

Research Article

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Single state feedback stabilization of unified chaotic systems and circuit implementation

Abstract: This paper focuses on the single state feedback stabilization problem of unified chaotic system and circuit implementation. Some stabilization conditions will be derived via the single state feedback control scheme. The robust performance of controlled unified chaotic systems with uncertain parameter will be investigated based on maximum and minimum analysis of uncertain parameter, the robust controller which only requires information of a state of the system is proposed and the controller is linear. Both the unified chaotic system and the designed controller are synthesized and implemented by an analog electronic circuit which is simpler because only three variable resistors are required to be adjusted. The numerical simulation and control in MATLAB/Simulink is then provided to show the effectiveness and feasibility of the proposed method which is robust against some uncertainties. The results presented in this paper improve and generalize the corresponding results of recent works.

Keywords: unified chaotic system; single state feedback control; robust stabilization; circuit implementation

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1 Introduction

In 1963, Lorenz found the first canonical chaotic system [1]. Chaos has been found to be useful in analysing many problems, due to the potential applications in economics, physics, control theory, secure communication, information processing, collapse prevention of power systems, high-performance circuits and device, and so on, which has been intensively studied in science and engineering. A chaotic system is a nonlinear deterministic system that displays complex, noisy-like and unpredictable behavior. These motions are trajectories in which infinite unstable periodic orbits (UPOs) are embedded. Chaos is generally undesirable in many fields. This irregular and complex phenomenon can lead systems to harmful or even catastrophic situations. In these troublesome cases chaos should be suppressed as much as possible or totally eliminated. In fact, this is particularly significant because most realistic chaotic systems need to be operated in regular regimes. Therefore controlling chaos has become one of the most considerable research area in the nonlinear problems ranging from biology, physics and chemistry to economics. An important challenge in chaos theory is the control of chaotic system, that is, to make the chaotic systems reach steady states or regular behavior [2–4].

In 1990, Ott, Grebogi, and Yorke proposed the first approach of chaos control, that is, OGY approach [5], thereafter enormous research activities have been carried out in chaos control by many researchers from different disciplines, and lots of successful experiments have been reported in nonlinear dynamics [6]. Indeed, chaos control theory has advanced far beyond theoretical perspectives to experimental realizations (see [7, 8] and the references therein). For this reason, designing simple and available control input is extremely relevant for experimental chaos control. Besides the OGY method, many other control algorithms have been proposed in recent years to control chaotic systems, such as PC method [9–11], feedback approach [12, 13], adaptive control [14–16], linear state space

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feedback [17], backstepping method [18], nonlinear feedback control [19], sliding mode control [20, 21], neural network control [22], fuzzy logic control [23], robust control [24–30], passivity theory control [31], adaptive passive control [32], time-delay feedback approach [33, 34], multiple delay feedback control [35], double delayed feedback control [36], hybrid control [37], etc. These control algorithms can be used to stabilize a desired unstable periodic orbit (UPO) embedded within a chaotic attractor.

An important aspect of the problem of chaos control is how to realize the control of chaotic system by designing a simple and physically available controller [38], which is particularly significant both for theoretical research and practical applications. In practice, the problem of controller complexity is a very crucial issue and two fundamental issues in this direction have been identified [38, 39]: one is that the cost implication and the density requirement for designing controllers, the other is that the need to make the complexity of the controller to be, at least comparable to, or less than, the device being controlled, if the controlling technique is desired to achieve a useful end far beyond mere scientific curiosity. Indeed, the entire control goal would be untenable if a simple chaotic system requires a massively complex controller like the controller proposed in [40]. In order to solve it, some efforts have been made to deal with this problem [41]. However, in the formulation of the control problem, the proposed controllers in previous works are, in most cases too complex both in design and implementation and too single. Outstanding among the various methods of chaos control is adaptive control [15, 42], where the authors did not give a condition to reduce the number of the feedback gains ($\varepsilon_i, i = 1, 2, 3$). Although, this method can contain single feedback gain in numerical simulations, it is only a numerical result. The method in [43] is a modification for the method in [42], but the controller obtained by using the method in [43] is more complex than that controller obtained by our method, we shall give a concrete example to demonstrate in Section 4.

In circuit design and implementation of aspects of chaotic systems, the circuit implementation of the individual Lorenz system, Chen's system and Lü system can be found in [44–46] and [47, 48], respectively. In this paper, we also focus on the circuit design and implementation of the unified chaotic system [49] so that different attractors can be obtained with simple adjusting. This new circuit is based on the design in [48, 50]. However, it is simpler because only three variable resistors are required to be adjusted. In our experiments, it is illustrated that different chaotic attractors can be duly obtained by adjusting three variable resistors in the circuit.

Motivated by the above discussion, the problem of stabilization of unified chaotic systems and circuit implementation will be investigated by using the single state feedback control and analog electronic circuit analysis in this paper. First, partly improving and extending the previous chaos control techniques (see [43, 51–59]), the single state feedback control is designed, some stabilization conditions will be derived, some simpler controllers are developed. Then, the robust performance of controlled unified chaotic systems with uncertain parameters will be investigated based on maximum and minimum analysis of the uncertain parameter, and the present controller only contains a single state feedback. Finally, a more standardized and more unified circuit of unified chaotic systems is developed. Furthermore, experiment of improved circuit and numerical simulations are applied to verify the effectiveness and feasibility of the proposed method. In fact, the proposed method in this paper at least has advantages over the method in the following two aspects: (a) The controller obtained in this paper is simpler than the controller obtained by previous method in [43]; (b) The utility of the proposed method is easier to physically realize than the method obtained in [50], that is, which is more suitable circuit implementation.

The rest of the paper is organized as follows. Section 2 gives a mathematical description and analysis of unified chaotic systems. Sections 3–4 propose the main results of this paper, the single state feedback controller is proposed to stabilize of unified chaotic systems in section 3, the robust stabilization of uncertain unified chaotic system is presented and investigated in section 4. Then, experiment of circuit and numerical simulations results are also given in Section 5 and 6, respectively. The paper will be closed by a conclusion in section 7.

2 System analysis

Based on the Lorenz system, Lü system and Chen system, the unified chaotic system was proposed by Lü [49]. The unified chaotic system is described by

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x), \\ \dot{y} = (28 - 35\alpha)x + (29\alpha - 1)y - xz, \\ \dot{z} = xy - \frac{8+\alpha}{3}z. \end{cases} \quad (1)$$

where $[x, y, z]^T$ is the state variables of unified chaotic system and $\alpha \in [0, 1]$ is a system parameter.

