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The Joint Philosophical Program of Russell and Wittgenstein and Its Demise

Abstract

Between April and November 1912, Bertrand Russell and Ludwig Wittgenstein were engaged in a joint philosophical program. Wittgenstein's meeting with Gottlob Frege in December 1912 led, however, to its dissolution – the joint program was abandoned. This paper outlines the key points of that program, identifying what Russell and Wittgenstein each contributed to it. It determines precisely those features of their collaborative work that Frege criticized. Finally, building upon the evidence developed in the first two sections, it recasts, along previously undeveloped lines, Wittgenstein's logical–philosophical discoveries in the two years following his encounter with Frege in 1912. The paper concludes with an overview of the dramatic consequences the Frege-Wittgenstein critique had for Russell's philosophical development.

1. Wittgenstein-Frege-Russell 1912-13

This paper investigates the interaction between the three founding fathers of analytic philosophy – Russell, Wittgenstein and Frege – during a formative period of their philosophical development. It sheds light on the joint program that Russell and Wittgenstein collaborated on from April till November 1912, as well as on its collapse after Wittgenstein visited Frege in Jena in December the same year. The key finding is that Frege's criticism of elements of the program both motivated and informed Wittgenstein's criticism

of Russell's approach to philosophy. This radical challenge culminated in May and June of 1913 when, facing Wittgenstein's criticism, Russell abandoned his book project on theory of knowledge. Frege's remarks also impelled Wittgenstein to rethink and reformulate his own philosophical ideas.

Among other matters, germane to this seminal development in the history of early analytic philosophy is an issue, addressed in section 4.2, that has been actively debated in the literature over the past thirty years: Wittgenstein's criticism of Russell's multiple-relation theory of judgment (Griffin 1985, Hanks 2007). Rather than explaining this criticism in the usual manner, i. e., on the basis of purely logical considerations, the move here is to disclose its ground in Frege's impact on Wittgenstein at a particular historical moment. The evidence adduced in this account implicitly discredits the "competitive interpretation" of the collaboration between Russell and Wittgenstein, an account that casts the purported competition in terms of who was the better philosopher, Russell or Wittgenstein. Nicholas Griffin and Gregory Landini, for example, criticize the alleged claim that in the months between April 1912 and June 1913 Wittgenstein repeatedly corrected mistakes of his teacher Russell, so that "if only Russell had been a better philosopher, he would have been Wittgenstein" (Griffin 1996: 222; Landini 2003/4, 2007).

By contrast with the latter view, this essay establishes that while between April and November 1912 Wittgenstein enhances Russell's philosophical development, this occurred only because the young student opened Russell to new perspectives on Russell's own philosophy. Moreover, the evidence makes it clear that between January and June of 1913 what Wittgenstein confronted Russell with were mainly changes in Wittgenstein's thinking, which trace directly to the influence of Frege's philosophical logic. Russell's putative "defeat" in the face of Wittgenstein's criticism in June 1913 thus merely signaled the former's realization that the idea of an exact philosophy, as he had initially conceived it, faced viable alternatives, and hence that the prospect of systematically articulating such a philosophy was fraught with much more complexity than he had anticipated.

2. The joint program (April–November 1912)

After the publication of Volume I of *Principia Mathematica* in 1910, Bertrand Russell concentrated his efforts mainly on questions of epistemology. In "Knowledge by Acquaintance and Knowledge by Description" (1911) and *The Problems of Philosophy* (written in July and August of 1911, and published in 1912), he addressed some of the epistemological problems that his new logic had raised. Late in October 1911, Russell first met Ludwig Wittgenstein. During the Winter Term at Cambridge a strong intellectual sympathy developed between the two men. Russell revealed as much in a remark that appears in a letter dated March 15, 1912, a week after the term ended: "His [Wittgenstein's] attitude justifies all I have hoped about my work" (Clark 1975: 172).

In the spring and Fall Terms of 1912, Russell and Wittgenstein collaborated intensively on what can be seen as a joint philosophical program. This was first brought to light in 1988 by the Wittgenstein biographer Brian McGuinness, who observed that "the two philosophers were concerned with the same problems" (McGuinness 1988: 159). Some years later, Ray Monk made explicit in his biography of Russell that "Russell and Wittgenstein regarded themselves as collaborators on the same project" (Monk 1996: 286).

During the months of their mutual engagement in this project, Russell and Wittgenstein often worked together. It is noteworthy that this was the only period in his career that Wittgenstein ever collaborated with another philosopher. While he frequently challenged Russell's conceptions, Wittgenstein limited himself to constructive criticism, concentrating his theoretical energies on developing ideas that supplemented those of Russell.

Apparently, the joint program featured a division of labor. While Russell concentrated on problems of epistemology, Wittgenstein focused mainly on problems of logic. As we shall see, however, each of them also introduced ideas into the other's field. This culminated in October–November 1912, during the last weeks of their collaboration, when Russell wrote a paper on logic – "What is Logic?" (see section 2.3, below) – and Wittgenstein authored one on epistemology: "What is Philosophy?", which he read at the

Moral Science Club in Cambridge on November 22. In it "philosophy was defined as all those primitive propositions which are assumed as true without a proof by the various sciences" (McGuinness 1988: 144).

