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Unsteady MHD boundary layer flow and heat transfer over the stretching sheets submerged in a moving fluid with Ohmic heating and frictional heating

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Abstract: This paper is devoted to the analysis of the unsteady magnetohydrodynamic (MHD) boundary layer flow and heat transfer on a permeable stretching sheet embedded in a moving incompressible viscous fluid. The combined effects of Ohmic heating, thermal radiation, frictional heating and internal heat absorption/generation are taken into account. The governing time dependent nonlinear boundary layer equations are converted into a system of nonlinear ordinary differential equations by similarity transformations. Some analytical results that give the characteristics of the velocity field in the boundary layer are presented and proved. The governing equations are then solved by using the shooting technique along with the fourth order Runge-Kutta method. The analytical properties proved in this paper are consistent with those obtained by the numerical method. Furthermore, the effects of the various parameters on the velocity and temperature fields are presented graphically and discussed in detail.

Keywords: Boundary layer; heat transfer; ohmic heating; analytical analysis

PACS: 44.20.+b, 47.15.-x, 47.65.-d

1 Introduction

The study of momentum and heat transfer induced by a continuous stretching heated sheet has gained tremendous interest among researchers in recent years because of industrial and engineering applications, such as aerodynamic extrusion of plastic sheets and fibers, drawing, annealing and thinning of copper wire, paper production, crystal growing, glass blowing, and so on. The steady two-dimensional flow over a stretching surface in a quiescent fluid was first analyzed by Crane [1]. The pioneering work of Crane was subsequently extended by many authors to explore various aspects of the flow and heat transfer on a steady stretching sheet and to obtain similarity solutions [2–8]. Recently, some research has focused on the steady MHD boundary layer flow and heat transfer of an electrically conducting fluid over a stretching sheet [9–16].

It is worth mentioning that the above studies deal with a steady flow. However, it is of great significance to include unsteadiness into the governing equations of any problem for the development of a more physically realistic characterization of the flow configuration. Therefore, the study of the unsteady flow and heat transfer phenomenon is very interesting. The research related to the unsteady boundary layer flow and heat transfer mainly focuses on the boundary layer problem where the fluid passes through an unmoving sheet or the sheet stretches in a quiescent mainstream. The unsteady flow and heat transfer over the stretching sheet submerged in a quiescent fluid were considered in some works in the literature [17–29]. The effects of thermal radiation, internal absorption/generation and frictional heating were respectively analyzed in the above works. According to the the literature that we can find, there are only a few results on unsteady boundary layer problems in the case of a stretching sheet submerged in a moving fluid. Moreover, it should be noted that there is a lack of research on the analytical properties of the velocity and temperature in boundary layers.

Motivated by the works mentioned above, we aim here to present research on the unsteady MHD boundary layer flow and heat transfer over the stretching sheets submerged in a moving fluid in the presence of thermal radiation, internal absorption/generation, Ohmic heating

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and frictional heating. The present paper is to extend the research on the unsteady flow and heat transfer over a stretching sheet in three directions (i) to present and prove analytical properties of the dimensionless velocity, (ii) to analyze the unsteady velocity and temperature boundary layers over a stretching permeable sheet submerged in a moving fluid, and (iii) to include the combined effects of external magnetic field, Ohmic heating,frictional heating and internal absorption/generation.

2 Mathematical formulation

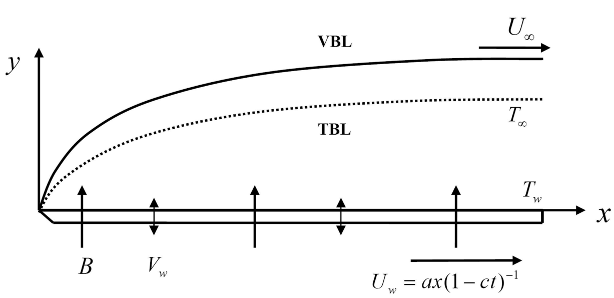

Figure 1: Unsteady MHD boundary flow model and coordinate system.