Due to the system (1) being chaotic for arbitrarily $\alpha \in [0, 1]$ and the system (1) belonging to the generalized Lorenz chaotic system for $0 \leq \alpha < 0.8$, the system (1)

belonging to Lü chaotic system at $\alpha = 0.8$ and the system (1) belonging to the generalized Chen chaotic system for $0.8 < \alpha \leq 1$, so the system (1) is regarded as unified chaotic system.

The equilibria of system (1) can be easily found by solving the three equations $\dot{x} = \dot{y} = \dot{z} = 0$, which lead to

$$\begin{cases} (25\alpha + 10)(y - x) = 0, \\ (28 - 35\alpha)x + (29\alpha - 1)y - xz = 0, \\ xy - \frac{8+\alpha}{3}z = 0. \end{cases}$$

it can be easily verified that there are three equilibria:

$$\begin{aligned} S_0(0, 0, 0), \\ S_-(-\sqrt{(8+\alpha)(9-2\alpha)}, -\sqrt{(8+\alpha)(9-2\alpha)}, 27-6\alpha), \\ S_+(\sqrt{(8+\alpha)(9-2\alpha)}, \sqrt{(8+\alpha)(9-2\alpha)}, 27-6\alpha). \end{aligned}$$

where two equilibria, S_- and S_+ , are symmetrically placed with respect to the z -axis.

The Jacobic matrix of system (1) at S_0 is given by

$$A_0 = \begin{bmatrix} -(25\alpha + 10) & 25\alpha + 10 & 0 \\ 28 - 35\alpha & 29\alpha - 1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}.$$

Eigenvalues of system (1) are

$$\begin{aligned} \lambda_1 &= -\frac{8+\alpha}{3}, \\ \lambda_2 &= -2\alpha - \frac{11}{2} + \frac{1}{2}\sqrt{-584\alpha^2 + 2372\alpha + 1201}, \\ \lambda_3 &= -2\alpha - \frac{11}{2} - \frac{1}{2}\sqrt{-584\alpha^2 + 2372\alpha + 1201} \end{aligned}$$

and the following characteristic equation is

$$f(\lambda) = \left(\lambda + \frac{8+\alpha}{3}\right) [\lambda^2 + (11-4\alpha)\lambda + (25\alpha+10)(6\alpha-27)] = 0.$$

Since $\alpha \in [0, 1]$, it always satisfies both $11 - 4\alpha > 0$ and $(25\alpha + 10)(6\alpha - 27) < 0$, and the two eigenvalues satisfy $\lambda_2 > 0 > \lambda_3$. So, the equilibrium S_0 is a saddle point in the three dimensional phase space and the system (1) is unstable.

Similarly, the system about the other equilibria yields the following characteristic equation:

$$\begin{aligned} f(\lambda) = & \lambda^3 + \frac{41-11\alpha}{3}\lambda^2 + \frac{(38-10\alpha)(\alpha+8)}{3}\lambda \\ & + 2(25\alpha+10)(\alpha+8)(9-2\alpha) = 0. \end{aligned}$$

Obviously, the two equilibria $S_{\pm}$ have the same characterization.

Since, the inequalities

$$\begin{cases} \frac{41-11\alpha}{3} > 0, \\ \frac{(38-10\alpha)(\alpha+8)}{3} > 0, \\ 2(25\alpha+10)(\alpha+8)(9-2\alpha) > 0, \\ \frac{(41-11\alpha)(38-10\alpha)(\alpha+8)}{9} > 2(25\alpha+10)(\alpha+8)(9-2\alpha), \\ 0 \leq \alpha \leq 1 \end{cases}$$

haven't solutions. So, according to Routh-Hurwitz criterion, the system (1) at $S_{\pm}$ is unstable, too.

In this paper, the goal of design problem for system (1), one use the single state feedback control approach by Yu [51], is to find the simplest control scheme for the controlled system, which ensure that the system (1) will gradually converge to the unsteady equilibrium position S_0 , there is no loss of generality in doing so because any equilibrium point can be shifted to the origin via a change of variables.

3 Single state feedback control

Some theorems stated below will enforce the desired gradually converge to the unstable equilibrium position S_0 . The control scheme for system (1) is presented in the following theorems:

Theorem 1. For the controlled unified chaotic system

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x) + kx, \\ \dot{y} = (28 - 35\alpha)x + (29\alpha - 1)y - xz, \\ \dot{z} = xy - \frac{8+\alpha}{3}. \end{cases} \quad (2)$$

If the feedback coefficient satisfies

$$k < \frac{(35\alpha - 28)(25\alpha + 10)}{1 - 29\alpha} + 25\alpha + 10, \text{ at } \alpha \in \left[0, \frac{1}{29}\right). \quad (3)$$

Then the controlled system (2) is asymptotically stable at the equilibrium S_0 .

Proof. The Jacobian matrix of controlled system (2) at equilibrium S_0 is

$$A_{c_1} = \begin{bmatrix} -(25\alpha + 10) + k & 25\alpha + 10 & 0 \\ 28 - 35\alpha & 29\alpha - 1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}.$$

The characteristic equation is

$$f(\lambda) = \left(\lambda + \frac{8+\alpha}{3}\right) \cdot g(\lambda) = 0, \quad (4)$$

where $g(\lambda) = \lambda^2 + (11 - 4\alpha - k)\lambda + (25\alpha + 10 - k)(1 - 29\alpha) + (35\alpha - 28)(25\alpha + 10)$.

According to the Routh-Hurwitz criterion, all roots of (4) have negative real parts if and only if

$$\begin{cases} 11 - k - 4\alpha > 0, \\ -(25\alpha + 10 - k)(29\alpha - 1) + (35\alpha - 28)(25\alpha + 10) > 0. \end{cases} \quad (5)$$

It is easy to know that the condition (5) is equivalent to condition (3). $\square$

Remark 1. Based on the Lyapunov stabilization theory and matrix measure, the strategies of speed feed back control of chaotic system to the unsteady equilibrium points S_- and S_+ were achieved [52]. However, the state of system (1) cannot converge to the unsteady equilibrium point $S_0(0, 0, 0)$. But, it can be achieved in this paper.

Theorem 2. For the controlled unified chaotic system

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x) + ky, \\ \dot{y} = (28 - 35\alpha)x + (29\alpha - 1)y - xz, \\ \dot{z} = xy - \frac{8+\alpha}{3}z. \end{cases} \quad (6)$$

If the feedback coefficient satisfies

$$k < \frac{(25\alpha + 10)(29\alpha - 1)}{35\alpha - 28} - 25\alpha - 10, \quad \text{at } \alpha \in [0, 0.8]. \quad (7)$$

or

$$k > \frac{(25\alpha + 10)(29\alpha - 1)}{35\alpha - 28} - 25\alpha - 10, \quad \text{at } \alpha \in (0.8, 1]. \quad (8)$$

Then the controlled system (6) is asymptotically stable at the equilibrium S_0 .

