

Research Article

Open Access

Kjell Prytz*

Sources of inertia in an expanding universe

Abstract: In a cosmological perspective, gravitational induction is explored as a source to mechanical inertia in line with Mach's principle. Within the standard model of cosmos, considering the expansion of the universe and the necessity of retarded interactions, it is found that the assumed dynamics may account for a significant part of an object's inertia.

Keywords: inertia; gravitational induction; Mach's principle; cosmology; general relativity

PACS: 04.20.Cv; 04.80.Cc; 95.30.Sf; 98.80.Es; 98.80.Jk

DOI 10.1515/phys-2015-0016

Received August 27, 2014; accepted October 28, 2014

1 Introduction

The present day limit on the equivalence between gravitational and inertial mass is one part in 10^{12} [1, 2]. There have been many attempts trying to explain this equivalence, the main stream being based on Mach's principle stating that inertia originates in a mutual interaction between bodies [3, 4]. Frequently it has been assumed that the gravitational counterpart to the electrodynamic inductive effect is the appropriate interaction [4–9].

The present analysis further develops these ideas. Since the inductive interaction is inversely proportional to distance, its contribution will increase with distance assuming a homogeneous and isotropic universe. Since the universe is expanding and continuously changing it is necessary to take into account a retarded interaction. Through the measurements of WMAP [10] and Planck [11] collaborations, the scale factor, mass densities and Hubble's constant are today known with remarkable accuracy, making it meaningful to explore the potential of detailed inertia analyses in a cosmological perspective.

Similar analyses taking into account retarded interactions have been done before [12, 13]. In the summary section below the crucial differences between those analyses and this one will be pointed out.

2 The gravitational inductive interaction

The interaction considered dominates at large distance. For that reason the gravitational field is weak and the linearised weak-field approximation of Einstein's general relativity may be utilized. These were first derived in 1922 by Einstein in his book 'The meaning of relativity' [4]. The approximation may also be formulated as a set of equations equivalent to Maxwell's electrodynamic equations, then called the Gravitational ElectroMagnetic theory (GEM) [14]:

$$\nabla \cdot \vec{E} = 4\pi G\rho, \quad (1)$$

$$\nabla \cdot \left(\frac{1}{2} \vec{B} \right) = 0, \quad (2)$$

$$\nabla \times \left(\frac{1}{2} \vec{B} \right) = \frac{1}{c} \frac{\partial}{\partial t} \vec{E} + \frac{4\pi G}{c} \vec{j}, \quad (3)$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{B} \right), \quad (4)$$

where $\vec{E}$ and $\vec{B}$ are the gravitational fields corresponding to the electric and magnetic fields, ρ is the mass density, $\vec{j}$ is the mass current density, G is the gravitational constant and c is the speed of mediation which is taken as speed of light. Apart from some factors GEM theory is equivalent to Maxwell theory. This is not a coincidence since the magnetic and inductive effects are kinematic effects arising through Lorentz transformations of the static interactions [15]. An essential difference between Maxwell's electrodynamic equations and GEM is that the Faraday-Henry induction formula in the former case is inferred from closed conductor dynamics with charges moving along the closed path. In this case, Weber showed [16, 17] that the electrodynamic induction effect may be formulated as a force action

$$\vec{f}_{ind} = -\frac{q_1 q_2}{4\pi\epsilon_0 c^2 r} \frac{d\vec{v}_1}{dt} \quad (5)$$

for an interaction between object 1 and 2 at a distance r . In 1953, Sciama used Maxwell's equations to propose an analogue in gravitation. His result turns out to be the direct gravitational equivalent of the Weber formula

$$\vec{f}_{g-ind}^{Sciama} = G \frac{m_1 m_2}{c^2 r} \frac{d\vec{v}_1}{dt}. \quad (6)$$

*Corresponding Author: Kjell Prytz: University of Gavle, SE-80176 Gavle, Sweden, E-mail: Kjell.Prytz@hig.se, gun4nu@telia.com

Sciama was then able to qualitatively demonstrate a possible gravitational origin of inertia [6].

