

## Research Article

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# Vibration analysis of functionally graded materials for cylinder liner used in agricultural engine part

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**Abstract:** The engine part is one of the major sources of vibration of agricultural machinery such as a tractor. Therefore, vibration analysis of agricultural engine part will improve the engine efficiency and agricultural performance. The main objective of present work was to study the dynamic behavior of functionally graded (FG) structural material for the application as cylinder liner as agricultural engine part. The vibration analysis of functionally graded (FG) beam was performed using Finite Element Method (FEM). A typical simply-supported FG beam was modeled in COMSOL Software, where the upper portion of the beam was alumina and the lower portion was steel. The basic properties of material such as Young's Modulus and mass density were varied along the thickness according to the power law. The boundary conditions were also modeled, and parametric study was carried out with mass density and young's modulus. Eigen value problem was solved and in turn natural frequency and mode shapes were obtained. The frequency ratio was calculated and compared for various boundary conditions. The finding of the results indicated that when power exponent was increased from 0 to 5, the nonlinear reduction in frequency was occurred but when power exponent was increased from 5 to 10, linear reduction in frequency was occurred. Also, the increase in power exponent caused the increase in frequency for Young's Modulus ratio of 0.25 and 0.5, decrease in frequency for Young's Modulus ratio of 2 and 4 and no change occurred for Young's Modulus ratio of 1. The first non-dimensional frequency for Clamped-Clamped boundary condition was comparatively more than other boundary conditions and lowest frequency is obtained for Clamped –Free boundary conditions.

**Keywords:** Functionally Graded Material, Finite Element Method, COMSOL

## 1 Introduction

Functionally Graded Materials (FGMs) have been developed as innovative materials in various applications such as aerospace, automotive industries and machine elements. The application of FGM in agricultural equipment is new and is very limited as the cost of production of functionally graded materials is high. However, the major features or functional properties of materials suitable for agricultural applications include higher load bearing capacity due to increased machine performance, lighter in weight to avoid soil compaction, more wear resistant, long life, and better ergonomics. In this regard, the better physical, mechanical, thermal and thermo-mechanical properties of FGMs justify their use in important parts and machinery in agriculture. The vibration analysis of agriculture machine and its various parts have been done by researchers (Gialamas et al. 2016; Cutini et al. 2017). The use of FGM as cylinder liner in engine part is a new approach as the function of cylinder liner is to reduce wear on cylinder, piston and transfer heat from piston to coolant. The properties of functionally graded materials change over a varying dimension. In general, FGMs are the advanced composite materials consisting of two or more materials and varying volume fraction along dimension. It can be mixture of metal and metal or ceramics. FGMs are preferred for the structure having very small thickness and subjected to high temperature gradient (Chakraborty et al. 2003). Under such extreme working condition, composite material leads to fail due to separation of fibers from the matrix called delaminations. Therefore, in the mid-1980s, Japanese researchers worked on hypersonic space plane project to develop thermal barrier coating of less than 10 mm thickness and outside temperature 2000K and inside temperature 1000K. In this regard, they developed a thermal barrier coating of a novel material called Functionally Graded Material (FGM). FGM occur in nature as bones, teeth etc. (Mehta and Balaji

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2013). Functionally Graded Beams have many applications in automotive industries, manufacturing, aerospace, and mechanical components. However, in comparison with FG plates and shells, there are few literatures working on FG beams (Aydogdu and Taskin 2007).

Rao et al. (Rao et al. 2012) analyzed free vibration of functionally graded (FG) shell structures under thermal and mechanical loading using finite element analysis. They compared the thermal and mechanical properties of FG shells, the homogeneous shells of pure ceramic ( $\text{Al}_2\text{O}_3$ ) and pure metal (steel) shells. It was revealed that the thermal and mechanical properties of the FGM shells lie between those of the homogeneous shells.

Ramu and Mohanty (Ramu and Mohanty 2014) evaluated natural frequencies and mode shapes of a functionally graded material (FGM) using Finite Element Method (FEM). It was observed that the mechanical properties of an FGM plate vary continuously from one surface to another surface according to power law. For modal analysis of FGM plate, program has been coded in MATLAB software. The modal shape and natural frequencies of rectangular FGM plate are found at different boundary conditions. In addition, the effects of power law index on the FGM plate natural frequency and mode shapes with different boundary conditions are studied.