Things changed radically after Wittgenstein traveled to Jena and met with Frege in December 1912. As we shall see, Frege convinced him that his joint project with Russell was based on flawed assumptions and hence was fundamentally misconceived.

The first sign that the collaboration with Russell was at an end was that Wittgenstein ceased seeking out Russell's for collaboration. He now felt that he could progress more profitably by thinking his way to logical–philosophical discoveries of his own, without consulting Russell (cf. Milkov 2007: 83–4; 2012). The dissolution of the collaborative spirit between the two philosophers is captured in a note by David Pinsent from February 4, 1913. Wittgenstein, we read, dictated "his latest discoveries in the Fundamentals of Logic. […] Russell acquiesced in what he said without a murmur" (Pinsent 1990: 44).

Concomitant with this breakdown of significant philosophical interaction was a palpable cooling of the personal ties between Russell and Wittgenstein. In March 1913 Russell "began to feel that Wittgenstein was narrow and uncivilized, 'rather too much the champion of a party'" (McGuinness 1988: 172, Russell's letter to Ottoline Morrell #717 6.3.1913[1]). By May of that year they had become so estranged that Russell did not bother to inform his younger colleague that he was composing a new book on epistemology: *Theory of Knowledge*.

The event that sealed the termination of their joint program was Wittgenstein's devastating criticism of Russell's *Theory of Knowledge* in late May 1913 (which will be discussed in section 4.2). The first positive result of this total break with Russell was Wittgenstein's "Notes on Logic", which he wrote in August and September of that year. In practical terms, the estrangement between the two

[1] Henceforth, letters from Russell to Lady Ottoline Morrell, as systematised at the Harry Ransom Centre, University of Texas at Austin will be referred to with their numbers preceded by #. In other cases, Russell's letters are identified by the date they were written.

philosophers was evident in Wittgenstein's decision in the summer of 1913 to leave Cambridge for Norway – he no longer needed to collaborate with Russell.

2.1 The impact of Wittgenstein on Russell's epistemology

Elements of the joint program appear in Russell's paper "On the Notion of Cause" (Russell 1912c), which he read at a meeting of the Aristotelian Society in early November of 1912. As merely a popular version of Russell's highly developed views on the relation between philosophy and physics, however, it hardly counts as a significant resource for understanding the Program. Be that as it may, Russell's criticism of the notion of causality in the paper clearly parallels Wittgenstein's approach to the topic in *Tractatus* 5.136–1: only logic is necessary; causality can convey neither regularity nor principle.

The most important document attesting to Russell and Wittgenstein's bona fide collaboration in a joint program is "On Matter" (Russell 1912b), a paper that Russell wrote in May 1912 and delivered that same month at a meeting of the Philosophical Society of University College, Cardiff. Russell revised the essay in October 1912 and read it near the end of that month before the Moral Science Club at Cambridge.[2]

Russell's declared aim in "On Matter" is to show

> (i) that all the arguments hitherto alleged by philosophers against matter are fallacious; (ii) that all the arguments hitherto alleged in favor of matter are fallacious; (iii) that, although there may perhaps be reason to suppose that there is matter, yet we can have no means of finding out anything whatever as to its intrinsic nature. (Russell 1912b: 80)

The first thing that strikes the reader of these lines is their close affinity with the highly truncated style of Wittgenstein's writings from the period 1912–16. As regards their content, Russell's radical skepticism reflects Wittgenstein's impact on his thought. The link between skepticism and Wittgenstein in Russell's thinking is borne

[2] This variant of the paper was published as Russell 1992b. In our analysis of the Joint Program below we will refer to it.

out by Russell himself in a remark from May 2, 1912: Wittgenstein, he confessed, "is the only man I have ever met with a real bias for philosophical scepticism; he is glad when it is *proved* that something can't be known" (McGuinness 1988: 106). Thus it is no surprise that, when Russell delivered the revised version of "On Matter" in October, his feeling was that "no one except Wittgenstein understood it at all" (#608). It is worth noting that along with Wittgenstein, G. E. Moore was also in the audience, but even he failed to make sense of Russell's new ideas. On the other hand, Wittgenstein thought that Russell's paper "On Matter" was the best thing he had done (#460) – not excluding *The Principles of Mathematics* and *Principia Mathematica*.

The results of Russell's newly adopted skeptical stance can be briefly outlined. A few months before he met Wittgenstein, Russell adopted the view, which he presented in *The Problems of Philosophy*. It maintained that we do not directly perceive physical objects *per se* – we perceive sense-data; the latter are qualities and relations, including qualities and relations of sense-data. Nevertheless, we *can* know physical objects, although only by description. This is because the sense-data with which we are acquainted help us logically to *infer* that there are physical objects. At that time, Russell believed that despite not being absolutely satisfactory, this conception is much more consistent with the facts of the external world than any competing philosophy of matter, especially *solipsism*, according to which there are no physical objects at all.

