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The analysis of the suction/injection on the MHD Maxwell fluid past a stretching plate in the presence of nanoparticles by Lie group method

Abstract: In this paper, the magnetohydrodynamic (MHD) Maxwell fluid past a stretching plate with suction/injection in the presence of nanoparticles is investigated. The Lie symmetry group transformations are used to convert the boundary layer equations into non-linear ordinary differential equations. The dimensionless governing equations are solved numerically using Bvp4c with MATLAB, which is a collocation method equivalent to the fourth order mono-implicit Runge-Kutta method. The effects of some physical parameters, such as the elastic parameter K, the Hartmann number M, the Prandtl number Pr, the Brownian motion Nb, the thermophoresis parameter Nt and the Lewis number Le, on the velocity, temperature and nanoparticle fraction are studied numerically especially when suction and injection at the sheet are considered.

Keywords: Lie group; Maxwell fluid; MHD; stretching surface; suction/injection

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1 Introduction

During last few years the boundary layer flow behaviours of different types of fluids attracted the interest of many researchers. Especially the interest of non-Newtonian fluids is increasing substantially due to the large number of practical applications in industrial and manufacturing processes. Because of the complexity of these fluids, many types of constitutive equations have been constructed to exhibit the properties of the non-Newtonian fluids [1–9]. For example, Maxwell model, a subclass of non-Newtonian fluids, can predict the stress relaxation and therefore has become more popular [10, 11]. Noor [12] presented analysis of the MHD flow of a Maxwell fluid past a vertical stretching sheet in the presence of thermophoresis and chemical reactions. Liu et al. [13] studied the time periodic electroosmotic flow (EOF) of generalized Maxwell fluids between two microparallel plates and an analytical solution of EOF velocity was presented. Hayat et al. [14] constructed an analytic solution for unsteady MHD flow in a rotating Maxwell fluid through a porous medium. Nadeem et al. [15] studied numerically two dimensional boundary-layer flows and the heat transfer of a Maxwell fluid past a stretching sheet.

The investigation of flow due to a stretching sheet also has received great attention due to the various industrial applications, such as in the manufacturing of polymer sheets, filaments and wires. During the manufacturing process, the moving sheet is assumed to stretch on its own plane, and the stretched surface interacts with the ambient fluid both mechanically and thermally. Initially, Sakiadis [16] introduced the concept of a boundary layer flow over a stretching surface. Crane [17] modified the idea introduced by Sakiadis and extended this idea for both linear and exponentially stretching sheets. In addition, a few recent investigations have been made for the low flow over a stretching surface including the various effects described in Refs. [18–21]. For example, Nadeem et al. [18] investigated the stagnation point flow of a viscous fluid towards a stretching sheet and obtained the analytical solution of the boundary layer equation by homotopy analysis method. Chen [20] analyzed magneto-hydrodynamic mixed convective flow and heat transfer of an electrically conducting,

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power-law fluid past a stretching surface in the presence of heat generation/absorption and thermal radiation.

Lie group analysis can be used to obtain similarity transformations that can reduce a system of governing partial differential equations and associated boundary conditions to a system of ordinary differential equations [22–24]. This technique has been applied by many researchers to solve some partial differential problems [25–29].

Motivated by above works, in this paper, we present a general procedure for applying the one-parameter group of transformations to the MHD Maxwell fluid past a stretching plate in the presence of nanoparticles. The present study is to extend the work done by Nadeem et al. [15] to the case of the flow past a porous plate with suction/injection. As a result, we obtain the similarity transformation by Lie group analysis. Finally, the results for some physical parameters of the velocity, temperature distribution and nanoparticle fraction are investigated numerically when the suction or injection at the wall are considered.

