

Research Article

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Hygrothermal bending analysis of sandwich nanoplates with FG porous core and piezomagnetic faces *via* nonlocal strain gradient theory

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Abstract: The bending of sandwich nanoplates made of functionally graded (FG) porous core and electromagnetic layers is explored for the first time through a nonlocal strain gradient theory and a four-unknown shear deformation theory. The proposed model can account for both nonlocal and strain gradient impacts. Therefore, the stiffness enhancement and stiffness reduction processes of sandwich nanoplates are observed. The porosities in the nanoplate are modeled with even and uneven distribution patterns. Six equations of equilibrium are constructed by using virtual work principle. The effects of the porosity factor, externally applied electric and magnetic fields, nonlocal parameter, strain gradient parameter, temperature and moisture parameters, aspect ratio, and side-to-thickness ratio on the static behaviors of FG sandwich nanoplates for simply supported boundary conditions are demonstrated using a parametric study. This article offers comparison treatments for the bending investigation of smart sandwich nanoplates, which can be used in a variety of computational methods. According to the results, deflections induced by negative electric and magnetic potentials behave differently than those brought on by positive electric and magnetic potentials. Other important findings are reached that should aid in the development and implementation of electromagnetic sandwich nanoplate structures.

Keywords: electro-magneto-mechanical loadings, smart FG sandwich nanoplates, porosities, hygrothermal environment, bending, strain gradient theory

1 Introduction

Many researchers are interested in developing materials with at least two simultaneous couplings between electric, magnetic, elastic, and thermal ones due to the possibility and inventive uses of multipurpose instruments in the fields of engineering and manufacturing. These types of structures can be used in micro- and nano-electro-mechanical systems for many smart device applications such as generators, sensors, resonators, transducers, and actuators [1–4]. Several research investigations [5–14] have been published on the mechanical properties of nanostructures.

Because experiments at the nanoscale are notoriously difficult and costly, the characteristics of nanodevices must be investigated theoretically. Three methods have been used to theoretically model nanomaterials: hybrid molecular continuum mechanics, molecular dynamics modeling, and classical continuum mechanics. The continuum mechanics strategy is easier to compute than the previous two concepts, and it is effective for analyzing large-scale nanostructures [15]. Due to the absence of a nonlocal elasticity theory, the minimal size impact of nanostructures cannot be predicted by the traditional elasticity theory. As a result, classical continuum theories must be adapted to account for the small-scale effect. To address this challenge, a number of nonlocal elasticity theories, involving Eringen's nonlocal theory, couple stress, strain gradient, surface stress, the modified couple stress, and integral type, have been developed for the study of nanostructures [16–27]. The nonlocal elasticity theory supposes that the nonlocal stress is impacted by the strain of every point in the body and has the ability of predicting the softening impact of the stiffness of the nanostructure. According to the strain gradient theory, stress is affected by strain and its gradient and can predict stiffness hardening. Because of the differences in scaling among nonlocal elasticity and strain gradient theories, a new theory capable

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of describing the two softening and hardening stiffness size impacts is needed.

Lim *et al.* [6] suggested a theory, referred to as nonlocal strain gradient theory (NSGT) that considers two material length-scale parameters (nonlocal parameter and strain gradient parameter). By setting the strain gradient parameter to zero, the NSGT can be transformed into the nonlocal elasticity theory, and *vice versa*.

Many researchers have used the NSGT to investigate the static and dynamic behaviors of nanostructures. Jiang *et al.* [28] introduced a physically based NSGT that presents the small-length parameters of the constitutive relation for polymer networks. Tang *et al.* [29] examined the bending behavior of micro-/nano-scale beams through a unified strain gradient beam model that incorporates the thickness and shear deformation coupling impacts.

To examine the vibration and bending of nanoplates, Aghababaei and Reddy [30] modified the third-order shear deformation plate theory through nonlocal linear elasticity theory. Huang *et al.* [31] explored the static and dynamic properties of hybrid plates made of a fiber-reinforced composite layer and a carbon nanotube-reinforced composite core supported by an elastic foundation.

Functionally graded (FG) materials have been extensively used in technical applications due to numerous exceptional benefits, including resistance to abrasion, high bearing strength, and high-temperature performance. These materials are composed of two or more element materials, the most common of which are ceramics and metals. Nanoscale structures made of FG materials are becoming more prevalent in practice, including solar cells, micro-/nanosensors, artificial structures, and micro-/nano-electro-mechanical systems. To examine the bending and free vibration behaviors of FG nanoplates, Hoa *et al.* [32] suggested a nonlocal theory that used a single-variable shear deformation theory plate model. Shahriar and Akgoz [33] explored the static and dynamic characteristics of FGM macro- and nanoplates by using three-dimensional elasticity theory in conjunction with Eringen's nonlocal theory. Garg *et al.* [34] conducted a comprehensive review of the literature on the mechanical properties of multiple nanostructures involving plates, beams, and shells. Porosities occur within the material throughout the production procedure of FGMs [35]. Porous FGMs that have high stiffness but low density serve a purpose within a variety of engineering applications involving aerospace, aviation, and military.

Alghanmi [36] introduced a recent study that involved a FG nanoplate and took the porosity factor into account using NSGT. Additional investigations have been conducted to explore FG structures under the impact of the porosity factor [37–48]. By using a refined sinusoidal plate theory, Ebrahimi and Barati [49] presented a free vibration issue

involving a magneto-electro-elastic (MEE)-FG nanoplate resting on an elastic basis. Esen and Ozmen [50] explored the free vibration and buckling behavior of a MEE-FG nanoplate under the impact of electric, magnetic, and thermal fields, implementing the porosity and small-scale effects of the materials. Additional literature on the mechanical properties of nanostructures exposed to magnetic and electric loads is available, which, for the sake of conciseness, can be seen in previous studies [51–53]. Arefi *et al.* [54] examined the impact of a neutral surface on the electro-elastic analysis of a functionally graded piezoelectric plate supported by a Pasternak foundation. Arefi *et al.* [55] proposed a neutral surface for free vibration analysis of a sandwich nanoplate with an FG nanocore and two piezoelectric nanofaces. Zhao *et al.* [56], Zhang *et al.* [57], and Zhang *et al.* [58] have published recent studies on the mechanical properties of micro-/nano-structures.

According to this review, although the publication of a number of significant works on the implementation of NSGT and the analysis of porous material structures have been published, there has not been a thorough study on the electro-elastic bending analysis of sandwich plates with an FG porous core integrated with two piezomagnetic faces subjected to electro-magneto-mechanical loads and exposed to hygrothermal conditions. NSGT and a higher-order shear deformation theory are used to describe the constitutive relations. The proposed model can capture both nonlocal and strain gradient effects in the sandwich nanoplate by incorporating two parameters, namely, nonlocal and strain gradient parameters, into the elastic constants of the sandwich nanoplate. The applied higher-order shear deformation theory with only four variables has been developed to overcome the shortcomings of classical plate theory and first-order shear deformation theory for a better representation of the bending of the FG composite plate. Although it is a two-dimensional theory, it can predict good results for the studied nanoplates. The sandwich nanoplates are put through to hygrothermal, electrical, magnetic, and mechanical loads. This structure can function as both a sensor and an actuator in nanostructures. Furthermore, the findings of this study can be implemented in a variety of applications of MEE nanoplates, including nanorobotics, the design of force measurement transducers, accelerometers, and soft robotic nano-grippers. Wearable technology, energy harvesting, surgical treatment applications, and other areas have significant application potential. The literature survey highlighted the current study's originality as well as the importance of this topic to investigators. This study conducts a comprehensive parametric analysis that addresses the impact of crucial parameters that involve the implemented electric and magnetic potentials, moisture and temperature rise, strain gradient and nonlocal parameters, and the porosity factor.

2 Methodology

2.1 Structural model

Figure 1 depicts a sandwich nanoplate with a porous FG core and two piezomagnetic faces. The sandwich plate in-plane dimensions a and b and total thickness h are provided in Cartesian coordinates (x, y, z) . The thicknesses of the FG core and each of the face sheets are denoted by h_c and h_p . The sandwich nanoplate is exposed to surrounding hygrothermal environment and mechanical loads as well as electromagnetic potentials (Φ and Ψ).

The FG porous core has been examined with two models of even and uneven porosity distribution throughout the plate thickness. In accordance with the modified power-law distribution, the material characteristics of the FG core are expected to vary within the direction of thickness of the constituents. For this reason, it is assumed that the core's material properties are expressed as follows [59,60]:

$$P(z) = P_m + (P_c - P_m) \left(\frac{z}{h} + \frac{1}{2} \right)^k - (P_c + P_m) \frac{\zeta}{2}, \text{ even porosity model} \quad (1)$$

$$P(z) = P_m + (P_c - P_m) \left(\frac{z}{h} + \frac{1}{2} \right)^k - (P_c + P_m) \frac{\zeta}{2} \left(1 - \frac{2|z|}{h} \right), \text{ uneven porosity model} \quad (2)$$

where the letters c and m stand for ceramic and metal, respectively. The porosity coefficient is indicated by the symbol ζ ($0 \leq \zeta \ll 1$), and placing $\zeta = 0$ results in the mechanical properties of the perfect FG porous core. The symbol k ($k \geq 0$) refers to the exponent of power law. The hygrothermal variations are presumptively linear along the thickness [61] as:

2.2 Nonlocal modeling of sandwich nanoplate

The stress field incorporates both the nonlocal elastic stress field and the strain gradient stress field, in accordance with NSGT introduced by Lim *et al.* [6]. As a result, the stress can be stated as:

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \sigma_{ij}^{(1)}, \quad (5)$$

in which the stresses $\sigma_{ij}^{(0)}$ and $\sigma_{ij}^{(1)}$ correspond to strain ε_{ij} and strain gradient $\nabla \varepsilon_{ij}$, respectively, and are determined as [6]:

$$\sigma_{ij}^{(0)} = \int_v c_{ijkl} \alpha_0(x, x', e_0 a) \varepsilon'_{kl}(x') dx', \quad (6)$$

$$\sigma_{ij}^{(1)} = l^2 \int_v c_{ijkl} \alpha_1(x, x', e_1 a) \varepsilon'_{kl}(x') dx', \quad (7)$$

where l refers to the length of the nanoplate, $e_0 a$ and $e_1 a$ denote the lower- and higher-order nonlocal parameters, and c_{ijkl} are the elastic coefficients. When the nonlocal functions $\alpha_0(x, x', e_0 a)$ and $\alpha_1(x, x', e_1 a)$ meet Eringen's conditions [17], the constitutive relation of NSGT takes the following form:

$$[1 - (e_1 a)^2 \nabla^2][1 - (e_0 a)^2 \nabla^2] \sigma_{ij} = c_{ijkl} [1 - (e_1 a)^2 \nabla^2] \varepsilon_{kl} - c_{ijkl} l^2 [1 - (e_0 a)^2 \nabla^2] \varepsilon_{kl}, \quad (8)$$

in which $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Assuming $e_1 = e_0 = e$, then the constitutive relation in equation (8) can be rewritten as (Lim *et al.* [6]):

$$[1 - \eta \nabla^2] \sigma_{ij} = c_{ijkl} [1 - \lambda \nabla^2] \varepsilon_{kl}, \quad (9)$$

