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Kantowski-Sachs Cosmological model with quark and strange quark matter in $f(R)$ theory of gravity

Abstract: In this paper we have studied the Kantowski-Sachs cosmological model with the quark and strange quark matter in the $f(R)$ theory of gravity. The general solutions of the field equations are obtained by assuming the physical condition shear scalar σ is proportional to scalar expansion θ, which leads to the relation $B = A^n$ between metric coefficients B and A. The physical and geometrical aspects of the model are also discussed.

Keywords: $f(R)$ gravity; quark and strange quark matter; Kantowski-Sachs space-time

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1 Introduction

A fundamental theoretical challenge to gravitational theories has been imposed by the recent observational data [1–6] on the late time acceleration of the universe and the existence of the dark matter. Carroll *et al.* [7] explained the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. The $f(R)$ gravity models are reviewed by Capozziello and Faraoni [8]. The $f(R)$ theory of gravity has also been helpful in describing the evolution of the universe.

$f(R)$ gravity is getting a lot of attention since it can describe early acceleration Theory [9]. Most of the work has been done in $f(R)$ gravity with different matter sources. On the other hand, a quark and strange quark matter solution in the framework of $f(R)$ gravity has not yet been investigated in detail. It will be interesting to study the $f(R)$ gravity model in the case of quark and strange quark matter. So, we are interested in behaviors of quark and strange quark matter in $f(R)$ gravity for a Kantowski-Sachs universe.

In this study, we will examine quark and strange quark matter in a Kantowski-Sachs space-time. It is well known that quark-gluon plasma existed during one of the phase transitions of the universe at the early time when the cosmic temperature was $T \sim 200\,\text{MeV}$. Typically, strange quark matter is modeled with an (EoS) equation of state $p = \frac{(\rho - 4B_c)}{3}$ based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this equation B_c is the difference between the energy density of the perturbative and nonperturbative QCD vacuum, known as the bag constant. ρ and p are the energy density and thermodynamic pressure of the quark matter respectively. In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (bag constant). Also, in the framework of this model, the quark matter is composed of massless u and d quarks, massive s quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are massless and non-interacting. Therefore, we have quark pressure $p_q = \frac{\rho_q}{3}$ (ρ_q is the quark energy density), the total energy density is $\rho = \rho_q + B_c$ and the total pressure is $p = p_q - B_c$. There are many studies on quark matter in general relativity. Mak and Harko [10] have studied charged strange quark matter in the spherically symmetric space-time admitting conformal motion. The study of strange quark matter attached to the string cloud in the spherical symmetric space-time admitting conformal motion have been done by Yavuz *et al.* [11]. Adhav *et al.* [12, 13] have discussed string cloud and domain walls with quark matter in an n-dimensional Kaluza-Klein cosmological model in general relativity, and strange quark matter attached to a string cloud in Bianchi type-III space time in general relativity. Katore [14] studied a cosmological model with strange quark matter attached to cosmic strings for FRW space-time in general relativity.

Yilmaz *et al.* [15] have discussed quark and strange quark matter in $f(R)$ gravity for Bianchi I and V space time models. They have concluded that Quark Matter may behave like phantom-type dark energy for $\varepsilon < -1$, and quark matter may be source of early dark energy, which causes

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early acceleration of the universe due to negative pressure. Furthermore, obtained $f(R)$ solutions represent early eras of the universe since $f(R)$ solutions for quark matter coincide with $f(R)$ equations for inflation. Adhav [16] discussed the Kantowski-Sachs string cosmological model in the $f(R)$ theory of gravity. Recently, Sahoo and Mishra [17] studied the axially symmetric space-time with strange quark matter attached to string clouds in bimetric theory.

The anisotropy, quark matter and strange quark matter play a significant role in the early stage of evolution of the universe and hence the study of anisotropic and homogeneous cosmological models becomes important. In this paper, we have studied the Kantowski-Sachs cosmological model with quark and strange quark matters in the $f(R)$ theory of gravity. The general solutions of the field equations are obtained by assuming the physical condition shear scalar σ is proportional to scalar expansion θ, which leads to the relation $B = A^n$ between metric coefficients B and A. The physical and geometrical aspects of the model are also discussed.

2 Metric and the field equations

The standard representation of Kantowski-Sachs space-time [18] is given by

$$ds^2 = dt^2 - A^2 dr^2 - B^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where A and B are cosmic scale factors and are functions of the cosmic time t.