Consider an unsteady two-dimensional MHD boundary layer flow and heat transfer over a continuous stretching permeable sheet embedded in a moving viscous, incompressible, electrically conducting fluid as shown in Figure 1. The sheet is stretching with a velocity $U_\mathrm{w} = ax(1-ct)^{-1}$ in the positive direction, where a and c are two constants with dimension time^{-1}. The free stream velocity far away from the sheet is $U_\infty = RU_\mathrm{w}$. Here $R \geq 0$, $a > 0$, and $ct < 1$. The fluid is under the influence of the magnetic field B, which acts in the direction normal to the stretching sheet. The induced magnetic field is negligible, which is valid under the assumption of small magnetic Reynolds number. It is also assumed that the external electric field is zero. Under these assumption along with the boundary layer approximations, the basic unsteady boundary layer equations governing for momentum and heat transfer in the presence of Ohmic heating, thermal radiation, frictional heating, and internal absorption/generation take the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \tag{2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) =$$
$$u\frac{\partial p}{\partial x} + \alpha\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + Q(T - T_\infty) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B^2 u^2 \tag{3}$$

subject to the following boundary conditions

$$u = U_\mathrm{w},\ v = V_\mathrm{w},\ T = T_\mathrm{w} \text{ at } y = 0, \tag{4}$$

$$u = U_\infty,\ T = T_\infty \text{ as } y = \infty, \tag{5}$$

where u and v are respectively the velocity components along x and y directions. Outside the boundary layer, Eq.(2) gives $\frac{-1}{\rho}\frac{\partial p}{\partial x} = U_\infty\frac{\partial U_\infty}{\partial x} + \frac{\partial U_\infty}{\partial t} + \frac{\sigma B^2 U_\infty}{\rho}$. T is the temperature of the fluid inside the boundary layer, α is the thermal conductivity, c_p is the specific heat at constant pressure, ρ is density of fluid, μ is the fluid viscosity, $\nu = \mu/\rho$ is the kinematics viscosity of the fluid, t is the time, $T_\mathrm{w} = T_\infty + ax^2(2\nu)^{-1}(1-ct)^{-2}$ is the temperature of the stretching sheet, and T_∞ is the temperature of the fluid far away from the stretching sheet. $V_\mathrm{w} = -C(\nu U_\infty)^{1/2}x^{-1/2}$ represents the mass transfer at the surface of the sheet with $C < 0$ for injection and $C > 0$ for suction. $Q = Q_0(1 - ct)^{-1}$ is the heat generation when $Q_0 > 0$ or heat absorption when $Q_0 < 0$, where Q_0 is a constant. The variable external magnetic field B is of the form $B = B_0 U_\mathrm{w}^{1/2}(\nu x)^{-1/2}$. By using the Rosseland approximation for radiation [21], the radiative heat flux q_r is simplified as

$$q_r = -\frac{4\sigma^\star}{3k^\star}\frac{\partial T^4}{\partial y}, \tag{6}$$

where $\sigma^\star$ and $k^\star$ are respectively the Stefan-Boltzman constant and the mean absorption coefficient. Assuming that the temperature differences within the fluid in the boundary layer are sufficiently small, we can express the term T^4 as a linear function of temperature. Hence, by means of expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we may obtain $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$. Then, Eq. (6) is expressed as

$$q_r = -\frac{16T_\infty^3\sigma^\star}{3k^\star}\frac{\partial T}{\partial y}. \tag{7}$$

In terms of Eqs. (1)-(3), the following transformations are introduced

$$\eta = U_\mathrm{w}^{1/2}(\nu x)^{-1/2}y,\ \psi(x, y) = (\nu x U_\mathrm{w})^{1/2}f(\eta)$$
$$\text{and } \theta(\eta) = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty}, \tag{8}$$

where $\psi(x, y)$ is the stream function defined as $u = \partial\psi/\partial y = U_w f'(\eta)$ and $v = -\partial\psi/\partial x = -(\nu U_w)^{1/2}x^{-1/2}f(\eta)$. Then the continuity Eq.(1) are automatically satisfied. The functions $f' = u/U_w$ and θ are the dimensionless velocity and temperature, respectively. Substituting Eqs. (7)-(8) into (2)-(5), we get

$$f''' + ff'' - f'^2 - A(f' + \frac{\eta}{2}f'' - R) - M(f' - R) + R^2 = 0, \quad (9)$$

$$Pr^{-1}(1 + Nr)\theta'' + (\lambda - 2A)\theta + Ec(f''^2 - R(R + A)f') + EcM(f'^2 - Rf') - \frac{A}{2}\eta\theta' - 2\theta f' + f\theta' = 0, \quad (10)$$

