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Effects of phase shift on bipartite and multipartite correlations of a spin chain under dephasing

Abstract: We investigate the effects of phase shift on entanglement, quantum discord, geometric discord, and spin-squeezing of a Heisenberg chain under dephasing. An analytical solution of the present model is obtained. Our results show that the initial correlations of the spin chain could be partially stored for a long time in the presence of dephasing and the amount of steady state correlations can be adjusted via phase shift. Particularly, we find the effects of phase shift on quantum discord and geometric discord are not always the same, i.e., the increase of geometric discord does not always imply the increase of quantum discord. Then, we calculate the spin-squeezing of the spin chain and find that spin-squeezing first increases with time and then reaches a plateau. The amount of spin-squeezing can be controlled via phase shift.

Keywords: dephasing; spin chain; phase shift

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1 Introduction

Entanglement plays a fundamental role in quantum information processing and quantum computation [1–4]. However, it is not the only kind of quantum correlation useful for quantum information processing [5–9]. It has been pointed out that some tasks can be sped up over their classical counterparts using fully separable and highly mixed states, i.e., states with no entanglement but have nonzero quantum discord [10–16]. Quantum discord is another kind of quantum correlation different from entanglement. Up to now, there are two kinds of quantum discord: measurement-based discord and distance-based discord [8]. The original definition of quantum discord [17, 18] is a typical measurement-based discord. This kind of discord is based on the observation that a local measurement performed upon a subsystem of a multipartite system with quantum correlations will inevitably disturb the whole system. Thus, it is impossible to obtain all information contained in a subsystem by performing only local measurements. The second kind of discord (distance-based discord) is usually defined as the minimal distance between a given quantum state and all states with zero discord. Similar to the geometric measures of entanglement, the square norm in the Hilbert-Schmidt space is adopted as a measure of distance between two states [19]. In this context, this kind of measure is also called the geometric discord. For arbitrary two-qubit systems, an analytical expression is obtained [19].

On the other hand, a quantum system is inevitably influenced by its surrounding environment [20]. In order to implement quantum information processing, one has to generate and transmit quantum correlations from one place to another in the presence of decoherence [2]. It is the building block of many quantum communication protocols such as quantum teleportation [21] and quantum key distribution [22]. Quantum correlations transfer can be accomplished using several quantum systems such as spin chains [23], optical systems [24, 25], and trapped atoms [26, 27]. Recently, the bipartite and multipartite correlations of spin chain have been studied by several authors [28–32]. The scaling of quantum discord in spin models was studied analytically and it was pointed out that at finite temperature the block scaling of quantum discord satisfies an area law for any two-local Hamiltonian [29]. The multipartite correlations such as multipartite entanglement and nonlocality in the quantum phase transition of an infinite XY chain was calculated by employing Bell-type inequalities [30]. Also, the steady state correlations of dissipative spin chain were investigated [31, 32].

In the present paper, we investigate the effects of phase shift [33–35] on the dynamics of bipartite correlations (measured by concurrence, discord, and geometric discord) and multipartite correlations (measured by spin-squeezing) of a spin chain under dephasing. We first derive an analytical solution of the model. Then, we calculate

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bipartite correlations of two spins. If there is no dephasing, bipartite correlations oscillate with time. When the dephasing is taken into accounted, the maximal amount of bipartite correlations decreases with the increase of dephasing rate. In particular, there are steady state bipartite correlations in the presence of dephasing. Our results show that the steady state bipartite correlations can be adjusted via phase shift [33–35]. Finally, we study the multipartite correlations measured by spin-squeezing of the whole spin chain. The spin-squeezing first increases with time and then reaches a plateau. The amount of spin-squeezing can be controlled by phase shift. We also discuss the influence of the number of spins on spin-squeezing and find that the spin-squeezing increases with the spin numbers.

The structure of the paper is as follows. In section 2, we briefly review the N-site Heisenberg chain with phase shift in the presence of dephasing and derive a analytical solution of the model. In section 3, we review several measures of bipartite and multipartite correlations including concurrence, quantum discord, geometric discord, and spin-squeezing. In section 4, we discuss the effects of phase shift on the dynamics of bipartite and multipartite correlations in the presence of dephasing. In section 5, some conclusive remarks are given.