The corresponding Ricci scalar is given by

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{2\ddot{B}}{B} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2}\right], \tag{2}$$

where dot $(\cdot)$ represents derivative with respect to t.

The field equations in the $f(R)$ theory of gravity are given by [19, 20].

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\nabla_\nu(R) + g_{\mu\nu}\Box F(R) = \kappa T_{\mu\nu}. \tag{3}$$

where $F(R) = df(R)/dR$, $\Box \equiv \nabla^\mu\nabla_\mu$, ∇_μ is the covariant derivative, $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m, and $\kappa\ (= \frac{8\pi G}{c^4} = 1)$ is the coupling constant in gravitational units. These are the fourth-order partial differential equations in the metric tensor $g_{\mu\nu}$. The fourth order is due to the last two terms on the left hand side of the equation. If we take $f(R) = R$, these equations reduce to the field equations of general relativity.

Now contracting the field equations (3), we get

$$F(R)R - 2f(R) + 3\Box F(R) = T. \tag{4}$$

From equation (4), we get

$$f(R) = \frac{1}{2}[F(R)R + 3\Box F(R) - T]. \tag{5}$$

This is used to analyze several features of $f(R)$ gravity.

The energy momentum tensor for quark matter is given as

$$T^{(\text{Quark})}_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu} \text{ or}$$
$$T^{(\text{Quark})}_{\nu\mu} = \text{diag}(\rho, -p, -p, -p), \tag{6}$$

where $\rho = \rho_q + B_c$, $p = p_q - B_c$ and $u^\mu = u_\mu = \delta^\mu_0$ is the four velocity in the comoving coordinates. Since quark matter behaves as a nearly perfect fluid [15, 21–24], we will use the following equation of state for quark matter

$$p_q = \varepsilon\rho_q, \quad 0 \le \varepsilon \le 1. \tag{7}$$

Also the linear equation of state for strange quark matter is [25, 26]

$$p = \varepsilon(\rho - \rho_0), \tag{8}$$

where ρ_0 is the energy density at zero pressure and ε is a constant.

When $\varepsilon = \frac{1}{3}$ and $\rho_0 = 4B_c$ the above linear equation of state is reduced to the following equation of state for strange quark matter in the bag model [15, 24]

$$p = \frac{(\rho - 4B_c)}{3}. \tag{9}$$

where B_c is the Bag constant.

In the comoving co-ordinate system, the field equations (3) for the metric (1) with the help of equation (6) can be written as

$$\left(\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B}\right)F + \frac{1}{2}f(R) - \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{F} = -(\rho_q + B_c), \tag{10}$$

$$\left(\frac{\ddot{A}}{A} + 2\frac{\dot{A}\dot{B}}{AB}\right)F + \frac{1}{2}f(R) - \left(\frac{2\dot{B}}{B}\right)\dot{F} - \ddot{F} = (p_q - B_c), \tag{11}$$

(12)

$$\left(\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2}\right)F + \frac{1}{2}f(R) - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{F} - \ddot{F} = (p_q - B_c), \tag{13}$$

where overhead dot $(\cdot)$ denotes derivative with respect to time t.

3 Solutions of the field equations

Subtracting equations (10) from (11) and (10) from (13), we get

$$\left(2\frac{\dot{A}\dot{B}}{AB}-\frac{2\dot{B}}{B}\right)F+\left(\frac{\dot{A}}{A}\right)\dot{F}-\ddot{F}=\rho_q+p_q, \tag{14}$$

$$\left(\frac{\dot{A}\dot{B}}{AB}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{B^2}\right)F+\left(\frac{\dot{B}}{B}\right)\dot{F}-\ddot{F}=\rho_q+p_q. \tag{15}$$

The field equations (14) and (15) are a system of two non-linear differential equations with five unknowns A, B, F, ρ and p. Hence, in order to solve system completely we assume a physical condition that expansion scalar θ is proportional to the shear scalar σ which gives the following relation between metric function as

$$B=A^n, \tag{16}$$

where $n \neq 1$ is an arbitrary constant.

Kantowaski and Sachs [18], Kristian and Sachs [27] and Thorne [28] suggested that the Hubble expansion of the universe is isotropic to within 30% (from observations of the velocity red-shift relation for extra-galactic sources). Collins *et al.* [29] have pointed out that for spatially homogeneous metric the normal congruence to the homogeneous hypersurface satisfies the condition $\frac{\sigma}{\theta}$ = constant. Banerjee and Santos [30] had obtained the solutions of Bianchi type II, VIII and IX under this condition. Recently, many authors have assumed this condition in order to obtain solutions of the field equations for different types cosmological models [31–38].