When Russell raised the problem of solipsism with Wittgenstein on 23 April 1912, however, the latter breezily dismissed its significance as a philosophical challenge, declaring that solipsism "doesn't hurt, since [even if there are no other minds,] physics and astronomy, and all the other sciences could still be interpreted so as to be true" (Monk 1996: 260). Indeed, we can imagine a private world – a world existing only in order to affect our senses – in which the laws of science are valid. This argument impelled Russell to abandon the view that we can infer matter from sense-data. Instead, he began regarding matter as a *logical construction* on the

basis of the objects of acquaintance (Russell 1912b: 84).[3] What underlay this radical shift in Russell's thinking was the idea that the world consists ultimately of independent atomic units (later called "logical atoms") – sense-data. These primitive elements could be ordered in many different, logically organized nets in which sense-data interrelate with one another.[4] The objects of common sense and those of the hypotheses of science could be seen, in this account, as alternative constructs of these and other units.

In the philosophy of science, Russell now subscribed to the view that

> physics may be studied, […] as a piece of pure mathematics; the space and matter concerned in this study are variables,[5] concerning which certain hypotheses are made; that is to say, they are not definite entities, but merely anything having certain properties. (Russell 1912b: 83)

This statement reveals how fundamentally Russell revised his philosophy during his collaboration with Wittgenstein. He now claimed that we can abandon the problem of the content of the beliefs of common sense, contending instead that its supposed content – the matter – and the objects of common sense are logically congruent.

Just how radical the change was in Russell's position is clear in that Russell, who had since 1898 considered himself a committed realist, saw his revised program as "The bankruptcy of Realism" (#423).

Not surprisingly, the new doctrine had consequences for Russell's worldview. Among other things, it brought with it an unsettling sense that nothing in this world is solid and secure.

[3] This was the first time ever Russell spoke about "logical constructions". The new concept was, however, related to ideas from *Principia Mathematica*, in particular, with the definition (in Introduction, chapter 3) of classes as "incomplete symbols". The term itself was first used in F. H. Bradley's *The Principles of Logic* where he spoke about "logical, or ideal, constructions" (Bradley 1883: 257).

[4] How exactly such nets are to be constructed Russell learned from Whitehead. In December 1912 the latter showed Russell the technique of defining points, instants and "things" as logical constructions (Russell 1914: 11). The idea of "logical constructing", however, was born in the discussions of Russell with Wittgenstein some weeks earlier.

[5] These variables can be replaced by the sense-data we perceive.

Russell himself intended his paper "On Matter" to be "a model of cold passionless analysis, setting forth the most painful conclusions with utter disregard of human feelings" (24.5.12).

2.2 Russell's new ontology and Wittgenstein's new logic

Even as Russell developed his philosophical position in writing and redrafting "On Matter", Wittgenstein was doing work in the philosophy of logic which exhibits demonstrable parallels with it. In fact, Russell's discussion of the problem of matter and Wittgenstein's of the nature of logic "proceeded *pari passu*" (McGuinness 1988: 160). There is concrete evidence testifying to this mutual engagement. On the verso of the folio 1 of Russell's project-paper "Matter, the Problem Stated" (Russell 1912/13: 96) we find notes on logic, some in Wittgenstein's hand and one in Russell's, most probably written before Wittgenstein visited Frege (before December 1912).[6] It is thus undeniable that between April and November of 1912, Russell and Wittgenstein collaborated closely.

Wittgenstein's jottings proved of considerable historical significance, including as they do the first sketch of a truth table device.[7] The latter schematized in a tabular form the truth-possibilities of the combinations (of the logical connections, or operations) of two propositions. This innovation had important consequences, most significantly in demonstrating that logical operations can be represented in a radically perspicuous way by means of *one* symbol. For example, 'p v q' can be presented as follows:

[6] Cf. n. 8.

[7] It is not to be confused with the propositional logic of truths introduced by Frege in his *Conceptual Script*. The difference between them is that whereas the latter is "a logical analysis of the truth-values of a proposition, the truth-table *device* is the presentation of this analysis in tabular or matrix form" (Anellis 2004: 57).

‘

p	q	
T	T	T
F	T	T
T	F	T
F	F	F

’.

In other words, this innovation helped “to explain the self-evidence of logical propositions” (Wittgenstein 1976: 177). As we are going to see in section 4.3, below, the truth table device proved to be a first step toward the Doctrine of Showing that Wittgenstein introduced a year later.

Besides the truth table device, in November 1912[8] Wittgenstein also originated the notion that there is only one logical constant. To be more specific, he determined that we can express truth-operations by employing a single sign for the logical connective. The idea was that logical constants can be reduced to a single logical operation, ‘ ( ) ’ (cf. McGuinness 1988: 161).