Before GEM theory was developed, Will and Nordtvedt expressed the linearised Einstein equations in a formalism called parametrised-post-Newtonian equations [18]. From these equations Nordtvedt derived in 1988 the gravitational inductive interaction in the weak-field limit [5]:

$$\vec{f}_{g-ind} = 4G \frac{m_1 m_2}{c^2 r} \frac{d\vec{v}_1}{dt} \quad (7)$$

valid for free masses. A similar formula is found in Einstein's book from 1922 [4], where Einstein claims the gravitational inductive dynamics to be the source of inertia, although no quantitative analysis was done. In the latest edition of the book from 1956 the discussion and conclusion about inertia are unchanged. Formula (7) may also be obtained from the GEM formulation of ref. [14].

In the present analysis, formula (7) will be used to estimate its contribution to inertial mass.

3 Inertia from Mach's principle

Frequently Mach's principle is discussed and analysed with respect to a rotating earth-bound frame exhibiting inertia through centrifugal effects. Through the interaction with the distant stars, which rotate in the rest frame of the observer, inertia is generated. This is qualitatively explained through an interaction via the inductive gravitational force, the acceleration being centripetal.

In this paper, the simpler case of linear motion is considered for which Mach's principle works in the following conceptual way. Consider the mutual interaction of a pair of uncharged massive objects with respect to the gravitational inductive force, Figure 1.

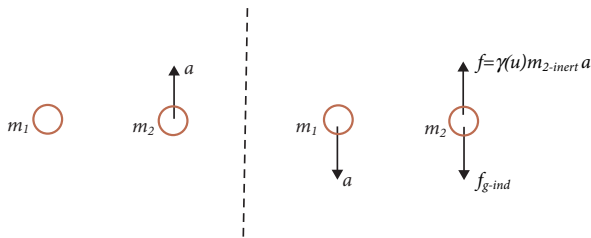


Figure 1: Left: Object m_2 accelerates by an external influence. Right: The inductive effect is obtained in the rest frame of object 2. The apparent acceleration of object 1 generates the inductive force on object 2.

Let object 2 accelerate externally (left figure). In the rest frame of object 2, object 1 accelerates in the opposite direction (right figure). The acceleration of object 1 causes a same directed inductive force on object 2. Since in the rest frame of object 2 the total force vanishes:

$$4G \frac{m_1 m_2}{c^2 r} a = \gamma(u) m_{2-inert} a \quad (8)$$

is obtained, where m_1 and m_2 are the gravitational masses, $m_{2-inert}$ is the inertial mass, and $\gamma(u)$ is the relativistic factor depending on the speed u of the object. While the speed due to the external acceleration is assumed to be small, there is no limit on the speed u (apart from being less than speed of light) since it is due to the expansion of the universe. The relativistic factor $\gamma(u)$ is therefore introduced treated as a constant for each elementary interaction to be considered in this context.

Apart from this relativistic factor $\gamma(u)$, Nordtvedt derived and used formula (8) to estimate the inertia contribution of the earth due to self-gravitation, i.e. the inductive interaction among the mass elements of the body when externally accelerated as a whole. It was found that the contribution amounts to one part in 10^9 and is therefore not negligible [5].

4 Cosmology

Nordtvedt's formula (8) will now be applied to estimate the contribution to inertia of a mass m_2 due to the gravitational inductive interaction from all the other masses of the known and casually connected universe. This complements Nordtvedt's self-gravitational calculation. The mass m_1 corresponds accordingly to the rest of the universe. When the mass m_2 is externally accelerated all other mass elements of the universe will act as in Figure 1.

The inertial mass is then given by

$$m_{2-inert} = \frac{4Gm_2}{c^2} \int_V \frac{\rho(t_e)dV}{\gamma(u)r(t_0)}, \quad (9)$$

where the mass density $\rho(t_e)$ and volume element dV are given at 'emission time' (retarded time) t_e . $r(t_0)$ is the distance between m_2 and the mass element $\rho(t_e)dV$ at observation time t_0 and V is the present volume of the universe. The speed u is obtained from Hubble's law, i.e. the distance between the objects at observation time t_0 times Hubble's constant. The time coordinates are defined as the time after Big Bang which appears at $t = 0$.