Enab (Enab 2014) predicted stress concentration factors for functionally graded material under uniaxial and biaxial loads. Finite element analysis for the plates was done using ANSYS Parametric Design Language (APDL). Various geometric and material parameters were taken as the elliptic hole major axis to plate width ratio, shape factor, the gradation direction of FGM. It was concluded that, by selecting the proper distribution of functionally graded materials, stress concentration factors can be controlled. Some of the important findings of reviews are presented in Table 1.

In comparison with functionally graded plate and shells, studies for functionally graded beams are relatively less and until now most of the works have been done through MATLAB or FEM method. However, in present work, COMSOL 4.4 version software has been used, which gives very effective result in less time as comparable to others. This would help itself to design the dynamically prone structures. The objective of this paper is to study the Structural vibration analysis of functionally graded Beams using Finite Element Method (FEM). The simply-supported FGM beam model was developed where upper surface of the beam was alumina and lower surface was steel, and material properties i.e. (young's modulus and mass density) varied along the thickness of the beam according to the power law. The different boundary

condition was applied, and finite element mesh size was taken for obtaining optimized result through analysis.

## 2 Materials and Method

For vibration analysis, the functionally graded material for hollow cylinder liner has been taken as a functionally graded simply-supported beam of length  $L$ , width  $b$  (i.e. thickness of liner, different of outer and inner diameter of cylinder), and thickness  $h$  (i.e. circumference of outer circle) (Figure 1).

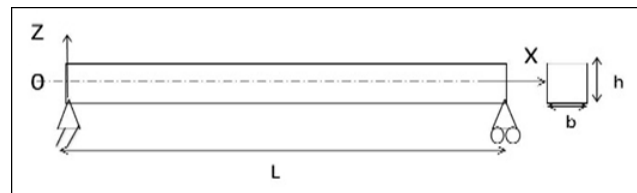


Figure 1: A functionally graded simply supported beam

### 2.1 Material properties

The material properties of steel and alumina in presented in Table 2. The material properties such as Young's modulus and mass density varied through the beam thickness in the power-law form as presented in Eq. (1).

$$P(z) = (P_U - P_L) \left( \frac{z}{h} + \frac{1}{2} \right)^k + P_L \quad (1)$$

where  $P_U$  is material property of the upper surface and  $P_L$  that of lower surface of the beam and  $k$  is the non-negative power-law exponent.  $k$  denotes the material variation of properties along the thickness of the beam.

### 2.2 Methodology

The following steps were followed in Eigen frequency analysis through Comsol Multiphysics 4.4

- The parameters ( $L$ ,  $b$ ,  $h$  and  $k$ ) were set in global definitions.
- The analytic functions of young's modulus and mass density were defined and expression according to power law under component was written.
- Geometry was defined, the width, depth, height of the block was set up, also set the position and finally build all objects.
- The boundary conditions and the mesh size were set to build the final mesh.