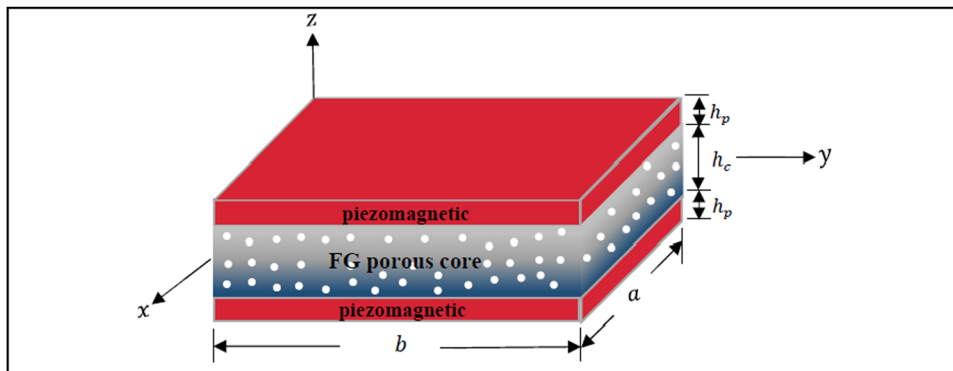


Figure 1: Schematic of sandwich nanoplates with FG porous core and piezomagnetic face sheets.

in which $\eta = (ea)^2$ and $\lambda = l^2$ refer to the parameters of nonlocality and strain gradient length size. According to NSGT, the stress σ_{ij} , electric displacement D_i , and magnetic induction B_i are defined as follows [36,49,62]:

$$[1 - \eta \nabla^2] \sigma_{ij} = c_{ijkl} [1 - \lambda \nabla^2] \varepsilon_{kl} - e_{kij} E_k - f_{kij} H_k, \quad (10a)$$

$$[1 - \eta \nabla^2] D_i = e_{ijkl} [1 - \lambda \nabla^2] \varepsilon_{kl} + \kappa_{ik} E_k + g_{ik} H_k, \quad (10b)$$

$$[1 - \eta \nabla^2] B_i = f_{ikl} [1 - \lambda \nabla^2] \varepsilon_{kl} + g_{ik} E_k + \mu_{ik} H_k, \quad (10c)$$

in which c_{ijkl} , e_{kij} , and f_{kij} symbolize the stiffness coefficients, the piezoelectric, and piezomagnetic coefficients, respectively. Also, κ_{ik} , g_{ik} , and μ_{ik} are the dielectric, electromagnetic, and magnetic coefficients, respectively. Considering the hygrothermal coefficients, the constitutive equation for the FG porous core $\left(\frac{-h_c}{2} \leq z \leq \frac{h_c}{2}\right)$ can be reported in the form [36,49,62]:

$$(1 - \eta \nabla^2) \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = (1 - \lambda \nabla^2) \begin{bmatrix} c_{11}^e & c_{12}^e & 0 & 0 & 0 \\ c_{12}^e & c_{22}^e & 0 & 0 & 0 \\ 0 & 0 & c_{44}^e & 0 & 0 \\ 0 & 0 & 0 & c_{55}^e & 0 \\ 0 & 0 & 0 & 0 & c_{66}^e \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} - \alpha^e \Delta T - \beta^e \Delta M \\ \varepsilon_{yy} - \alpha^e \Delta T - \beta^e \Delta M \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}, \quad (11)$$

where α^e and β^e are the hygrothermal effects for the FG porous core. $\Delta T = T - T_0$, $\Delta M = M - M_0$ in which $T_0 = 25^\circ\text{C}$, $M_0 = 0\%$ are the initial hygrothermal effects, and the stiffness coefficients c_{ij}^e are defined as:

$$c_{11}^e = c_{22}^e = \frac{E(z)}{1 - \nu^2(z)}, c_{12}^e = \frac{\nu(z)E(z)}{1 - \nu^2(z)}, c_{44}^e = c_{55}^e = c_{66}^e = \frac{E(z)}{2(1 + \nu(z))}, \quad (12)$$

where $\nu(z)$ and $E(z)$ are Poisson's ratio and Young's modulus. For the piezomagnetic faces $\left(\frac{-h_e}{2} - h_p \leq z \leq \frac{-h_e}{2}, \frac{h_e}{2} \leq z \leq \frac{h_e}{2} + h_p\right)$, the constitutive relations are expressed by [47,50,63]:

$$(1 - \eta \nabla^2) \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = (1 - \lambda \nabla^2) \begin{bmatrix} c_{11}^p & c_{12}^p & 0 & 0 & 0 \\ c_{12}^p & c_{22}^p & 0 & 0 & 0 \\ 0 & 0 & c_{44}^p & 0 & 0 \\ 0 & 0 & 0 & c_{55}^p & 0 \\ 0 & 0 & 0 & 0 & c_{66}^p \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_1^p \Delta T - \beta_1^p \Delta M \\ \varepsilon_y - \alpha_2^p \Delta T - \beta_2^p \Delta M \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} \quad (13)$$

$$- \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} - \begin{bmatrix} 0 & 0 & f_{31} \\ 0 & 0 & f_{32} \\ 0 & f_{24} & 0 \\ f_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix},$$

$$(1 - \eta \nabla^2) \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = (1 - \lambda \nabla^2) \begin{bmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_x - \alpha_1^p \Delta T - \beta_1^p \Delta M \\ \varepsilon_y - \alpha_2^p \Delta T - \beta_2^p \Delta M \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} \quad (14)$$

$$+ \begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} + \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix},$$

$$\begin{aligned}
(1 - \eta \nabla^2) \begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} &= (1 - \lambda \nabla^2) \begin{bmatrix} 0 & 0 & 0 & f_{15} & 0 \\ 0 & 0 & f_{24} & 0 & 0 \\ f_{31} & f_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_1^p \Delta T - \beta_1^p \Delta M \\ \varepsilon_y - \alpha_2^p \Delta T - \beta_2^p \Delta M \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \\
&+ \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} + \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix},
\end{aligned} \quad (15)$$

in which c_{ij}^p symbolize the stiffness coefficients. α_i^p , α_2^p , β_1^p , and β_2^p are the hygrothermal effects for the piezomagnetic faces. The electrical E_i and magnetic H_i field components are expressed as [50]:

$$E_x = -\frac{\partial \Phi}{\partial x}, E_y = -\frac{\partial \Phi}{\partial y}, \text{ and } E_z = -\frac{\partial \Phi}{\partial z}, \quad (16)$$

$$H_x = -\frac{\partial \Psi}{\partial x}, H_y = -\frac{\partial \Psi}{\partial y}, \text{ and } H_z = -\frac{\partial \Psi}{\partial z}. \quad (17)$$

2.3 Displacement field

A shear deformable model for plates that includes the following displacement field is adopted in this investigation [64]:

$$\begin{aligned}
u_1(x, y, z) &= u(x, y) - z \frac{\partial w_b}{\partial x} - f(z) \frac{\partial w_s}{\partial x}, \quad u_2(x, y, z) = v(x, y) - z \frac{\partial w_b}{\partial y} - f(z) \frac{\partial w_s}{\partial y}, \\
u_3(x, y, z) &= w_b(x, y) + w_s(x, y),
\end{aligned} \quad (18)$$

where w_b and w_s are the lateral displacements brought on by bending and shear effects and u and v represent the in-plane displacements. The following is the form of the shear strain function [65]:

$$f(z) = -\frac{z}{4} + \left(\frac{5}{3}\right) \left(\frac{z^3}{h^2}\right). \quad (19)$$

In accordance with the displacement field provided in equation (18), the strain relationships may be stated as:

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} - z \frac{\partial^2 w_b}{\partial x^2} - f(z) \frac{\partial^2 w_s}{\partial x^2}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} - z \frac{\partial^2 w_b}{\partial y^2} - f(z) \frac{\partial^2 w_s}{\partial y^2}, \quad \gamma_{yz} = \frac{\partial w_s}{\partial y} [1 - f'(z)], \quad \gamma_{xz} = \frac{\partial w_s}{\partial x} [1 - f'(z)], \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2 \left[z \frac{\partial^2 w_b}{\partial x \partial y} + f(z) \frac{\partial^2 w_s}{\partial x \partial y} \right].
\end{aligned} \quad (20)$$

In this investigation, the electric and magnetic potentials are characterized as [50]:

$$\Phi(x, y, z) = \frac{2z}{h_p} \varphi_0 - \varphi(x, y) \cos\left(\frac{\pi z}{h_p}\right), \quad (21)$$

$$\Psi(x, y, z) = \frac{2z}{h_p} \psi_0 - \psi(x, y) \cos\left(\frac{\pi z}{h_p}\right), \quad (22)$$

where $\varphi(x, y)$ and $\psi(x, y)$ indicate the mid-plane electric and magnetic potentials and φ_0 and ψ_0 are the external electric and magnetic values. The components of the electric and magnetic fields are as follows:

$$E_x = \frac{\partial \varphi}{\partial x} \cos\left(\frac{\pi z}{h_p}\right), \quad E_y = \frac{\partial \varphi}{\partial y} \cos\left(\frac{\pi z}{h_p}\right), \quad E_z = -\frac{2\varphi_0}{h_p} - \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \varphi, \quad (23)$$

$$H_x = \frac{\partial \psi}{\partial x} \cos\left(\frac{\pi z}{h_p}\right), \quad H_y = \frac{\partial \psi}{\partial y} \cos\left(\frac{\pi z}{h_p}\right), \quad H_z = -\frac{2\psi_0}{h_p} - \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \psi. \quad (24)$$

3 Governing equations

Through the application of Hamilton's rule, the next governing equations and associated boundary conditions were generated [50,63]:

$$\iiint_V (\sigma_{ij} \delta \varepsilon_{ij} - D_i \delta E_i - B_i \delta H_i) dv - \iint_{\Omega} q (\delta w_b + \delta w_s) d\Omega = 0, \quad (25)$$

in which $q(x, y)$ denotes the transverse load. Inserting equations (20–24) into equation (25) yields

$$\begin{aligned} \iint_{\Omega} \left\{ N_{xx} \frac{\partial \delta u}{\partial x} - M_{xx} \frac{\partial^2 \delta w_b}{\partial x^2} - S_{xx} \frac{\partial^2 \delta w_s}{\partial x^2} + N_{yy} \frac{\partial \delta v}{\partial y} - M_{yy} \frac{\partial^2 \delta w_b}{\partial y^2} - S_{yy} \frac{\partial^2 \delta w_s}{\partial y^2} + N_{xy} \frac{\partial \delta u}{\partial y} + N_{xy} \frac{\partial \delta v}{\partial x} - 2M_{xy} \frac{\partial^2 \delta w_b}{\partial x \partial y} \right. \\ \left. - 2S_{xy} \frac{\partial^2 \delta w_s}{\partial x \partial y} + Q_{yz} \frac{\partial \delta w_s}{\partial y} + Q_{xz} \frac{\partial \delta w_s}{\partial x} + \tilde{D}_x \frac{\partial \delta \varphi_0}{\partial x} + \tilde{D}_y \frac{\partial \delta \varphi_0}{\partial y} \right. \\ \left. - \tilde{D}_z \delta \varphi_0 + \tilde{B}_x \frac{\partial \delta \psi_0}{\partial x} + \tilde{B}_y \frac{\partial \delta \psi_0}{\partial y} - \tilde{B}_z \delta \psi_0 - q \delta w_b - q \delta w_s \right\} d\Omega = 0, \end{aligned} \quad (26)$$

in the above equation N_{ij} , M_{ij} , S_{ij} , Q_{iz} , $\tilde{D}_i$, and $\tilde{B}_i$ are defined by:

$$\begin{aligned} \{N_{ij}, M_{ij}, S_{ij}\} &= \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \sigma_{ij}^p \{1, z, f(z)\} dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \sigma_{ij}^e \{1, z, f(z)\} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2} + h_p} \sigma_{ij}^p \{1, z, f(z)\} dz, \\ Q_{iz} &= \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \tau_{iz}^p [1 - f'(z)] dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \tau_{iz}^e [1 - f'(z)] dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2} + h_p} \tau_{iz}^p [1 - f'(z)] dz, \\ \{\tilde{D}_i, \tilde{D}_z\} &= \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \left\{ D_i \cos\left(\frac{\pi z}{h_p}\right), D_z \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \right\} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2} + h_p} \left\{ D_i \cos\left(\frac{\pi z}{h_p}\right), D_z \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \right\} dz, \\ \{\tilde{B}_i, \tilde{B}_z\} &= \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \left\{ B_i \cos\left(\frac{\pi z}{h_p}\right), B_z \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \right\} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2} + h_p} \left\{ B_i \cos\left(\frac{\pi z}{h_p}\right), B_z \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \right\} dz. \end{aligned} \quad (27)$$

