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Pulse Propagation in a Non-Linear Medium

Abstract: This paper considers a novel approach to solving the general propagation equation of optical pulses in an arbitrary non-linear medium. Using a suitable change of variable and applying the Adomian decomposition method to the non-linear Schrödinger equation, an analytical solution can be obtained which takes into account-parameters such as attenuation factor, the second order dispersive parameter, the third order dispersive parameter and the non-linear Kerr effect coefficient. By analysing the solution, this paper establishes that this method is suitable for the study of light pulse propagation in a non-linear optical medium.

Keywords: Adomian method, non-linear Schrödinger equation, optical pulse propagation

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1 Introduction

The Non-Linear Schrödinger Equation (NLSE) remains an important fundamental equation to decribe optical pulse propagation in non-linear dispersion media. The NLSE is obtained from Maxwell's equations under the slowly varying envelope approximation. Owning to the complexity of the partial differential equation, only a few exact solutions to the NLSE are known.

In the absence of a general solution it is often convenient to experiment with models to obtain information on the envelope. In a series of remarkable papers [1–3] solutions to NLSEs are considered using a variety of approaches. These methods can be classified into two broad categories known as the finite-difference methods and the pseudo spectral methods. The split-step Fourier method is a finite-difference method which has recently been used extensively due to its faster calculation speedcompared to other methods. This paper considers the use of the Adomian decomposition method for solving the NLSE for pulse propagation in non-linear media. It has previously been used to solve a wide range of physical problems [4–7] and is a semi-exact method which does not require linearization or discretization. The method has gone through a series of modification in the following references [8–16]. An advantage of this method is that it can provide analytical approximation or an approximated solution to a wide class of non-linear equations without linearization, perturbation, closure approximation or discretization methods. Its abilities attracted many authors to use this method for solving physical problems.

In this paper analytical solutions to NLSEs are investigated by using the Adomian decomposition method, taking into account the input pulse propagation and parameters such as attenuation factor, the second order dispersive parameter, the third order dispersive parameter and the non-linear Kerr effect coefficient. The rest of this paper is organized as follows: the next section presents details of the model and the Adomian decomposition method, section 3 compares the results using this method to classic analytical solutions and the paper is finished with concluding remarks.

2 General theory and analytical results

The optical pulse propagation in optical medium including loss, chromatic dispersion and non-linear effects will follow the NLSE [1]

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} = i\gamma|A|^2 A, \quad (1)$$

$$A(z,T)|_{z=0} = A(0,T), \quad (2)$$

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where A is $A(z, T)$ which represents the slowly varying envelope of the optical pulse, $A(0, T)$ is the original input optical pulse and z is the spatial coordinate i.e. the distance of transmission. T denotes the temporal coordinate in the so-called retarded frame that moves at the speed of the group velocity, $T = t - z/v_g$, and v_g is the group velocity. The following parameters $\alpha, \beta_2, \beta_3, \gamma$ represent the attenuation factor, the second order dispersive parameter, the third order dispersive parameter and the non-linear Kerr effect coefficient respectively.

For simplicity, the following normalization processing is applied to $A(z, T)$, z and T; $A(z, T) = \sqrt{P_0}U(z_0, \tau)$, $z_0 = \frac{z}{L_D}$, $\tau = \frac{T}{T_0}$ $L_D = \frac{T_0^2}{|\beta_2|}$, $L_N = \frac{1}{\gamma P_0}$, $N^2 = \frac{L_D}{L_N}$ where L_D is the dispersion length and L_N is the non-linear length. By using these transformations, U is found to satisfy the following equation, with the initial pulse profile given by:

$$\frac{\partial U}{\partial z_0} + \frac{\alpha L_D}{2}U + \frac{i\text{sgn}(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2} - \frac{\text{sgn}(\beta_3)}{6T_0}\frac{|\beta_3|}{|\beta_2|}\frac{\partial^3 U}{\partial \tau^3} = iN^2|U|^2U; \quad U(z_0, \tau)|_{z_0=0} = e^{-(1+iC)\frac{\tau^2}{2}}; \tag{3}$$

where $sgn()$ stands for signum function and $U(0, \tau)$ the corresponding Gaussian input optical pulse, C a chirp parameter.