2 The model

2.1 Heisenberg chain with phase shift

The Hamiltonian of the N-site quantum Heisenberg chain with nearest-neighbor coupling and phase shift is [33–35]:

$$H = \frac{J}{2}\sum_{i=1}^{N}(e^{i\phi}\sigma_i^+\sigma_{i+1}^- + e^{-i\phi}\sigma_i^-\sigma_{i+1}^+) + \frac{J}{4}\sum_{i=1}^{N}(\sigma_i^z\sigma_{i+1}^z - 1), \quad (1)$$

where ϕ is the phase shift induced by the Aharonov-Casher effect [34], J is the exchange interaction constant, σ_i^x , σ_i^y and σ_i^z are the Pauli matrices of the ith qubit, and $\sigma_i^\pm = \frac{1}{2}(\sigma_i^x \pm i\sigma_i^y)$ are the raising and lowering operators, respectively. The chain is said to be antiferromagnetic for $J > 0$ and ferromagnetic for $J < 0$. Here, we adopt the periodic boundary condition, i.e., $\sigma_{N+1} = \sigma_1$.

We assume that the system is cooled to state $|\widetilde{0}\rangle = |00\cdots 0\rangle$ with $|0\rangle$ denoting the spin down state and introduce the class of states (one spin up):

$$|\widetilde{i}\rangle = |00\cdots 010\cdots 0\rangle = \sigma_i^+|\widetilde{0}\rangle. \quad (2)$$

It is easy to see that any state $|\widetilde{i}\rangle$ of this class obeys the following identity:

$$H|\widetilde{i}\rangle = \frac{J}{2}(e^{i\phi}|\widetilde{i-1}\rangle + e^{-i\phi}|\widetilde{i+1}\rangle) - J|\widetilde{i}\rangle. \quad (3)$$

Thus the eigenvectors of the class are given by:

$$|k\rangle' = \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\exp(in\varphi_k)|\widetilde{n}\rangle, \quad (4)$$
$$\varphi_k = 2\pi k/N, \quad k = 1, 2, \cdots, N,$$

with eigenvalues:

$$H|k\rangle' = E_k|k\rangle', \quad E_k = J[\cos(\varphi_k + \phi) - 1]. \quad (5)$$

2.2 Dephasing and solution

Now, we consider the dephasing mechanism. Based on the assumption that on sufficiently short time steps, a quantum system evolves in a stochastic sequence of identical unitary transformations, Milburn proposed a master equation to describe the effects of dephasing (intrinsic decoherence) on the time evolution of the system (we set $\hbar = 1$) [36–39]:

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{\gamma}{2}[H,[H,\rho]], \quad (6)$$

where γ is the dephasing rate. The quantum coherence of the system is automatically destroyed as the quantum system evolves and there is no dissipation of energy. The above equation was used to study the time evolution of a single trapped ion under phase fluctuations in the exciting laser pulses [37]. When $\gamma = 0$ there is no phase decoherence and the above equation reduces to the ordinary von Neuman equation. The formal solution of the master equation can be expressed as [38]:

$$\rho(t) = \sum_{k=0}^{\infty}(\gamma t)^k\frac{1}{k!}M^k(t)\rho(0)M^{\dagger k}(t), \quad (7)$$

where $\rho(0)$ is the initial density operator and $M^k(t)$ is defined by:

$$M^k(t) = H^k\exp(-iHt)\exp(\frac{-\gamma t}{2}H^2). \quad (8)$$

We assume the qubit j is in the spin up state and the others are in the spins down state $|0\rangle^{\otimes(N-1)}$, so the initial state is:

$$|\widetilde{j}\rangle = \sigma_j^+|\widetilde{0}\rangle = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}e^{-ij\varphi_k}|k\rangle'. \quad (9)$$