Most significantly, Wittgenstein’s logical innovations at this time were closely tied to the ontology Russell advanced in “On Matter”, which Wittgenstein thus evidently embraced. As remarked above,

[8] I.e. before H. M. Sheffer, who first presented a similar idea to the American Mathematical Society on December 31, 1912. This shows Gregory Landini’s claim that in the jottings Russell and Wittgenstein discussed Sheffer’s stroke to be mistaken. As Landini himself notes, Russell received a copy of Sheffer’s paper on April 15, 1913 (Landini 2007: 107 f.). (The paper itself was published in October 1913.) After Wittgenstein returned from Vienna at the very end of January 1913, however, he did not worked together with Russell: he *dictated* his “new ideas” (cf. section 2, above). And subsequent to the “terrific contest” between Russell and Wittgenstein from 6.3.1913 (Monk 1996: 291), *tête-à-tête* collaborative work between them was unthinkable.

in section 2.1, Russell's ontology assumed that the world consists of complex units, sense-data, which physics represents as variables and which we can order, or compose, in many different logically organized nets: either hypotheses of science, or "things" of common sense. We could call this Russell's Compositionality Thesis. Apparently, Russell's minimalist ontology, without physical objects and without objects of common sense, went hand in hand with Wittgenstein's novel minimalist logic, with its single logical constant.[9] Indeed, both maintained a parsimonious form of compositionality: Russell in ontology, Wittgenstein in logic.

Corroborating this interconnection is the fact that Russell initially employed the sign ')(' to refer to the interweaving of the elements of ontological complexes with which we are acquainted (Russell 1905: 169, cf. section 2.3 below). Wittgenstein, on his side, used it to symbolize the only logical constant and, so, the interweaving of the logical atoms ("atomic propositions").

Wittgenstein himself appears to have been cognizant of the interconnection between Russell's constructivist ontology and the assumption that there is only one logical constant, which he eventually expressed this way: "Wherever there is compositeness […] we already have all the logical constants", and this means that there is a "sole logical constant" (TLP 5.47).

2.3 Harmony between logic and ontology

The foregoing points cast additional light on Russell's motivation to address directly the relatedness between logic and ontology in the Joint Program. This he did in the days immediately after 13 October 1912, when he wrote "What is Logic?" (cf. section 2, above), a piece merely two pages in length. Its premise is that "logic is the study of the forms of complexes" (Russell 1912b: 55).[10] Logic, it declares, does not deal with judgments, something it

[9] We can see this fact as an example of how Russell's "ideal of eliminativistic reconstruction" (Landini 2003/4: 118) – in this case, the elimination of physical objects – was also embraced by Wittgenstein in the form of elimination of logical objects. Cf. also Landini 2007.

[10] An echo of this program is found in *Our Knowledge*, where Russell claimed that "the first business of logic [is ...] a classification of logical forms of facts" (Russell 1914: 60).

consigns to psychology; nor does logic concern propositions, which can be false and hence in Russell's view cannot be anything objective but merely forms of words. Indeed, "*true* and *false*", asserts Russell, "are extra-logical" (ibid.).

What evidently inspired Russell's paper were ideas that Wittgenstein had formulated even prior to their collaboration. In November 1911, Wittgenstein took as his own philosophical point of departure the position from which Moore and Russell himself began in 1899:[11] "there is nothing in the world except asserted propositions which are complexes of concepts" (McGuinness 1988: 89). In the Spring and Fall Terms of 1912, Russell readily embraced Wittgenstein's (and Russell's old) position, and he had compelling reasons for doing so: it harmonized with a tendency in Russell to restore to his philosophy the notion of complexes which he had championed between 1898 and 1900, but which he had more or less repudiated between 1900 and 1905 under the influence of Peano and Frege. (In § 3 below, we will call the impact of Frege's logic on Russell immediately after August 1900 the "first lesson" Russell received from Frege.)

The year 1905, however, saw Russell adopting (arguably, under Alexius Meinong's influence) once again, an ontology of complexes. After assigning "knowledge by acquaintance" a primary role in epistemology in "On Denoting", Russell contended that complexes are among the things with which we are acquainted (Milkov 2003: 50). This move partly restored to his philosophy the realistic mereology (i.e., part/whole "logic") of complex and simples that was an ingredient in his philosophy prior to August 1900.

This tendency persisted. Shortly after he formulated the theory of descriptions, Russell eliminated classes from his logic: classes are "incomplete symbols". There are only propositions and propositional functions. Two years later, in 1907, Russell discovered that propositions produce paradoxes of their own. In consequence, he came to maintain that propositions too are

[11] Apparently, Moore and Russell did so under the influence of the Brentanist school. Cf. Bell 1999.

incomplete symbols. To be more exact, they were eliminated with the help of the multiple relation theory of judgment which claims that propositions only receive meaning (and unity) through the judging mind (Stevens 2005: 79). Truth-bearers are judgments, not propositions.