Proof. The Jacobian matrix of controlled system (6) at equilibrium S_0 is

$$A_{c_2} = \begin{bmatrix} -(25\alpha + 10) & 25\alpha + 10 + k & 0 \\ 28 - 35\alpha & 29\alpha - 1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}.$$

The characteristic equation is

$$f(\lambda) = \left(\lambda + \frac{8+\alpha}{3} \right) \cdot g(\lambda) = 0, \quad (9)$$

where $g(\lambda) = \lambda^2 + (11 - 4\alpha)\lambda - (25\alpha + 10)(29\alpha - 1) + (35\alpha - 28)(25\alpha + 10 + k)$.

According to the Routh-Hurwitz criterion, all roots of (9) have negative real parts if and only if

$$\begin{cases} 11 - 4\alpha > 0, \\ -(25\alpha + 10)(29\alpha - 1) + (35\alpha - 28)(25\alpha + 10 + k) > 0. \end{cases} \quad (10)$$

It is easy to find that the condition (10) is equivalent to condition (7) or (8). $\square$

Remark 2. The same problem is investigated in [51]. In [51], there are multiple hypothesis and there must be a nonsingular coordinate transformation $y = Tx$. The original system is transformed to the cascade form by the $y = Tx$. The nonlinear function of driving subsystem must be smooth and the driving subsystem must be uniformly exponentially stable

about the origin. However, these restricted conditions are removed in this paper. Therefore, the results presented in this paper improve the corresponding results of recent works.

Theorem 3. For the controlled unified chaotic system

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x), \\ \dot{y} = (28 - 35\alpha)x + (29\alpha - 1)y - xz + kx, \\ \dot{z} = xy - \frac{8+\alpha}{3}z. \end{cases} \quad (11)$$

If the feedback coefficient satisfies

$$k < 6\alpha - 27. \quad (12)$$

Then the controlled system (11) is asymptotically stable at the equilibrium S_0 .

Proof. The Jacobian matrix of controlled system (11) at equilibrium S_0 is

$$A_{c_3} = \begin{bmatrix} -(25\alpha + 10) & 25\alpha + 10 & 0 \\ 28 - 35\alpha + k & 29\alpha - 1 & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}.$$

The characteristic equation is

$$f(\lambda) = \left(\lambda + \frac{8+\alpha}{3} \right) \cdot g(\lambda) = 0, \quad (13)$$

where $g(\lambda) = \lambda^2 + (11 - 4\alpha)\lambda - (25\alpha + 10)(29\alpha - 1) + (35\alpha - 28 - k)(25\alpha + 10)$.

According to the Routh-Hurwitz criterion, all roots of (13) have negative real parts if and only if

$$\begin{cases} 11 - 4\alpha > 0, \\ -(25\alpha + 10)(29\alpha - 1) + (35\alpha - 28 - k)(25\alpha + 10) > 0. \end{cases} \quad (14)$$

It is easy to know that the condition (14) is equivalent to condition (12). $\square$

Theorem 4. For the controlled unified chaotic system

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x), \\ \dot{y} = (28 - 35\alpha)x + (29\alpha - 1)y - xz + ky, \\ \dot{z} = xy - \frac{8+\alpha}{3}z. \end{cases} \quad (15)$$

If the feedback coefficient satisfies

$$k < 6\alpha - 27.$$

Then the controlled system (15) is asymptotically stable at the equilibrium S_0 .

Proof. The Jacobian matrix of controlled system (15) at equilibrium S_0 is

$$A_{c_4} = \begin{bmatrix} -(25\alpha + 10) & 25\alpha + 10 & 0 \\ 28 - 35\alpha & 29\alpha - 1 + k & 0 \\ 0 & 0 & -\frac{8+\alpha}{3} \end{bmatrix}.$$

The characteristic equation is

$$f(\lambda) = \left(\lambda + \frac{8+\alpha}{3} \right) \cdot g(\lambda) = 0, \quad (16)$$

where $g(\lambda) = \lambda^2 + (11 - 4\alpha - k)\lambda + (25\alpha + 10)(1 - 29\alpha - k) + (35\alpha - 28)(25\alpha + 10)$.

According to the Routh-Hurwitz criterion, all roots of (16) have negative real parts if and only if

$$\begin{cases} 11 - 4\alpha - k > 0, \\ (25\alpha + 10)(1 - 29\alpha - k) + (35\alpha - 28)(25\alpha + 10) > 0. \end{cases} \quad (17)$$

that is, $k < 6\alpha - 27$. $\square$

Remark 3. The same problem is investigated in [54]. The specific control scheme in [54] requires three controllers that can be stabilizing the unified chaotic system. However, one only uses the single state feedback control approach that can be stabilizing the unified chaotic system, the physical implementation of controller is also convenience and the question has also been much simplified in this paper.

4 Robust stabilization of uncertain unified chaotic system

In this section, the robust controller is proposed and designed to achieve the global stabilization of the unstable equilibria of uncertain unified chaotic system.

Consider the controlled unified chaotic system as follows:

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x), \\ \dot{y} = (28 - 35\alpha)x + (29\alpha - 1)y - xz + u, \\ \dot{z} = xy - \frac{8+\alpha}{3}z, \end{cases} \quad (18)$$

where $\alpha \in [0, 1]$ is a shifty parameter, u is designed controller.

Theorem 5. The single-state feedback controller

$$u = -(27 + \varepsilon)x \quad \text{or} \quad u = -(27 + \varepsilon)y \quad (19)$$

can be robust stabilization the controlled unified chaotic system (18), where $\varepsilon > 0$ is an arbitrarily constant.

Proof. Combined Theorem 3 with Theorem 4, the proved is quite simple, so, it is omitted. $\square$

Remark 4. The same problem is investigated in [53], the designed robust controller is nonlinear and associated with all states, that is, to involve product cross terms, and it is

also required that the uncertainty matrix is a linear combination of some constant matrixes. However, the robust controller in this paper is only associated with one state x or y , and it is linear. It is only required that uncertain parameter α is belong to a set $[0, 1]$. So, the results presented in this paper improve and generalize the corresponding results of recent works.

5 Circuit implementation and experiment

In order to verify the application of the proposed control law in this paper, we give an illustrative example of circuit experiment and simulation results on unified chaotic system (6) with $\alpha \in [0, 1]$, currently in its chaotic state.