We define the spatial volume $V$and average scale factor a of the universe as

$$V=AB^2, \quad a=(AB^2)^{\frac{1}{3}}. \tag{17}$$

The directional Hubble parameters in the directions of r, θ and ϕ axes respectively are defined as

$$H_r=\frac{\dot{A}}{A}, \quad H_\theta=H_\phi=\frac{\dot{B}}{B}, \tag{18}$$

The mean Hubble parameter H is defined as

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{2\dot{B}}{B}\right), \tag{19}$$

Subtracting equations (15) from equation (14) and using equation (16), we obtain

$$\left(\frac{\ddot{A}}{A}+2n\frac{\dot{A}^2}{A^2}+\frac{\dot{A}\dot{F}}{AF}\right)=\frac{1}{(1-n)A^{2n}}. \tag{20}$$

We know that the power-law relation between scale factor and scalar field has been used by Johri and Desikan [39], whereas, Kotub Uddin *et al.* [40] have established a result in the context of $f(R)$ gravity which shows

$$F \propto a^n,$$

where m is an arbitrary constant.

Recently, this has been used by Sharif and Shamir [19, 20].

Now, we solve equation (20) by using a power-law relation between F and a [19, 20, 39, 40]

$$F=ka^m, \tag{21}$$

where k is the constant of proportionality and m is any integer.

Using equation (17) and (21) in equation (20) we obtain

$$\frac{\ddot{A}}{A}+\left(\frac{6n+m(2n+1)}{3}\right)\frac{\dot{A}^2}{A^2}=\frac{1}{(1-n)A^{2n}}. \tag{22}$$

Let $\dot{A}=S(A)$ then $\ddot{A}=\dot{S}S$, where $\dot{S}=\frac{dS}{dA}$.

Using this in above equation (22) and integrating we get

$$\dot{A}^2=\frac{3A^{2(1-n)}}{(1-n)[3(n+1)+m(2n+1)]}+c_1. \tag{23}$$

Choosing integration constant $c_1 = 0$ and again integrating, we obtain the following scale factors

$$A=(\alpha t+\beta)^{\frac{1}{n}}, \tag{24}$$

$$B=(\alpha t+\beta), \tag{25}$$

where $\alpha = \left\{\frac{3n^2}{(1-n)[3(n+1)+m(2n+1)]}\right\}^{\frac{1}{2}}$, $\beta = c_2\alpha$ and c_2 is integration constant.

The metric (1) with the help of equations (24), (25) can be written as

$$ds^2=dt^2-(\alpha t+\beta)^{\frac{2}{n}}dr^2-(\alpha t+\beta)^2[d\theta^2+\sin^2\theta d\phi^2]. \tag{26}$$

3.1 Some physical properties

Using equations (24) and (25) in equation (17), the volume scale factor V of the universe is given by

$$V=(\alpha t+\beta)^{\frac{(2n+1)}{n}}. \tag{27}$$

Using equations (24) and (25) in equation (17), the directional Hubble parameters in the directions of r, θ and ϕ axes are given by

$$H_r=\frac{1}{n(\alpha t+\beta)}, \quad H_\theta=H_\phi=\frac{\alpha}{(\alpha t+\beta)}. \tag{28}$$

The mean Hubble parameter H is obtained by using equations (19) and (28) as

$$H = \frac{1}{3n}\left(\frac{2n\alpha+1}{\alpha t+\beta}\right). \tag{29}$$

The expansion scalar $\theta = 3H$ is given by

$$\theta = \frac{1}{n}\left(\frac{2n\alpha+1}{\alpha t+\beta}\right). \tag{30}$$

The mean anisotropy parameter of the expansion $\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2$ is given as

$$\Delta = 2\left(\frac{\alpha n-1}{2n\alpha+1}\right)^2. \tag{31}$$

The shear scalar is defined as $\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3}H_i^2 - 3H^2\right)$ and found to be

$$\sigma^2 = \frac{1}{3n^2}\left(\frac{\alpha n-1}{\alpha t+\beta}\right)^2. \tag{32}$$