2.1 Analytical solution for $\beta_3 = 0$

In this subsection, the NLSE is solved for the particular case when $\beta_3 = 0$. In this case, setting: $\eta = \varepsilon z_0 + \tau$, $\eta_0 = \eta_{z_0=0} = \tau$, $\eta - \eta_0 = \varepsilon z_0$, $f(\eta) = U(z_0, \tau)$ for small positive ε, equation (3) reduces to the following non-linear functional equation:

$$f - G(f) = p; \tag{4}$$

where

$$G(f) = 2i\varepsilon \int_{\eta_0}^{\eta} f(\xi)d\xi\text{sgn}(\beta_2) + \int_{\eta_0}^{\eta}\int_{\xi_0}^{\xi}\left[i\alpha L_D f(\sigma) + 2N^2|f(\sigma)|^2 f(\sigma)\right]d\sigma d\xi\text{sgn}(\beta_2)$$
$$p = p(\eta) = f(\eta_0) + \left[f'(\eta_0) - 2i\varepsilon f(\eta_0)\text{sgn}(\beta_2)\right](\eta - \eta_0). \tag{5}$$

In order to obtain the analytical solution of the equation (4) the Adomian method is used, where G is a nonlinear operator from a Hilbert space H into H, p is a given function in H. In the solution f satisfies (4) and it is assumed that (4) has a unique solution for every p belonging to H. The Adomian method consists in representing f as follows [4–8]

$$f = \sum_{n=0}^{+\infty} f_n. \tag{6}$$

The nonlinear operator G is decomposed as follows

$$G(f) = \sum_{n=0}^{+\infty} A_n, \tag{7}$$

where the A_n are functions (Adomian's polynomials) of $f_0, \ldots, f_n$ that are obtained by writing:

$$q = \sum_{n=0}^{+\infty}\lambda^n f_n, \quad G\Big(\sum_{n=0}^{+\infty}\lambda^n f_n\Big) = \sum_{n=0}^{+\infty}\lambda^n A_n. \tag{8}$$

λ is a parameter introduced for convenience. From (8) the values of A_n are deduced by the formulae

$$n!A_n = \frac{d^n}{d\lambda^n}\left[G\Big(\sum_{i=0}^{+\infty}\lambda^i f_i\Big)\right]_{\lambda=0}, \quad n = 0, 1, 2, 3, \ldots \tag{9}$$

Thus, f_n and A_n can be computed recurrently using the following reletionships:

$$\begin{cases} f_0 = p \\ f_1 = A_0 \\ \vdots \\ f_n = A_{n-1} \\ \vdots \end{cases}$$

For reasons of simplicity, if $f'(\eta_0) = i\varepsilon f(\eta_0)\text{sgn}(\beta_2)$ is chosen as the integral then $f(\eta_0) = U(0, \tau) = e^{-(1+iC)\frac{\tau^2}{2}}$ and f_0, f_1 and the 2-term approximation of $f = \sum_{n\geq 0} f_n$ can be written in the form:

$$f(\eta) = e^{-(1+iC)\frac{\tau^2}{2}}\Bigg\{1 + \varepsilon^4 z_0^2 + \frac{\alpha L_D}{6}\varepsilon^4 z_0^3 + N^2 e^{-\tau^2}\left(\varepsilon^2 z_0^2 + \frac{\varepsilon^6}{6}z_0^4\right)\text{sgn}(\beta_2) + \text{i}\left[\left(\varepsilon^2 \text{z}_0 + \frac{\alpha}{2}\text{L}_\text{D}\varepsilon^2\text{z}_0^2\right)\text{sgn}(\beta_2) + N^2 e^{-\tau^2}\left(-\frac{1}{3}\varepsilon^4 z_0^3 - \frac{\varepsilon^8}{10}z_0^5\right)\right]\Bigg\}. \tag{10}$$

Since $A(z_0, \tau) = \sqrt{P_0}U(z_0, \tau)$, it follows

$$|A(z_0, \tau)|^2 = P_0 e^{-\tau^2}\Bigg\{\left[1 + \varepsilon^4 z_0^2 + \frac{\alpha L_D}{6}\varepsilon^4 z_0^3 - N^2 e^{-\tau^2}\left(\varepsilon^2 z_0^2 + \frac{\varepsilon^6}{6}z_0^4\right)\right]^2 + \varepsilon^4\left[z_0 + \frac{\alpha L_D}{2}z_0^2 + N^2 e^{-\tau^2}\left(\frac{1}{3}\varepsilon^2 z_0^3 + \frac{\varepsilon^6}{10}z_0^5\right)\right]^2\Bigg\}. \tag{11}$$

