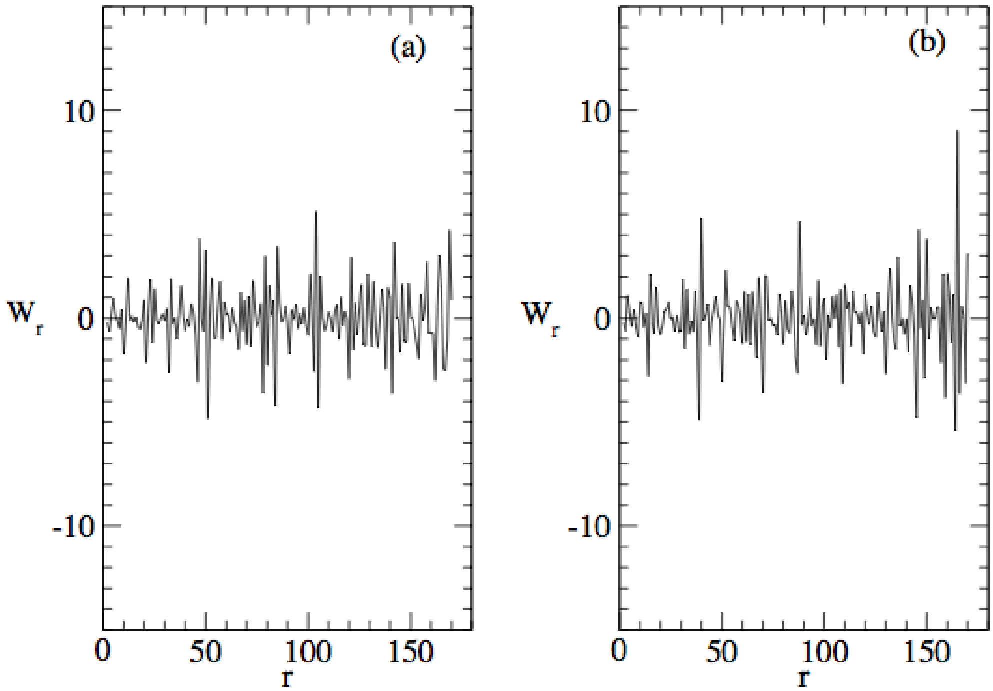

Figure 6: W-measures, Eq. (5), for the system of the largest 170 USA cities: (a) 1990; (b) 2010

4 Conclusions

As a concluding remark, we point out to the following. Two questions, among others, surely, were still in need of data analysis in order to understand hierarchy causes and primacy reversal in city systems. In this paper, a city primacy concept, usually to be representing a country hierarchical organization, has been investigated for two well defined groups of Bulgarian cities: first, on the basis of the conventional index of Sheppard, next generalizing it to avoid numerical difficulties. Finally introducing a new concept, the "length ratio", for monitoring the evolution of the city hierarchy away from the classical (but often incorrect) rank-size power law, seems to be a new and simple measure of power law deviations.

Somewhat showing that the hierarchy reversal might *not* be due to an external cause (colonisation or its reverse) but can be due to an internal cause found in the country history, we stress that we have indicated that generalized primacy measures should be useful for discriminating between cities with similar population sizes. It is obvious that the new indices are more sensitive, thus can be better investigated in monitoring (other) cases. We have given definitions and subsequent numerical results for proving so. In particular, we have defined and discussed results obtained by measures called V_r, W_r, L_β, L_N and R_N. Moreover, these measures can be used quite generally to quantify the deviation of any rank-size distribution from a power-law relationship, i.e. not only for a group of cities but also for any group of objects that can be ranked on the basis of some quantitative characteristics.

We have also shown that one can observe the advantage of being a capital, - from an increase of population size point of view!

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References

[1] M. Jefferson, The Law of the Primate City, Geographical Review 29(4): 226-232 (1939).

[2] G. K. Zipf, Human Behavior and the Principle of Least Effort: An Introduction to Human Ecology, (Addison Wesley,Cambridge, MA, 1949).

[3] H. Kantz, T. Schreiber, Nonlinear time series analysis, (Cambridge University Press, Cambridge, UK, 1997).

[4] N. K. Vitanov, Upper bounds on convective heat transport in a rotating fluid layer of infinite Prandtl number: Case of intermediate Taylor numbers, Phys. Rev. E 62(3): 3581-3591 (2000).
[5] R. Axelrod, M. D. Cohen, Harnessing complexity: Organizational implications on a scientific frontier, (Free Press, New York, 1999).
[6] T. Boeck, N. K. Vitanov, Low-dimensional chaos in zero-Prandtl-number Benard–Marangoni convection, Phys. Rev. E 65(3): 037203 (2002).
[7] T. Puu, A. Panchuk, Nonlinear economic dynamics, (Springer, Berlin, 1991).
[8] N. K. Vitanov, E. Yankulova, Multifractal analysis of the long-range correlations in the cardiac dynamics of Drosophila melanogaster, Chaos, Solitons & Fractals 28(3): 768-775 (2006).
[9] J. M. T. Thomson, H. B. Stewart, Nonlinear dynamics and chaos: Geometrical methods for scientists, (Wiley, New York, 1986)
[10] J. S. Durbin, J. Koopman, Time series analysis by state space methods, (Oxford University Press, Oxford, 2012).
[11] A. Vespignani, Predicting the behavior of techno-social systems, Science 325(5939): 425-428 (2009).
[12] N. K. Vitanov, M. Ausloos, G. Rotundo, Discrete model of ideological struggle accounting for migration, Advances in Complex Systems 15(S1): 1250049 (2012).
[13] M. E. J. Newman, Power laws, Pareto distributions and Zipf's law, Contemporary Physics 46(5): 323-351 (2005).
[14] J.C. Córdoba, On the distribution of city sizes, Journal of Urban Economics 63(1): 177–197 (2008).
[15] C. Mladenov, E. Dimitrov, Development of the urbanization process in Bulgaria during the period between the Liberation and the end of World War Two, Geography (in Bulgarian) 1(1): 13-17 (2009).
[16] B. J. L. Berry, Cities as systems within systems of cities, Papers in Regional Science 13(1): 147–163 (1964).
[17] S. El-Shakhs, Development, primacy, and systems of cities, The Journal of Developing Areas 7(1): 11–36 (1972).
[18] B. Lyman, Colonial governance in the development of urban primacy, Studies in Comparative International Development 27(2): 24–38 (1992).
[19] C. A. Smith, Types of city-size distributions: a comparative analysis, (Clarendon Press, Oxford, 1990).
[20] E. Sheppard, City size distributions and spatial economic change. WP-82-31, Working papers of the International Institute for Applied System Analysis, Laxenburg, Austria (1982).
[21] C. Chase-Dunn, The effects of international economic dependence on development and inequality: a cross-national study, American Sociological Review 40(6):720-738 (1975).
[22] H. W. Richardson, Theory of the distribution of city sizes: Review and prospects, Regional Studies 7(3): 239-251 (1973).
[23] P. Krugman, Confronting the mystery of urban hierarchy, Journal of the Japanese and International Economies 10(4): 99-418 (1996).
[24] N.K. Vitanov, Z.I. Dimitrova, Bulgarian cities and the New Economic Geography, (Vanio Nedkov, Sofia, 2014).