with the following boundary conditions

$$f(0) = C, \ f'(0) = 1, \ f'(\infty) = R, \quad (11)$$

$$\theta(0) = 1, \ \theta(\infty) = 0. \quad (12)$$

Here the prime indicates differentiation with respect to η, $Pr = \mu c_p/\alpha$ is the Prandtl number, $A = c/a$ is a parameter that measures the unsteadiness of the flow of fluids, and $Nr = 16k^{\star}\alpha/(3\sigma^{\star}T_{\infty}^3)$ is the thermal radiation parameter, $M = \sigma B_0^2(\rho\nu)$ is the magnetic parameter, $Re_x = U_w x/\nu$ is the local Reynolds number, $Re_\alpha = U_w\alpha^{1/2}/\nu$, $Ec = U_w^2/(c_p(T_w - T_\infty))$ is the local Eckert number, and $\lambda = Q\alpha Re_x/(\mu c_p Re_\alpha^2)$ is the dimensionless heat absorption/generation parameter. Further, $\lambda > 0$ corresponds to heat generation, $\lambda < 0$ corresponds to heat absorption.

The skin friction coefficient C_f and local Nusselt number Nu_x at the sheet are respectively given by

$$C_f = 2Re_x^{-1/2}f''(0), \ Nu_x = -Re_x^{-1/2}\theta'(0)$$

.

3 Analytical properties of dimensionless velocity

In this section, analytical properties of the dimensionless velocity $f'(\eta)$ are to be presented and proved.

Lemma 3.1 $f'(\eta)$ is not identically a constant which is not equal to R on each subinterval of $(0, +\infty)$, and has the following characteristics (i) if $R > 1$, then $1 < f' \leq R$ in $(0, +\infty)$; (ii) if $0 \leq R < 1$, then $R \leq f' < 1$ in $(0, +\infty)$.

Proof. Suppose that there is an interval Ω_0 in which f' is identically a constant which is not R, then we have $f'' \equiv 0$ in Ω_0. In terms of Eq.(9), when $f''(\eta) = 0$, we have

$$f'''(\eta) = (f'(\eta) - R)(f'(\eta) + R + A + M). \quad (13)$$

Eq.13 implies that $f'''(\eta) \neq 0$ when $\eta \in \Omega_0$, which contradicts $f'' \equiv 0$. This indicates that $f'(\eta)$ can not be equal to the constant (except for R) in any subinterval of $(0, +\infty)$.

(i) Suppose the value of $f'(\eta)$ is greater than R at some points in $(0, +\infty)$. In view of the boundary conditions $f'(0) = 1$, $f'(+\infty) = R$, and the continuity of the function $f'(\eta)$, we can draw the conclusion that $f'(\eta)$ must have a local maximum value $f'(\eta_0)$, which is greater than R and satisfies $f''(\eta_0) = 0$. Then Eq.(13) reveals $f'''(\eta_0) > 0$, which contradicts the result that $f'(\eta_0)$ is a local maximum value of $f'(\eta)$.

As the value of $f'(\eta)$ is less than 1 at some points, the function $f'(\eta)$ should have a local minimum value point η_1 such that $f'(\eta_1)$ is less than 1. Consequently, $f''(\eta_1) = 0$ holds and then Eq.(13) shows $f'''(\eta_1) < 0$. This contradicts that $f'(\eta_1)$ is a local minimum value of $f'(\eta)$. Thus, it is proved that $f'(\eta)$ satisfies $f'(\eta) \geq 1$ when $\eta \in [0, +\infty)$. On the other hand, we can prove that $f'(\eta) \neq 1$ when $\eta \in (0, +\infty)$. Because $f'(\eta)$ is not identically equal to a constant which does not equal R in any subinterval of $(0, +\infty)$, when there is a point $\eta_1 \in (0, +\infty)$ satisfying $f'(\eta_1) = 1$, η_1 will be a minimum point of $f'(\eta)$. However, that is contradictory to $f'''(\eta_1) < 0$. Hence $f'(\eta)$ satisfies $f'(\eta) > 1$ in $(0, +\infty)$. Further the inequality $1 < f' \leq R$ is established in $(0, +\infty)$. The proof of case (ii) proceeds in a manner similar to the above case (i). □

Lemma 3.2 If $f'(\eta_e) = R$, then $f''(\eta_e) = 0$ and $f'(\eta) = R$ holds identically in $(\eta_e, +\infty)$.