Combing Equations (4)–(5) with Equations (7)–(9), we obtain the following transformations:

$$|\widetilde{j}\rangle\langle\widetilde{j}| \rightarrow \sum_{n=1}^{N}\sum_{m=1}^{N}\alpha_{nm}(j,t)|\widetilde{n}\rangle\langle\widetilde{m}|, \quad (10)$$

$$|\widetilde{j}\rangle\langle\widetilde{0}| \to \sum_{n=1}^{N} \beta_n(j,t)|\widetilde{n}\rangle\langle\widetilde{0}|, \tag{11}$$

where:

$$\begin{aligned}\alpha_{nm}(j,t) &= \frac{1}{N^2}\sum_{k,k'=1}^{N} \exp\left\{-\frac{\gamma t}{2}(E_k - E_{k'})^2\right.\\ &- \left. i(E_k - E_{k'})t + i[(n-j)\varphi_k - (m-j)\varphi_{k'}]\right\},\\ \beta_l(j,t) &= \frac{1}{N}\sum_{k=1}^{N} \exp\left[-\frac{\gamma t}{2}E_k^2 - iE_k t + i(l-j)\varphi_k\right].\end{aligned} \tag{12}$$

These transformations will be used in the next section.

3 Measures of bipartite and multipartite correlations

Now, we briefly review some measures of bipartite and multipartite correlations for quantum systems including concurrence, discord, geometric discord, and spin-squeezing.

3.1 Measure of bipartite correlations: concurrence

For a 2×2 system described by the density matrix ρ, concurrence is a good measure of entanglement which is defined as [40]:

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{13}$$

where $\lambda_i (i = 1, 2, 3, 4)$ are the square roots of the eigenvalues in decreasing order of magnitude of the "spin-flipped" density matrix operator $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ and σ_y is the Pauli Y matrix.

3.2 Measure of bipartite correlations: quantum discord

Suppose the density matrix of a bipartite system is ρ_{AB}; $\rho_A = Tr_B(\rho_{AB})$ and $\rho_B = Tr_A(\rho_{AB})$ are the reduced density matrix of subsystem A and subsystem B. Then, we perform a set of local projective measurements (von Neumann measurements) $\{\Pi_B^{(j)}\} = \{|j_B\rangle\langle j_B|\}$ on subsystem B. The state related to the measurement $\Pi_B^{(j)}$ is [17, 18]:

$$\rho_{AB|j} = \frac{1}{p_j}\left(I_A \otimes \Pi_B^{(j)}\right)\rho_{AB}\left(I_A \otimes \Pi_B^{(j)}\right), \tag{14}$$

with I_A being the identity matrix of subsystem A. Note that we perform measurements on subsystem B only and $p_j = Tr[(I_A \otimes \Pi_B^{(j)})\rho_{AB}(I_A \otimes \Pi_B^{(j)})]$ is the probability of obtaining the outcome j. The conditional entropy is $S(\rho_{AB}|\{\Pi_B^{(j)}\}) = \sum_j p_j S(\rho_{A|j})$, where $\rho_{A|j} = Tr_B(\rho_{AB|j})$ is the reduced density matrix of subsystem A after measurements and $S(\rho) = -Tr(\rho \log \rho)$ is the von Neumann entropy of density matrix ρ. Thus, the two expressions of quantum mutual information are defined by [17, 18]:

$$\begin{aligned}\mathcal{I}(\rho_{AB}) &= S(\rho_A) + S(\rho_B) - S(\rho_{A,B}), \\ \mathcal{J}(\rho_{AB}) &= S(\rho_A) - S(\rho_{AB}|\{\Pi_B^{(j)}\}).\end{aligned} \begin{matrix}(15)\\(16)\end{matrix}$$

The quantum discord is defined as [17, 18]:

$$D(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \max_{\Pi_B}\{\mathcal{J}(\rho_{AB})\}, \tag{17}$$

where the maximization is taken over all possible measurements $\{\Pi_B^{(j)}\}$. It is introduced to eliminate the dependence of discord upon measurements. The first term is the total correlations (quantum and classical correlations) of the whole system. The second term of the above equation states that all information can be obtained by performing local measurements on subsystem B only. It is often referred to as the classical correlations [18]. From the above discussion, we see that quantum discord is the difference between the total correlations and classical correlations.