**Table 1:** Important findings of studies on vibration analysis of functionally graded materials

Sl. no.	Reference detail	Materials and Methods	Research findings
1	Ramu and Mohanty, 2012	They studied free vibration analysis of rectangular plate structures and determined the natural frequencies of an isotropic thin plate using FEM.	By varying the thickness of the plate, it has been concluded that the frequency parameter is constant. Increase in thickness of the plate does not affect the frequency parameter.
2	Xiao and Yue, 2012	They studied stress and displacement fields in a functionally graded material of semi-infinite extent induced by rectangular loading. Variation of elastic modulus was varied along the depth but kept constant in lateral directions.	It was found that the heterogeneity of FGM influences elastic fields of the semi-infinite elastic solids
3	Vimal et al. 2014	The functionally graded materials (FGMs) are assumed to be graded across their thicknesses according to a power law distribution of the volume fractions and the Poisson's ratio is taken as constant. Convergence study with respect to the number of nodes has been carried out and the results are compared with those from past investigations available in the literature. The effects of parameters such as cutout, cutout size, volume fraction index, boundary conditions and thickness ratio on the natural frequencies are studied. Free vibration analysis of functionally graded skew plates with circular cut outs based on the finite element approach using ANSYS.	The natural frequency of moderately thick functionally graded skew plates with circular cut outs decreases as the volume fraction index increases and increases as the thickness ratio increases. It is observed that by increasing the distance between the centers of two circular holes in a FG skew plate, the frequency of the first mode, decreases. The variation in the non-dimensional frequencies is less when the skew angle varies from 0° to 30°, but the variation in the non-dimensional frequencies is more when the skew angle rises from 30° to 45°.
4	Anandrao et al. 2012	They studied free vibration analysis of Functionally Graded Beams.	Considering transverse shear in the design and formulation increased the flexibility and reduced the frequencies of short beam.
5	Alshorbagy, 2011	They studied the effect of material-temperature dependent on the vibration characteristics of a functionally graded thick beam using FEM. The beam was modeled by higher order shear deformation theory (HOBT), which was accommodated for a thick beam.	The natural frequencies increase with an increase in power exponent, and decrease with an increase in power exponent.
6	Wei et al. 2012	Obtained an analytical method for solving the free vibration of cracked functionally graded material (FGM) beams with axial loading, rotary inertia and shear deformation	The presence of cracks reduces the frequencies and changes the vibration mode shapes of FGM beams. Increase in crack diminishes the natural frequency ratios. The slenderness ratio and Young's modulus ratio are more sensitive to the free vibration of cracked FGM beams than the presence of cracks.
7	Khan and Parhi, 2013	Estimated the effects of crack depth on natural frequency and mode shape of beam	Natural frequency increases and Mode shape decreases as the crack depth increases.
8	Rao et al. 2018	Proposed a new extended isogeometric hybrid collocation–Galerkin method to obtain natural frequencies of a homogeneous and functionally graded material plate with internal defects of varying gradient index, sizes and shapes and compared with FEM result using COMSOL.	The obtained results from the proposed method are found to be in good agreement with FEM results (COMSOL).

**Table 2:** Material properties of FGM beam (Alshorbagy 2011)

Properties	unit	steel	Alumina
Young modulus	GPa	210	390
Mass density	Kg/m <sup>3</sup>	7800	3960

- Set the parameter sweep and desired Eigen frequency under study tab.
- Final step was to compute.
- Different Eigen frequencies and mode shapes for different parameter were obtained.

### 2.2.1 The condition of geometric fit (Alshorbagy 2011)

If  $u_0$  be the axial and  $w_0$  the transverse displacement of any point on the mid-plane at time  $t$ , then, the axial and transverse displacement of any point of the beam are given by Euler–Bernoulli beam theory (Eq. (2) and Eq. (3) respectively).

$$u(x, z, t) = u_0(x, t) - z \frac{\partial w_0(x, t)}{\partial x} \quad (2)$$

$$w(x, z, t) = w_0(x, t) \quad (3)$$

Eqs. (2) and (3) can be rewritten as Eq. (4) to define displacement vector  $d_s$ .

$$\{d_s\} = \begin{Bmatrix} u \\ w \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -z \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ w_0 \\ \frac{\partial w_0}{\partial x} \end{Bmatrix} \quad (4)$$

Assuming small deformations, the relationship between displacement ( $d_s$ ) and strain ( $\epsilon_{xx}$ ) can be given by Eq. (5)

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0(x, t)}{\partial x} - z \frac{\partial^2 w_0(x, t)}{\partial x^2} \quad (5)$$

### 2.2.2 The law of material (Alshorbagy 2011)

According to Hooke's law, the relationship between stress  $\sigma_{xx}$  and strain ( $\epsilon_{xx}$ ) is given by Eq. (6).

$$\sigma_{xx} = E(z)\epsilon_{xx} = E(z) \begin{bmatrix} 1 & -z \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial^2 w_0}{\partial x^2} \end{Bmatrix} \quad (6)$$

Where young modulus varies through the thickness direction.

### 2.2.3 Equilibrium condition (Alshorbagy 2011)

According to the principle of virtual work, the equilibrium condition of functionally graded beam for free vibration can be stated by Eq. (7).

$$\delta W_s - \delta W_i = 0 \quad (7)$$

Where  $\delta W_s$  and  $\delta W_i$  virtual work done by the stress field and inertia forces respectively. The virtual work done by a stress field  $\delta W_s$  on a virtual strain field is given by

$$\delta W_s = \iiint_V \sigma^T \delta \epsilon dx dy dz \quad (8)$$

$$\delta W_s = b \int_0^L \left\{ \frac{\partial u_0}{\partial x} \quad \frac{\partial^2 w_0}{\partial x^2} \right\} \left[ \int_{-h/2}^{h/2} \begin{bmatrix} E & -ZE \\ -ZE & Z^2 E \end{bmatrix} dz \right] \delta \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial^2 w_0}{\partial x^2} \end{Bmatrix} dx \quad (9)$$