Proof. Lemma 3.1 gives that a point $\eta_e \in (0, +\infty)$ satisfying $f'(\eta_e) = R$ must be a maximum point of $f'(\eta)$ when $R > 1$. Suppose there is another point $\eta_1 \in (\eta_e, +\infty)$ such that $f'(\eta_1) < R$ holds. In view of the continuity of $f'(\eta)$ and the inequality $f'(\eta_1) < f'(+\infty) = R$, the function $f'(\eta)$ has at least one local minimum value which is less than R in $(\eta_e, +\infty)$. Supposing that $f'(\eta_2)$ is a local minimum value of $f'(\eta)$ in the interval $(\eta_e, +\infty)$, it is clear that $f''(\eta_2) = 0$ and $f'''(\eta_2) \geq 0$ hold. However, according to Eq. (13), we get the inequality $f'''(\eta_2) < 0$ which leads to a contradiction. In addition, Lemma 3.2 can also be proved for the case $0 \leq R < 1$ with a proof similar to the above. □

Lemma 3.3 $f'(\eta)$ has no extreme value in $(0, +\infty)$ and satisfies (i) $f''(0) > 0$ and $f'' \geq 0$ in $(0, +\infty)$ when $R > 1$; (ii) $f''(0) < 0$ and $f'' \leq 0$ in $(0, +\infty)$ when $0 \leq R < 1$.

Proof. We only prove that Lemma 3.3 holds when $R > 1$ because the proof of Lemma 3.3 when $0 \leq R < 1$ is very similar to that of the case $R > 1$. Firstly, supposing that $f'(\eta)$ has a local minimum value $f'(\eta_m)$ in $(0, +\infty)$, we immediately get the inequalities $f'(\eta_m) < R$ and $f'''(\eta_m) < 0$ from Lemma 3.1 and Eq.(13). However, it is contrary to the above supposition that $f'(\eta_m)$ is a local minimum value. Hence $f'(\eta)$ has no local minimum value in $(0, +\infty)$. On the other hand, if $f'(\eta)$ has a local maximum value $f'(\eta_M)$ in $(0, +\infty)$, due to the boundary conditions (11), there will be a positive constant δ_1 satisfying $f''(\eta) < 0$ in $(\eta_M, \eta_M + \delta_1)$. At the same time, the boundary condition $f'(+\infty) = R$ and the inequality $f'(\eta_M) < R$, which can be obtained by Lemma 3.2, indicate that $f''(\eta) > 0$ in a subinterval of $(\eta_M + \delta_1, +\infty)$. Moreover, it is clear from Lemma 3.1 that $f''(\eta) \equiv 0$ and $f'(\eta) < R$ are incompatible in each subinterval of $(0, +\infty)$. Thus there are a positive constants δ_2 and a point $\eta_1 \in (\eta_M + \delta_2, +\infty)$ such that $f'(\eta_1) = 0$, $f''(\eta) < 0$ in (η_M, η_1) and $f''(\eta) > 0$ in $(\eta_1, \eta_1 + \delta_2)$ hold. As a result, $f'(\eta_1)$ is a local minimum value of the function $f'(\eta)$, which yields a contradiction. It is thus established that $f'(\eta)$ has no extreme value in $(0, +\infty)$.

(i) The above conclusion, that $f'(\eta)$ has no extreme value in $(0, +\infty)$, combined with the condition $f'(+\infty) = R$ reveal $f''(0) > 0$, so there is a point η_0 such that $f''(\eta) > 0$ holds in $(0, \eta_0)$ when $R > 1$. If inequality $f''(\eta) > 0$ in $(0, +\infty)$ is not true, there will be another point $\eta_1 \in (\eta_0, +\infty)$ satisfying $f''(\eta_1) < 0$. Consequently, $f''(\eta) < 0$ holds in a subinterval of $(\eta_0, +\infty)$. From Lemmas 3.1 and 3.2, the above results indicate that $f'(\eta)$ have at least one local maximum value, which also leads to a contradiction. Hence we have $f''(\eta) > 0$ in $(0, +\infty)$ when $R > 1$. For $0 \le R < 1$, namely the case (ii), the proof is similar to that of the case (i). □

Lemma 3.4 If $f''(\eta_0) = 0$, then $f'''(\eta_0) = 0$ and $f'(\eta_0) = R$. If $f'''(\eta_0) = 0$, then $f''(\eta_0) = 0$ and $f'(\eta_0) = R$.