2 Preliminaries

Consider a two-dimensional steady incompressible fluid flowing past a stretching plate with suction/injection. Here we assume that $\bar{x}$-axis is measured along the horizontal stretching surface and the flow is assumed to be confined to $\bar{y} > 0$. The sheet is stretched with the linear velocity $\bar{u}(\bar{x}) = a\bar{x}$, where $a > 0$ is constant. A uniform constant magnetic surface field is applied normal to the stretching surface. The effects of the induced magnetic field are negligible. The boundary layer equations of the Maxwell fluid with nanoparticles are [15]:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{2.1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \kappa_0\left(\bar{u}^2\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \bar{v}^2\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 2\bar{u}\bar{v}\frac{\partial^2 \bar{u}^2}{\partial \bar{x}\partial \bar{y}}\right) - \frac{\sigma B_0^2}{\rho}(\bar{u} + \kappa_0\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}), \tag{2.2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) + \tau\left\{D_B(\frac{\partial C}{\partial \bar{x}}\frac{\partial T}{\partial \bar{x}} + \frac{\partial C}{\partial \bar{y}}\frac{\partial T}{\partial \bar{y}}) + \left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial \bar{x}}\right)^2 + \left(\frac{\partial T}{\partial \bar{y}}\right)^2\right]\right\}, \tag{2.3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D_B\left(\frac{\partial^2 C}{\partial \bar{x}^2} + \frac{\partial^2 C}{\partial \bar{y}^2}\right) + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right), \tag{2.4}$$

where $\bar{u}$ and $\bar{v}$ denote the velocities in the $\bar{x}$ and $\bar{y}$ directions, respectively, ν is the kinematic viscosity of the fluid, κ_0 is the relaxation time of the Upper-Convected Maxwell fluid, σ is the electrical conductivity, B_0 is the magnetic induction, ρ is the density of the fluid, α is the thermal diffusivity, T is the fluid temperature, C is the nanoparticle faction, T_w and C_w are the temperature of fluid and nanoparticle faction at the wall, respectively, D_B is the Brownian diffusion, D_T is the thermophoretic diffusion coefficient, and τ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid. When $\bar{y}$ tends toward infinity, the ambient values of T and C are denoted by T_∞ and C_∞, respectively.

The corresponding boundary conditions are:

$$\begin{aligned} &\bar{u} = a\bar{x}, \bar{v} = v_w, \quad T = T_w, \quad C = C_w \ \ at \ \ \bar{y} = 0, \\ &\bar{u} = 0, \quad T = T_\infty, C = C_\infty \ as \ \bar{y} \to \infty, \end{aligned} \tag{2.5}$$

where v_w is the injection or suction velocity at the wall.

The following non-dimensional variables can be introduced,

$$\begin{aligned} &x = \frac{\bar{x}}{\sqrt{\nu/a}}, y = \frac{\bar{y}}{\sqrt{\nu/a}}, u = \frac{\bar{u}}{\sqrt{a\nu}}, v = \frac{\bar{v}}{\sqrt{a\nu}}, \\ &\Theta = \frac{T - T_\infty}{T_w - T_\infty}, \Phi = \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned} \tag{2.6}$$

and the stream function Ψ defined by $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$ leads to Eqs.(2.1)-(2.4) taking the following non-dimensional form

$$\begin{aligned} &\Psi_y\Psi_{xy} - \Psi_x\Psi_{yy} - \Psi_{yyy} - K\big((\Psi_y)^2\Psi_{xxy} + (\Psi_x)^2\Psi_{yyy} \\ &- 2\Psi_x\Psi_y\Psi_{xyy}\big) + M^2(\Psi_y - K\Psi_x\Psi_{yy}) = 0, \end{aligned} \tag{2.7}$$

$$\begin{aligned} &\Psi_y\Theta_x - \Psi_x\Theta_y - \frac{1}{Pr}(\Theta_{xx} + \Theta_{yy}) - Nb(\Phi_x\Theta_x + \Phi_y\Theta_y) \\ &- Nt((\Theta_x)^2 + (\Theta_y)^2) = 0, \end{aligned} \tag{2.8}$$

$$LePr(\Psi_y\Phi_x - \Psi_x\Phi_y) - (\Phi_{xx} + \Phi_{yy}) - \frac{Nt}{Nb}(\Theta_{xx} + \Theta_{yy}) = 0, \tag{2.9}$$

where $K = a\kappa_0(\geq 0)$ is the elastic parameter, $M^2 = \sigma B_0^2/\rho a$ is the Hartmann number, $Pr = \nu/\alpha$ is the Prandtl number, $Nb = \tau D_B(C_w - C_\infty)/\nu$ is Brownian motion parameter, $Nt = \tau D_T(T_w - T_\infty)/(\nu T_\infty)$ is the thermophoresis parameter, and $Le = \alpha/D_B$ is the Lewis number.

The boundary conditions can be written as

$$\begin{aligned} &\Psi_y = x, \Psi_x = S, \Theta = 1, \Phi = 1 \ \ at \ \ y = 0, \\ &\Psi_y = 0, \Theta = 0, \Phi = 0 \ \ at \ \ y \to \infty, \end{aligned} \tag{2.10}$$

where $S = -v_w/\sqrt{a\nu}$ ($S > 0$ corresponds to suction and $S < 0$ corresponds to injection).