3.3 Measure of bipartite correlations: geometric discord

Geometric discord or square norm-based discord is similar to the geometric measure of entanglement and is defined as [19]:

$$D^G(\rho) = \min_{\chi} ||\rho - \chi||^2, \tag{18}$$

where the minimum is taken over all possible classical states χ whose density matrix can be written as $p_1|\psi_1\rangle\langle\psi_1| \otimes \rho_1 + p_2|\psi_2\rangle\langle\psi_2| \otimes \rho_2$ with $p_1 + p_2 = 1$. $|\psi_1\rangle$ and $|\psi_2\rangle$ are two orthonormal basis of subsystem A; ρ_1 and ρ_2 are two density matrices of subsystem B; $||\rho - \chi||^2 = Tr(\rho - \chi)^2$ is the square norm of Hilbert-Schimidt space. For arbitrary two-qubit systems, an analytical expression of geometric discord is found [19]:

$$D^G = \frac{1}{4}(||\vec{x}||^2 + ||T||^2 - \lambda_{\max}), \tag{19}$$

where $\lambda_{\max}$ is the maximal eigenvalues of matrix $\Omega = \vec{x}\vec{x}^t + TT^t$. Here, the superscript t stands for transpose of a vector or matrix, $\vec{x}$ is a column vector with $||\vec{x}||^2 = x_1^2 + x_2^2 + x_3^2$, and $T = (t_{ij})$ is a 3×3 matrix. Note that $x_i = Tr[\rho(\sigma_i \otimes I_B)]$, $y_i = Tr[\rho(I_B \otimes \sigma_i)]$, $t_{ij} = Tr[\rho(\sigma_i \otimes \sigma_j)]$, σ_i are three Pauli matrices.

3.4 Measure of multipartite correlations: spin-squeezing

Here, we study multipartite quantum correlations using spin squeezing introduced by Kitagawa and Ueda [41]. The main advantage of spin squeezing as a measure of multipartite correlations is that it is relatively easy to generate and measure spin squeezing experimentally due to the fact that spin-squeezing parameters only involve the first and second moments of the collective angular momentum operators. The spin squeezing is defined by [41, 42]:

$$\xi_s^2 = \frac{4(\Delta J_\perp)^2_{min}}{N}, \tag{20}$$

where N is the number of particles and the minimization in the above equation is taken over all directions denoted by $\perp$, which are perpendicular to the mean spin direction $\langle\vec{J}\rangle/|\langle\vec{J}\rangle|$. After some algebra, the spin squeezing can be derived as [42]:

$$\begin{aligned}
\xi_s^2 &= \frac{2}{N}\left[\langle(\vec{J}^2_{\vec{n}_1} + \vec{J}^2_{\vec{n}_2}\rangle\right. \\
&\quad \left. - \sqrt{\langle(\vec{J}^2_{\vec{n}_1} - \vec{J}^2_{\vec{n}_2})\rangle + 4cov(\vec{J}_{\vec{n}_1}, \vec{J}_{\vec{n}_2})}\right], \\
\vec{n}_1 &= (-\sin\phi, \cos\phi, 0), \vec{n}_2 \\
&= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta), \\
\theta &= \arccos\frac{\langle J_z\rangle}{|\vec{J}|}, \\
cov(x,y) &= \frac{1}{2}(\langle xy\rangle + \langle yx\rangle) - \langle x\rangle\langle y\rangle,
\end{aligned} \tag{21}$$

with:

$$\phi = \begin{cases} \arccos\frac{\langle J_x\rangle}{|\vec{J}|\sin\theta}, & \langle J_y\rangle > 0, \\ 2\pi - \arccos\frac{\langle J_x\rangle}{|\vec{J}|\sin\theta}, & \langle J_y\rangle \le 0. \end{cases} \tag{22}$$

If $\xi_s^2 < 1$, then we can conclude that there is multipartite quantum correlations.