Proof. Lemma 3.1 and 3.3 show that if there is a point $\eta_0 \in (0, +\infty)$ satisfying $f''(\eta_0) = 0$, the $f''(\eta_0)$ is respectively the minimum value and the maximum value of $f''(\eta)$ under the cases of $R > 1$ and $0 \le R < 1$. Thus $f'''(\eta_0) = 0$ immediately follows from $f''(\eta_0) = 0$. Then we obtain $f'(\eta_0) = R$ from Eq. (13). Next we will show that $f'''(\eta_0) = 0$ leads to $f''(\eta_0) = 0$ and $f'(\eta_0) = R$. When , Eq. (9) gives

$$f^{(4)}(\eta_0) = f''(\eta_0)(f'(\eta_0) + \frac{3A}{2} + M). \tag{14}$$

$f''(\eta_0)$ will be a local minimum value of $f''(\eta)$ in $(0, +\infty)$ when $f''(\eta_0) > 0$. That is because Eq.(14) insures $f^{(4)}(\eta_0) > 0$. However, the boundary condition $f''(+\infty) = 0$ and the continuity of $f''(\eta)$ show that if $f''(\eta_0)$ is a positive local minimum value of $f''(\eta)$, $f''(\eta)$ will have at least one local maximum value $f''(\eta_M)$ satisfying $\eta_M \in (\eta_0, +\infty)$ and $f''(\eta_M) > 0$, which contradicts $f^{(4)}(\eta_M) > 0$. Similarly, it is also can proved that $f''(\eta_0) < 0$ does not hold when $f'''(\eta_0) = 0$. As a result, $f''(\eta_0) = 0$ also follows from $f'''(\eta_0) = 0$. So, in conclusion, $f'''(\eta_0) = 0$ will lead to $f'(\eta_0) = R$ for $\eta_0 \in (0, +\infty)$. □

Theorem 3.1 The dimensionless velocity $f'(\eta)$ has the following characteristics: (i) for $R > 1$, $f'(\eta)$ is convex, monotonically increasing and bounded in $(0, +\infty)$; (ii) for $0 \le R < 1$, $f'(\eta)$ is concave, monotonically decreasing and bounded in $(0, +\infty)$.

Proof. (i) Due to $f''(0) > 0$, which is obtained by Lemma 3.3, and the boundary condition $f''(+\infty) = 0$, there should be a point $\eta_1 \in (0, +\infty)$ which satisfies $f'''(\eta_1) < 0$. Suppose there is another point $\eta_2 \in (0, +\infty)$ satisfies $f'''(\eta_2) > 0$. Without loss of generality, Let $\eta_1 < \eta_2$ then there are at least one point $\eta_3 \in (\eta_1, \eta_2)$ satisfying $f'''(\eta_3) = 0$ which leads to $f'(\eta_3) = R$ from Lemma 3.4. In view of $\eta_2 > \eta_3$ and Lemma 3.2, we have $f'''(\eta_2) = 0$ which contradicts the above assumption $f'''(\eta_2) > 0$. Hence, $f'''(\eta) \le 0$ holds in $(0, +\infty)$, which implies that $f'(\eta)$ is convex in $(0, +\infty)$. From Lemma 3.1 and 3.3, it is clear that $f'(\eta)$ is monotonically increasing and bounded. In a similar manner to the above, the theorem can also be proved in Case (ii), and therefore holds in all cases. Now, the proof of the theorem has been completed. □

4 Results and discussion

We also solved the BVP (9)-(12) numerically using the fourth-order Runge-Kutta and shooting method offered in the literature [13] for the various values of the parameters such as the unsteadiness parameter A, suction /injection parameter C, magnetic parameter M, velocity ratio parameter R, radiation parameter Nr, heat absorption /generation parameter λ, local Eckert number Ec, and Prandtl number Pr. The values of the local Nusselt number $-\theta'(0)$ presented by this method are compared with previously published results for some simple cases. From the comparison listed in Table 1, we note that the present results agree well with those in Ref. [23], which verifies the accuracy of the method used. The solutions obtained numerically are graphically presented in Figures 2–15. From these figures, it is found that the dimensionless velocity $f'(\eta)$ obtained by the numerical method is consistent with the analytical properties proved in Theorem 3.1.

Figures 2 and 3 display the effects of the suction /injection parameter C on the velocity and temperature profiles. The gradients of the fluid velocity and the temperature both are found to increase with increasing value of C. The injection at the sheet surface increases the velocity and the thermal boundary layer thickness, and gives rise to decreasing the velocity and temperature gradients. By contrast, the suction at the sheet surface produces the opposite effect on the boundary layer.

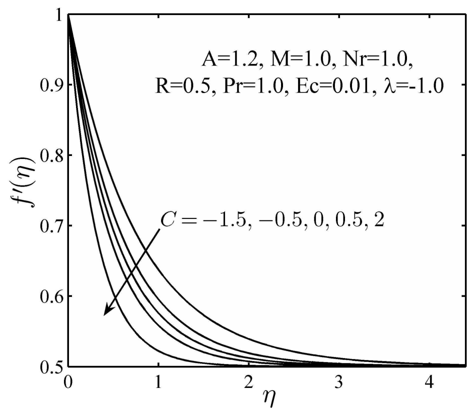

Figure 2: Velocity distribution against η for various values of C.