5.1 System diagram

In order to implement the above system (6) with above-derived controller gain (7), the first step must be to recognize the main functional block of system (6), as presented in Figure 1, when the switch is on, the feedback control signal is applied.

5.2 Circuit design

Each block of this system can be realized by analog electronic components. However, it is required to scale the system in its state variables to slow down, and in order to avoid saturations of operational amplifiers. Hence, all the state variables are reduced by $k = 10$ times, that is, one using a nonsingular coordinate transformation $\frac{[x, y, z]^T}{10} = [u_1, u_2, u_3]^T$, then the system (1) is equivalent to

$$\begin{cases} \dot{u}_1 = (25\alpha + 10)(u_2 - u_1), \\ \dot{u}_2 = 100 \left[\left(\frac{28}{100}u_1 - \frac{1}{100}u_2 \right) - \frac{u_1 u_3}{10} - \left(\frac{35\alpha}{100}u_1 - \frac{29\alpha}{100}u_2 \right) \right], \\ \dot{u}_3 = 100 \left[\frac{u_1 u_2}{10} - \frac{8+\alpha}{300}u_3 \right]. \end{cases} \quad (20)$$

First of all, one defines $\alpha = \frac{R}{10k\Omega}$.

5.2.1 Circuit design of state equation u_1

Consider a rewrite u_1 state equation as follows:

$$\dot{u}_1 = -\frac{1}{10^{-1}} \left[-(u_2 - u_1) - \frac{25}{10}\alpha(u_2 - u_1) \right].$$

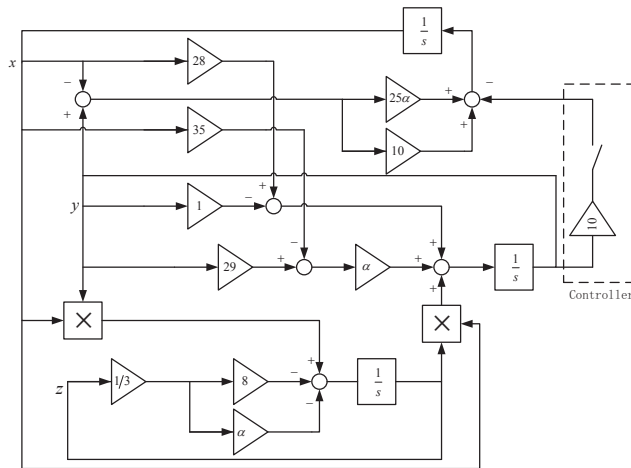


Figure 1: Blocks diagram for the controlled unified chaotic system (6) with controller (7).

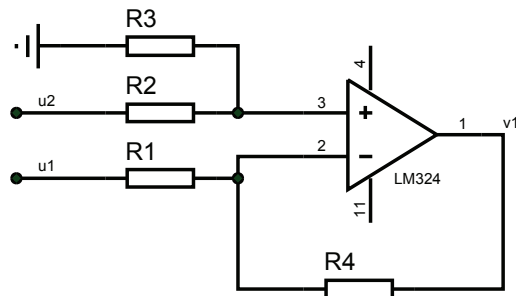


Figure 2: Circuit implementation of equation (21-1).

Outline 1.

- The coefficient of integral circuit is $-\frac{1}{RC}$, so, the constant $-\frac{1}{10^{-1}}$ can be realized by an integrator.
- The argument $\alpha \in [0, 1]$ can be realized by employing a slide rheostat.
- As a matter of convenience, one can be defined

$$v_1 \triangleq u_2 - u_1. \quad (21-1)$$

$$v_2 \triangleq \frac{25}{10} \alpha (u_2 - u_1). \quad (21-2)$$

$$\dot{u}_1 \triangleq \frac{-1}{10^{-1}} (-v_2 - v_1). \quad (21-3)$$

The equation (21-1) can be implemented by a circuit which as shown in Figure 2. From the Figure 2, it is easy to know that

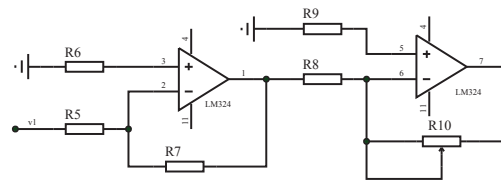


Figure 3: Circuit implementation of equation (21-2).

$$v_1 = \left(1 + \frac{R_4}{R_1}\right) \cdot \frac{R_3}{R_2 + R_3} u_2 - \frac{R_4}{R_1} u_1$$

and if we choose resistors $R_1 = R_2 = R_3 = R_4 = 10k\Omega$, then one can obtain equation (21-1), that is,

$$v_1 = \left(1 + \frac{10k\Omega}{10k\Omega}\right) \cdot \frac{10k\Omega}{10k\Omega + 10k\Omega} u_2 - \frac{10k\Omega}{10k\Omega} u_1 = u_2 - u_1.$$

The equation (21-2) can be implemented by a circuit which as shown in Figure 3. From Figure 3, it is easy to know that

$$v_2 = -\frac{R_{10}}{R_8} \cdot \left(-\frac{R_7}{R_5} v_1\right) = \frac{R_{10} R_7}{R_8 R_5} v_1$$

and if we choose resistors $R_5 = R_6 = R_8 = R_9 = 1k\Omega$, $R_7 = 25k\Omega$, $R_{10} = R$, then one can obtain equation (21-2), that is,

$$v_2 = \frac{R \times 25k\Omega}{10k\Omega \times 10k\Omega} v_1 = \frac{25}{10} \alpha (u_2 - u_1).$$

The equation (21-3) can be implemented by a circuit which as shown in Figure 4, where u is the controller input terminal in Theorem 2.

From the Figure 4, it is easy to know that

$$\dot{u}_1 = -\frac{1}{R_{15} C_1} \left(-\frac{R_{14}}{R_{12}} v_1 - \frac{R_{14}}{R_{11}} v_2\right)$$

and if we choose resistors $R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = 10k\Omega$, and $C_1 = 10\mu F$, then one can obtain equation (21-3), that is

$$\begin{aligned} \dot{u}_1 &= -\frac{1}{10^4 \times 10 \times 10^{-6}} \left(-\frac{10k\Omega}{10k\Omega} v_1 - \frac{10k\Omega}{10k\Omega} v_2\right) \\ &= -10 (-v_1 - v_2). \end{aligned}$$

So much for that, the state equation u_1 can be realized by circuit in figures 2-4.