4 Dynamics of quantum correlations

In this section, we consider the transmission of an entangled state through dephasing channel. Let us investigate the effect of the dephasing rate γ on this processing. The entangled state to be transferred is stored in spin 0 and spin 1 and the initial state of these two spins is of the form:

$$|\psi\rangle_{01} = (\cos\frac{\theta}{2}|0\rangle_0|0\rangle_1 + \sin\frac{\theta}{2}|1\rangle_0|1\rangle_1). \tag{23}$$

Here, we assume that the spin 0 has no direct interaction with the spin chain. If all other spins (from spin 2 to spin N) are in spin down states ($|0\cdots0\rangle_{23\ldots N}\langle0\cdots0|$), then the initial state of the total system is:

$$\begin{aligned}
\rho(0) &= (|\psi\rangle_{01}\langle\psi|) \otimes (|0\cdots0\rangle_{23\ldots N}\langle0\cdots0|) \\
&= \cos^2\frac{\theta}{2}|0\rangle_0\langle0| \otimes |\widetilde{0}\rangle\langle\widetilde{0}| + \sin^2\frac{\theta}{2}|1\rangle_0\langle1| \otimes |\widetilde{1}\rangle\langle\widetilde{1}| \\
&+ \frac{\sin\theta}{2}(|1\rangle_0\langle0| \otimes |\widetilde{1}\rangle\langle\widetilde{0}| + h.c),
\end{aligned} \tag{24}$$

where h.c stands for Hermitian conjugate, $|\widetilde{1}\rangle = \sigma_1^+|\widetilde{0}\rangle$, and $|\widetilde{0}\rangle = |0\ldots0\rangle$ are the state of the spin chain (note that spin 0 doesn't belong to the spin chain since it has no direct interaction with other spins as we have assumed). Using the above equation and Equation (12), we can obtain the time evolution density matrix as follows:

$$\begin{aligned}
\rho(t) &= \cos^2\frac{\theta}{2}|0\rangle_0\langle0| \otimes |\widetilde{0}\rangle\langle\widetilde{0}| + \sin^2\frac{\theta}{2}|1\rangle_0\langle1| \\
&\otimes \sum_{n,m=1}^{N}\alpha_{nm}(1,t)|\widetilde{n}\rangle\langle\widetilde{m}| + \frac{\sin\theta}{2}\left\{|1\rangle_0\langle0|\right. \\
&\otimes \left.\sum_{n=1}^{N}\beta_n(1,t)|\widetilde{n}\rangle\langle\widetilde{0}| + h.c\right\}.
\end{aligned} \tag{25}$$

The reduced density matrix of spin 0 and spin j (denoted by $\rho_{0j}(t)$) can be obtained by tracing out the degrees of freedom of all other spins. Then, we can calculate the concurrence, quantum discord, and geometric discord of $\rho_{0j}(t)$ using the results of the previous section. Here, we plot the results of bipartite correlations as functions of time t.

Figure 1 is the schematic picture of the present model with $N = 6$. Note that spin 0 has no direct interaction with the spin chain formed by spins 1, 2, ..., 6. In Figure 2, we plot the concurrence, discord, and geometric discord of spin 0 and spin 3 as functions of time t with $N = 6, J = 1, \theta = \pi/2, \phi = 0$ for different values of γ. In the case of $\gamma = 0$, the bipartite correlations are periodic functions of time. Now, we discuss the correlations propagation in the spin chain. The quantum correlations which were initially stored in spin 0 and spin 1 will move around

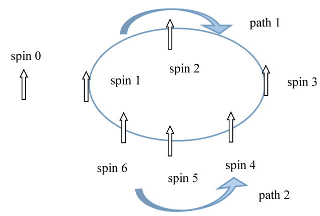

Figure 1: The schematic picture of the present model for the case of $N = 6$. Note that spin 0 doesn't interact with other spins directly. Spin 0 and spin 1 are entangled initially.