The virtual work done by the inertia forces through the virtual displacement field i.e.  $\delta W_i$  can be represented as Eq. (10).

$$\delta W_i = - \iiint_V \rho d_s^T \delta d_s dx dy dz \quad (10)$$

Considering Eq. (4), Eq. (10) can be expressed as Eq. (11)

$$\delta W_i = b \int_0^L \left\{ \dot{u}_0 \quad \dot{w}_0 \quad \frac{\partial \dot{w}_0}{\partial x} \right\} \left[ \int_{-h/2}^{h/2} \begin{bmatrix} \rho & 0 & -\rho z \\ 0 & \rho & 0 \\ -\rho z & 0 & \rho z^2 \end{bmatrix} dz \right] \delta \begin{Bmatrix} \dot{u}_0 \\ \dot{w}_0 \\ \frac{\partial \dot{w}_0}{\partial x} \end{Bmatrix} dx \quad (11)$$

### 2.2.4 The finite element analysis (Alshorbagy 2011)

The displacement function of functionally graded beam at the mid-plane can be represented in plane component (Figure 2)

$$U_0^e(x, t) = N_1 U_1(t) + N_4 U_2(t) \quad (12)$$

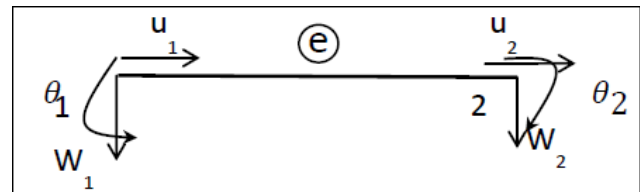


Figure 2: Beam element  
Transverse components

$$W_0^e(x, t) = N_2 W_1(t) + N_3 \theta_1(t) + N_5 W_2(t) + N_6 \theta_2(t) \quad (13)$$

Where,  $N$  denotes the shape functions or interpolating function.

$$N_1 = 1 - \frac{x}{L} \quad N_2 = \frac{1}{L^3} [L^3 - 3Lx^2 + 2x^3]$$

$$N_3 = \frac{1}{L^2} [L^2 x - 2Lx^2 + x^3] \quad N_4 = \frac{x}{L}$$

$$N_5 = \frac{1}{L^3} [3Lx^2 - 2x^3] \quad N_6 = \frac{1}{L^2} [-L^2 x + x^3]$$

From Eq. (9), Eq. (10), Eq. (11), Eq. (12), and Eq. (13), we have Eq. (14)

$$M\ddot{d}_s + K d_s = 0 \quad (14)$$

Where  $M$  and  $K$  is the global mass-matrix and global stiffness matrix of beam respectively.

Let  $n$  be the number of discretized elements. Then, the global stiffness matrix and element stiffness matrix are given by Eq. (15) and Eq. (16) respectively.

$$K = \sum_0^n k^e \quad (15)$$

$$K^e = b \int_0^1 B^T D_E B dx \quad (16)$$

Where,

$$D_E = \int_{-h/2}^{h/2} \begin{bmatrix} E & -ZE \\ -ZE & Z^2 E \end{bmatrix} dz$$

$$D_R = \int_{-h/2}^{h/2} \begin{bmatrix} \rho & 0 & -\rho z \\ 0 & \rho & 0 \\ -\rho z & 0 & \rho z^2 \end{bmatrix} dz N = \begin{bmatrix} N_1 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_2 & N_3 & 0 & \frac{\partial N_5}{\partial X} & \frac{\partial N_6}{\partial X} \\ 0 & \frac{\partial N_2}{\partial X} & \frac{\partial N_3}{\partial X} & 0 & \frac{\partial^2 N_5}{\partial X^2} & \frac{\partial^2 N_6}{\partial X^2} \end{bmatrix}$$