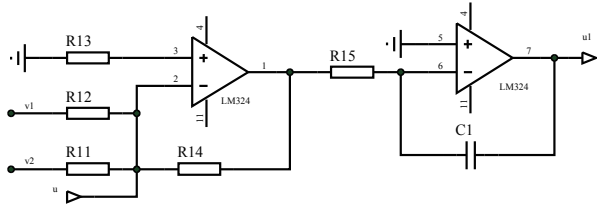
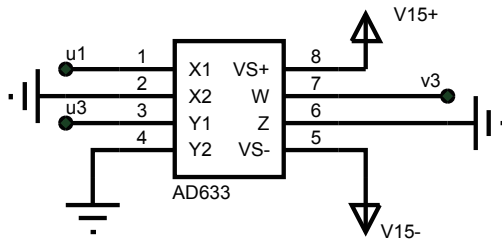
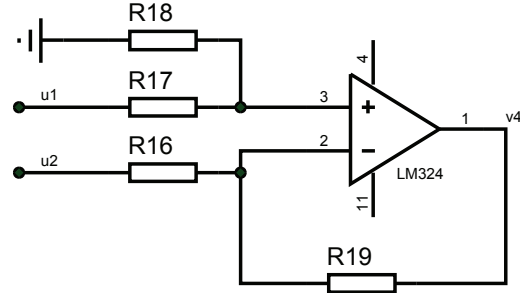
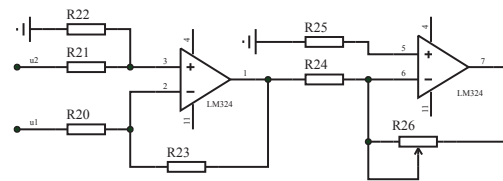


Figure 4: Circuit implementation of equation (21-3).

Figure 5: Implementation of v_3 circuit.Figure 6: Implementation of v_4 circuit.Figure 7: Implementation of v_5 circuit.

5.2.2 Circuit design of state equation u_2

We can require the state equation u_2 as follows:

$$\dot{u}_2 = -\frac{1}{100^{-1}} \left[\alpha \left(\frac{35}{100} u_1 - \frac{29}{100} u_2 \right) + \frac{u_1 u_3}{10} - \left(\frac{28}{100} u_1 - \frac{1}{100} u_2 \right) \right].$$

Outline 2.

- The coefficient of integral circuit is $-\frac{1}{RC}$, so, the constant $-\frac{1}{100^{-1}}$ can be realized by a integrator.
- The transmission gain of AD633 multiplier is $\frac{1}{10}$, so, the $\frac{u_1 u_3}{10}$ can be simply realized by a AD633 multiplier.
- If one defines

$$v_3 \triangleq \frac{u_1 u_3}{10}. \quad (22-1)$$

$$v_4 \triangleq \frac{28}{100} u_1 - \frac{1}{100} u_2. \quad (22-2)$$

$$v_5 \triangleq \alpha \left(\frac{35}{100} u_1 - \frac{29}{100} u_2 \right). \quad (22-3)$$

$$\dot{u}_2 \triangleq \frac{-1}{100^{-1}} (v_3 - v_4 + v_5). \quad (22-4)$$

Implementation of v_3 circuit is as shown in Figure 5. Implementation of v_4 circuit is as shown in Figure 6. From

Figure 6, it is easy to know that

$$v_4 = \left(1 + \frac{R_{19}}{R_{16}} \right) \cdot \frac{R_{18}}{R_{17} + R_{18}} u_1 - \frac{R_{19}}{R_{16}} u_2$$

and if we choose resistors $R_{16} = 100k\Omega$, $R_{17} = 73k\Omega$, $R_{18} = 28k\Omega$, $R_{19} = 1k\Omega$, then we have

$$\begin{aligned} v_4 &= \left(1 + \frac{1k\Omega}{100k\Omega} \right) \cdot \frac{28k\Omega}{73k\Omega + 28k\Omega} u_1 - \frac{1k\Omega}{100k\Omega} u_2 \\ &= \frac{28}{100} u_1 - \frac{1}{100} u_2. \end{aligned}$$

Implementation of v_5 circuit is as shown in Figure 7. From the Figure 7, it is easy to know that

$$v_5 = -\frac{R_{26}}{R_{24}} \left[-\frac{R_{23}}{R_{20}} u_1 + \left(1 + \frac{R_{23}}{R_{20}} \right) \left(\frac{R_{22}}{R_{21} + R_{22}} \right) u_2 \right]$$

and if we choose resistors $R_{20} = 100k\Omega$, $R_{21} = 106k\Omega$, $R_{22} = 29k\Omega$, $R_{23} = 35k\Omega$, $R_{24} = 10k\Omega$, $R_{25} = 10k\Omega$, $R_{26} = R$, then we have

$$\begin{aligned} v_5 &= -\frac{R}{10k\Omega} \left[-\frac{35k\Omega}{100k\Omega} u_1 \right. \\ &\quad \left. + \left(1 + \frac{35k\Omega}{100k\Omega} \right) \left(\frac{29k\Omega}{106k\Omega + 29k\Omega} \right) u_2 \right] \\ &= \alpha \left(\frac{35}{100} u_1 - \frac{29}{100} u_2 \right). \end{aligned}$$

Implementation of u_2 circuit is as shown in Figure 8. From

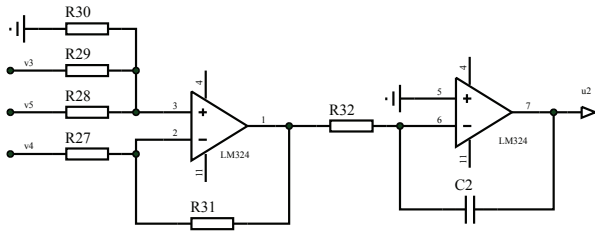
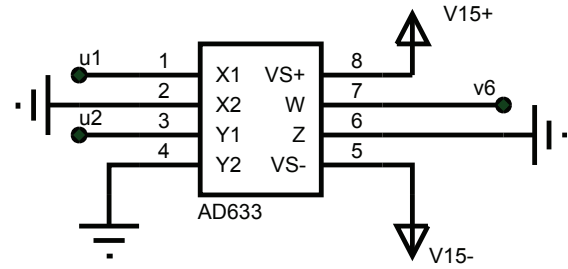
Figure 8: Implementation of u_2 backbone circuit.