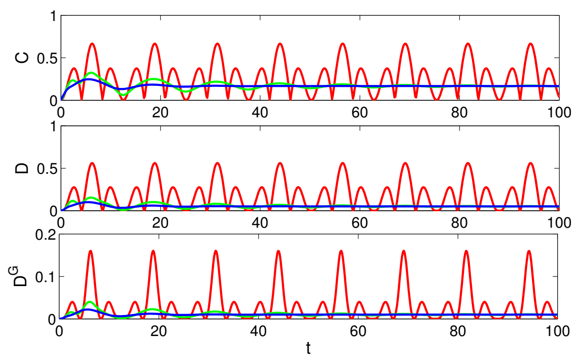

Figure 2: The concurrence, quantum discord, and geometric discord of spin 0 and spin 3 are plotted as functions of time t with $N = 6, J = 1, \theta = \pi/2, \phi = 0$ for $\gamma = 0$ (red lines), $\gamma = 0.3$ (green lines), and $\gamma = 1$ (blue lines). We set $\hbar = 1$ in this paper.

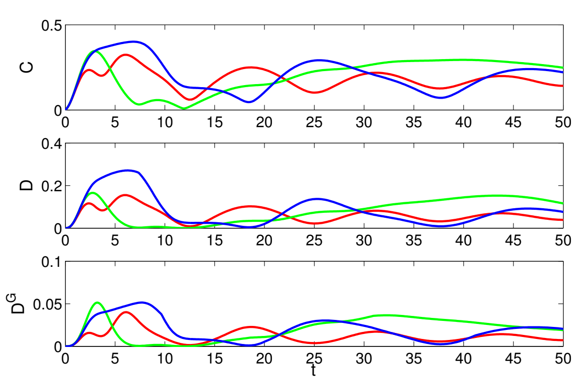

Figure 3: The concurrence, quantum discord, and geometric discord of spin 0 and spin 3 are plotted as functions of time t with $N = 6, J = 1, \theta = \pi/2, \gamma = 0.3$ for $\phi = 0$ (red lines), $\phi = \pi/5$ (green lines), and $\phi = 2\pi/5$ (blue lines).

the spin chain as the system evolves. There are two paths for spreading correlations as we have shown in Figure 1. The first path is $1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 1$. The second path is $1 \to 6 \to 5 \to 4 \to 3 \to 2 \to 1$. The dynamics of quantum correlations of spin 0 and spin 3 are determined by the correlations propagation along these two paths. The numbers of spins between spin 0 and spin 3 are different for the two paths. For instance, there are two spins between spin 0 and spin 3 along path 1. However, there are four spins between spins 0 and 3 through path 2. It takes more time to spread correlations along path $1 \to 6 \to 5 \to 4 \to 3$. The maximal values of correlations in Figure 2 could be different due to the geometric forms of the spin chain and the propagation of quantum correlations along the spin chain. If the dephasing is taken into account, there are steady state correlations. The bigger the dephasing rate γ is, the quicker the bipartite correlations reach their stationary values. However, the number of steady state correlations does not depend upon the dephasing rate γ as one can see from Equation (12) in the limit $t \to \infty$.

In Figure 3, the concurrence, discord, and geometric discord of spin 0 and spin 3 are plotted as functions of time with $N = 6, J = 1, \theta = \pi/2, \gamma = 0.3$ for different values of phase shift ϕ. From this figure, we see the number of bipartite correlations can be controlled by the phase shift. In particular, we find that the response of different measures of bipartite correlations to phase shift could be different. For instance, if we increase ϕ from $\pi/5$ to $2\pi/5$ at $t \approx 2$, the geometric discord D^G decreases while the quantum discord D increases. This result shows that the geometric discord and quantum discord are qualitatively different as has been pointed out in [43]. The authors of [43] have shown that the behaviors of the quantum discord and geometric discord under decoherence could be different, i.e., a sudden change in the decay rate of the quantum discord does not always imply that of the geometric discord and vice versa. Our results are consistent with that of [43].

The bipartite correlations as functions of time are plotted in Figure 4. From this figure, we see there are steady state correlations in the presence of dephasing. The dynamics of quantum correlations depends on the energy spectrum $E_k = J[cos(\varphi_k + \phi) - 1]$ as one can see from Equations (12) and (25). If the eigenvalues are changed by adjusting the phase shift ϕ, then the number of steady state correlations are changed as one can see from Figure 4.