3 Lie point symmetries of the problem

Let us consider a one-parameter Lie group of infinitesimal transformations

$$\begin{aligned} x^{\star} &= x + \varepsilon\xi(x, y, \Psi, \Theta, \Phi) + O(\varepsilon^2),\\ y^{\star} &= y + \varepsilon\zeta(x, y, \Psi, \Theta, \Phi) + O(\varepsilon^2),\\ \Psi^{\star} &= \Psi + \varepsilon\psi(x, y, \Psi, \Theta, \Phi) + O(\varepsilon^2),\\ \Theta^{\star} &= \Theta + \varepsilon\theta(x, y, \Psi, \Theta, \Phi) + O(\varepsilon^2),\\ \Phi^{\star} &= \Phi + \varepsilon\phi(x, y, \Psi, \Theta, \Phi) + O(\varepsilon^2), \end{aligned} \tag{3.1}$$

where ε is a small parameter.

A system of partial differential equations (2.7)-(2.9) is said to admit a symmetry generated by the vector field

$$\Gamma \equiv \xi\frac{\partial}{\partial x} + \zeta\frac{\partial}{\partial y} + \psi\frac{\partial}{\partial \Psi} + \theta\frac{\partial}{\partial \Theta} + \phi\frac{\partial}{\partial \Phi}, \tag{3.2}$$

if it is left invariant by the transformation $(x, y, \Psi, \Theta, \Phi) \to (x^{\star}, y^{\star}, \Psi^{\star}, \Theta^{\star}, \Phi^{\star})$ given by (3.1).

The vector Γ given by (3.2) is said to be a Lie point symmetry vector field for (2.7)-(2.9) if

$$\begin{aligned} &\Gamma^{[3]}(\Psi_y\Psi_{xy} - \Psi_x\Psi_{yy} - \Psi_{yyy} - K((\Psi_y)^2\Psi_{xxy} + (\Psi_x)^2\Psi_{yyy}\\ &- 2\Psi_x\Psi_y\Psi_{xyy}) + M^2(\Psi_y - K\Psi_x\Psi_{yy})) = 0, \end{aligned} \tag{3.3}$$

$$\begin{aligned} &\Gamma^{[3]}(\Psi_y\Theta_x - \Psi_x\Theta_y - \frac{1}{Pr}(\Theta_{xx} + \Theta_{yy}) - Nb(\Phi_x\Theta_x + \Phi_y\Theta_y)\\ &- Nt((\Theta_x)^2 + (\Theta_y)^2)) = 0, \end{aligned} \tag{3.4}$$

$$\Gamma^{[3]}(LePr(\Psi_y\Phi_x - \Psi_x\Phi_y) - (\Phi_{xx} + \Phi_{yy}) - \frac{Nt}{Nb}(\Theta_{xx} + \Theta_{yy})) = 0, \tag{3.5}$$

where

$$\begin{aligned} \Gamma^{[3]} \equiv{}& \xi\frac{\partial}{\partial x} + \zeta\frac{\partial}{\partial y} + \psi\frac{\partial}{\partial \Psi} + \theta\frac{\partial}{\partial \Theta} + \phi\frac{\partial}{\partial \Phi} + \psi^x\frac{\partial}{\partial \Psi_x}\\ &+ \psi^y\frac{\partial}{\partial \Psi_y} + \psi^{xy}\frac{\partial}{\partial \Psi_{xy}} + \psi^{yy}\frac{\partial}{\partial \Psi_{yy}} + \psi^{xxy}\frac{\partial}{\partial \Psi_{xxy}}\\ &+ \psi^{xyy}\frac{\partial}{\partial \Psi_{xyy}} + \psi^{yyy}\frac{\partial}{\partial \Psi_{yyy}} + \theta^x\frac{\partial}{\partial \Theta_x} + \theta^y\frac{\partial}{\partial \Theta_y}\\ &+ \theta^{xx}\frac{\partial}{\partial \Theta_{xx}} + \theta^{yy}\frac{\partial}{\partial \Theta_{yy}} + \phi^x\frac{\partial}{\partial \Phi_x} + \phi^y\frac{\partial}{\partial \Phi_y}\\ &+ \phi^{xx}\frac{\partial}{\partial \Phi_{xx}} + \phi^{yy}\frac{\partial}{\partial \Phi_{yy}}, \end{aligned} \tag{3.6}$$

is the third prolongation of the vector Γ.