In order to evaluate the error of Adomian method, the second order differential of the pulse amplitude is calculated, giving:

$$f_2 = f_1 G'(f_0) = G(f_0)G'(f_0), \tag{12}$$

Without loss of generality, G is differentiable when f is a real function. The amplitude of the pulse is given by: $|B(z_0,\tau)^2| = |f_0 + f_1 + f_2|^2$ whereas $|A(z_0,\tau)|^2 = |f_0 + f_1|^2$. The error is given by $\frac{|B-A|}{|B|} \times 100$.

$$\begin{aligned}|B(z_0,\tau)|^2 = P_0 e^{-\tau^2} \Bigg\| \Bigg[& 1 - \varepsilon^2 z_0^2\left(\frac{a}{2b} + \frac{1}{2b^2}\right) - \frac{a\varepsilon^3 z_0^3}{6b^2} \\ & + \frac{i}{b}N^2 e^{-\tau^2}\left(\frac{\varepsilon^5 z_0^5}{20b|b|^2} + \frac{\varepsilon^4 z_0^4}{12|b|^2} + \right. \\ & \left. \frac{\varepsilon^3 z_0^3}{6b} + \frac{\varepsilon^2 z_0^2}{2}\right)\Bigg] \times \Bigg[-\frac{\varepsilon z_0}{b} - \frac{a\varepsilon^2 z_0^2}{2b} \\ & + \frac{3i}{b}N^2 e^{-\tau^2}\left(\frac{\varepsilon^4 z_0^4}{12b^2} + \frac{\varepsilon^3 z_0^3}{3b} + \frac{\varepsilon^2 z_0^2}{2}\right)\Bigg]\Bigg\|^2\end{aligned}$$

2.2 Analytical solution for $\beta_3 \neq 0$

In this subsection, the analytical solution of the NLSE for $\beta_3 \neq 0$ is calculated. In this case, setting $a = \frac{1}{2}\alpha L_D$, $b = \frac{1}{2}\mathrm{sgn}(\beta_2)$, $\mathrm{c} = \frac{|\beta_3|}{6\mathrm{T}_0|\beta_2|}\mathrm{sgn}(\beta_3)$, $\eta = \varepsilon \mathrm{z}_0 + \tau$, $\mathrm{f}(\eta) = \mathrm{U}(\mathrm{z}_0,\tau)$ for small positive ε. The equation (3) reduces to the following non-linear functional equation $f - G(f) = p$, where

$$\begin{aligned} f &= f(\eta) \\ G(f) &= \frac{ib}{c}\int_{\eta_0}^{\eta} f(\xi)d\xi + \frac{\varepsilon}{c}\int_{\eta_0}^{\eta}\int_{\xi_0}^{\xi} f(\sigma)d\sigma d\xi \\ &+ \frac{1}{c}\int_{\eta_0}^{\eta}\int_{\xi_0}^{\xi}\int_{\sigma_0}^{\sigma}\left[af(y) - iN^2|f(y)|^2 f(y)\right]dyd\sigma d\xi \\ p &= p(\eta) = f(\eta_0) + \left[f'(\eta_0) - \frac{ib}{c}f(\eta_0)\right](\eta - \eta_0) \\ &+ \frac{1}{2}\left[f''(\eta_0) - \frac{ib}{c}f'(\eta_0) - \frac{\varepsilon}{c}f(\eta_0)\right](\eta - \eta_0)^2. \end{aligned}$$