The global mass-matrix is

$$M = \sum_0^n M^e$$

And the element mass matrix is

$$M^e = b \int_0^1 N^T D_R N dx$$

Where,

$$D_R = \int_{-h/2}^{h/2} \begin{bmatrix} \rho & 0 & -\rho z \\ 0 & \rho & 0 \\ -\rho z & 0 & \rho z^2 \end{bmatrix} dz N = \begin{bmatrix} N_1 & 0 & 0 & N_4 & 0 & 0 \\ 0 & N_2 & N_3 & 0 & \frac{\partial N_5}{\partial X} & \frac{\partial N_6}{\partial X} \\ 0 & \frac{\partial N_2}{\partial X} & \frac{\partial N_3}{\partial X} & 0 & \frac{\partial^2 N_5}{\partial X^2} & \frac{\partial^2 N_6}{\partial X^2} \end{bmatrix}$$

**Ethical approval:** The conducted research is not related to either human or animal use.

## 3 Results and discussion

### 3.1 Effect of mass density and young modulus variation

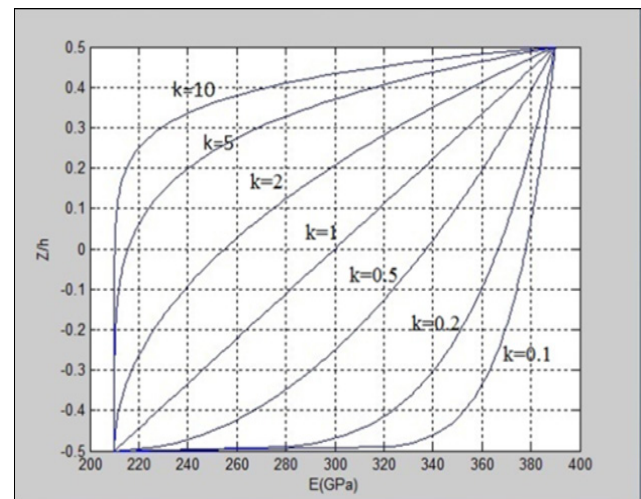
The variation in young modulus and mass density of functionally graded material with respect to thickness directions as well as in power law exponent were shown in Figure 3 and Figure 4 respectively. From Figure 3, it can be revealed that at the value of  $z/h$  as  $-0.5$ , young modulus

is 210 GPa and at  $z/h$  as  $0.5$ , young modulus is 390 GPa. Similarly, from Figure 4, it can be revealed that at  $z/h$  as  $-0.5$ , the mass density is  $4000 \text{ kg/m}^3$  and at  $z/h$  as  $0.5$ , the young modulus is  $7800 \text{ kg/m}^3$ .

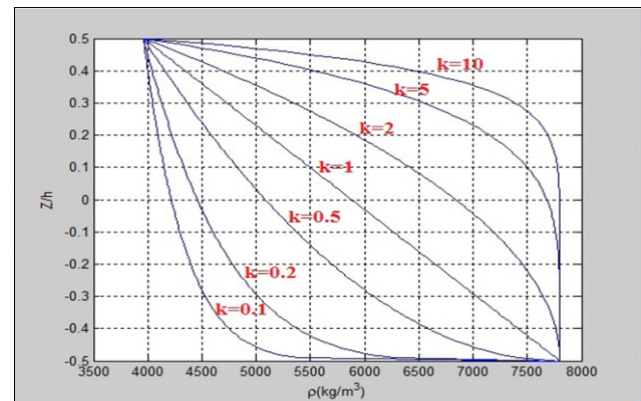
Different value of power law exponent ( $k$ ) showed different physical significance given below:

- (1) If the value of power law exponent ( $k$ ) is zero, then the beam is full alumina i.e. ( $E(z) = E_{\text{alumina}}$ ,  $\rho(z) = \rho_{\text{alumina}}$ )
- (2) If the value of power law exponent ( $k$ ) is 1000, then the beam is full steel i.e. ( $E(z) = E_{\text{steel}}$ ,  $\rho(z) = \rho_{\text{steel}}$ ).
- (3) If the value of power exponent ( $k$ ) is between 0 and 1000, the beam is functionally graded beam and their composition changes from alumina to steel.