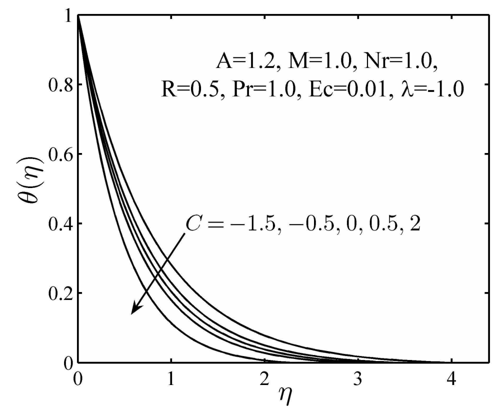

Figure 3: Temperature distribution against η for various values of C.

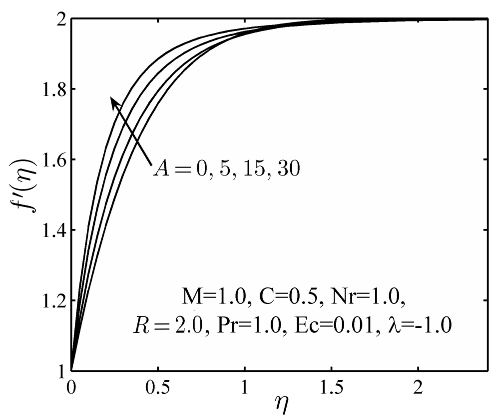

Figure 4: Velocity distribution against η for various values of A.

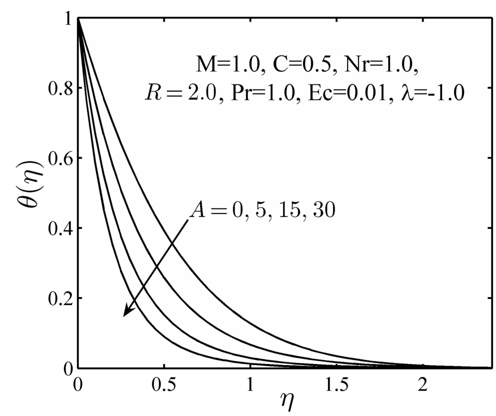

Figure 5: Temperature distribution against η for various values of A.

Table 1: Comparison of values of $-\theta'(0)$ for $M = 0$, $R = 0$, $C = 0$, $N = 0$, $A = 0$ and $\lambda = -1$.

Pr	Ec	Present results	Ref. [23]
3	0	−3.082164	−3.082174
3	0.02	−3.069175	−3.069188
4	0	−3.585180	−3.585191
4	0.02	−3.569325	−3.569339
5	0	−4.028431	−4.028530
5	0.02	−4.010079	−4.010089

In Figures 4 and 5, the velocity and temperature profiles are shown for different values of the unsteadiness parameter A. It is seen that an increase in A leads to an increase of the gradient of the velocity and a decrease of the velocity boundary layer thickness. From Figure 5, it is observed that the increase of A has the tendency to reduce the thermal boundary layer thickness which results in reduction in the gradient of temperature in the thermal boundary layer.

Figures 6 and 7 show the influences of the magnetic parameter M on the velocity and temperature profiles in the two cases $0 < R < 1$ and $R > 1$. An increase in the magnetic parameter M is to decrease the momentum boundary layer thickness and to increase the velocity gradient in the boundary layer when $0 < R < 1$ or $R > 1$. Figure 7 indicates the Ohmic heating due to the electromagnetic work increases the thermal boundary layer thickness and reduces the temperature gradient at the sheet surface with the increasing of the magnetic parameter M when $R < 1$. In addition, the opposite effects on the heat transfer are observed when M increases in the case $R > 1$.

Figures 8 and 9 illustrate the effects of the velocity ratio parameter R on the velocity and temperature. It can be observed that with increasing values of R, the fluid velocity decreases when $R \in [0, 1)$, however, have an opposite trend in the case $R > 1$. Figure 9 shows that the

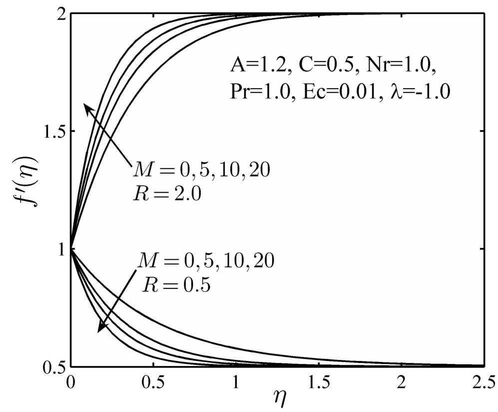

Figure 6: Velocity distribution against η for various values of M and R.