The signal result of this development was that the ontology of complexes came to play an important role in *Principia Mathematica*, something clearly attested to in the following statement from that work:

> … the universe consists of objects having various qualities and standing in various relations. Some of the objects which occur in the universe are complex. When an object is complex, it consists of interrelated parts. (Russell & Whitehead 1910: 43)

This ontology is clearly close to that of Russell in 1898–1900. Be this as it may, till 1912, Russell's realistic mereology was no more than a focal tendency in his ontology. With "What is Logic?", however, Russell recast his mereology as a consistent program.

3. Frege's criticism

In December 1912 Wittgenstein met with Frege in Jena to debate philosophical logic, and subsequently reported that Frege "absolutely wiped the floor with [him]" (Goodstein 1972: 272). What followed was that Russell received his "second lesson in logic" from Frege.

Russell got his "first lesson in logic" in 1900. Up to that time, his logic followed the relational theory of judgment elaborated in Moore (1899). According to the latter, judgments and propositions are composed of complexes consisting of concepts and relations between them. This was a program for part/whole "logic", or mereology, in which logical implication is possible both between terms and between propositions. Russell's doctrine was also in conformity with the logic of classes, as well as with Boole's algebra of thought. However, at the International Philosophy Congress in Paris, in August 1900, Russell learned from Peano (something the latter learned from Frege) that besides the relations between parts and whole there is also a relation of implication which holds

between propositions, not between individuals, and that this second relation is more fundamental.[12]

Other intensionalities (unanalyzed units) which Russell introduced into his logic after August 1900 were the concepts of "proposition" and "denoting phrase". It was in accordance with Frege's context principle that Russell now subscribed to the view that we employ a proposition's terms within the frame of the entire proposition, not as autonomous, discrete units.[13] As for the intensionality of the denoting phrase, Russell maintained that even when singular, a denoting phrase refers to a collection, which may be either finite or infinite (Milkov 2003: 50, 63). In short, denoting phrases and propositions signify holistically, and do not require the availability of all their elements in order to have a sense.

While Russell did not want to understate the importance of analysis and of relations, incorporating intensionalities in his logic in 1903 led him to recognize two kinds of wholes: aggregates and units. An aggregate is definite only when all its constituents are known. Units, by contrast, have no such requirement; what's more, we can know a unit when we know merely a part of it. The paradigm of the unit in Russell's logic is the proposition (Russell 1903: § 135; Stevens 2005).

F. H. Bradley correctly observed that this assumption contradicts Russell's defense of "strict pluralism, for which nothing is admissible beyond single terms and external relations" (Bradley 1910: 179). It appears that Russell was conscious of this inconsistency, for in a letter to Bradley dated March 2, 1911, he implicitly conceded Bradley's point: "With regard to unities, I have

[12] Russell embraced a rudimentary form of what a year later was called "material implication" already in Russell (1899–1900). He did so under the continuing influence of Moore's work in philosophical logic, especially of his paper "Necessity" (Moore 1900).

[13] In fact, Russell accepted this position only with reservation – a point set out by Peter Hylton who insisted that "Russellian propositions [of 1903] are hybrid entities. On the one hand, they are, like Fregean *Gedanken*, abstract entities representing or embodying the content of a declarative sentence. On the other hand, unlike their Fregean analogues, these abstract entities can contain concrete entities, such as people and moments of time." (Hylton 2005: 35) This ambiguity in Russell's 1903 position on propositions came to light in his rehabilitation of the mereological conception of propositions after 1905.

nothing short to say. The subject is difficult […] & I do not pretend to have solved all its problems." (Bradley 1999: 145)

As we noted in § 2.3, aggregates gradually came to assume an ever greater importance at the expense of units in Russell's work after 1905. This development culminated in the Joint Program, which saw units virtually eliminated. And it was then that Russell got his second lesson in intensionality in logic – from Frege again but this time via Wittgenstein.

Before exploring this, it should be said that the only place where Wittgenstein explicitly acknowledges what he took from his meeting with Frege in December 1912 is in a fragment published both in his *Philosophical Remarks* and *Philosophical Grammar*. Hence it is that fragment that serves here as the starting point for the analysis of the Joint Program's demise. Frege criticized Russell's and Wittgenstein's move to identify complexes with facts, pointing out that a "complex is not like a fact. For I can, for example, say of a complex that it moves from one place to another, but not of a fact" (Wittgenstein 1964: 301, Wittgenstein 1974: 199). Frege also questioned Wittgenstein about whether if an object were a part of a fact about it, the fact would be larger than the object. Frege obviously held that whereas "a complex is a spatial object, composed of spatial objects" (ibid.: 302; 200), a fact is not.

At first sight it is surprising that Frege spoke about facts at all. Usually, he restricted facts to the realm of sense (or thought), so they did not play a significant role in his ontology (Dummett 1981: 177). It seems that Frege started to think about facts more intensively only after his meeting with Wittgenstein in December 1912. What especially struck Frege as mistaken was the idea that when we understand propositions we grasp spatial complexes. He argued, instead, that in such cases, we understand *one* thing that is not spatial, namely the sense of the proposition that we grasp, which can be either true or false. It really differs from the spatial complex, which is segmented. We find an echo of these considerations in Frege's paper "Thoughts" (1918/19) where he defined "fact" as "a thought that is true" (Frege 1918/19: 368). It deserves notice that this was the only place in Frege's corpus where he discussed facts.