From Eqs. (3.3)-(3.5), the following system of linear partial differential equations is given

$$\begin{aligned} &\psi^y\Psi_{xy} + \psi^{xy}\Psi_y - \psi^x\Psi_{yy} - \psi^{yy}\Psi_x - \psi^{yyy}\\ &- K(2\psi^y\Psi_y\Psi_{xxy} + \psi^{xxy}(\Psi_y)^2 + 2\psi^x\Psi_x\Psi_{yyy}\\ &+ \psi^{yyy}(\Psi_x)^2 - 2\psi^x\Psi_y\Psi_{xyy} - 2\psi^y\Psi_x\Psi_{xyy} - 2\psi^{xyy}\Psi_x\Psi_y)\\ &+ M^2(\psi^y - K\psi^x\Psi_{yy} - K\psi^{yy}\Psi_x) = 0, \end{aligned} \tag{3.7}$$

$$\begin{aligned} &\psi^y\Theta_x + \theta^x\Psi_y - \psi^x\Theta_y - \theta^y\Psi_x - \frac{1}{Pr}(\theta^{xx} + \theta^{yy})\\ &- Nb(\phi^x\Theta_x + \theta^x\Phi_x + \phi^y\Theta_y + \theta^y\Phi_y)\\ &- Nt(2\theta^x\Theta_x + 2\theta^y\Theta_y) = 0, \end{aligned} \tag{3.8}$$

$$\begin{aligned} &LePr(\psi^y\Phi_x + \phi^x\Psi_y - \psi^x\Phi_y - \phi^y\Psi_x) - (\phi^{xx} + \phi^{yy})\\ &- \frac{Nt}{Nb}(\theta^{xx} + \theta^{yy}) = 0. \end{aligned} \tag{3.9}$$

The components $\psi^x, \psi^y, \psi^{xy}, \psi^{yy}, \psi^{xxy}, \psi^{xyy}, \psi^{yyy}$, θ^x, θ^y, θ^{xx}, θ^{yy}, ϕ^x, ϕ^y, ϕ^{xx}, ϕ^{yy} can be determined from the following expressions

$$\begin{aligned} \psi^S &= D_S\psi - \Psi_x D_S\xi - \Psi_y D_S\zeta,\\ \theta^S &= D_S\theta - \Theta_x D_S\xi - \Theta_y D_S\zeta,\\ \phi^S &= D_S\phi - \Phi_x D_S\xi - \Phi_y D_S\zeta,\\ \psi^{JS} &= D_S\psi^J - \Psi_{Jx} D_S\xi - \Psi_{Jy} D_S\zeta,\\ \theta^{JS} &= D_S\theta^J - \Theta_{Jx} D_S\xi - \Theta_{Jy} D_S\zeta,\\ \phi^{JS} &= D_S\phi^J - \Phi_{Jx} D_S\xi - \Phi_{Jy} D_S\zeta, \end{aligned} \tag{3.10}$$

where S,J stand for x, y and the total derivatives D_x, D_y are

$$\begin{aligned} D_x \equiv{}& \partial_x + \Psi_x\partial_\Psi + \Theta_x\partial_\Theta + \Phi_x\partial_\Phi + \Psi_{xx}\partial_{\Psi_x} + \Theta_{xx}\partial_{\Theta_x}\\ &+ \Phi_{xx}\partial_{\Phi_x} + \Psi_{xy}\partial_{\Psi_y} + \ldots,\\ D_y \equiv{}& \partial_y + \Psi_y\partial_\Psi + \Theta_y\partial_\Theta + \Phi_y\partial_\Phi + \Psi_{yy}\partial_{\Psi_y} + \Theta_{yy}\partial_{\Theta_y}\\ &+ \Phi_{yy}\partial_{\Phi_y} + \Psi_{xy}\partial_{\Psi_x} + \ldots. \end{aligned} \tag{3.11}$$

Substituting (3.10) into (3.7) and then assuming the coefficients of $\Psi_x\Psi_{xy}$, $\Psi_y\Psi_{yy}$, $\Psi_x(\Psi_y)^2\Psi_{xxy}$, $(\Psi_y)^3\Psi_{xxy}$, $\Psi_x\Psi_{yy}$ to be zero, one obtains

$$\xi_y = 0,\ \ \zeta_x = 0,\ \ \xi_\Psi = 0,\ \ \zeta_\Psi = 0,\ \ \psi_\Psi - \xi_x = 0. \tag{3.12}$$