After integrating by parts equation (26) and considering the arbitrariness of the coefficients δu , δv , δw_b , δw_s , $\delta \varphi_0$, and ψ_0 , the governing equations are as follows:

$$\begin{aligned} \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0, \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0, \\ \frac{\partial^2 M_{xx}}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + q &= 0, \\ \frac{\partial^2 S_{xx}}{\partial x^2} + 2 \frac{\partial^2 S_{xy}}{\partial x \partial y} + \frac{\partial^2 S_{yy}}{\partial y^2} + \frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} + q &= 0, \\ \frac{\partial \tilde{D}_x}{\partial x} + \frac{\partial \tilde{D}_y}{\partial y} + \tilde{D}_z &= 0, \\ \frac{\partial \tilde{B}_x}{\partial x} + \frac{\partial \tilde{B}_y}{\partial y} + \tilde{B}_z &= 0. \end{aligned} \quad (28)$$

Combined with the NSGT included in equation (11) and equations (13–15), the stress resultants in equation (26) can be structured as follows:

$$\begin{aligned}
N_{xx} - \eta \nabla^2 N_{xx} &= (1 - \lambda \nabla^2) \left(A_{11} \frac{\partial u}{\partial x} + A_{12} \frac{\partial v}{\partial y} - A_{13} \frac{\partial^2 w_b}{\partial x^2} - A_{14} \frac{\partial^2 w_b}{\partial y^2} - A_{15} \frac{\partial^2 w_s}{\partial x^2} - A_{16} \frac{\partial^2 w_s}{\partial y^2} \right) - N_{xx}^h + A_{17} \varphi + A_{18} \psi + N_{xx}^p, \\
N_{yy} - \eta \nabla^2 N_{yy} &= (1 - \lambda \nabla^2) \left(A_{12} \frac{\partial u}{\partial x} + A_{22} \frac{\partial v}{\partial y} - A_{23} \frac{\partial^2 w_b}{\partial x^2} - A_{24} \frac{\partial^2 w_b}{\partial y^2} - A_{25} \frac{\partial^2 w_s}{\partial x^2} - A_{26} \frac{\partial^2 w_s}{\partial y^2} \right) - N_{yy}^h + A_{27} \varphi + A_{28} \psi + N_{yy}^p, \\
M_{xx} - \eta \nabla^2 M_{xx} &= (1 - \lambda \nabla^2) \left(A_{13} \frac{\partial u}{\partial x} + A_{23} \frac{\partial v}{\partial y} - A_{33} \frac{\partial^2 w_b}{\partial x^2} - A_{34} \frac{\partial^2 w_b}{\partial y^2} - A_{35} \frac{\partial^2 w_s}{\partial x^2} - A_{36} \frac{\partial^2 w_s}{\partial y^2} \right) - M_{xx}^h + A_{37} \varphi + A_{38} \psi + M_{xx}^p, \\
M_{yy} - \eta \nabla^2 M_{yy} &= (1 - \lambda \nabla^2) \left(A_{14} \frac{\partial u}{\partial x} + A_{24} \frac{\partial v}{\partial y} - A_{34} \frac{\partial^2 w_b}{\partial x^2} - A_{44} \frac{\partial^2 w_b}{\partial y^2} - A_{45} \frac{\partial^2 w_s}{\partial x^2} - A_{46} \frac{\partial^2 w_s}{\partial y^2} \right) - M_{yy}^h + A_{47} \varphi + A_{48} \psi + M_{yy}^p, \\
S_{xx} - \eta \nabla^2 S_{xx} &= (1 - \lambda \nabla^2) \left(A_{15} \frac{\partial u}{\partial x} + A_{25} \frac{\partial v}{\partial y} - A_{35} \frac{\partial^2 w_b}{\partial x^2} - A_{45} \frac{\partial^2 w_b}{\partial y^2} - A_{55} \frac{\partial^2 w_s}{\partial x^2} - A_{56} \frac{\partial^2 w_s}{\partial y^2} \right) - S_{xx}^h + A_{57} \varphi + A_{58} \psi + S_{xx}^p, \\
S_{yy} - \eta \nabla^2 S_{yy} &= (1 - \lambda \nabla^2) \left(A_{16} \frac{\partial u}{\partial x} + A_{26} \frac{\partial v}{\partial y} - A_{36} \frac{\partial^2 w_b}{\partial x^2} - A_{46} \frac{\partial^2 w_b}{\partial y^2} - A_{56} \frac{\partial^2 w_s}{\partial x^2} - A_{66} \frac{\partial^2 w_s}{\partial y^2} \right) - S_{yy}^h + A_{67} \varphi + A_{68} \psi + S_{yy}^p, \\
N_{xy} - \eta \nabla^2 N_{xy} &= (1 - \lambda \nabla^2) \left[B_{11} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - 2 \left(B_{12} \frac{\partial^2 w_b}{\partial x \partial y} + B_{13} \frac{\partial^2 w_s}{\partial x \partial y} \right) \right], \\
M_{xy} - \eta \nabla^2 M_{xy} &= (1 - \lambda \nabla^2) \left[B_{12} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - 2 \left(B_{22} \frac{\partial^2 w_b}{\partial x \partial y} + B_{23} \frac{\partial^2 w_s}{\partial x \partial y} \right) \right], \\
S_{xy} - \eta \nabla^2 S_{xy} &= (1 - \lambda \nabla^2) \left[B_{13} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - 2 \left(B_{23} \frac{\partial^2 w_b}{\partial x \partial y} + B_{33} \frac{\partial^2 w_s}{\partial x \partial y} \right) \right], \\
Q_{yz} - \eta \nabla^2 Q_{yz} &= (1 - \lambda \nabla^2) C_{11} \frac{\partial w_s}{\partial y} + C_{12} \frac{\partial \varphi}{\partial y} + C_{13} \frac{\partial \psi}{\partial y}, \\
Q_{xz} - \eta \nabla^2 Q_{xz} &= (1 - \lambda \nabla^2) C_{21} \frac{\partial w_s}{\partial x} + C_{22} \frac{\partial \varphi}{\partial x} + C_{23} \frac{\partial \psi}{\partial x}, \\
\tilde{D}_x - \eta \nabla^2 \tilde{D}_x &= (1 - \lambda \nabla^2) C_{22} \frac{\partial w_s}{\partial x} + D_{11} \frac{\partial \varphi}{\partial x} + D_{12} \frac{\partial \psi}{\partial x}, \\
\tilde{D}_y - \eta \nabla^2 \tilde{D}_y &= (1 - \lambda \nabla^2) C_{12} \frac{\partial w_s}{\partial y} + D_{21} \frac{\partial \varphi}{\partial y} + D_{22} \frac{\partial \psi}{\partial y}, \\
\tilde{D}_z - \eta \nabla^2 \tilde{D}_z &= (1 - \lambda \nabla^2) \left(A_{17} \frac{\partial u}{\partial x} + A_{27} \frac{\partial v}{\partial y} - A_{37} \frac{\partial^2 w_b}{\partial x^2} - A_{47} \frac{\partial^2 w_b}{\partial y^2} - A_{57} \frac{\partial^2 w_s}{\partial x^2} - A_{67} \frac{\partial^2 w_s}{\partial y^2} \right) - D_z^h + E_{11} \varphi + E_{12} \psi - D_z^p, \\
\tilde{B}_x - \eta \nabla^2 \tilde{B}_x &= (1 - \lambda \nabla^2) C_{23} \frac{\partial w_s}{\partial x} + D_{12} \frac{\partial \varphi}{\partial x} + F_{11} \frac{\partial \psi}{\partial x}, \\
\tilde{B}_y - \eta \nabla^2 \tilde{B}_y &= (1 - \lambda \nabla^2) C_{13} \frac{\partial w_s}{\partial y} + D_{22} \frac{\partial \varphi}{\partial y} + F_{22} \frac{\partial \psi}{\partial y}, \\
\tilde{B}_z - \eta \nabla^2 \tilde{B}_z &= (1 - \lambda \nabla^2) \left(A_{18} \frac{\partial u}{\partial x} + A_{28} \frac{\partial v}{\partial y} - A_{38} \frac{\partial^2 w_b}{\partial x^2} - A_{48} \frac{\partial^2 w_b}{\partial y^2} - A_{58} \frac{\partial^2 w_s}{\partial x^2} - A_{68} \frac{\partial^2 w_s}{\partial y^2} \right) - B_z^h + E_{12} \varphi + E_{22} \psi - B_z^p,
\end{aligned} \tag{29}$$

in which the quantities in the preceding equations are identified in Appendix.

4 Solution approach

The Navier approach is implemented to solve the problem analytically. This option enables us to analyze the plate through just simply supported boundary conditions; yet, the solution functionality is extremely quick and dependable, and it can be implemented as a benchmark. The Navier expansion for all the parameters that govern the current problem is as follows:

$$\begin{Bmatrix} u \\ v \\ w_b, w_s, \varphi, \psi \end{Bmatrix} = \begin{Bmatrix} U \cos \alpha x \sin \beta y \\ V \sin \alpha x \cos \beta y \\ (W_b, W_s, \tilde{\varphi}, \tilde{\psi}) \sin \alpha x \sin \beta y \end{Bmatrix}, \quad (30)$$

in which $\alpha = \pi/a$, $\beta = \pi/b$ and $(U, V, W_b, W_s, \tilde{\varphi}, \tilde{\psi})$ are the unknown coefficients. In addition, the mechanical, electrical, magnetic, and hygrothermal loads are expanded as follows:

$$(q, \varphi_0, \psi_0, T_n, M_n) = (q_0 \tilde{\varphi}_0, \tilde{\psi}_0, \tilde{T}_n, \tilde{M}_n) \sin \alpha x \sin \beta y, \quad n = 1, 2. \quad (31)$$

The boundary conditions of the sandwich porous nanoplate are stated as follows:

$$\begin{aligned} v = w_b = w_s = \frac{\partial w_b}{\partial y} = \frac{\partial w_s}{\partial y} = N_x = M_x = S_x = \varphi = \psi = 0, \text{ at } x = 0, a, \\ u = w_b = w_s = \frac{\partial w_b}{\partial x} = \frac{\partial w_s}{\partial x} = N_y = M_y = S_y = \varphi = \psi = 0, \text{ at } y = 0, b. \end{aligned} \quad (32)$$

Replacing equations (30) and (31) into equation (28) yields the following:

$$[A]\{\Delta\} = \{F\}, \quad (33)$$

in which

$$\begin{aligned} \{\Delta\} &= \{U, V, W_b, W_s, \tilde{\varphi}, \tilde{\psi}\}, \\ \{F\} &= \{F_1, F_2, F_3, F_4, F_5, F_6\}, \end{aligned} \quad (34)$$

where the coefficients of the matrix $[A]$ are specified in Appendix.