Since the universe is expanding during the interaction, the distance at observation time depends on the ex-

pansion rate. Therefore a scale factor $a(t)$ is introduced describing the expansion rate, defined as

$$r(t_0) = \int_{t_e}^{t_0} \frac{cdt}{a(t)}, \quad (10)$$

where the integral ranges from emission to observation time [19]. The scale factor is obtained from the standard cosmological model, the concordance Λ CDM, based on the Friedman-Robertson-Walker metric.

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda} \right)^{1/3} \left[\sinh\left(\frac{3}{2} \sqrt{\Omega_\Lambda} H_0 t\right) \right]^{2/3} \quad (11)$$

is obtained, valid from the time of matter domination to infinite time [20]. This is relevant in our case since the radiation dominated period is so short and far away in time that it gives negligible contribution. $H_0 = 2.17 \cdot 10^{-18} \text{ s}^{-1}$ is Hubble's constant. Ω is the fractional density of respective source consisting of

- Radiation $\rho_r(t_0) = 8.4 \cdot 10^{-31} \text{ kg/m}^3$
- Matter (including dark matter)
 $\rho_m(t_0) = 0.32 \cdot 10^{-26} \text{ kg/m}^3$
- Dark energy $\rho_\Lambda(t_0) = 0.68 \cdot 10^{-26} \text{ kg/m}^3$,

where all data are taken from the results of the Planck collaboration [11]. The evolutions of the densities are controlled by the scale factor such that [19]

$$\rho(t_e) = \frac{\rho_r(t_0)}{a(t_e)^4} + \frac{\rho_m(t_0)}{a(t_e)^3} + \rho_\Lambda(t_0) \quad (12)$$

so that $a(t_0) = 1$. The density of dark energy is constant over time.

It is assumed that space is flat in accordance with observation [11]. The physical volume element at the retarded time dV is then given by

$$dV = a(t_e)^2 r(t_0)^2 a(t_e) dr(t_0) d\Psi, \quad (13)$$

where $d\Psi$ is the element of solid angle. Assuming an isotropic universe $\int d\Psi = 4\pi$. The formula under consideration (9) may then be written

$$\begin{aligned} m_{2\text{-inert}} &= \frac{4Gm_2}{c^2} \int_V \frac{\frac{\rho_r(t_0)}{a(t_e)^4} + \frac{\rho_m(t_0)}{a(t_e)^3} + \rho_\Lambda(t_0)}{\gamma(u)r(t_0)} dV = \\ &= \frac{4Gm_2}{c^2} \int_V \frac{\frac{\rho_r(t_0)}{a(t_e)^4} + \frac{\rho_m(t_0)}{a(t_e)^3} + \rho_\Lambda(t_0)}{\gamma(u)r(t_0)} \\ &\times a(t_e)^2 r(t_0)^2 a(t_e) dr(t_0) d\Psi \approx \frac{4Gm_2}{c^2} \\ &\times \int_V \frac{\left[\frac{\rho_m(t_0)}{a(t_e)^3} + \rho_\Lambda(t_0) \right] a(t_e)^3 r(t_0)}{\gamma(u)} dr(t_0) d\Psi, \end{aligned}$$

where the radiation density has been neglected since it gives negligible contribution. The integration over the distance is taken from 0 to the Hubble radius R_H since at this distance the objects move at speed of light relative each other and no graviton can be absorbed. The fractional contribution φ to inertia due to gravitational induction becomes

$$\varphi = \frac{16\pi G}{c^2} \int_0^{R_H} \frac{[\rho_m(t_0) + \rho_\Lambda(t_0)a(t_e)^3] r(t_0)}{\gamma(u)} dr(t_0). \quad (14)$$

5 Results

The integral (14) was computed in Matlab in conjunction with formula (10) to get the emission time t_e . Technically this was done by searching for the solution w.r.t. t_e of the equation

$$r(t_0) - \int_{t_e}^{t_0} \frac{cdt}{a(t)} = 0 \quad (15)$$

using the Matlab function 'fzero', for each value of $r(t_0)$. The age of the universe t_0 was taken as [19]

$$t_0 = \frac{1}{H_0} \int_0^1 \frac{ada}{(\Omega_m a + \Omega_\Lambda a^4)^{1/2}} = 4.36 \cdot 10^{17} \text{ s}. \quad (16)$$

The fractional contribution φ to inertia due to gravitational induction was then calculated as

$$\varphi = 1.15, \quad \varphi_m = 0.76, \quad \varphi_\Lambda = 0.39$$

where φ_m and φ_Λ are the contributions from matter (including dark matter) and dark energy respectively. The graph shows the relative contribution to inertia versus distance between objects.