Then substituting (3.12) and (3.10) into (3.7) and letting the coefficients of $(\Psi_y)^2$, Ψ_y, Ψ_{xy}, Ψ_{yy} be zero, one can obtains the following results again.

$$\begin{aligned} &\psi_{x\Psi} = 0,\ \ \psi_{xy} - \psi_{yy\Psi} + M^2\zeta_x = 0,\\ &(M^2 + K)\psi_{yy} + \xi_{yyy} + M^2\xi_y = 0,\\ &\psi_y + 3\xi_{yy} + M^2\psi_{x\Psi} = 0,\\ &(M^2 + K)\psi_x + \psi_{y\Psi} = 0. \end{aligned} \tag{3.13}$$

Similarly, substituting (3.10) into the energy and nanoparticle fraction equations (3.8) and (3.9), and then assuming the coefficients of Ψ_x and Ψ_y to be zero yields

$$\theta_x = 0, \ \ \theta_y = 0, \ \ \phi_x = 0, \ \ \phi_y = 0. \tag{3.14}$$

From (3.12),(3.13) and (3.14) we have

$$\xi = a_1 x, \ \ \zeta = a_2, \ \ \psi = a_1 \Psi + a_3, \ \ \theta = a_4, \ \ \phi = a_5. \tag{3.15}$$

If one assumes $a_1 = 1, a_2 = a_3 = a_4 = a_5 = 0$, the characteristic equations for the similarity transformations will be

$$\frac{\mathrm{d}x}{x} = \frac{\mathrm{d}y}{0} = \frac{\mathrm{d}\Psi}{\Psi} = \frac{\mathrm{d}\Theta}{0} = \frac{\mathrm{d}\Phi}{0}, \tag{3.16}$$

then the similarity variable and functions can be obtained from (3.16)

$$\Psi = xG(y), \ \ \Theta = \Theta(y), \ \ \Phi = \Phi(y). \tag{3.17}$$

Substituting (3.17) into (2.7)-(2.9) obtains the following ordinary equations

$$\begin{aligned} &\frac{\mathrm{d}^3 G}{\mathrm{d}y^3} + KG^2 \frac{\mathrm{d}^3 G}{\mathrm{d}y^3} + (1 + M^2 K) G \frac{\mathrm{d}^2 G}{\mathrm{d}y^2} - 2KG \frac{\mathrm{d}G}{\mathrm{d}y} \frac{\mathrm{d}^2 G}{\mathrm{d}y^2} \\ &- \left(\frac{\mathrm{d}G}{\mathrm{d}y} \right)^2 - M^2 \frac{\mathrm{d}G}{\mathrm{d}y} = 0, \end{aligned} \tag{3.18}$$

$$\frac{\mathrm{d}^2 \Theta}{\mathrm{d}y^2} + Pr[G \frac{\mathrm{d}\Theta}{\mathrm{d}y} + Nb \frac{\mathrm{d}\Theta}{\mathrm{d}y} \frac{\mathrm{d}\Phi}{\mathrm{d}y} + Nt \left(\frac{\mathrm{d}\Theta}{\mathrm{d}y} \right)^2] = 0, \tag{3.19}$$

$$\frac{\mathrm{d}^2 \Phi}{\mathrm{d}y^2} + LePrG \frac{\mathrm{d}\Phi}{\mathrm{d}y} + \frac{Nt}{Nb} \frac{\mathrm{d}^2 \Theta}{\mathrm{d}y^2} = 0. \tag{3.20}$$

The boundary conditions (2.13) become

$$\begin{aligned} &\frac{\mathrm{d}G(0)}{\mathrm{d}y} = 1, \ G(0) = S, \ \Theta(0) = 1, \ \Phi(0) = 1, \\ &\frac{\mathrm{d}G(\infty)}{\mathrm{d}y} = 0, \ \ \Theta(\infty) = 0, \ \Phi(\infty) = 0. \end{aligned} \tag{3.21}$$

4 Numerical solutions and discussion

Since equations (3.18)-(3.20) together with the boundary conditions (3.21) are coupled nonlinear boundary value problem, a numerical treatment would be more appropriate. Thus, these equations are solved numerically by Bvp4c with MATLAB. Since the velocity changes sharply in the boundary layer near the plate, this region with sharp change makes this boundary value problem a relatively difficult one. In order to resolve better the boundary layer and obtain a more accurate solution, the relative error tolerance on the residuals is defined to be 10^{-6}(i.e. RelTol=10^{-6}) during the process of numerical computation. The results are presented graphically and in tables.