Figure 9: Implementation of equation (23-1).

the figures 5-8, it is easy to know that

$$\dot{u}_2 = -\frac{1}{R_{32}C_2} \left[\left(1 + \frac{R_{31}}{R_{27}} \right) \frac{R_{30}}{R_{29} + R_{30}} v_3 - \frac{R_{31}}{R_{27}} v_4 + \left(1 + \frac{R_{31}}{R_{27}} \right) \frac{R_{30}}{R_{28} + R_{30}} v_5 \right]$$

and if we choose resistors $R_{27} = R_{28} = R_{29} = R_{30} = R_{31} = 10k\Omega$, $R_{32} = 1k\Omega$, capacitance $C_2 = 10\mu F$, then we have

$$\begin{aligned} \dot{u}_2 &= -\frac{1}{10^3 \times 10 \times 10^{-6}} \left[\left(1 + \frac{10k\Omega}{10k\Omega} \right) \frac{10k\Omega}{10k\Omega + 10k\Omega} v_3 \right. \\ &\quad \left. - \frac{10k\Omega}{10k\Omega} v_4 + \left(1 + \frac{10k\Omega}{10k\Omega} \right) \frac{10k\Omega}{10k\Omega + 10k\Omega} v_5 \right] \\ &= -100(v_3 - v_4 + v_5). \end{aligned}$$

From Figure 8, it is easy to design a circuit of state equation u_2 .

5.2.3 Circuit design of state equation u_3

Consider u_3 of rewrite state equation as follows:

$$\dot{u}_3 = -100 \left(-\frac{u_1 u_2}{10} + \frac{8}{300} u_3 + \frac{\alpha}{300} u_3 \right).$$

Outline 3.

- The coefficient of integral circuit is $-\frac{1}{RC}$, so, the constant -100 can be realized by a integrator.
- The argument $\alpha \in [0, 1]$, can be also realized by employing a slide rheostat.
- The transmission gain of AD633 multiplier is also $\frac{1}{10}$, so the $\frac{u_1 u_2}{10}$ can be simply realized by a AD633 multiplier.
- As a matter of convenience, one can be defined

$$v_6 \triangleq \frac{u_1 u_2}{10}, \quad (23-1)$$

$$v_7 \triangleq \frac{8}{300} u_3, \quad (23-2)$$

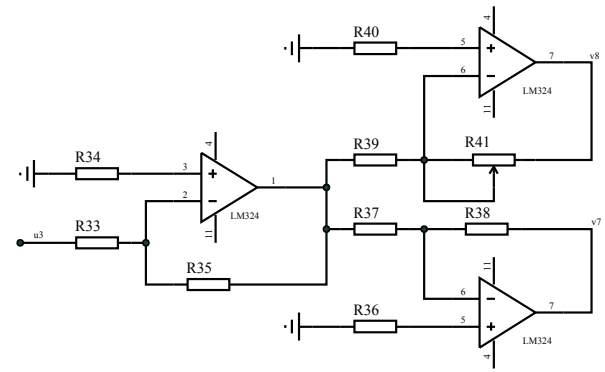


Figure 10: Implementation of equations (23-2) and (23-3).

$$v_8 \triangleq \frac{\alpha}{300} u_3, \quad (23-3)$$

$$\dot{u}_3 \triangleq -100(-v_6 + v_7 + v_8). \quad (23-4)$$

The equation (23-1) can be implemented by a circuit which as shown in Figure 9. The equations (23-2) and (23-3) can be implemented by a circuit which as shown in Figure 10. From Figure 10, it is easy to know that

$$v_7 = -\frac{R_{38}}{R_{37}} \left(-\frac{R_{35}}{R_{33}} u_3 \right) = \frac{R_{38} \cdot R_{35}}{R_{37} \cdot R_{33}} u_3,$$

$$v_8 = -\frac{R_{41}}{R_{39}} \left(-\frac{R_{35}}{R_{33}} u_3 \right) = \frac{R_{41} \cdot R_{35}}{R_{39} \cdot R_{33}} u_3$$

and if we choose resistors $R_{33} = 300k\Omega$, $R_{34} = R_{36} = R_{39} = R_{40} = 10k\Omega$, $R_{35} = R_{37} = 1k\Omega$, $R_{38} = 8k\Omega$, $R_{41} = R$, then one can obtain equations (23-2) and (23-3), that is

$$v_7 = \frac{8k\Omega \times 1k\Omega}{1k\Omega \times 300k\Omega} u_3 = \frac{8}{300} u_3,$$

$$v_8 = \frac{R \times 1k\Omega}{10k\Omega \times 300k\Omega} u_3 = \frac{\alpha}{300} u_3.$$

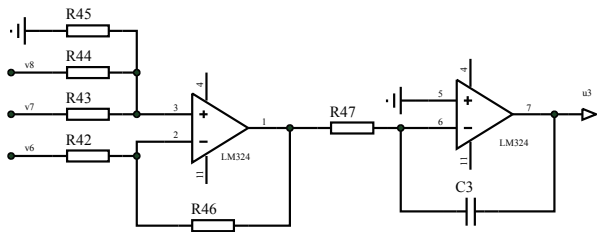


Figure 11: Circuit implementation of equation (23-4).

The equation (23-4) can be implemented by a circuit which as shown in Figure 11. From Figure 11, it is easy to know that

$$\dot{u}_3 = -\frac{1}{R_{47}C_3} \left[-\frac{R_{46}}{R_{42}}v_6 + \left(1 + \frac{R_{46}}{R_{42}}\right) \frac{R_{45}}{R_{43} + R_{45}}v_7 + \left(1 + \frac{R_{46}}{R_{42}}\right) \frac{R_{45}}{R_{44} + R_{45}}v_8 \right]$$

and if we choose resistors $R_{42} = R_{43} = R_{44} = R_{45} = R_{46} = 10k\Omega$, $R_{47} = 1k\Omega$, capacitance $C_3 = 10\mu F$, then one can obtain equation (23-4), that is

$$\begin{aligned} \dot{u}_3 &= -\frac{1}{10^3 \times 10 \times 10^{-6}} \left[-\frac{10k\Omega}{10k\Omega}v_6 + \left(1 + \frac{10k\Omega}{10k\Omega}\right) \frac{10k\Omega}{10k\Omega + 10k\Omega}v_7 + \left(1 + \frac{10k\Omega}{10k\Omega}\right) \frac{10k\Omega}{10k\Omega + 10k\Omega}v_8 \right] \\ &= -100(-v_6 + v_7 + v_8). \end{aligned}$$

So much for that, the state equation u_3 can be realized by circuits 9-11.