The components of F are outlined as follows:

$$\begin{aligned} F_1 &= \frac{N_{xx}^p}{\partial x} - \frac{N_{xx}^h}{\partial x}, \quad F_2 = \frac{\partial N_{yy}^p}{\partial y} - \frac{\partial N_{yy}^h}{\partial y}, \quad F_3 = q_0 + \frac{\partial^2 M_{xx}^p}{\partial x^2} + \frac{\partial^2 M_{yy}^p}{\partial y^2} - \frac{\partial^2 M_{xx}^h}{\partial x^2} - \frac{\partial^2 M_{yy}^h}{\partial y^2}, \\ F_4 &= q_0 + \frac{\partial^2 S_{xx}^p}{\partial x^2} + \frac{\partial^2 S_{yy}^p}{\partial y^2} - \frac{\partial^2 S_{xx}^h}{\partial x^2} - \frac{\partial^2 S_{yy}^h}{\partial y^2}, \quad F_5 = D_z^p + D_z^h, \quad F_6 = B_z^p + B_z^h. \end{aligned} \quad (35)$$

5 Numerical results

The results are presented to demonstrate the impact of the length-scale and nonlocal parameters on the bending of sandwich nanoplates with FG porous core under a combination of electric, magnetic, and hygrothermal loads. The sandwich nanoplates have a dimension of $a = 10$ nm, and the FG core characteristics are assumed as:

Table 1: Material characteristics of the piezomagnetic faces

$c_{11}^p = c_{22}^p$ (GPa)	226
c_{12}^p (GPa)	125
$c_{44}^p = c_{55}^p$ (GPa)	44.2
c_{66}^p (GPa)	50.5
$e_{31} = e_{32}$ (C/m ²)	-2.2
$e_{15} = e_{24}$ (C/m ²)	5.8
$\eta_{11} = \eta_{22}$ (Ns ² /CV)	297×10^{-6}
η_{33} (Ns ² /CV)	83.5×10^{-6}
$f_{31} = f_{32}$ (N/A m)	290.5
$f_{15} = f_{24}$ (N/A m)	275
$\kappa_{11} = \kappa_{22}$ (C/V m)	5.64×10^{-9}
κ_{33} (C/V m)	6.35×10^{-9}
$g_{11} = g_{22}$ (C/V m)	5.367×10^{-12}
g_{33} (Ns/CV)	2737.5×10^{-12}
$\alpha_1 = \alpha_2$ (1/K)	12.3×10^{-6}
α_2 (1/K)	1.99×10^{-6}
β_1 (m ³ /kg)	0
β_2 (m ³ /kg)	1.1×10^{-4}

Table 3: Displacement $\bar{w}$ of a square sandwich nanoplates for different values of η , λ , and ζ

η	λ	Perfect	Even		Uneven	
		$\zeta = 0$	$\zeta = 0.15$	$\zeta = 0.25$	$\zeta = 0.15$	$\zeta = 0.25$
0	0	0.4678	0.3432	0.2481	0.4047	0.3597
	1	0.4526	0.3279	0.2328	0.3894	0.3444
	2	0.4416	0.3169	0.2219	0.3785	0.3335
1	0	0.4861	0.3615	0.2664	0.4230	0.3780
	1	0.4678	0.3432	0.2481	0.4047	0.3597
	2	0.4547	0.3301	0.2350	0.3916	0.3466
2	0	0.5044	0.3798	0.2847	0.4413	0.3963
	1	0.4831	0.3585	0.2634	0.4200	0.3750
	2	0.4678	0.3432	0.2481	0.4047	0.3597

$$\begin{aligned}
 E_m &= 70 \times 10^9 \text{ N/m}^2, \alpha_m = 23 \times 10^{-6} \text{ 1/K}, \\
 \beta_m &= 0.44 \text{ m}^3/\text{kg}, \nu_m = 0.3, \\
 E_c &= 380 \times 10^9 \text{ N/m}^2, \alpha_c = 8.1 \times 10^{-6} \text{ 1/K}, \\
 \beta_c &= 0.001 \text{ m}^3/\text{kg}, \text{ and } \nu_c = 0.3.
 \end{aligned} \tag{36}$$

Table 2: Displacement $\bar{w}$ of a square FG nanoplate for different values of η

a/h	η	Method	k				
			0	0.5	1	4	10
4	0	Hoa <i>et al.</i> [66]	3.7905	5.6097	7.1689	11.0892	13.5096
		Present	3.7864	5.6546	7.2842	11.5987	13.9086
	0.5	Hoa <i>et al.</i> [66]	3.9775	5.8866	7.5227	11.6364	14.1762
		Present	3.9732	5.9336	7.6437	12.1711	14.5949
	1	Hoa <i>et al.</i> [66]	4.5387	6.7171	8.5840	13.2781	16.1762
		Present	4.5338	6.7708	8.7221	13.8881	16.6540
	1.5	Hoa <i>et al.</i> [66]	5.4740	8.1012	10.3528	16.0142	19.5096
		Present	5.4680	8.1660	10.5194	16.7500	20.0858
	10	Hoa <i>et al.</i> [66]	2.9607	4.5292	5.8701	8.7307	10.0194
		Present	2.9606	4.5371	5.8895	8.8148	10.0870
	0.5	Hoa <i>et al.</i> [66]	3.1068	4.7527	6.1598	9.1615	10.5139
		Present	3.1067	4.7610	6.1802	9.2498	10.5848
100	0	Hoa <i>et al.</i> [66]	3.5451	5.4233	7.0289	10.4540	11.9972
		Present	3.5450	5.4327	7.0521	10.5547	12.0781
	1.5	Hoa <i>et al.</i> [66]	4.2756	6.5408	8.4773	12.6082	14.4694
		Present	4.2755	6.5522	8.5053	12.7297	14.5670
	0	Hoa <i>et al.</i> [66]	2.8042	4.3255	5.6252	8.2859	9.3613
		Present	2.8042	4.3255	5.6254	8.2868	9.3620
	0.5	Hoa <i>et al.</i> [66]	2.9426	4.5389	5.9028	8.6948	9.8232
		Present	2.9426	4.5390	5.9030	8.6957	9.8239
	1	Hoa <i>et al.</i> [66]	3.3577	5.1793	6.7356	9.9215	11.2091
		Present	3.3577	5.1794	6.7358	9.9225	11.2099
	1.5	Hoa <i>et al.</i> [66]	4.0496	6.2466	8.1236	11.9660	13.5189
		Present	4.0496	6.2467	8.123939	11.9671	13.5199

Table 4: Displacement $\bar{w}$ of a square sandwich nanoplates for different values of η , λ , and $\bar{\varphi}_0$

η	λ	Even			Uneven		
		$\bar{\varphi}_0 = 100$	$\bar{\varphi}_0 = 0$	$\bar{\varphi}_0 = 100$	$\bar{\varphi}_0 = 100$	$\bar{\varphi}_0 = 0$	$\bar{\varphi}_0 = 100$
0	0	2.2184	1.5933	0.9682	2.1648	1.5781	0.9913
	1	2.2031	1.5780	0.9530	2.1495	1.5628	0.9761
	2	2.1922	1.5671	0.9420	2.1385	1.5518	0.9651
1	0	2.2367	1.6116	0.9865	2.1831	1.5963	1.0096
	1	2.2184	1.5933	0.9682	2.1648	1.5781	0.9913
	2	2.2053	1.5802	0.9551	2.1517	1.5649	0.9782
2	0	2.2550	1.6299	1.0048	2.2014	1.6146	1.0279
	1	2.2337	1.6086	0.9835	2.1800	1.5933	1.0066
	2	2.2184	1.5933	0.9682	2.1648	1.5781	0.9913

Table 5: Displacement $\bar{w}$ of a square sandwich nanoplates for different values of η , λ , and $\bar{\psi}_0$

η	λ	Even			Uneven		
		$\bar{\psi}_0 = 0.1$	$\bar{\psi}_0 = 0$	$\bar{\psi}_0 = 0.1$	$\bar{\psi}_0 = 0.1$	$\bar{\psi}_0 = 0$	$\bar{\psi}_0 = 0.1$
0	0	0.0613	0.1437	0.2262	0.1400	0.2174	0.2948
	1	0.0459	0.1284	0.2109	0.1247	0.2021	0.2795
	2	0.0350	0.1175	0.1999	0.1138	0.1912	0.2686
1	0	0.0796	0.1620	0.2445	0.1583	0.2357	0.3131
	1	0.0613	0.1437	0.2262	0.1400	0.2174	0.2948
	2	0.0482	0.1306	0.2131	0.1269	0.2043	0.2817
2	0	0.0979	0.1803	0.2628	0.1766	0.2540	0.3314
	1	0.0765	0.1590	0.2415	0.1553	0.2327	0.3101
	2	0.0613	0.1437	0.2262	0.1400	0.2174	0.2948

Table 1 lists the material characteristics of the piezomagnetic faces [53,63].

5.1 Comparison and verification

To validate the current investigation, a comparison with the work of Hoa *et al.* [66] is provided in Table 2 after removing the piezomagnetic faces and analyzing only the perfect FG nanoplate. Table 2 demonstrates how the non-local factor and side-to-thickness proportion affect the variation of the nondimensional displacement $\bar{w}$ in a square FG nanoplate. The current results support those stated by Hoa *et al.* [66].

5.2 Numerical results analysis

This section analyzes the numerical findings for sandwich nanoplates with piezomagnetic faces and FG material core using two different porosity distributions and depending on NSGT. The outcomes are offered in the nondimensional form $\bar{w} = \frac{1,000u_3}{h}$. Unless otherwise specified, the parameters implemented are $k = 2$, $a/h = 10$, $\bar{\varphi}_0 = 200$, $\bar{\psi}_0 = 1$, $\bar{T}_2 = 200$, $\bar{M}_2 = 10$, $\eta = 2$, $\lambda = 1$, and $\zeta = 0.15$.