For the verification of accuracy of the applied numerical scheme, a comparison of the present results corresponding to $-\Theta'(0)$ and $-\Phi'(0)$ with the ones obtained by Nadeem et al. [15] and Khan et al. [30] are presented in Table 1, which shows a favorable agreement. Table 2 presents the numerical values of $-\Theta'(0)$ and $-\Phi'(0)$ for various values of the Brownian motion parameter Nb and the thermophoresis parameter Nt when other parameters are fixed. It can be observed that $\Theta'(0)$ is an increasing function of the thermophoresis parameter Nt and the Brownian motion parameter Nb, respectively. However, $\Phi'(0)$ is the increasing function of the thermophoresis parameter Nt. When we consider the influence of the Brownian motion parameter Nb on $\Phi'(0)$, the case is different. $\Phi'(0)$ is the decreasing function of the Brownian motion parameter Nb.

In the following section, we will investigate the influence of different physical parameters on the velocity, temperature distribution and nanoparticle fraction. To begin with, we discuss the effects of the elastic parameter K and the Hartmann number M on the velocity $G'(y)$. Figure 1 shows that the velocity boundary layer for injection is thicker than the one for suction. For the case of injection, the boundary layer thickness reduces with the greater elastic parameter K. However, the boundary layer thickness has increasing behavior with increasing the elastic parameter K when there exists suction. Figure 1 also reveals that the velocity $G'(y)$ is sensitive to the tiny change of the elastic parameter K as there is suction at the wall. Figure 2 illustrates the effects of the Hartmann number on the velocity. The boundary layer thickness decreases with the increasing Hartmann number M for both injection and suction. Physically, the magnetic field is normal to the fluid, so for greater values of the Harmann number, it increases the resistance of the fluid flow.

Figures 3, 4 illustrate the effects of the thermophoresis parameter Nt and the Brownian motion parameter Nb on the temperature distribution and the nanoparticle fraction. No matter there exists suction or injection at the wall, the values of temperature $\Theta(y)$ and nanoparticle fraction $\Phi(y)$ tend to zero as both of them are far away from the wall. Furthermore, temperature $\Theta(y)$ is the increasing function of the thermophoresis parameter Nt and the Brownian motion parameter Nb. In physical meaning, the enhancement of the Brownian motion can improve the heat transfer.

Figure 5 shows the influence of Prandtl number on the temperature distribution. When there is injection at

Table 1: The comparison of $-\Theta'(0)$ and $-\Phi'(0)$ for $Pr = 10$, $Le = 1$ and $Nb = 0.1$

	Present results,	$K = M = S = 0$	$K = M = 0$	Nadeem et al. [15]	Khan et al. [30]	
Nt	$-\Theta'(0)$	$-\Phi'(0)$	$-\Theta'(0)$	$-\Phi'(0)$	$-\Theta'(0)$	$-\Phi'(0)$
0.1	0.952368	2.129391	0.9524	2.1294	0.9524	2.1294
0.2	0.693152	2.274048	0.6932	2.2732	0.6932	2.2740
0.3	0.520053	2.528702	0.5201	2.5286	0.5201	2.5286
0.4	0.402556	2.795254	0.4026	2.7952	0.4026	2.7952
0.5	0.321031	3.035231	0.3211	3.0351	0.3211	3.0351

Table 2: The values of $-\Theta'(0)$ and $-\Phi'(0)$ for $S = 1$, $M = 0.5$, $K = 0.5$,$Pr = 3$, $Le = 5$

	Nt=0.2		Nt=0.4		Nt=0.6		Nt=0.8	
Nb	$-\Theta'(0)$	$-\Phi'(0)$	$-\Theta'(0)$	$-\Phi'(0)$	$-\Theta'(0)$	$-\Phi'(0)$	$-\Theta'(0)$	$-\Phi'(0)$
0.2	1.713793	14.649150	1.365813	14.072039	1.109905	13.856683	0.919093	13.840343
0.4	1.017482	15.594233	0.810089	15.506259	0.657893	15.515114	0.544569	15.573292
0.6	0.595470	15.818806	0.473826	15.839852	0.384663	15.891683	0.318325	15.956364
0.8	0.344464	15.888933	0.274003	15.939527	0.222391	15.998269	0.184011	16.057833