For reasons of simplicity $f'(\eta_0) = i\varepsilon f(\eta_0)\mathrm{sgn}(\beta_2)$, $\mathrm{f}''(\eta_0) = -\varepsilon^2 \mathrm{f}(\eta_0)$ is chosen, taking in to account $f(\eta_0) = U(0,\tau)$; then f_0, f_1 and the 2-term approximation of $f = \sum_{n\geq 0} f_n$ in the form

$$\begin{aligned} f(\eta) = e^{-(1+iC)\frac{\tau^2}{2}} \Bigg\{ & 1 + i\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right)(\eta - \eta_0) \\ & + \frac{1}{2}\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right)(\eta - \eta_0)^2 \\ & + \frac{ib}{c}\Bigg[\eta - \eta_0 + \frac{i}{2}\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right)(\eta - \eta_0)^2 \\ & + \frac{1}{6}\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right)(\eta - \eta_0)^3\Bigg] \\ & + \frac{\varepsilon}{c}\Bigg[\frac{1}{2}\left(\eta - \eta_0\right)^2 + \frac{i}{6}\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right)(\eta - \eta_0)^3 \\ & + \frac{1}{24}\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right)(\eta - \eta_0)^4\Bigg] \\ & + \frac{a}{c}\Bigg[\frac{1}{6}\left(\eta - \eta_0\right)^3 + \frac{i}{24}\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right)(\eta - \eta_0)^4 \\ & + \frac{1}{120}\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right)(\eta - \eta_0)^5\Bigg] \\ & - \frac{1}{c}iN^2 e^{-\tau^2}\Bigg[\frac{1}{6}(\eta - \eta_0)^3 + \frac{1}{24}i\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right) \\ & \times(\eta - \eta_0)^4 \\ & + \frac{1}{120}\left(-\varepsilon^2 - \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{3\varepsilon}{\mathrm{c}} + 2\frac{\mathrm{b}^2}{\mathrm{c}^2}\right)(\eta - \eta_0)^5 \\ & + \frac{1}{120}i\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right) \\ & \times\left(-\frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}} + \frac{\mathrm{b}^2}{\mathrm{c}^2}\right)(\eta - \eta_0)^6 \\ & + \frac{1}{840}\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right) \\ & \times\left(-\varepsilon^2 - \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - 3\frac{\varepsilon}{\mathrm{c}} + 2\frac{\mathrm{b}^2}{\mathrm{c}^2}\right)(\eta - \eta_0)^7 \\ & + \frac{1}{1344}i\left(\varepsilon\mathrm{sgn}(\beta_2) - \frac{\mathrm{b}}{\mathrm{c}}\right) \\ & \times\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right)^2(\eta - \eta_0)^8 \\ & + \frac{1}{4032}\left(-\varepsilon^2 + \frac{b\varepsilon}{c}\mathrm{sgn}(\beta_2) - \frac{\varepsilon}{\mathrm{c}}\right)^3(\eta - \eta_0)^9\Bigg]\Bigg\}. \end{aligned}$$

3 Results and Discussion

3.1 Case of the third order dispersion coefficient $\beta_3 = 0$

In Figure 1 a Gaussian pulse is evaluated using $A(0,\tau) = A_0 \exp\left(-\tau^2/2(1 + iC)\right)$ with a pulse width of $T_0 = 20$ ps and a chirp parameter, C, of 0. The other basic parameters of the pulse are provided as follows: the pulse peak power $P_0 = 10$ mW, the fiber loss $\alpha = 0$ dB/km, the second order dispersion coefficient is $\beta_2 = -4\ ps^2/km$, 100 km, 200 km; the non-linear Kerr coefficient $\gamma = 0\ W^{-1}km^{-1}$, the third order dispersion coefficient $\beta_3 = 0\ ps^3/km$. The propagation distances are 0km, 100km and 200km and the control parameter is $\varepsilon = 0.5$. Figure 1 shows the following plots of the calculated results using the Adomian decomposition technique: the input pulse and the output pulse for 100

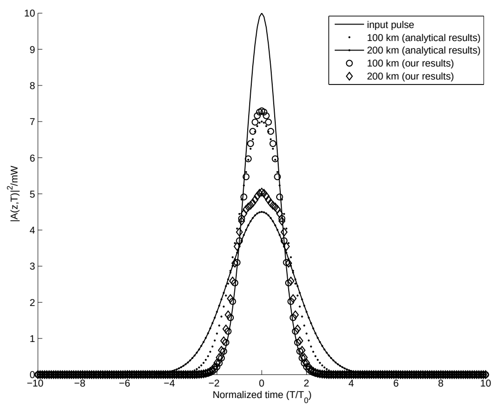

Figure 1: The input and output intensity profiles with respect to normalized time for propagating distances z=100 km, 200 km respectively obtained by the Adomian decomposition method and the classical analytical solution.