The deceleration parameter is defined as $q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1$ and found to be

$$q = \left(\frac{\alpha n-1}{2n\alpha+1}\right). \tag{33}$$

3.1.1 Quark matter for Kantowski-Sachs model

Using equations (21), (24) and (25) in equation (14) with the help of linear equation of state (7) for $\varepsilon = \frac{1}{3}$, we obtain the energy density and pressure of the quark matter as

$$\rho_q = \frac{\alpha k}{12n^2}\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right] \times (\alpha t+\beta)^{\frac{(2n+1)m-6n}{3n}}, \tag{34}$$

$$p_q = \frac{\alpha k}{(6n)^2}\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right] \times (\alpha t+\beta)^{\frac{(2n+1)m-6n}{3n}}, \tag{35}$$

Using equations (21), (24), (25) and (34) in equation (10), the $f(R)$ function for quark matter is found to be

$$f(R) = \frac{\alpha k}{6n^2}\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right] \times (\alpha t+\beta)^{\frac{(2n+1)m-6n}{3n}}. \tag{36}$$

Using equations (24) and (25) in equation (2), the scalar curvature R for the quark matter is found to be

$$R = -2\left[\frac{\alpha^2(1-n) + 2n\alpha + n^2(\alpha^2+1)}{n^2(\alpha t+\beta)^2}\right], \tag{37}$$

which clearly indicates that $f(R)$ cannot be explicitly written in terms of R.

However, using this value of R, the $f(R)$ function turns out to be

$$f(R) = \frac{\lambda_2\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right]}{\left[\alpha^2(1-n) + 2n\alpha + n^2(\alpha^2+1)\right]^{-\lambda_1}} R^{\lambda_1} \tag{38}$$

where $\lambda_1 = \dfrac{6n-(2n+1)m}{3n}$, $\lambda_2 = \dfrac{\alpha k(n)^{\lambda_1-2}}{6(2)^{\lambda_1}}$ are the constants.

This gives $f(R)$ only as a function of R.

3.1.2 Strange quark matter for Kantowski-Sachs model

Using equations (21), (24) and (25) in equation (14) with the help of a linear equation of state (9), we obtain the energy density and pressure of the strange quark matter as

$$\rho = \frac{\alpha k}{12n^2}\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right] \times (\alpha t+\beta)^{\frac{(2n+1)m-6n}{3n}} + B_c, \tag{39}$$

$$p = \frac{\alpha k}{(6n)^2}\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right] \times (\alpha t+\beta)^{\frac{(2n+1)m-6n}{3n}} - B_c. \tag{40}$$

Using equations (21), (24), (25) and (39) in equation (10), the $f(R)$ function for strange quark matter is found to be

$$f(R) = \frac{\alpha k}{6n^2}\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right] \times (\alpha t+\beta)^{\frac{(2n+1)m-6n}{3n}} - 2B_c. \tag{41}$$

Thus, the $f(R)$ function can be written in term of R as

$$f(R) = \frac{\lambda_2\left[3m(2n+1)(n\alpha+1) - m^2\alpha(2n+1)^2 + 18n\right]}{\left[\alpha^2(1-n) + 2n\alpha + n^2(\alpha^2+1)\right]^{-\lambda_1}} \times R^{\lambda_1} - 2B_c, \tag{42}$$

where $\lambda_1 = \dfrac{6n-(2n+1)m}{3n}$, $\lambda_2 = \dfrac{\alpha k(n)^{\lambda_1-2}}{6(2)^{\lambda_1}}$ are the constants.

4 Discussion and conclusion

In this paper we have studied the Kantowski-Sachs cosmological model with quark and strange quark matter in the $f(R)$ theory of gravity. We have obtained solutions of the field equations by assuming the physical condition shear scalar σ is proportional to scalar expansion θ, which leads to the relation $B = A^n$ between metric coefficients B and A. We have also evaluated the function of the Ricci scalar R, and $f(R)$, by using an equation of state (EoS) for quark and strange quark matters. In this model we observed that the spatial volume V is finite at $t = 0$ and it expands as t increase and becomes infinitely large as $t \to \infty$ as shown in Figure 1. From Figure 2, it is observed that the expansion scalar θ starts with a finite value at $t = 0$, and as time increases it decreases to a constant value and remains constant as $t \to \infty$. The energy density of the quark matter ρ_q is finite at $t = 0$ and then decreases to become constant as $t \to \infty$, provided that $m < \frac{6n}{(2n+1)}$ and $n > 0$, as shown in Figure 3. For $0 < n < 1$, the deceleration parameter q of the universe is in the range $-1 < q < 0$, as shown in Figure 4, which matches with the observations made by Riess *et al.*, Perlmutter *et al.* and Knop *et al.* [1, 2, 41, 42] that the present day universe is undergoing accelerated expansion.