Remark 5. Reviewing of the existing research results. The circuit implementation of the individual Lorenz system, Chen's system and Lü system can be found in [44–46] and [47, 48], respectively. An electronic circuit of the unified chaotic system has been considered by Li [50], it is only illustrated that Lorenz ($\alpha = 0$), Lü ($\alpha = 0.8$), Chen ($\alpha = 1$) chaotic attractors can be duly obtained by controlling two switches in the circuit, but this circuit can't reflect and depict that the parameter $\alpha \in [0, 1]$ and the continuous change of chaotic attractors. However, a new circuit has been designed in this paper, it is simpler so that only three variable resistors are required to be adjusted. In our experiments, it is illustrated that different chaotic attractors can be duly obtained by adjusting the three variable resistors in the circuit.

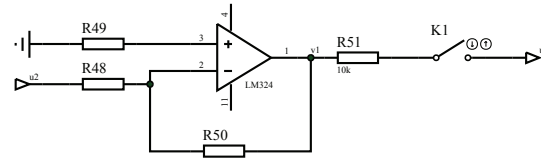


Figure 12: Controller implementation circuit.

In a nutshell, a more standardized and more unified circuit of the unified chaotic system has been developed in this paper.

5.3 Real-time unified chaotic circuit stabilization via single state feedback control

In order to verify the effectiveness of the proposed controller in this paper, without loss of generality, let $R = 0k\Omega$, currently in its Lorenz chaos, that is, $\alpha = 0$.

Under the Theorem 2, one can be let $u = -10y$, that is, the controller in circuit equation (20) is $u = -10u_2$. The controller can be realized in a circuit as follows Figure 12. From Figure 12, it is easy to know that

$$u = \begin{cases} -\frac{R_{50}}{R_{48}}u_2, & \text{Switch K1 is turned on,} \\ 0, & \text{Switch K1 is turned off} \end{cases}$$

and if we choose resistors $R_{48} = R_{49} = 1k\Omega$, $R_{50} = R_{51} = 10k\Omega$, then one can obtain the controller, that is

$$u = \begin{cases} -\frac{10k\Omega}{1k\Omega}u_2 = -10u_2, & \text{Switch K1 is turned on,} \\ 0, & \text{Switch K1 is turned off.} \end{cases} \quad (24)$$

5.4 Unified chaotic circuit experiments

In order to test and verify unified chaotic systems, the circuit was implemented in a printed board as shown in the Figure 13. The printed circuit board is a double-sided circuit, low cost and robust. The resistors which satisfied the EIA E96 standard with 1% error were chosen in the circuit design. Four LM324N operational amplifiers were used in the circuit. Two AD633 multipliers have been used in circuit implementation. Three variable resistors can be easily changed in order to scale the system in its time and state variables.

We tested the unified chaotic system. The 3A DC power supply was used to supply the circuit board. The

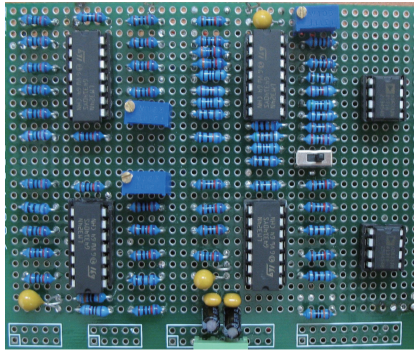


Figure 13: Implementation circuit of unified chaotic systems.

DC voltages are +15V and -15V. A 20MHz analog oscilloscope was used to observe waveforms. Figures 14-16 show the phase portraits of the unified chaotic system with $\alpha = 0$, $\alpha = 0.8$, $\alpha = 1$, respectively. In the circuit experiments, the resistors R_{10} , R_{26} and R_{41} were set to 0Ω , we got the Figure 14. The variable resistors R_{10} , R_{26} and R_{41} were adjusted to $8k\Omega$, the Figure 15 was observed. The resistors R_{10} , R_{26} and R_{41} were adjusted to $10k\Omega$, the oscilloscope displays were shown as Figure 16. From the figures 14-16, it is easy to know that the circuit designed in this paper is effective. Figure 17 is pictures of the chaotic circuit stabilization process under the control (24). In the experiments, the switch K1 was used to apply the control signal for stabilizing the chaotic circuit. As expected, the chaotic orbits of the system (20) is quickly driven to the unstable equilibrium S_0 . The real-time series u_1 state with controller is shown in Figure 17(a). The real-time phase portrait of $u_1 - u_2$ with controller is shown in Figure 17(b). The control signed $u = -10u_2$ is shown as Figure 17(c). From the Figure 17(c), it is easy to visualize and verify the satisfactory performance of the controller proposed in this paper.

Remark 6. Reviewing of the existing research results, Cho only presents a simulation for the control of a Lorenz system [59]. The chaos control of uncertain unified chaotic systems has been considered by adaptive control scheme [55]. A simple adaptive-feedback for controlling the unified chaotic system has been proposed [43]. Output feedback control of the unified chaotic systems has been investigated based on feedback passivity [56]. A feedback controller has been proposed to realize the stability control of a unified chaotic system [57]. Ablay investigates the chaos control of the uncertain unified chaotic systems by means of sliding mode control [58]. However, the proposed controllers in previous works are too complex both in design and implementation, and the circuit implementation of its controllers have not also been reported.

6 System simulation in MATLAB/Simulink

In this section, in order to demonstrate that the robust result of this paper is effective and convenient for the unified chaotic system with uncertain parameter. Without loss of generality, let α is an uncertain parameter, which is generated by the Uniform Random Number in MATLAB/Simulink, let Minimum= 0, Maximum= 1, the sample period is 0.01, the initial value of simulation is $[x, y, z]^T = [20, -10, -20]^T$ with the controller of Theorem 5, where $\varepsilon = 1$, that is, $u = -28y$.

The states responses of the controlled unified chaotic system (18) are shown in Figure 18. Demonstrating that the control scheme in this paper is effective.

7 Conclusion

This paper proposes some single state feedback controllers for the stabilization of unified chaotic systems and the circuit implementation problem of unified chaotic system has been considered. Some stabilization conditions have been derived via the single state feedback control. The robust performance of controlled unified chaotic systems with uncertain parameter has been investigated, the robust linear controller which only requires information of a state of the system is proposed. Both unified chaotic systems and the designed controller are synthesized and implemented by an analog electronic circuit which is simpler because only three variable resistors are required to be adjusted. The numerical simulation and control in MATLAB/Simulink is then provided to show the effectiveness and feasibility of the proposed method which is robust to some uncertainties. In fact, the proposed method in this paper at least has two advantages: (a) The controller obtained in this paper is simpler than the controller obtained by previous methods, the utility of the proposed method is easy to realize; (b) A more standardized and more unified circuit of the unified chaotic system has been developed in this paper. The results presented in this paper improve and generalize the corresponding results of recent works.