Thus, the properties of material vary in proportion in the direction of thickness. In the present work, functionally graded material (FGM) consisted of steel (lower part) and alumina (upper part). Therefore, the lowest part is pure



**Figure 3:** Variation of young modulus through the thickness direction of the beam



**Figure 4:** Variation of mass density through the thickness direction of the beam





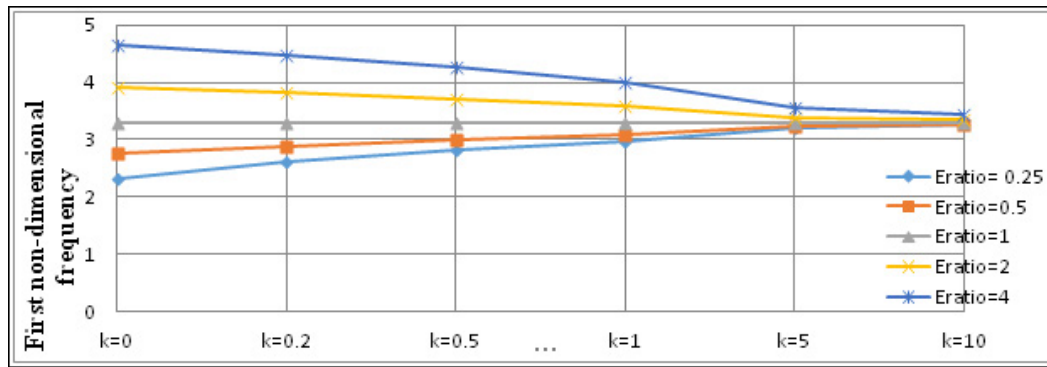


Figure 6: variation of the first non- dimensional frequency with Power Exponent (k)

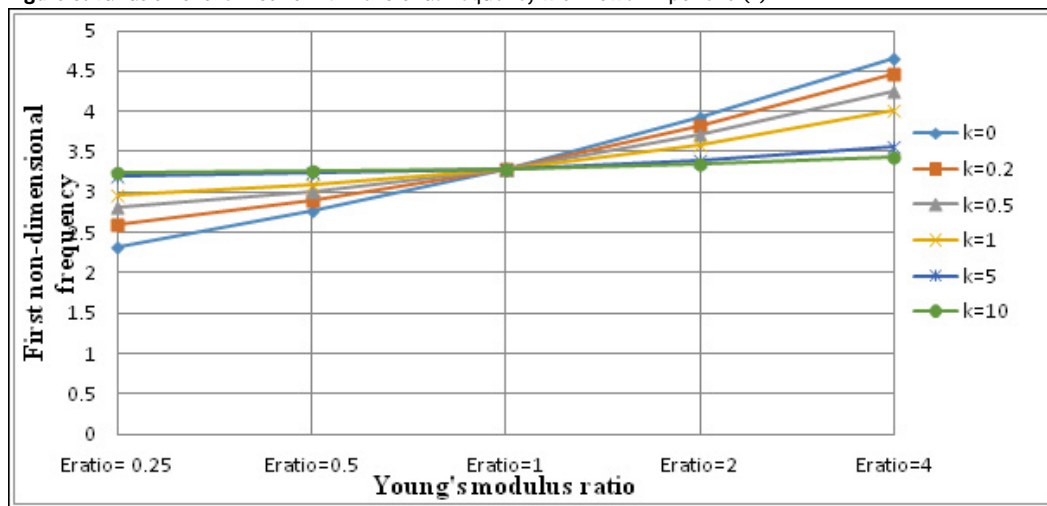


Figure 7: variation of the first non- dimensional frequency with Young's Modulus Ratio

was increased with the increase in power exponent. Further, for young's modulus greater than one, the Eigen frequencies was decreased with an increase in power exponent. However, no change was obtained for young's modulus ratio equals to 1. Above conclusions are also valid for second and third non dimensional frequencies with different young modulus ratio variation and non-negative power law index.

Figure 6 showed the graph between the first non-dimensional frequency and power law exponent for different young's modulus ratio. X-axis represents power law exponent and Y-axis represents first non-dimensional frequency. The increase in power exponent causes the increase in frequency for  $E_{ratio} = 0.25$  and  $0.5$ , decrease in frequency for  $E_{ratio} = 2$  and  $4$  and no change occurs for  $E_{ratio} = 1$ . For both situations when  $E_{ratio}$  is less than one or greater than one it is clear that the first Eigen frequency of the FG beam approaches the first Eigen frequency of the homogeneous beam as the power-law exponent  $k$  increases.

Figure 7 showed the graph between the first non-dimensional frequency and young's modulus ratio for different power law exponent. X-axis represents young's modulus ratio and Y-axis represents first non-dimensional frequency. The frequency was increased significantly with increase in young's modulus ratio for lower value of power exponent. Conversely, there were no significantly changes obtained in frequency for different value of young's modulus ratios for higher value of power exponent.