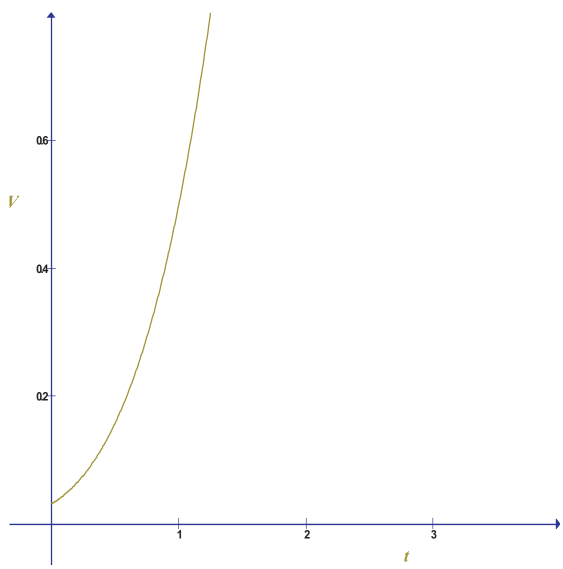

Figure 1: The variation of V vs. t.

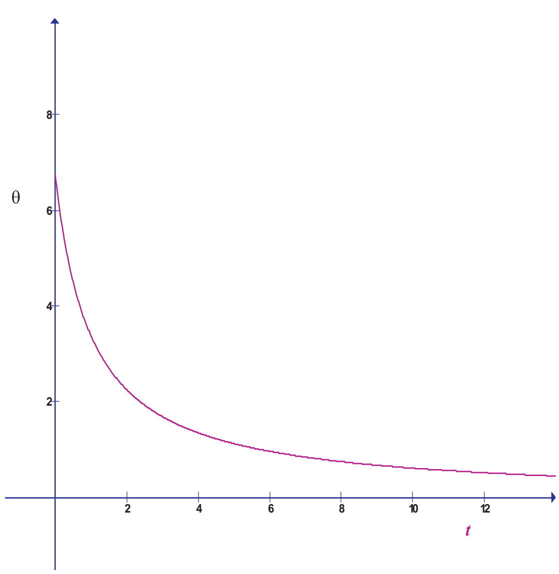

Figure 2: The variation of θ vs. t.

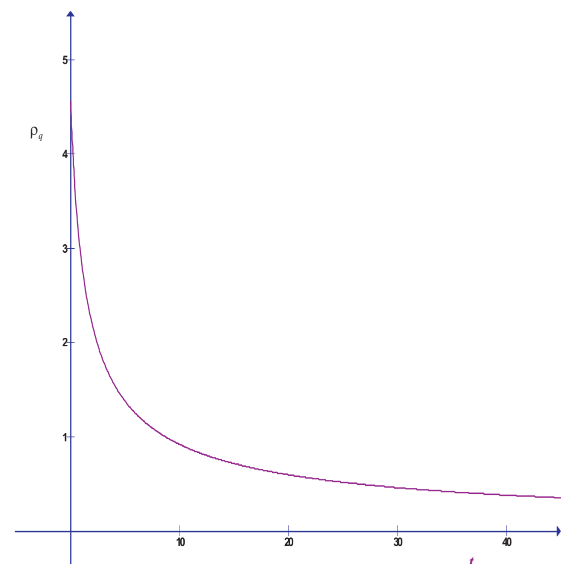

Figure 3: The variation of ρ_q vs. t.

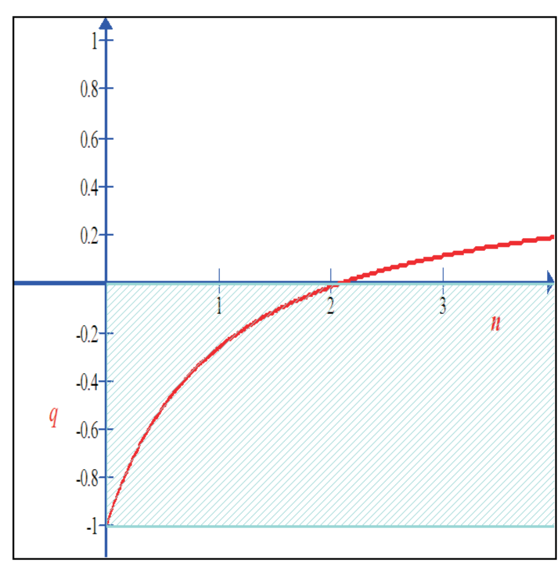

Figure 4: The variation of q vs. n.

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