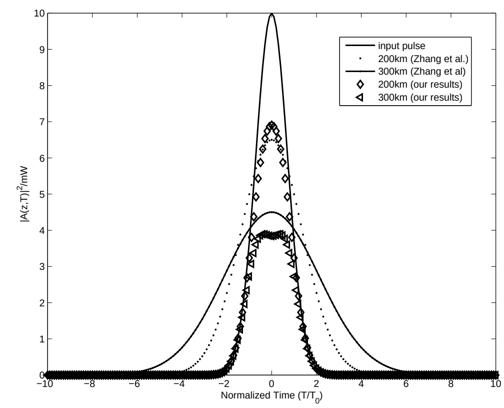

Figure 2: The input and output intensity profiles with respect to normalized time for propagating distances z=200 km, 300 km respectively obtained by the Adomian decomposition method and the rapid numerical difference recurrent formula of NLSE.

and 200 km propagation distances. It is seen that the peak power of the pulse is decreasing and the pulse is spreading.

The shape of the pulses obtained by numerical results calculated by the Adomian decomposition technique for z=100 km by setting $\varepsilon = 0.5$ have the same behavior as the analytical results and those obtained by [1]; however the FWHM of the pulse for our calculated results is less than the analytical one. As the propagation distance increases and becomes large for example when z=200 km), the shape of the optical pulse changes considerably inside the non-linear optical medium. The Gaussian shape is not conserved. In the analytical results, at this distance, the shape is conserved. When the propagation distance is larger, the chosen control parameter must be very small, which is the real purpose of introducing this control parameter. Note, the intensity of the output can not exceed that of the input pulse; there is no gain in wave propagation. In our calculations, $L_D = L_N = 100$ km. The dispersion effect will dominate if $z \sim L_D$ and $z \ll L_N$, but when the propagation distance is such that $z \sim L_N$ and $z \ll L_D$ only non-linear effects dominate.

In Figure 2 the pulse has the following basic parameters: $T_0 = 20$ ps, $P_0 = 10$ mW, $\alpha = 0$ dB/km, $\beta_2 = -4$ ps^2/km, $\beta_3 = 0$ ps^3/km, the $\gamma = 1.0$ $W^{-1}km^{-1}$. The transmission distances are z=200 km, 300 km respectively and the control parameter is $\varepsilon = 6.5 \times 10^{-3}$. Figure 2 shows the results obtained by the Adomian decomposition technique and via the rapid numerical difference recurrent formula of NLSE and its application developed in [1]. For both values of propagation distances, the output pulses are contained inside the input pulse.

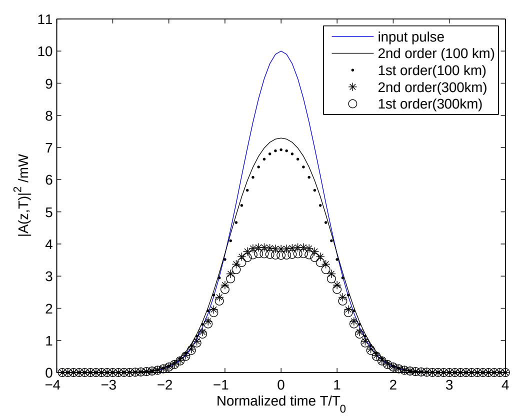

Figure 3: First order and second order approximations with respect to normalized time for propagating distances z=100 km, 300 km respectively obtained by the Adomian decomposition method.

Figure 3 shows the plot of the error in Adomian approximation as a function of the normalized time. The maximal error calculated numerically is 5%.

3.2 Case of the third order dispersion coefficient $\beta_3 \neq 0$

Figure 4 shows the plot of the calculated results using the following parameters: $T_0 = 20$ ps, $P_0 = 10$ mW, $\alpha = 0.2$ dB/km, $\beta_2 = -4$ ps^2/km, $\gamma = 1$ W^{-1}km^{-1}, $\beta_3 = 0.01$ ps^3/km. The propagation distances inside the medium are 300km, 1000km and 2000km and the control parameter $\varepsilon = 10^{-7}$. It can been seen that the pulse shape is contained in the envelope of the initial Gaussian pulse.