4. Wittgenstein's transformation

Wittgenstein did not take long to assimilate Frege's insight, process which McGuinness describes thus:

> At the time [Wittgenstein] thought the remark [of Frege on propositions and complexes] silly, but later he came to see the point of it. It was in fact an attack on the whole notion of explaining the meaning of propositions by saying that there were complexes corresponding to them. (McGuinness 1988: 164)

However, between Frege and Wittgenstein significant differences remained. Above all, Frege continued to consign facts to the realm of sense (thought). Wittgenstein, on the other hand, claimed that whereas a proposition's sense is the possible fact we grasp when we understand it (Wittgenstein 1914: 112), and which can be true or false, its meaning, by contrast, "is the [real] fact which actually corresponds to it" (ibid.: 94). In a word, Wittgenstein persisted in being a realist in logic. In this respect, at least, he remained true to Russell and to the Joint Program.

Wittgenstein's taking up Frege's point was followed by a series of discoveries Wittgenstein made in the next twenty months.

4.1 Truth-making

The upshot of the analyses described in the preceding section is that Frege urged Wittgenstein to conceive truth as a correspondence of propositions to singular objects of the external world such as facts. Facts, for their part, either exist or are merely possible. This means that every correctly constructed proposition, i.e. every proposition with sense, either does or does not correspond to a fact in the real world, which is its meaning.[14]

This, by the way, was a position that Russell opposed in the period of 1907–12. If we accept it, Russell objected, we must also accept the existence of counterfeit objects, such as "Charles I's death in his bed". This conception contradicted Russell's "robust sense of reality" and he sharply repudiated it. He concluded that

[14] "The *Bedeutung* of a proposition is the fact that corresponds to it" (Wittgenstein 1914: 112).

"*no* judgment consists in a relation to a single object" (Russell 1910a: 120). Instead, Russell embraced the view that both the judgment and the proposition are relations between the judging subject and the different particular objects of the judgment or proposition, with which the subject forms a complex. This was his famous multiple relation theory of judgment we already spoke about in section 2.3.[15]

By contrast, in 1913 Wittgenstein had come to advocate the view that propositions correspond to those facts which are the meanings of the proposition.

In this way he introduced an important refinement to the conventional correspondence theory of truth, the theory Russell defended after 1907. Wittgenstein now held that the real world *makes* some of the possible worlds of the sentences we use true, or "real". This was nothing less than the theory of truth-making, which Wittgenstein launched in "Notes on Logic" (Wittgenstein 1913: 95). He would later speak of truth-making in the *Tractatus* (5.101). Russell embraced the theory of truth-making only in "The Philosophy of Logical Atomism" (Russell 1918a: 182 ff.).

Significantly, this newly adopted theory of truth was consistent with the truth table device that Wittgenstein initially sketched in his jottings on logic of November 1912, albeit with some modifycations. He now conceived of the truth table as schematizing possible meanings or grounds – not just the truth-possibilities – of propositions in the sense of facts that make propositions true or false.

Incidentally, Wittgenstein's transition from truth-possibilities to truth-grounds explains a fact to which McGuinness first called attention. After November 1912, Wittgenstein did not discuss the truth table device. Neither in the "Notes on Logic" nor in the

[15] Russell, not always being careful about the terminology he employed, sometimes, for example, in *The Problems of Philosophy*, also defined truth as "some form of correspondence between belief and fact" (Russell 1912a: 190).

Notebooks 1914–1916 is there any trace of it. It does subsequently show up, however, in the *Tractatus* (4.31, 4.442, and 5.101).[16]

What likely explains Wittgenstein's silence on the truth table device is that after the change in his outlook following his encounter with Frege in December 1912, he began thinking about how this innovation might serve roles quite different from those it had played during his collaboration with Russell. Indeed, as reintroduced in the *Tractatus* the truth table device serves a new function. While in 1912 it was a purely symbolic figure that helps to grasp the truth-dependence of the propositions, in the *Tractatus* (5.101) it referred to the real world. Thus if initially the truth table schematically identified only truth-possibilities, by the time of the *Tractatus* it identified truth-grounds, or truth-makers.

4.2 Criticism of the multiple relation theory of judgment

In May and June 1913, when Russell showed Wittgenstein the first parts of *Theory of Knowledge*, Wittgenstein had the opportunity to criticize Russell's multiple relation theory of judgment directly. In light of the analysis in the preceding sections, it is reasonable to conclude that the gist of Wittgenstein's criticism was addressed against taking the ontology of complexes as fundamental in logic. Indeed, Wittgenstein was in effect to argue that Russell's theory was built on the ontology of complexes, according to which, for example, "'*C*'s belief that *A* hates *B*' is a complex in which *belief* combines *A* and *B* and *C* and hatred into one whole" (Russell 1911: 169).