The main contribution originates at an emission time more than 4 Gyears ago when matter dominated the universe. The reason is that the inductive force varies as $1/r$, making the inertia integral to increase linearly with distance. Accordingly, the past long distant sources dominate over those at close distances up to the point where the gamma factor starts to suppress the integrand regulating the necessary vanishing at the Hubble radius.

Although the dark energy density dominates today, it did not do so in the past since its density stays constant

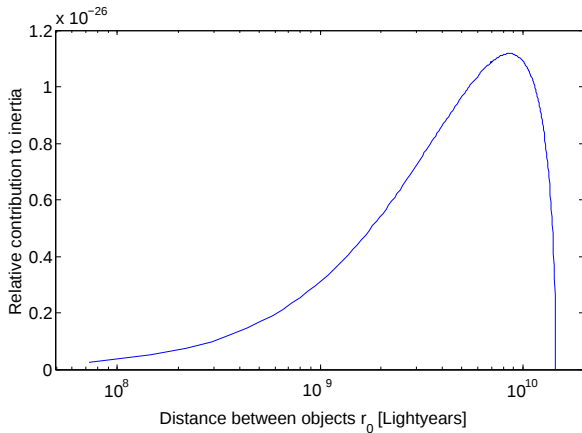


Figure 2: Relative contribution to inertia versus distance at observation time based on Formula (14).

while the matter density varies as $1/a^3$. Therefore the matter source gives the main contribution to inertia.

Within the assumed cosmological model, the concordance Λ CDM, the errors are due to experimental uncertainties of the densities and Hubble's constant together with the modelling uncertainties of the scale factor.

1. Scale factor

An overall uncertainty of 10% is assumed. This affects the result by 8%. A cross check with a different suggested parametrisation was done [19] where the Friedman formula is solved assuming single dominating sources of mass density giving different parametrisations for the different eras. Using these parametrisations gives almost the same result.

2. Mass density

Planck coll. gives [11]

$$\rho_m(t_0) = (0.32 \pm 0.02) \cdot 10^{-26} \text{ kg/m}^3,$$

$$\rho_\Lambda(t_0) = (0.68 \pm 0.02) \cdot 10^{-26} \text{ kg/m}^3,$$

resulting in an error of 7% of the fractional inertia.

3. Hubble's constant

Planck coll. gives

$$H_0 = (2.17 \pm 0.04) \cdot 10^{-18} \text{ s}^{-1}$$

giving an error of 7%.

Summing these sources in quadrature gives a total error of 13 %. Within the assumed model the gravitational inductive contribution to inertia finally becomes

$$\varphi = 1.15 \pm 0.15.$$

6 Summary and discussion

The existence of gravitational inductive effects within general relativity is well established. Since their nature is to counteract an external acceleration of a single object they contribute to inertia. The interaction cannot be identical to the electrodynamic counterpart since in this case the constituents are like charges and therefore statically repelling. Additionally, in electrodynamics the inductive force is inferred from studies of closed conductors. However, there ought to be an analogue since the inductive effects are kinematic and arise from a Lorentz transformation of the static interaction. Indeed, for free masses, statically attracting, the weak-field low velocity approximation of general relativity provides an inductive interaction similar to the Weber force in electrodynamics forming a basis for an inertia analysis in a cosmological perspective. To this end, a cosmological model is needed. Through the recent measurements of the satellite telescopes WMAP and Planck, a standard model of cosmology has emerged, the so-called concordance Λ CDM model utilized in this paper. This model is based on Big Bang and assumes the existence of dark matter and dark energy. The latter accounts for a universe in accelerated expansion and is related to Einstein's cosmological constant Λ . There are still many unanswered questions within this model such as the true nature of dark matter and dark energy. In this analysis these were treated as ordinary energies contributing to the inertia integral in the same manner as ordinary matter.