Tables 3–7 show the variance of the nondimensional displacement $\bar{w}$ of squared sandwich nanoplates with even and uneven porosity models, as well as different length-scale and nonlocal parameter values. These tables indicate that lowering the length-scale parameter λ and raising the

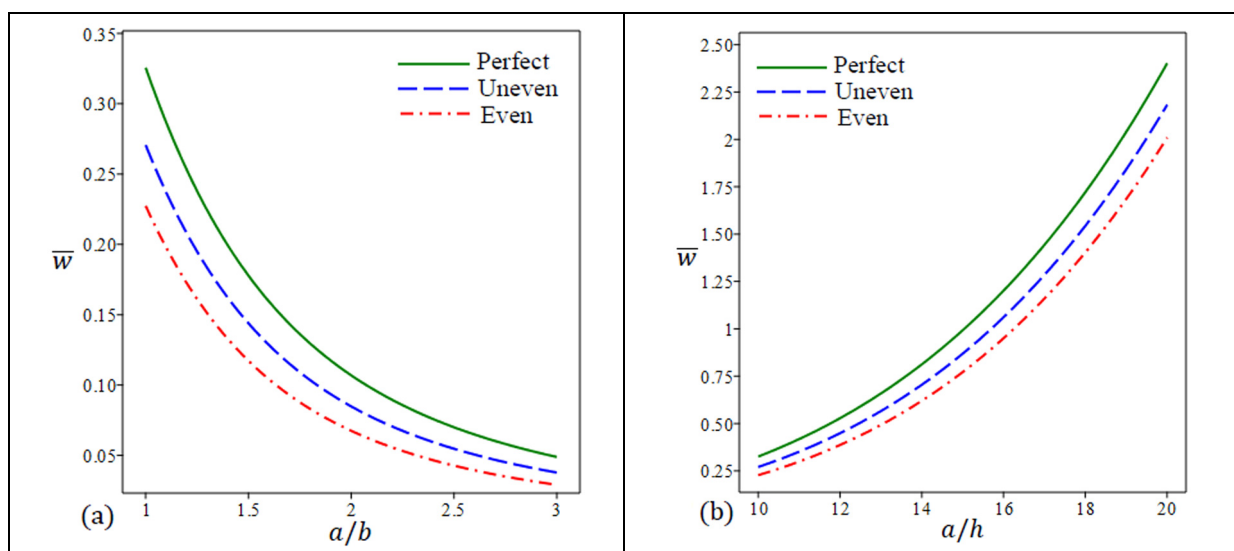
**Figure 2:** Displacement $\bar{w}$ of sandwich nanoplates as a function of (a) a/b and (b) a/h .

Table 6: Displacement $\bar{w}$ of a square sandwich nanoplates for different values of η , λ , and $\bar{T}_2$

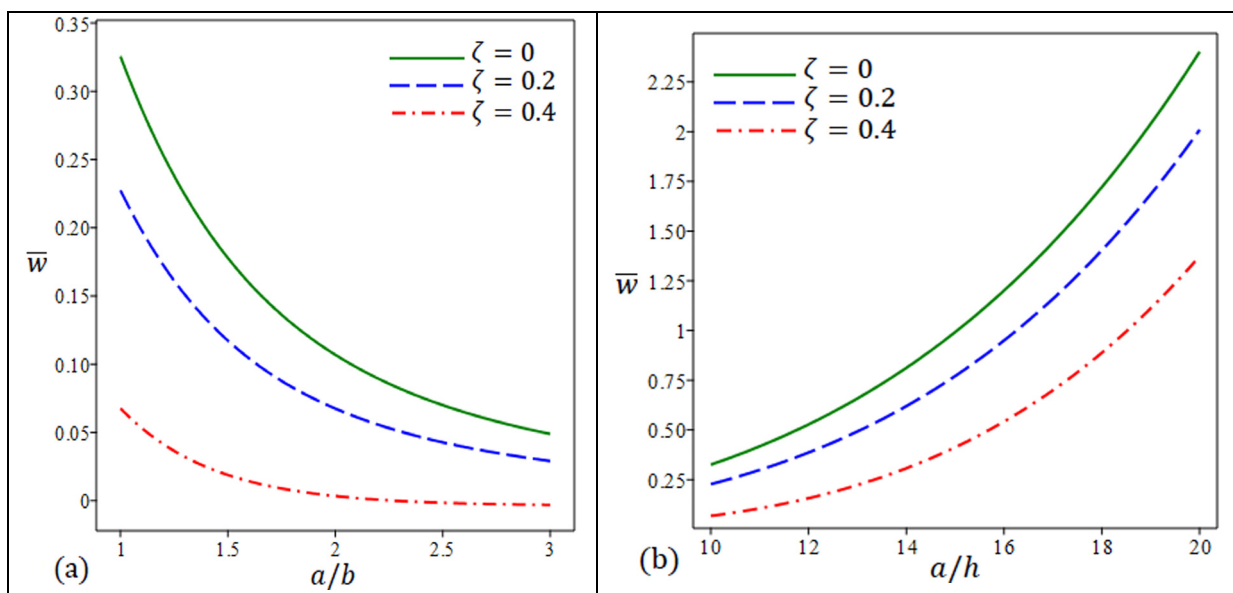
η	λ	Even			Uneven		
		$\bar{T}_2 = 100$	$\bar{T}_2 = 200$	$\bar{T}_2 = 300$	$\bar{T}_2 = 100$	$\bar{T}_2 = 200$	$\bar{T}_2 = 300$
0	0	0.6871	0.9682	1.2493	0.7102	0.9913	1.2724
	1	0.6719	0.9530	1.2341	0.6949	0.9761	1.2572
	2	0.6609	0.9420	1.2231	0.6840	0.9651	1.2462
1	0	0.7054	0.9865	1.2676	0.7285	1.0096	1.2908
	1	0.6871	0.9682	1.2493	0.7102	0.9913	1.2724
	2	0.6740	0.9551	1.2362	0.6971	0.9782	1.2593
2	0	0.7237	1.0048	1.2859	0.7468	1.0279	1.3091
	1	0.7024	0.9835	1.2646	0.7255	1.0066	1.2877
	2	0.6871	0.9682	1.2493	0.7102	0.9913	1.2725

Table 7: Displacement $\bar{w}$ of a square sandwich nanoplates for different values of η , λ , and $\bar{M}_2$

η	λ	Even			Uneven		
		$\bar{M}_2 = 1$	$\bar{M}_2 = 5$	$\bar{M}_2 = 10$	$\bar{M}_2 = 1$	$\bar{M}_2 = 5$	$\bar{M}_2 = 10$
0	0	0.5846	0.6302	0.6871	0.5759	0.6355	0.7101
	1	0.5693	0.6149	0.6719	0.5606	0.6203	0.6948
	2	0.5584	0.6040	0.6609	0.5497	0.6093	0.6839
1	0	0.6029	0.6485	0.7054	0.5942	0.6538	0.7284
	1	0.5846	0.6302	0.6871	0.5759	0.6355	0.7101
	2	0.5715	0.6171	0.6740	0.5627	0.6224	0.6970
2	0	0.6212	0.6668	0.7237	0.6125	0.6721	0.7467
	1	0.5999	0.6455	0.7024	0.5912	0.6508	0.7254
	2	0.5846	0.6302	0.6871	0.5759	0.6355	0.7101

nonlocal parameter η raise the center deflection, regardless of the used porosity model or the applied loads. Table 3 demonstrates how variations in porosity values affect the variation of $\bar{w}$. It can be noted that higher porosity results in smaller deflections for both the even and uneven models. Moreover, sandwich nanoplates that have uneven porosity distributions exhibit larger deflections than the ones with even porosity distributions.

To further comprehend the impact of the electric potential on the mechanical response of FG sandwich nanoplates, the deflections corresponding to three values of applied electric voltages are given in Table 4. Moreover, Figure 6c depicts the deflections of sandwich nanoplates with even porosity distribution *versus* length-to-width ratio for various positive and negative electric potentials. It is obvious that changing the applied voltages from positive

**Figure 3:** Displacement $\bar{w}$ of sandwich nanoplates with even porosity distribution as a function of (a) a/b and (b) a/h .

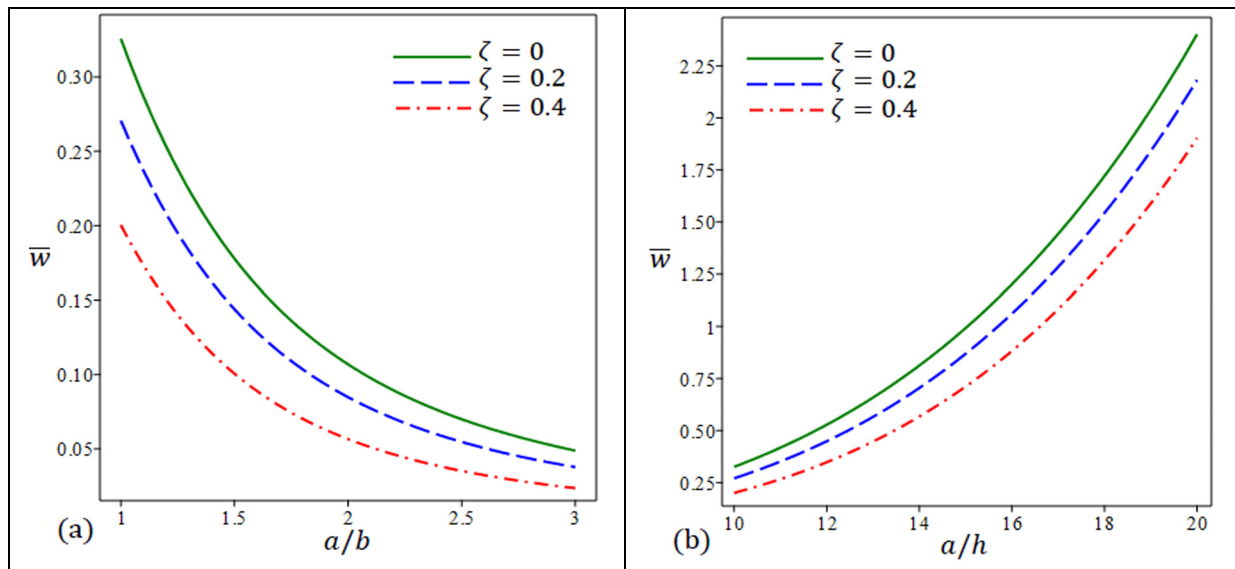


Figure 4: Displacement $\bar{w}$ of sandwich nanoplates with uneven porosity distribution as a function of (a) a/b and (b) a/h .

to negative has an opposite impact on the deflections. Furthermore, the deflections increase by increasing the negative external applied voltages while the deflections decrease by increasing the positive external applied voltages. Table 5 illustrates the impact of the magnetic potentials on the variation of nondimensional displacement $\bar{w}$. Adjusting the magnetic potentials from positive to negative leads to an opposite behavior on the deflections. In addition, the nondimensional displacement $\bar{w}$ distribution as a function of length-to-width ratio in the sandwich

nano-plates with even porosities is shown in Figure 6d. It can be noted that increasing the magnetic field intensity increases the deflections in the case of sandwich nanoplates with evenly distributed porosities. The sandwich nanoplates deform more easily in a negative magnetic field than in a positive magnetic field.