### 3.2 The effect of boundary conditions

The boundary conditions (BCs) affects the vibration behavior of functionally graded beam materials. Table 5 indicates the effects of four different boundary conditions on the fundamental frequencies of the functionally graded beam. Four different boundary conditions are simply-clamped (SC), clamped-clamped (CC), simply-supported (SS) and clamped-free (CF).

Figure 8 showed the graph between the first non-dimensional frequency and power law exponent for different boundary conditions. X-axis represents first non-dimensional frequency and Y-axis represents first power law exponent. It can be observed that for all boundary conditions, increase in power law exponent ( $k$ ) led to decrease in frequencies. The reduction in frequencies was attributed to increase in steel part as compared to alumina part in the beam composition. The first non-dimensional frequency for Clamped-Clamped boundary condition is comparatively more than other boundary conditions and lowest frequency is obtained for Clamped-Free boundary conditions.

## 4 Conclusions

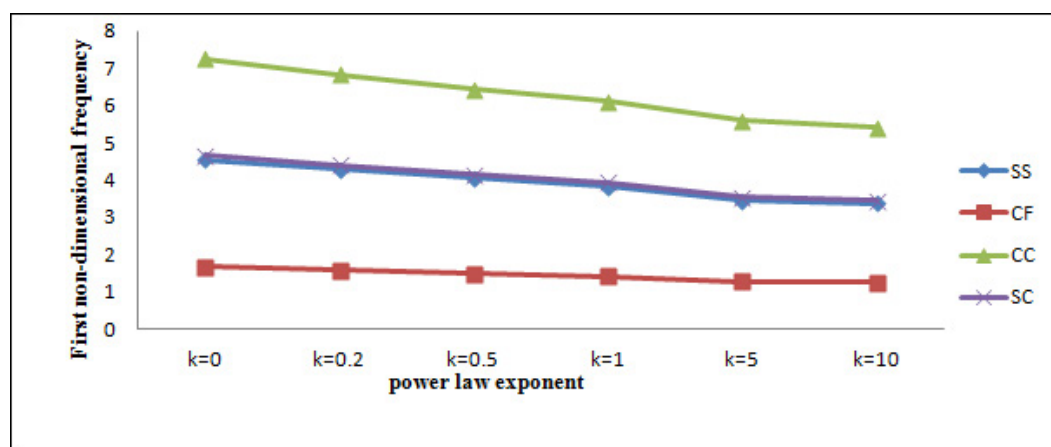
Using Finite Element Analysis, the vibration analysis of functionally graded beams for cylinder liner in agricultural application has been studied. The natural frequency of the FGM beam is affected by two parameters such as Young's modulus ratio and power law exponent. It can be concluded from the above analysis that the effect of Young's modulus ratio is more profound than the effect of power law exponent. The material behavior

is characterized by the power law exponent. The natural frequency and stiffness elevated when power law exponent approached zero shows the interdependency of natural frequency and stiffness. Hence, performance of FGM beam was similar to the alumina beam.

Conversely, the behavior of FGM beam was similar to the steel beam, when power law exponent was equal to or greater than 1000. The natural frequency increases with the increase of power exponent only when the Young's modulus ratio is less than one.

In contrast to this, the natural frequency decreased as the power exponent was increased, when the Young's modulus ratio was greater than one. The natural frequency was invariant, when Young's modulus ratio was unity. The initial non-dimensional frequency for CC boundary condition was relatively more compared to other BCs and lowest frequency was obtained for CF BCs. As the fixed-fixed beam has lesser moments and deflection at mid-span in comparison to other boundary conditions. The frequency is inversely related to the deflection. The frequency of CC boundary condition is moderately more than other BCs.

**Conflict of interest:** Authors declare no conflict of interest.



**Figure 8:** variation of first non- dimensional frequency with different boundary conditions

**Table 5:** First non-dimensional frequency ( $\lambda$ ) for different boundary conditions

L/h	B.C'S	K=0	K=0.2	K=0.5	K=1	K=5	K=10
20	SS	4.55	4.29	4.04	3.82	3.45	3.37
	SC	4.66	4.4	4.15	3.93	3.55	3.46
	CF	1.67	1.57	1.49	1.41	1.27	1.24
	CC	7.25	6.83	6.44	6.11	5.6	5.44



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