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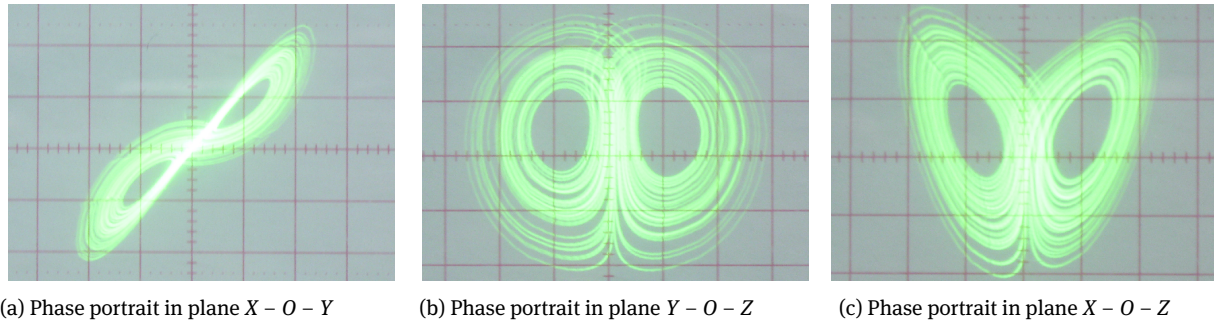


Figure 14: Phase portrait of unified chaotic system with $\alpha = 0$.

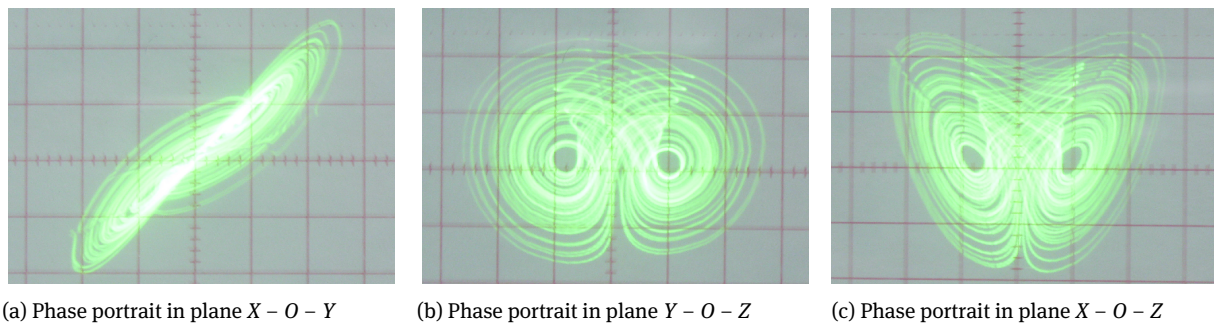


Figure 15: Phase portrait of unified chaotic system with $\alpha = 0.8$.

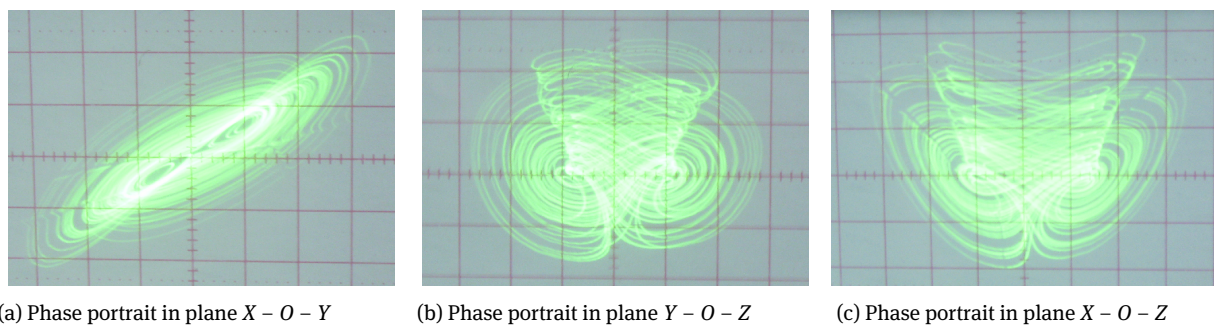


Figure 16: Phase portrait of unified chaotic system with $\alpha = 1$.

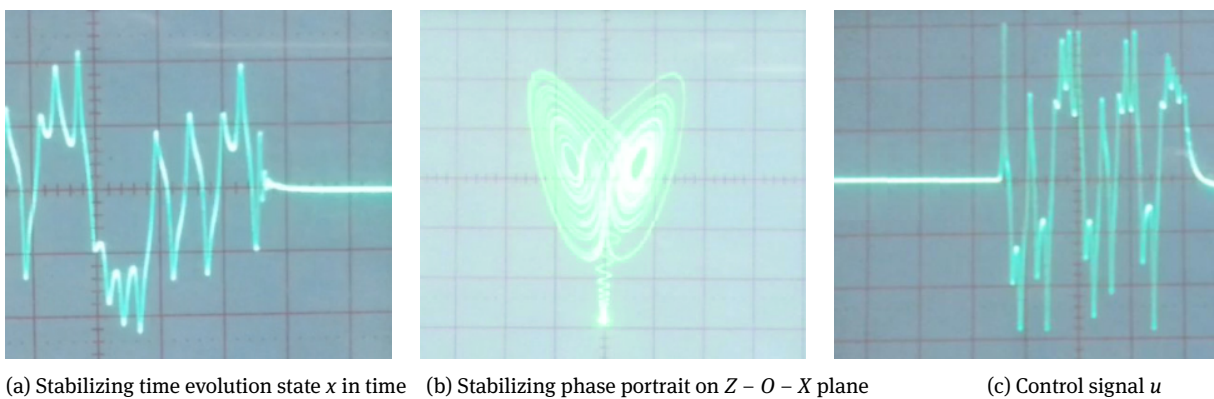


Figure 17: Real-time chaotic circuit stabilization with $\alpha = 0$.

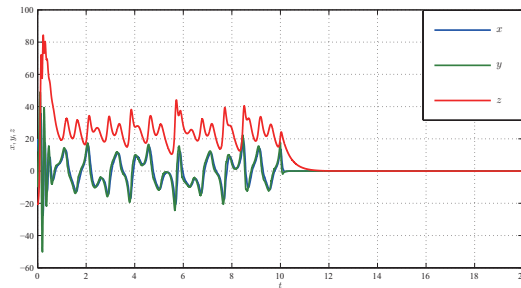


Figure 18: Dynamic response of the controlled unified chaotic system (18).

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