The well-known cosmological scale factor was used to find the density of matter/energy at the retarded time, i.e. time of graviton emission, which provides the mass of the cosmic element interacting with the considered object. In this way all mass elements of the known universe could be integrated. The total interaction appearing when the object under consideration is externally linearly accelerated was then found to be able to account for a substantial part of inertial mass of the object with a significant accuracy.

It was also found that far distant objects, i.e. objects more than 5 Gly away, contribute mostly to inertia, corresponding to the era of the matter dominated universe. This far distant interaction together with low externally generated velocity of the considered object are the necessary conditions to ensure the validity of the GEM approximation of general relativity. It is also in accordance with Mach's principle stating that the interactions causing inertia arise from the most distant objects, plausible since inertia is directionally independent.

Finally we comment on previous analyses in the retarded interaction picture [12, 13]. These are both based

on the Sciama force (6) differing by a factor of 4 from formula (7) derived within general relativity and used here. Furthermore, both previous works assume a causal sphere equal to the particle horizon whereas there is no need for an upper limit of the integral in (9) since it is regulated by the gamma factor appearing in the right hand side of formula (8). The cosmological model used in this analysis is equivalent to that used in ref. [13], although the latter did not have access to the recent cosmological data and therefore lacks an error analysis. However, ref. [12] assumes the Einstein-deSitter cosmological model with the critical density taken as the present day matter density, which bears very little resemblance with the contemporary understanding of the cosmos.

Acknowledgement: I have benefited from many useful comments from professors Johan Hansson at Lulea Technical University and Andre Koch Torres Assis at University of Campinas.

References

- [1] V.B. Braginsky, V.I. Panov, *Sov. Phys. JETP-USSR* 34, 463 (1972)
- [2] H.C. Ohanian, *Gravitation and Spacetime* (Norton & Company, New York and London, 1976)
- [3] E. Mach, *The Science of Mechanics* (The Open Court Publishing, Chicago and London, 1919)
- [4] A. Einstein, *The meaning of relativity* (Ed. 1: Methuen & Co Ltd 1922, Ed. 6: Chapman and Hall, London, 1956)
- [5] K. Nordtvedt, *Int. J. Theor. Phys.* 27, 1395 (1988)
- [6] D.W. Sciama, *Mon. Not. R. Astron. Soc.* 113, 34 (1953)
- [7] A.P. French, *Newtonian Mechanics* (The MIT Introductory Physics Series, New York and London, 1971)
- [8] A.K.T. Assis, *Found. Phys. Lett.* 26, 271 (1996)
- [9] D.J. Raine, *Rep. Prog. Phys.* 44, 1151 (1981)
- [10] N. Jarosik et al. (WMAP Collaboration), *Astrophys. J. Suppl. S.* 192, 14 (2011)
- [11] P.A.R. Ade et al. (Planck Collaboration), *Astron. Astrophys.* 571, A16 (2014)
- [12] L.R. Signore, *Nuovo. Ciment. B* 111, 1087 (1996)
- [13] J. Sultana, D. Kazanas, *Int. J. Mod. Phys. D* 20, 1205 (2011)
- [14] D. Bini et al., *Classical Quant. Grav.* 25, 1 (2008)
- [15] L. Page, *Am. J. Sci.* 34, 57 (1912)
- [16] W. Weber, *Ann. Phys-Berlin* 73, 229 (1848)
- [17] C.F. Gauss, *Zur mathematischen Theorie der elektrodynamische Wirkung* (Werke Göttingen 5, 1867)
- [18] C.M. Will, K. Nordtvedt, *Astrophys. J.* 177, 757 (1972)
- [19] B. Ryden, *Introduction to Cosmology* (Addison-Wesley, San Francisco, 2003)
- [20] E. Kolb, M. Turner, *The Early Universe* (Westview Press, Boulder, Colorado, 1994)