Now, we turn to investigate the influence of phase shift on the dynamics of the spin-squeezing of the whole spin chain whose density matrix is given by Equation (25). In Figure 5, we plot the spin-squeezing as functions of time with $J = 1, \theta = \pi/2, \gamma = 1$ for $\phi = \pi/6$ (red lines),

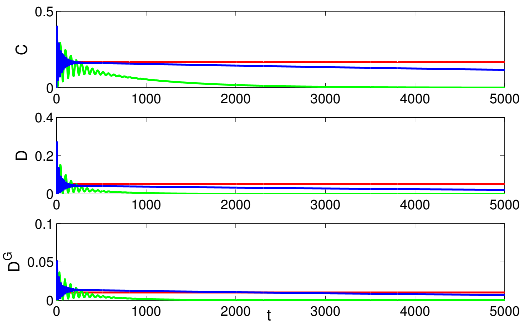

Figure 4: The concurrence, quantum discord, and geometric discord of spin 0 and spin 3 are plotted as functions of time t with $N = 6$, $J = 1$, $\theta = \pi/2$, $\gamma = 0.3$ for $\phi = 0$ (red lines), $\phi = \pi/5$ (green lines), and $\phi = 2\pi/5$ (blue lines).

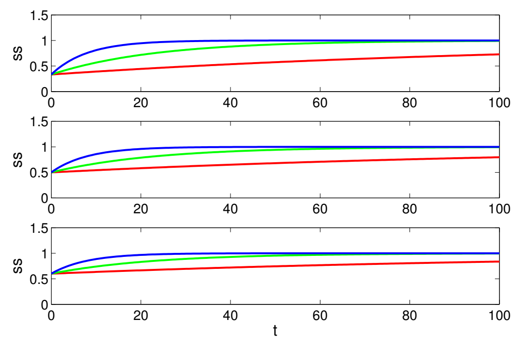

Figure 5: The spin-squeezing (denoted by ss) of the whole chain are plotted as functions of time t with $J = 1$, $\theta = \pi/2$, $\gamma = 1$ for $\phi = \pi/6$ (red lines), $\phi = \pi/4$ (green lines), and $\phi = \pi/3$ (blue lines). Top to bottom: (a) $N = 3$; (b) $N = 4$; (c) $N = 5$.

$\phi = \pi/4$ (green lines), and $\phi = \pi/3$ (blue lines). One can find that for a given spin chain the spin-squeezing can be decreased by increasing the phase shift by comparing the red, green, and blue lines of each panel. The spin-squeezing first increases with the increase of time and then reaches a plateau. The steady state value of spin-squeezing is 1. If we increase the number of spins N, the spin-squeezing is increased.

5 Conclusions

In the present paper, we have studied the influence of phase shift upon the dynamics of bipartite and multipartite correlations of a spin chain under dephasing. First, we derived an analytical solution of the present model. Then, we calculated bipartite correlations (measured by concurrence, discord, and geometric discord) of two spins. The behaviors of these measures of bipartite correlations are qualitatively similar in the present system. In the case of no dephasing, bipartite correlations are periodic functions of time. However, when the dephasing is considered, the maximal number of bipartite correlations decreases with the increase of dephasing rate. The response of different measures of bipartite correlations to phase shift could be different under decoherence, i.e., the geometric discord D^G will increase if we increase the phase shift ϕ from $\pi/4$ to $\pi/2$ while the quantum discord will decrease. There are steady state bipartite correlations in the presence of dephasing, i.e., the bipartite correlations including concurrence and discord can be stored partially for a long time in the presence of dephasing. Our results show that the steady state bipartite correlations can be adjusted by the phase shift [33–35]. Finally, we investigated the dynamics of the multipartite correlations of the whole spin chain. We adopted the spin-squeezing as a measure of multipartite correlations of the model. The spin-squeezing first increases with time and then reaches a plateau. Our results show that the number of spin-squeezing can be controlled by phase shift. We also discussed the influence of the number of spins on spin-squeezing and found that the spin-squeezing increases with the increase of spin numbers.

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