The effect of varying moisture and temperature values on the nondimensional displacement $\bar{w}$ is given in Tables 6 and 7 for FG sandwich nanoplates with even and uneven porosities. In the case of sandwich nanoplates with an even

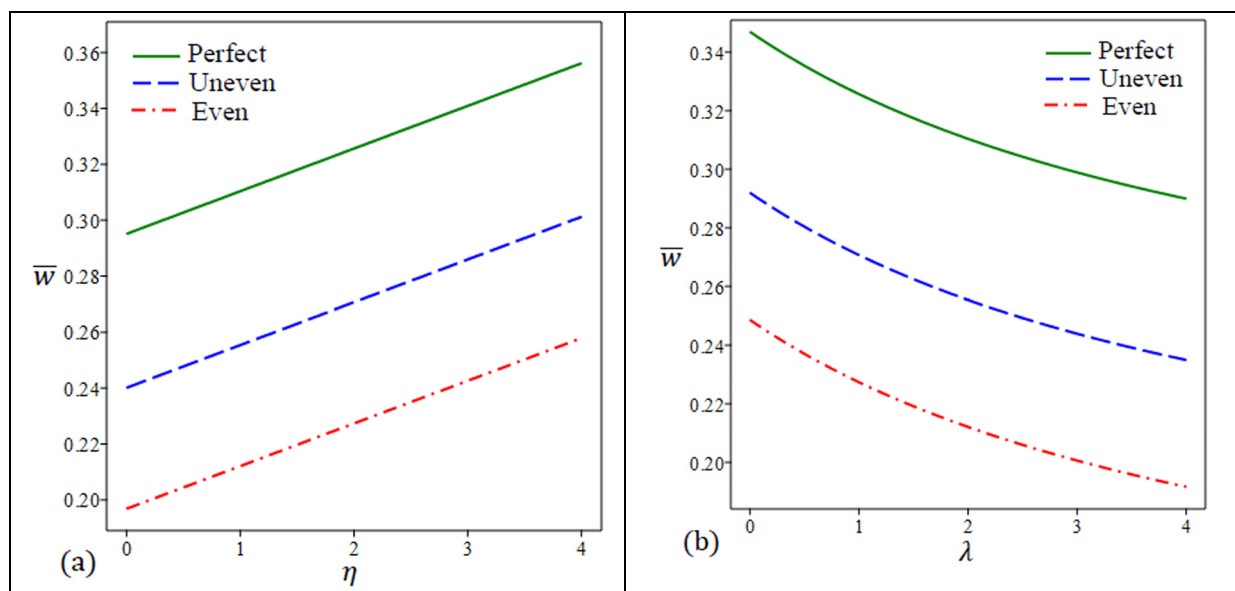


Figure 5: Displacement $\bar{w}$ of sandwich porous nanoplates ($\zeta = 0.2$) versus the variation of: (a) λ , (b) η .

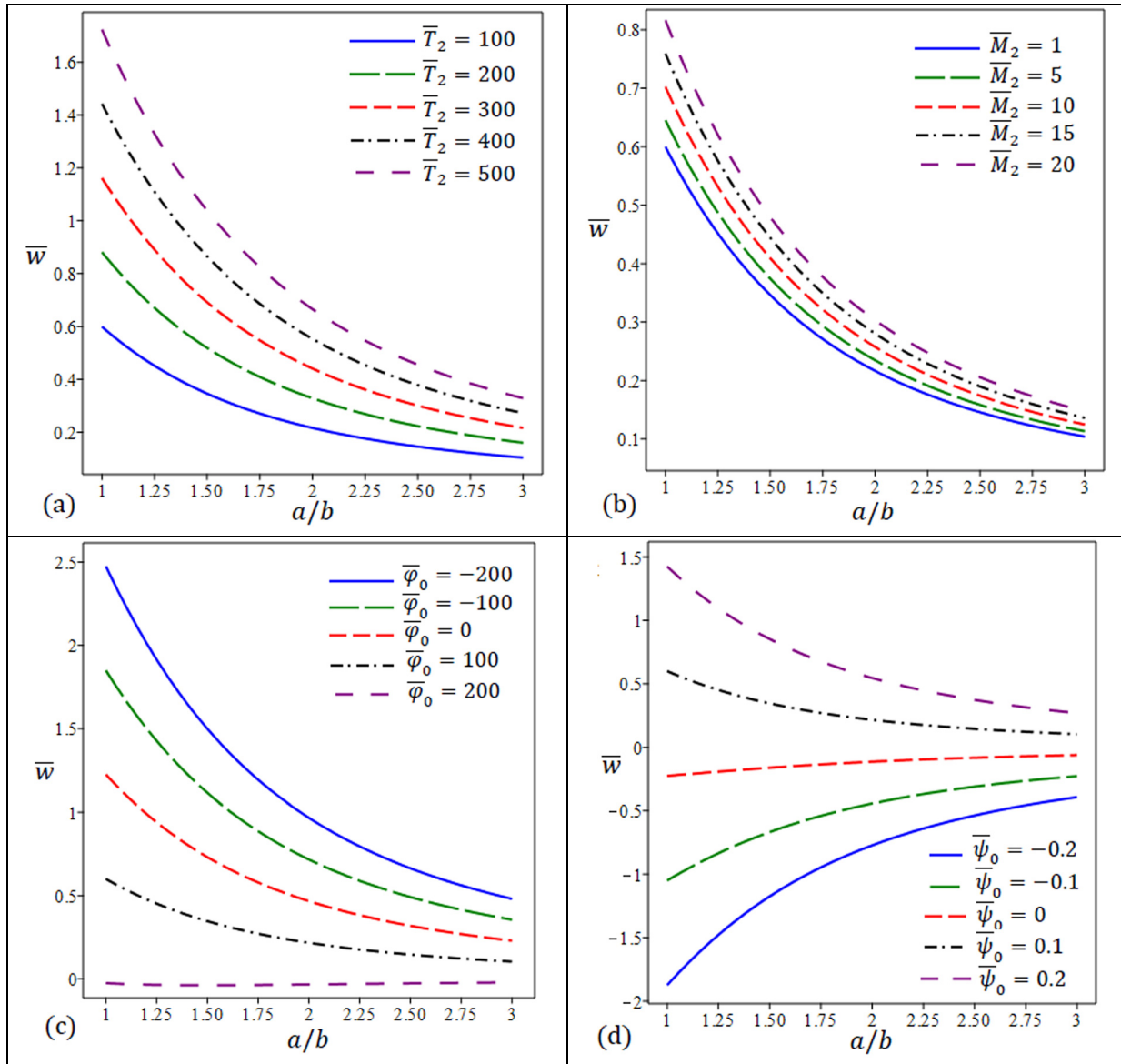


Figure 6: Displacement $\bar{w}$ of sandwich nanoplates with even porosity distribution ($\zeta = 0.2$) as a function of a/b for different values of: (a) $\bar{\varphi}_0$, (b) $\bar{\varphi}_0$, (c) $\bar{T}_2$, (d) $\bar{M}_2$.

porosity model, the impact of various moisture and temperature values on the nondimensional displacement $\bar{w}$ as a function of the aspect ratio a/b is shown in Figure 6a and b, respectively. Increasing the moisture and temperature values led to an increment in the deflections in both FG sandwich nanoplates with even and uneven porosity models. It is clear that the impact of thermal loads on the deflection variation is much larger than the moisture influence. The deflections are seen to decrease as the side-to-width ratio a/b increases.

Figure 2a and b shows the nondimensional displacement $\bar{w}$ variation for three models of FG sandwich nanoplates (perfect, uneven, and even) with respect of side-to-width ratio

a/b and side-to-thickness ratio a/h , respectively. These two figures indicate that the existence of porosity reduces the deflections when compared with nonporous FG sandwich nanoplates under the same electromagnetic and hygrothermal conditions. Additionally, evenly distributed porosities in FG sandwich nanoplates have less deflections compared to those with uneven porosities.

The nondimensional displacement $\bar{w}$ variation for FG sandwich nanoplates with an even porosity scheme in terms of aspect ratio is depicted in Figure 3a. This figure shows that a higher porosity value leads to smaller deflections. However, higher porosity value causes larger deflections in the case of the variation of nondimensional

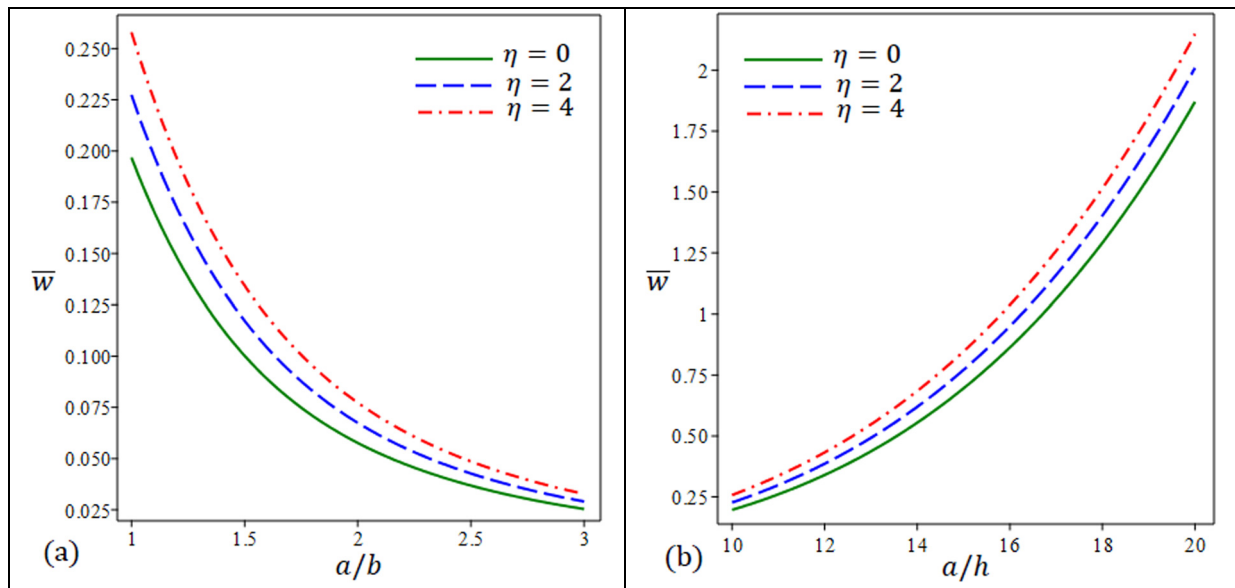


Figure 7: Displacement $\bar{w}$ of sandwich nanoplates with even porosity distribution as a function of (a) a/b and (b) a/h .

displacement $\bar{w}$ with respect to side-to-thickness ratio as shown in Figure 3b.

Compared to the deflections exhibited in Figure 3, a similar behavior for the deflections of FG sandwich nanoplates with uneven porosities is depicted in Figure 4 except that the uneven porosities model causes larger values than those with even porosity type. Figure 5a and b demonstrates the nondimensional displacement $\bar{w}$ variance for all three kinds of FG sandwich nanoplates against the nonlocal

parameter η and the length-scale parameter λ . As already stated, for the three schemes of FG sandwich nanoplates, the deflections increase by decreasing λ and increasing η .