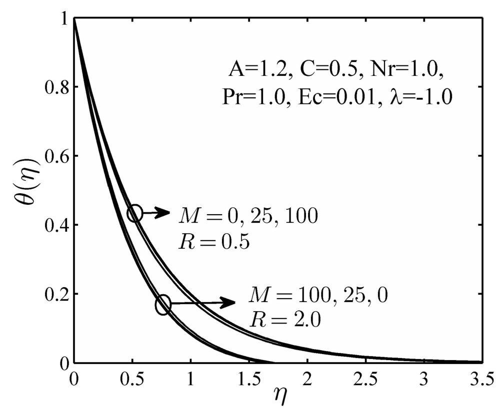

Figure 7: Temperature distribution against η for various values of M and R.

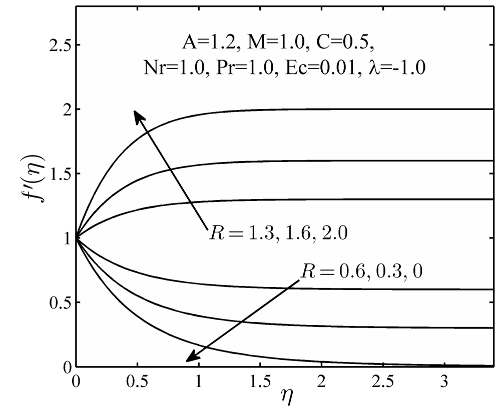

Figure 8: Velocity distribution against η for various values of R.

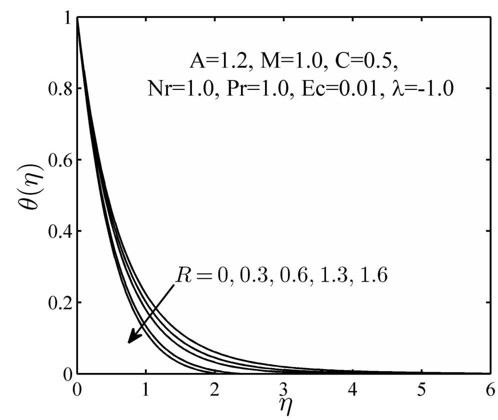

Figure 9: Temperature distribution against η for various values of R.

temperature decreases with increasing values of R in both $R \in [0, 1)$ and $R > 1$ cases.

Figures 10–13 aim to explore the effects of the radiation parameter Nr, local Eckert number Ec, Prandtl number Pr, and heat absorption/generation λ on the temperature boundary layer, respectively. These four figures exhibit that an increase in the radiation parameter Nr, heat Eckert number Ec, or in the absorption/generation parameter λ leads to a reduction in the temperature gradient at the sheet, while the opposite effect is found for the Prandtl number Pr.

Figures 14–15 show that the variations of the parameters M and R bring essential effects on the wall shear stress $f''(0)$ and the local Nusselt number, in terms of $-\theta'(0)$. It is seen from Figure 14 that when $R > 1$ the wall shear stress $f''(0)$ increases with the increase of M, but when $R < 1$, $f''(0)$ decreases with the increase of M. Figure 15 reveals that the local Nusselt number $-\theta'(0)$ decreases with the increase in the magnetic parameter M. In addition, it is found that an increase in the velocity ratio parameter R leads to the increase of the wall shear stress $f''(0)$ and the reduction of the local Nusselt number $-\theta'(0)$.

5 Conclusions

In this study, we have examined the effects of Ohmic heating , thermal radiation, frictional heating and internal absorption/generation on unsteady MHD boundary layer flow and heat transfer over a continuous stretching permeable sheet in a moving viscous, incompressible, electrically conducting fluid. A theorem on the analytical properties of the dimensionless velocity has been presented and proved. Then the similarity equations are solved by the fourth-order Runge-Kutta with shooting method. The dimensionless velocity obtained by the numerical method is consistent with the analytical properties prensented and proved in this paper. The effects of all the various parameters that appear in the governing equations on the temper-

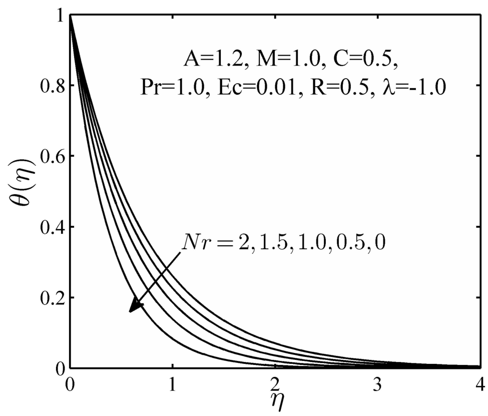

Figure 10: Temperature distribution against η for various values of Nr.