In fact, in "The Philosophy of Logical Atomism" (1918) Russell himself suggested an interpretation in this direction. He stated that what Wittgenstein showed him in 1913 was that we cannot make a geometrically articulated map of a belief since we cannot present a belief in Euclidean space. This is the case because in propositions such as "Othello believes that Desdemona loves Cassio" the subordinate verb ("loves") does not function as a verb when the

[16] Brian McGuinness commented on this fact in this way: "These jottings [on logic] are a valuable reminder of how little we know about the genesis of the *Tractatus* and how misleading the fragmentary preliminary work we have can be" (McGuinness 1988: 162).

judgment happens to be false, as in this example.[17] Russell concluded that "you cannot get in space any occurrence which is logically of the same form as belief" (Russell 1918a: 225). He was explicit that "the discovery of this fact is due to Mr. Wittgenstein" (ibid.: 226).

In a sense, Russell was right – we cannot make a map of a belief. Still, this interpretation shows that he failed to grasp the full force of Wittgenstein's argument with all its consequences. It is not just of beliefs that we cannot make maps; we cannot make maps of any fact whatsoever. This is because maps are articulated in space and so cannot communicate facts. Facts can be "modelled", not mapped (cf. section 4.3 below). We can map only complexes.

More specifically, Wittgenstein showed Russell's multiple relation theory of judgment to be invalid in light of a corollary of his own (Wittgenstein's) position, after December 1912, on the status of complexes. What we judge, asserted Wittgenstein, are propositions' senses which are singular objects; we do not judge complexes. One of Wittgenstein's elliptical pronouncements on the subject supports this reading of his new doctrine: "When we say A judges that etc., then we have to mention a *whole proposition* which A judges" (Wittgenstein 1913: 94, my italics). It does not suffice merely to enumerate the elements of the judgment, as the multiple relation theory prescribes. Wittgenstein concluded that "the proper theory of judgment must make it impossible to judge nonsense [such as] 'this table penholders the books'" (ibid.: 95). The latter phrase is not a proposition with sense but a heap (concatenation) of words.

We hold that this interpretation of Wittgenstein's critique of Russell's multiple relation theory of judgment is much simpler and has greater explanatory power than the competing ones. For example, it comfortably explains the "directional problem" of Russell's theory that Griffin (1985) sees as the main point attacked by Wittgenstein. According to Griffin, Russell's theory of judgment

[17] In fact, here Russell harks back to his argument against the correspondence theory of truth from 1910 which, he claimed, is incorrect since when the proposition happens to be false, it must correspond to cases of nonsense such as "the present King of France" (Russell 1910a).

lacks the resources to distinguish between "Cassio loves Desdemona" and "Desdemona loves Cassio"; it also fails to exclude cases where instead by a verb, the elements of a relation are connected by a substantive, as it is in the aforementioned example "This table penholders the books". If we accept that judgments signify facts, however, then all these difficulties instantly disappear; all constituents of the judgment come in their proper places and are unambiguously directed.

The same is true of the interpretation presented in Hanks (2007). According to it, in May 1913 Wittgenstein drew to Russell's attention that the content of judgment is something that must be true or false. This something, we would like to add, is nothing but the sense of the judgment that a fact makes true, or false.

4.3 The Picture Theory and the Doctrine of Showing: criticism of the theory of types

In a letter to Russell dated December 26, 1912, Wittgenstein dropped a cryptic remark about his encounter with Frege:

> I had a long discussion with Frege about our Theory of Symbolism of which, I think, he roughly understood the general outline. He said he would think the matter over. The complex-problem is now clearer to me and I hope very much that I may solve it. (Wittgenstein 1995: 21)

Having considered the complex-problem in §§ 4.1–2, we turn here to the theory of symbolism that Russell and Wittgenstein worked on together in 1912, and to its transformation after December 1912.

Recall that Frege's main idea was to advance a concept-script that would serve as a "perspicuous representation of the forms of thought" (Frege 1881: 89). This was to be something like a visual instrument (similar to a microscope) with the help of which we would immediately grasp the logic of propositions. Hence, it comes as no surprise that what Wittgenstein primarily wished to discuss with him in December 1912 was the theory of symbolism – all the more so given that Wittgenstein felt he was making significant progress in this direction (cf. section 2.2). During the course of discussion, Wittgenstein likely showed Frege, among other things,

his newly invented truth table device, because till then it had been his major achievement in this area of research.

As we observed, by early 1913 Wittgenstein understood facts and propositions each as singular objects. By the summer of 1913, he realized that "the meaning of a proposition is the fact which actually corresponds to it" (Wittgenstein 1913: 94). Working out these insights Wittgenstein had in effect adumbrated the picture theory of language, according to which propositions are facts which picture facts.[18] On this view, sentences are pictures, or facts, that do not map facts but rather model them. Wittgenstein's "Notes on Logic" articulates this idea as follows: "in 'aRb' it is not the complex that symbolizes but the fact that the symbol 'a' stands in a certain relation to the symbol 'b'. […] Thus facts are symbolized by facts" (Wittgenstein 1913: 96, cf. TLP 3.1432).