Figures 7–9 show the nondimensional displacement $\bar{w}$ variation for sandwich nanoplates with an even porosity scheme in terms of aspect ratio and side-to-thickness ratio. The impact of different values of the nonlocal parameter η on the variation of the nondimensional displacement $\bar{w}$ is illustrated in Figure 7. The larger deflection values

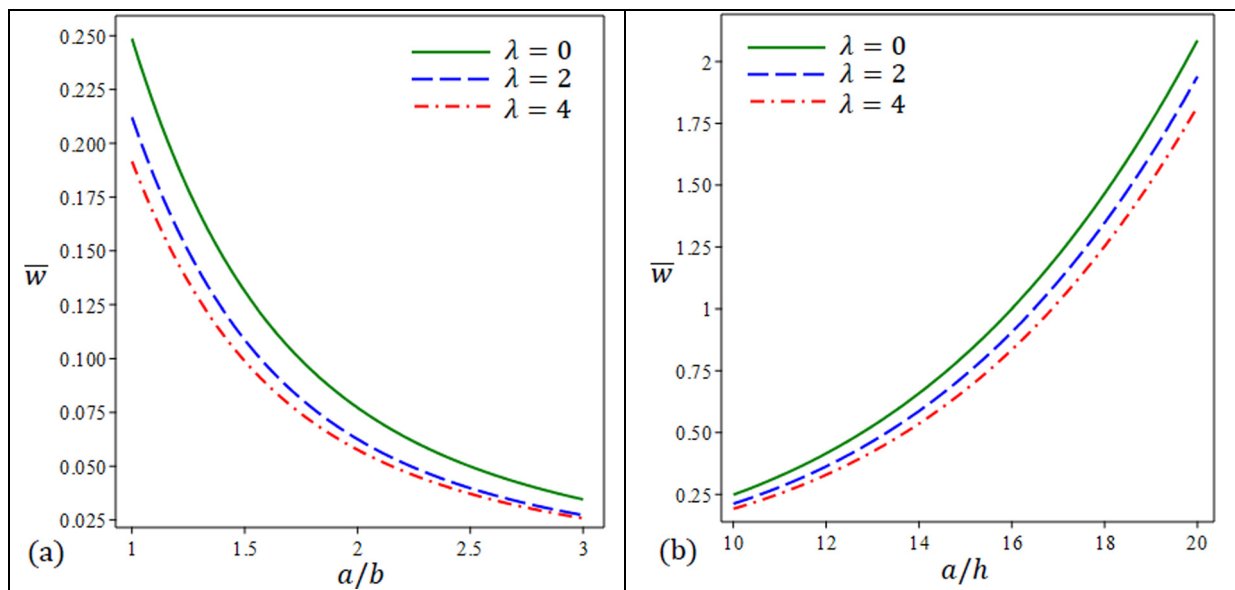


Figure 8: Displacement $\bar{w}$ of sandwich nanoplates with even porosity distribution as a function of (a) a/b and (b) a/h .

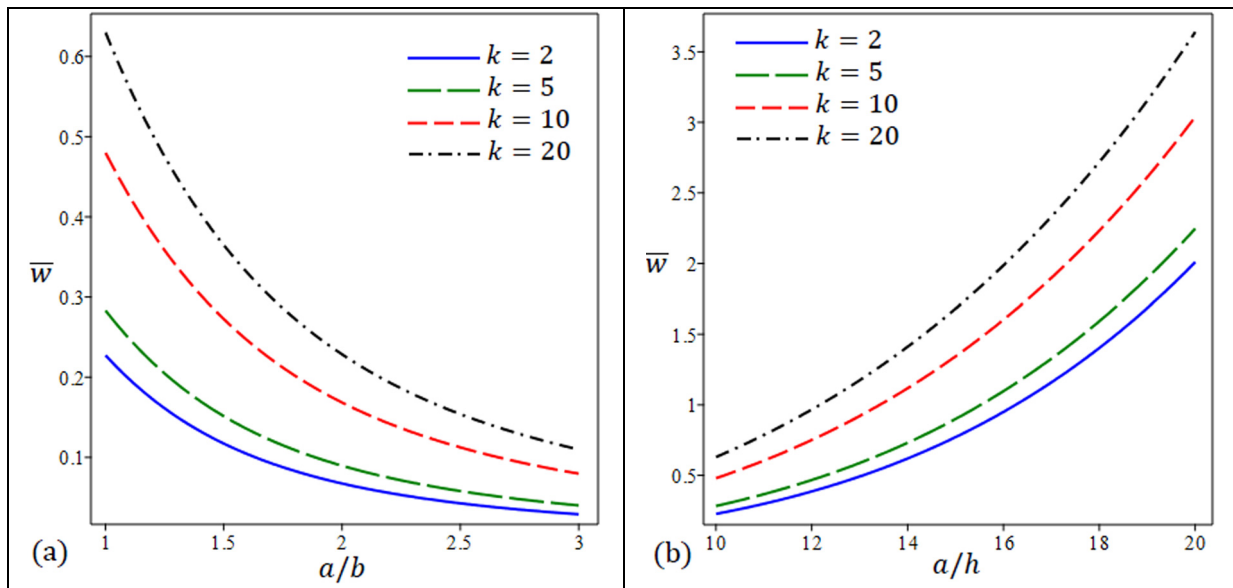


Figure 9: Displacement $\bar{w}$ of sandwich nanoplates with even porosity distribution as a function of (a) a/b and (b) a/h .

correspond to the highest value of the nonlocal parameter ($\eta = 4$). Figure 8 shows how different length-scale parameter values λ affect the variation of the nondimensional displacement $\bar{w}$. The lower deflection values appear with the highest value of the nonlocal parameter ($\lambda = 4$). The deflections increase as the power law exponent k increases as illustrated in Figure 9.

By comparing Figure 9a and b, it can be concluded that the deflections decrease with increasing the aspect ratio and decreasing the side-to-thickness ratio.

The two-dimensional distribution of the nondimensional displacement $\bar{w}$ of FG sandwich nanoplates in terms of η and λ with even and uneven porosity models is examined in Figure 10a and b, respectively. Again, for both cases, the increase of η and the decrease of λ increased deflections. As can be seen, sandwich nanoplates with uneven porosity distribution exhibit greater deflections than those with even porosity distribution under the same conditions.

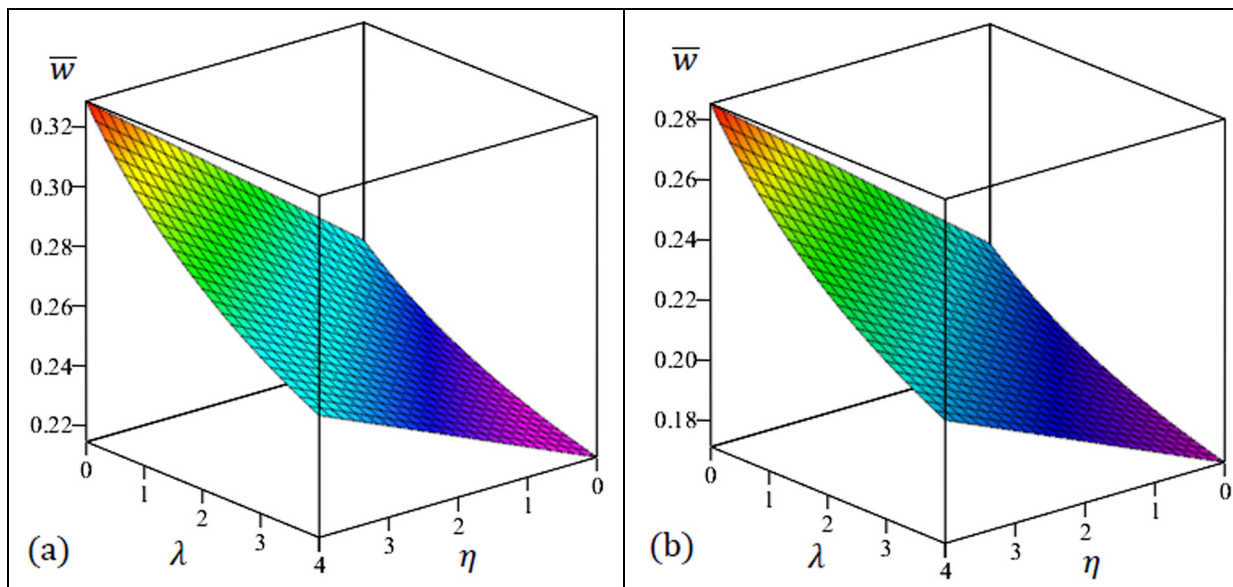


Figure 10: The two-dimensional variation of the displacement $\bar{w}$ of sandwich porous nanoplates ($\zeta = 0.2$) in terms of λ and η for (a) uneven porosity distribution and (b) even porosity distribution.

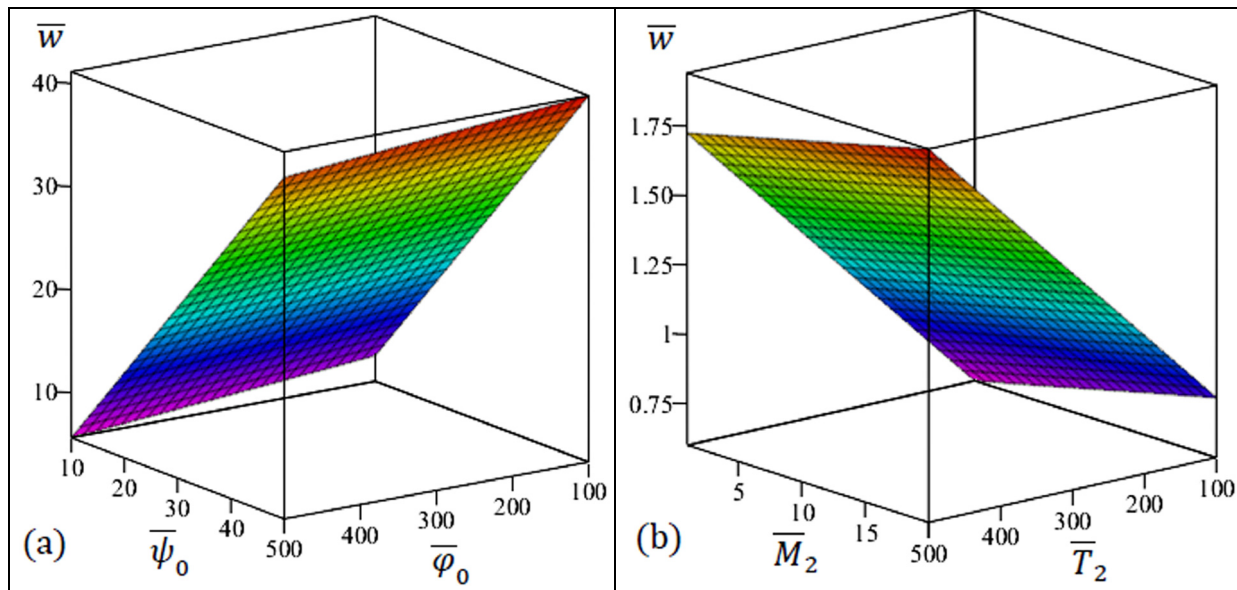


Figure 11: Displacement $\bar{w}$ of sandwich nanoplates with even porosity distribution ($\zeta = 0.15$): (a) in terms of $\bar{\varphi}_0$ and $\bar{\psi}_0$ and (b) in terms of $\bar{T}_2$ and $\bar{M}_2$.

Figure 11a depicts the two-dimensional distribution of the nondimensional displacement $\bar{w}$ of FG sandwich nanoplates with regard to magnetic and electric potentials, respectively. Deflections decrease with increasing external applied voltages and increase rapidly with increasing magnetic potential values. Finally, it is obvious from Figure 11b that increasing moisture and temperature values increases the deflections. However, thermal loads have a greater effect on deflections than moisture conditions.

6 Conclusions

The NSGT is being developed in this article to analyze the bending of sandwich nanoplates with FG porous cores and electromagnetic layers. The FG sandwich nanoplates are modeled through a four-variable shear deformation theory. In this study, two distinct porosity distribution models are taken into account. The virtual work principle and Navier's process are used to derive the equilibrium equations, which are described in detail. Investigations are conducted into the porosity factor, strain gradient and nonlocal parameters, side-to-thickness ratio, and side-to-width ratio. Externally applied electric and magnetic field potentials are also investigated. Studies of contrast are offered. To facilitate comparison, additional results are provided. These are the main conclusions.