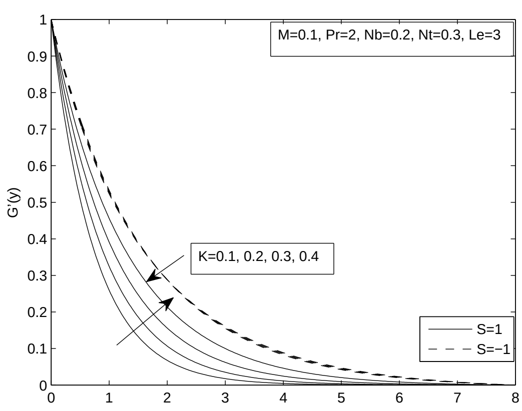

Figure 1: Effects of K on the velocity $G'(y)$.

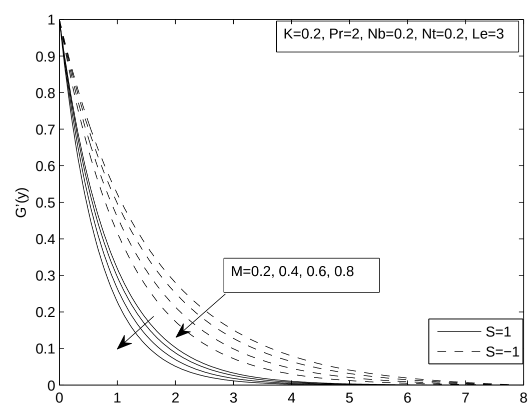

Figure 2: Effects of M on the velocity $G'(y)$.

the wall, the temperature boundary layer becomes thinner and is a decreasing function of Prandtl number. However, when there is suction at the wall, the temperature is increasing function of Prandtl number. Physically, the increase of Prandtl number also means the enhancement of the heat transfer, which leads to the thermal boundary layer becomeing thinner.

The effects of Lewis number Le, Prandtl number Pr and thermophoresis parameter Nt on the nanoparticle fraction $\Phi(y)$ are illustrated in Figures 6 – 8. It is observed that the effects of above parameters exhibit different trends as there is suction or injection at the plate. The nanoparticle fraction is the decreasing function of Lewis number and the Prandtl number, but the increasing function of the thermophoresis parameter when there is suction at the wall. However, when there is injection at the wall, the nanoparticle fraction changes its trends as the variable y changes from zero to the infinity.

5 Conclusion

Using group-theoretical methods, we have investigated the similarity solutions of MHD Maxwell fluid past a stretching plate with suction/injection in the presence of nanoparticles. By determining the transformation group, we can obtain the information about the invariants and symmetries of these equations. In turn, with the assistance of these invariants and symmetries, the governing equa-

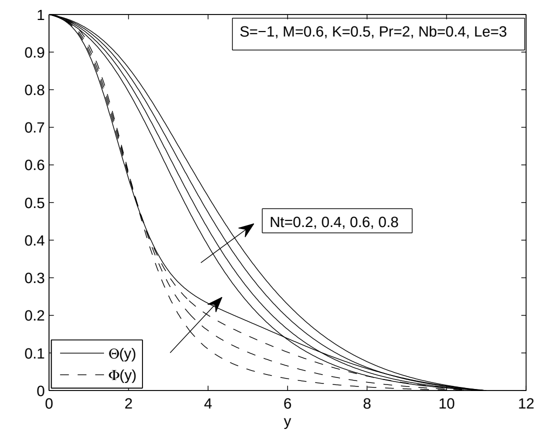

Figure 3: Effects of Nt on the temperature $\Theta(y)$ and nanoparticle fraction $\Phi(y)$.

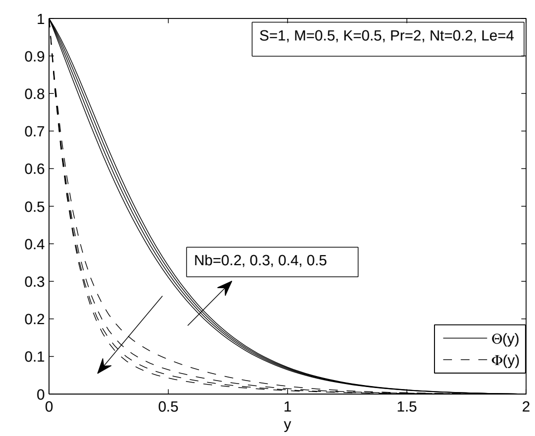

Figure 4: Effects of Nb on the temperature $\Theta(y)$ and nanoparticle fraction $\Phi(y)$.