Armed with this conception, in December 1913 Wittgenstein visited Frege again, and this time "he wiped the floor with Frege" (Goodstein 1972: 272). Apparently, what most impressed Frege, so that he did not challenge the argument straightaway, was Wittgenstein's newly devised picture theory, which introduced the conception that facts symbolize facts.

The story, however, does not end here. Wittgenstein's second discussion with Frege evidently made him confident that he should continue down the path he had taken on his own, completely independent of Russell. Indeed by April 1914, he concluded that his picture theory made the theory of types impossible. Wittgenstein found that we cannot *say* what a type is since the character of the type is *shown* by the signs themselves:

> [The Theory of Types] tries to say something about the types, when you can only talk about the symbols. But *what* you say about the symbols is not that this symbol has that type, which would be nonsense for [the] same reason: but you simply say: *This* is the symbol, to prevent misunderstanding. E.g., in 'aRb', 'R' is *not* a symbol, but *that* 'R' is between one name and another symbolises. Here we have *not*

[18] On this point we agree with Thomas Ricketts (2002: 227) that the picture theory (Ricketts called it "model theory") was introduced by Wittgenstein in 1913. To remind the reader, the picture theory of language was explicitly formulated in September 1914 (Wittgenstein 1979: 7).

> said: this symbol is not of this type but of that, but only: *This* symbolises and not that. (Wittgenstein 1914: 109)

Wittgenstein formulated the Doctrine of Showing this way: "logical so-called propositions *shew* [the] logical properties of language" (Ibid.: 108). This discovery had a transformative impact upon Wittgenstein's philosophy, changing it in ways that Russell never completely understood.[19]

5. The effects of the Frege-Wittgenstein critique on Russell

Russell had great hopes with the Joint Project. "[He] aspired to nothing less than a revolution in the aims and methods of philosophy, a transformation of the whole discipline" (Monk 1996: 282). His feeling was that there is "a whole new science to be created" (# 628, 09.11.1912).

The reintroduction of new intensionalities in logic had dramatic effects upon his philosophical development. In short, it persuaded Russell that the project for a new, "scientific" philosophy that he also called "analytic philosophy" cannot be realized in the form in which he initially envisaged it. Here is the story told in Russell's words:

> [Wittgenstein's criticism] was an event of first-rate importance in my life, and affected everything I have done since. I saw he was right, and I saw that I could not hope ever again to do fundamental work in philosophy. My impulse was shattered, like a wave dashed to pieces against a breakwater. Wittgenstein persuaded me that what wanted doing in logic was too difficult for me. So there was no really vital satisfaction of my philosophical impulse in that work, and philosophy lost its hold on me. That was due to Wittgenstein more than to the war. What the war has done is to give me a new and less difficult ambition, which seems to be quite as good as the old one.[20] (Russell 1968: 57)

Some interpreters try to downplay this avowal. For Gregory Landini, for example, these remarks are to be read against the background of "Russell's personal and emotional turmoil over

[19] For the effects of Wittgenstein's discussion with Frege on Frege's philosophy see Milkov (1999).
[20] Letter to Lady Ottoline Morrell #1,123 from 4.3.1916.

failed relationships with Ottoline [Morrell] and his wife Alys" (Landini 2003/4: 104). In contrast, we take it at its face-value.

Russell's hopes with the Joint Program were connected with the introduction of a new, "scientific" method in philosophy. In short, the idea was to treat problems of the external world and of other minds with logical means, and more precisely, with the help of ideas of Russell's analytic logic of relations that is based on the ontology of simple and complex. Most generally, this was a program for a radical (reductive and constructive) analysis of any subject-matter in philosophy.[21] This explains Russell's profound disappointment when he was confronted with the fact that this program could not be carried out in its full form in all areas. To be more exact, after Wittgenstein's criticism, Russell saw that "analysis is not enough", and this for the second time.

In our interpretation, however, what really happened was not that the impossibility of radically analytic philosophy was demonstrated. Rather, Russell realized that his program for exact philosophy has alternatives and is not as simple as he initially believed.[22]

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[21] And not only in it; Russell often made statements which show that he believed to follow the method of analysis also when discussing problems of public or personal interest.

[22] A first version of this paper was presented at the Kirchberg (Austria) International Wittgenstein Symposium in 2002, a second at the Lunch Time Colloquium at the Center for Philosophy of Science, University of Pittsburgh, and at the Bertrand Russell Society Session at the Central Division Meeting of the APA in Chicago in 2006. In 2009 I read it at a Finnish–German Workshop in Paderborn. The final version of the paper was prepared at McMaster University, Ontario, where I also had the opportunity to check some of the documents in The Bertrand Russell Archives. In March 2012 I presented it at McMaster, as well as at SUNY at Buffalo. Kenneth Blackwell and Nicholas Griffin were most helpful by improving the whole format of the paper.

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