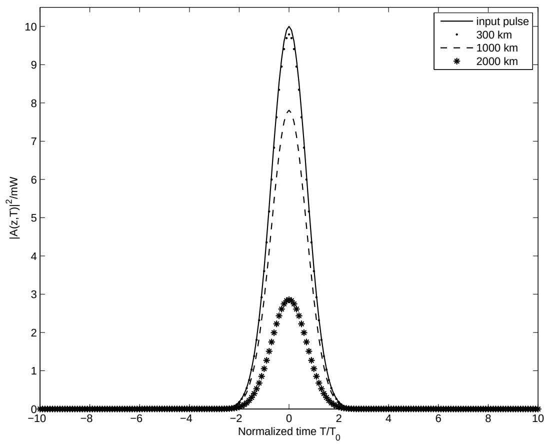

Figure 4: The input and output intensity profiles with respect to normalized time for propagating distances z=300 km, 1000 km, 2000 km respectively obtained by the Adomian decomposition method.

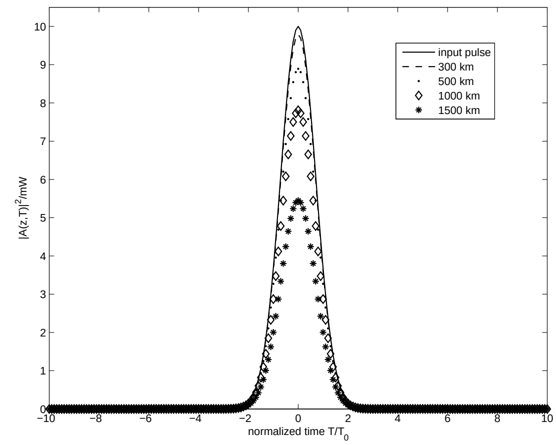

Figure 5: The input and output intensity profiles with respect to normalized time for propagating distances z=300 km, 500 km, 1000 km, 1500 km respectively obtained by the Adomian decomposition method.

It is expected that the pulse would spread out as propagation distance increases.

In Figure 5 all parameters are the same as Figure 3 except the third order dispersion term which is $\beta_3 = 1\ ps^3/km$. The propagation distances are 300 km, 500 km, 1000 km and 1500 km and the control parameter is $\varepsilon = 10^{-6}$. As propagation distance increases, the intensity of the pulse decreases.

Considering Figure 1, the criteria for choosing the control parametres are as follows:

$$0 \le z < 100\ km, \quad \varepsilon \le 0.75$$
$$100 \le z < 200\ km, \quad \varepsilon \le 0.56$$
$$200 \le z < 300\ km, \quad \varepsilon \le 0.43$$
$$300 \le z < 400\ km, \quad \varepsilon \le 0.335$$
$$400 \le z < 500\ km, \quad \varepsilon \le 0.273$$

For Figure5, the criteria are:

$$0 \le z \le 100\ km, \quad \varepsilon = 10^{-5}$$
$$z > 100\ km, \quad \varepsilon = 10^{-6}$$

In Figure 1 and Figure 2, the attenuation is zero so it is expected that the pulse energy is conserved throughout propagation. Using the results given in Figure 1 and Figure 2 the calculated energy of the input pulse is equal to 12.53 units using the Simpson quadrature Matlab function. This energy is the same as the energy of the output pulses with propagation distances of 10 km, 50 km, 75 km, and 100 km. The control parameter is equal to 0.5 as in Figure 1. In conclusion, the Adomian method used is well suited for solving NLSEs. It is evident that the energy of the pulse is conserved when the propagation distance is less than 100 km. In Figure 2, which shows propagtion distances greater than 100 km, energy is not conserved. This concern will form the basis of future study.

4 Concluding remarks

This work applies the Adomian decomposition technique to solve the NLSE for propagation of a short optical pulse inside a non-linear medium by taking into account attenuation factor, second order dispersive parameter, third order dispersive parameter and non-linear Kerr effect. These parameters influence the shape of the pulse during propagation. The established Adomian decomposition formula is a scientific, reasonable and effective analytical method for the study of light pulse propagation in an optical medium even though in our case the solution depends strongly on the control parameter and the propagation distance. These parameters have important effects on the propagation of the pulse in the medium. When the width of the pulse is small, these parameters become more important. Further study will consider pulses containing a few optical cycles by solving the generialized NLSE.

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