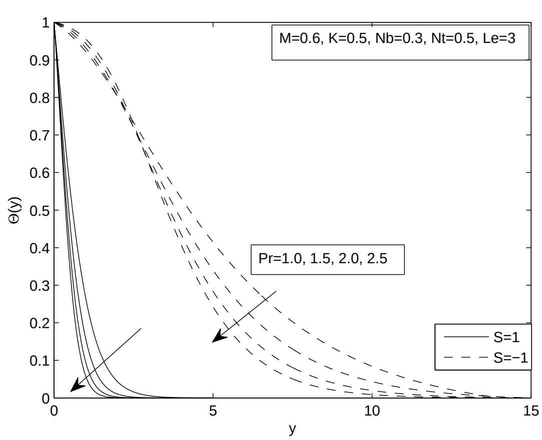

Figure 5: Effects of Pr on the temperature $\Theta(y)$.

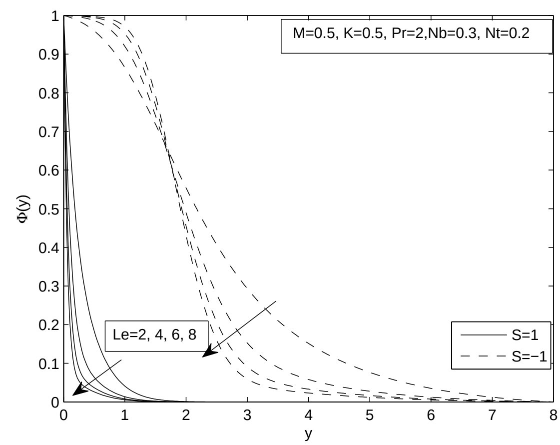

Figure 6: Effects of Le on the nanoparticle fraction $\Phi(y)$.

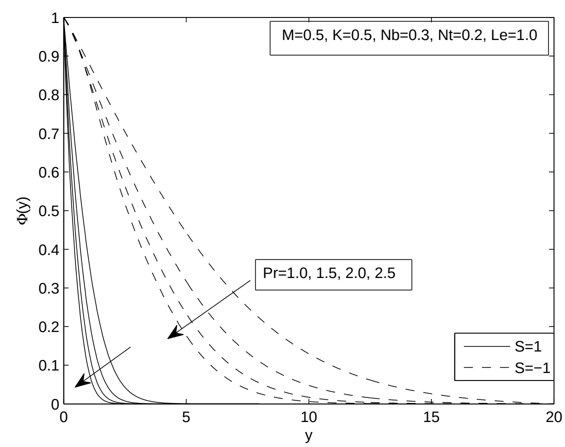

Figure 7: Effects of Pr on the nanoparticle fraction $\Phi(y)$.

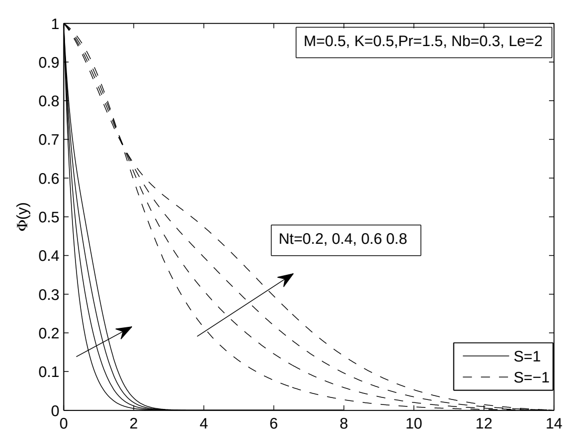

Figure 8: Effects of Nt on the nanoparticle fraction $\Phi(y)$.

tions are reduced to a system of coupled nonlinear differential equations. The physical effects of different parameters, such as the Hartmann number M,the elastic parameter K, the Prandtl number Pr, the Lewis number Le, the Brownian motion parameter Nb, and the thermophoresis parameter Nt, on velocity, temperature and nanoparticle fraction are analyzed by graphs and discussed in detail. Furthermore, one important conclusion can be drawn that the influence of these parameters on the velocity, temperature distribution and nanoparticle fraction is obviously different when there is suction or injection at the plate.

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