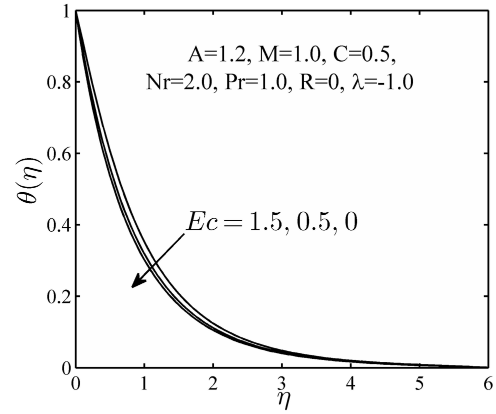

Figure 11: Temperature distribution against η for various values of Ec.

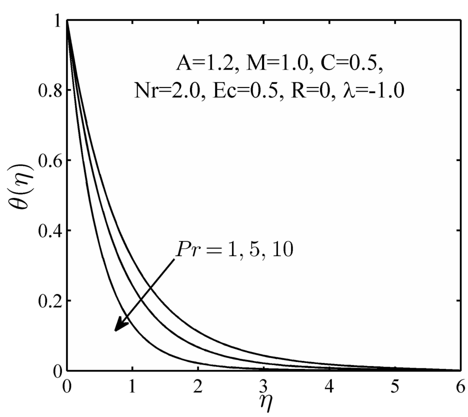

Figure 12: Temperature distribution against η for various values of Pr.

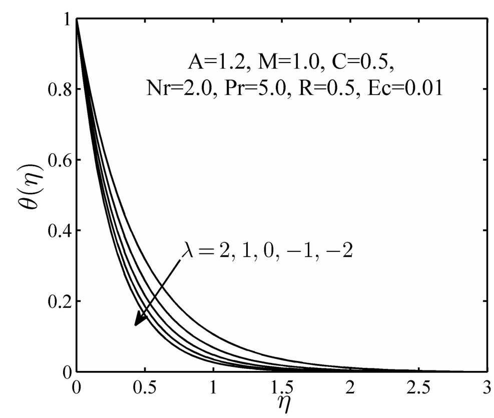

Figure 13: Temperature distribution against η for various values of λ.

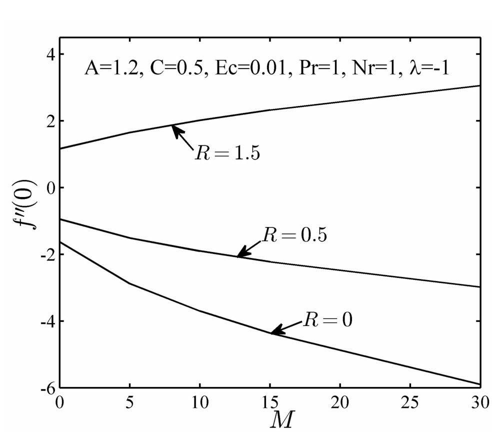

Figure 14: Variations of $f''(0)$ with M for various values of R.

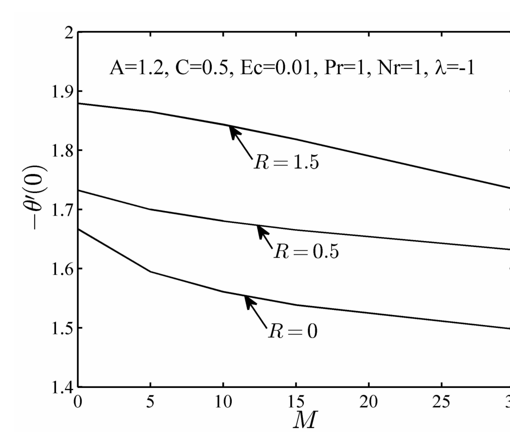

Figure 15: Variations of $-\theta'(0)$ with M for various values of R.

ature field and velocity field in the momentum and thermal boundary layers are revealed.

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