- 1) The bending of FG sandwich nanoplates is greatly influenced by the presence of a porosity factor.

- 2) When negative voltages are applied to the sandwich nanoplate, the deflections appear to behave in the opposite way to the deflections brought on by applying positive voltages.
- 3) In contrast to those brought on by applying positive magnetic potential, deflections caused by negative magnetic potential values seem to behave differently.
- 4) As temperature and moisture values increase, the deflections increase. Therefore, the presence of hygrothermal environments can affect the behavior of sandwich nanoplates.
- 5) For every type of FG porous sandwich nanoplates, the deflections are increased by the increment in the nonlocal parameter while they are decreased by the length-scale parameter. The nonlocal parameter could be able to lower the sandwich nanoplates' stiffness, according to this inference.
- 6) In order to stay within the deflection nanoplate's accepted range, there must be limitations on the electrical and magnetic loads, moisture and temperature values, and geometry of the nanoplate.

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Appendix

$$\begin{aligned}
 \begin{bmatrix} A_{11} & A_{13} & A_{15} \\ A_{12} & A_{14} & A_{16} \\ A_{22} & A_{24} & A_{26} \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \begin{bmatrix} c_{11}^p \\ c_{12}^p \\ c_{22}^p \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \begin{bmatrix} c_{11}^e \\ c_{12}^e \\ c_{22}^e \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \begin{bmatrix} c_{11}^p \\ c_{12}^p \\ c_{22}^p \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz, \\
 \{A_{23}, A_{25}, A_{45}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} c_{12}^p \{z, f(z), zf(z)\} dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} c_{12}^e \{z, f(z), zf(z)\} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} c_{12}^p \{z, f(z), zf(z)\} dz, \\
 \begin{bmatrix} A_{33} & A_{35} & A_{55} \\ A_{34} & A_{36} & A_{56} \\ A_{44} & A_{46} & A_{66} \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \begin{bmatrix} c_{11}^p \\ c_{12}^p \\ c_{22}^p \end{bmatrix} \begin{bmatrix} z^2 & zf(z) & f^2(z) \end{bmatrix} dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \begin{bmatrix} c_{11}^e \\ c_{12}^e \\ c_{22}^e \end{bmatrix} \begin{bmatrix} z^2 & zf(z) & f^2(z) \end{bmatrix} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \begin{bmatrix} c_{11}^p \\ c_{12}^p \\ c_{22}^p \end{bmatrix} \begin{bmatrix} z^2 & zf(z) & f^2(z) \end{bmatrix} dz, \\
 \begin{bmatrix} A_{17} & A_{37} & A_{57} \\ A_{27} & A_{47} & A_{67} \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \begin{bmatrix} e_{31} \\ e_{32} \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \begin{bmatrix} e_{31} \\ e_{32} \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz, \\
 \begin{bmatrix} A_{18} & A_{38} & A_{58} \\ A_{28} & A_{48} & A_{68} \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \begin{bmatrix} e_{31} \\ e_{32} \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \frac{\pi}{h_p} \sin\left(\frac{\pi z}{h_p}\right) \begin{bmatrix} e_{31} \\ e_{32} \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz, \\
 \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{22} & B_{23} & B_{33} \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} c_{66}^p \begin{bmatrix} 1 & z & f(z) \\ z^2 & zf(z) & f^2(z) \end{bmatrix} dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} c_{66}^e \begin{bmatrix} 1 & z & f(z) \\ z^2 & zf(z) & f^2(z) \end{bmatrix} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} c_{66}^p \begin{bmatrix} 1 & z & f(z) \\ z^2 & zf(z) & f^2(z) \end{bmatrix} dz, \\
 \{C_{11}, C_{21}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \{c_{44}^p, c_{55}^p\} [1 - f'(z)]^2 dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} \{c_{44}^e, c_{55}^e\} [1 - f'(z)]^2 dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \{c_{44}^p, c_{55}^p\} [1 - f'(z)]^2 dz, \\
 \{C_{12}, C_{22}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \{e_{24}, e_{15}\} [1 - f'(z)] \cos\left(\frac{\pi z}{h_p}\right) dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \{e_{24}, e_{15}\} [1 - f'(z)] \cos\left(\frac{\pi z}{h_p}\right) dz, \\
 \{C_{13}, C_{23}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \{f_{24}, f_{15}\} [1 - f'(z)] \cos\left(\frac{\pi z}{h_p}\right) dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \{f_{24}, f_{15}\} [1 - f'(z)] \cos\left(\frac{\pi z}{h_p}\right) dz, \\
 \{D_{11}, D_{21}, E_{11}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \left\{ \kappa_{11} \cos^2\left(\frac{\pi z}{h_p}\right), \kappa_{22} \cos^2\left(\frac{\pi z}{h_p}\right), \kappa_{33} \left(\frac{\pi}{h_p}\right)^2 \sin^2\left(\frac{\pi z}{h_p}\right) \right\} dz \\
 &\quad + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \left\{ \kappa_{11} \cos^2\left(\frac{\pi z}{h_p}\right), \kappa_{22} \cos^2\left(\frac{\pi z}{h_p}\right), \kappa_{33} \left(\frac{\pi}{h_p}\right)^2 \sin^2\left(\frac{\pi z}{h_p}\right) \right\} dz, \\
 \{D_{12}, D_{22}, E_{12}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \left\{ g_{11} \cos^2\left(\frac{\pi z}{h_p}\right), g_{22} \cos^2\left(\frac{\pi z}{h_p}\right), g_{33} \left(\frac{\pi}{h_p}\right)^2 \sin^2\left(\frac{\pi z}{h_p}\right) \right\} dz \\
 &\quad + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \left\{ g_{11} \cos^2\left(\frac{\pi z}{h_p}\right), g_{22} \cos^2\left(\frac{\pi z}{h_p}\right), g_{33} \left(\frac{\pi}{h_p}\right)^2 \sin^2\left(\frac{\pi z}{h_p}\right) \right\} dz, \\
 \{F_{11}, F_{22}, E_{22}\} &= \int_{-\frac{h_e}{2}-h_p}^{\frac{h_e}{2}} \left\{ \mu_{11} \cos^2\left(\frac{\pi z}{h_p}\right), \mu_{22} \cos^2\left(\frac{\pi z}{h_p}\right), \mu_{33} \left(\frac{\pi}{h_p}\right)^2 \sin^2\left(\frac{\pi z}{h_p}\right) \right\} dz \\
 &\quad + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \left\{ \mu_{11} \cos^2\left(\frac{\pi z}{h_p}\right), \mu_{22} \cos^2\left(\frac{\pi z}{h_p}\right), \mu_{33} \left(\frac{\pi}{h_p}\right)^2 \sin^2\left(\frac{\pi z}{h_p}\right) \right\} dz,
 \end{aligned}$$

$$\begin{aligned}
\{D_z^p, B_z^p\} &= \int_{-\frac{h_e}{2}-h_p}^{-\frac{h_e}{2}} \left\{ \kappa_{33}, g_{33} \right\} \frac{2\pi\phi_0}{h_p^2} \sin\left(\frac{\pi z}{h_p}\right) + \{g_{33}, \mu_{33}\} \frac{2\pi\psi_0}{h_p^2} \sin\left(\frac{\pi z}{h_p}\right) \Bigg| dz \\
&\quad + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \left\{ \kappa_{33}, g_{33} \right\} \frac{2\pi\phi_0}{h_p^2} \sin\left(\frac{\pi z}{h_p}\right) + \{g_{33}, \mu_{33}\} \frac{2\pi\psi_0}{h_p^2} \sin\left(\frac{\pi z}{h_p}\right) \Bigg| dz, \\
\begin{bmatrix} N_{xx}^h & M_{xx}^h & S_{xx}^h \\ N_{yy}^h & M_{yy}^h & S_{yy}^h \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{-\frac{h_e}{2}} \begin{bmatrix} c_{11}^p H_1^p & c_{12}^p H_2^p \\ c_{12}^p H_1^p & c_{22}^p H_2^p \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \\ 1 & z & f(z) \end{bmatrix} dz + \int_{-\frac{h_e}{2}}^{\frac{h_e}{2}} H^e \begin{bmatrix} c_{11}^e & c_{12}^e \\ c_{12}^e & c_{22}^e \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \\ 1 & z & f(z) \end{bmatrix} dz \\
&\quad + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \begin{bmatrix} c_{11}^p H_1^p & c_{12}^p H_2^p \\ c_{12}^p H_1^p & c_{22}^p H_2^p \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \\ 1 & z & f(z) \end{bmatrix} dz, \\
H_1^p &= \alpha_1^p \Delta T + \beta_1^p \Delta M, \quad H_2^p = \alpha_2^p \Delta T + \beta_2^p \Delta M, \quad H^e = \alpha^e \Delta T + \beta^e \Delta M, \\
\begin{bmatrix} N_{xx}^p & M_{xx}^p & S_{xx}^p \\ N_{yy}^p & M_{yy}^p & S_{yy}^p \end{bmatrix} &= \int_{-\frac{h_e}{2}-h_p}^{-\frac{h_e}{2}} \frac{2}{h_p} \begin{bmatrix} \phi_0 e_{31} + \psi_0 f_{31} \\ \phi_0 e_{32} + \psi_0 f_{32} \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix} dz + \int_{\frac{h_e}{2}}^{\frac{h_e}{2}+h_p} \frac{2}{h_p} \begin{bmatrix} \phi_0 e_{31} + \psi_0 f_{31} \\ \phi_0 e_{32} + \psi_0 f_{32} \end{bmatrix} dz, \\
a_{11} &= A_{11}\alpha^2 + B_{11}\beta^2, \quad a_{12} = (A_{12} + B_{11})\alpha\beta, \quad a_{13} = -\alpha[A_{13}\alpha^2 + (A_{14} + 2B_{12})\beta^2], \\
a_{14} &= -\alpha[A_{15}\alpha^2 + (A_{16} + 2B_{13})\beta^2], \quad a_{15} = -A_{17}\alpha, \quad a_{16} = -A_{18}\beta, \\
a_{22} &= B_{11}\alpha^2 + A_{22}\beta^2, \quad a_{23} = -\beta[(A_{23} + 2B_{12})\alpha^2 + A_{24}\beta], \quad a_{24} = -\beta[(A_{25} + 2B_{13})\alpha^2 + A_{26}\beta], \quad a_{25} = -A_{27}\beta, \quad a_{26} = -A_{28}\beta, \\
a_{33} &= A_{33}\alpha^4 + 2(A_{34} + 2B_{22})\alpha^2\beta^2 + A_{44}\beta, \quad a_{34} = A_{35}\alpha^4 + 2(A_{36} + A_{45} + 2B_{23})\alpha^2\beta^2 + A_{46}\beta^4, \quad a_{35} = A_{38}\alpha^2 + A_{48}\beta^2, \quad a_{36} = A_{37}\alpha^2 \\
&\quad + A_{47}\beta^2, \\
a_{44} &= A_{55}\alpha^4 + 2(A_{56} + 2B_{33})\alpha^2\beta^2 + A_{66}\beta^4 + C_{21}\alpha^2 + C_{11}\beta^2, \quad a_{45} = (A_{58} + C_{23})\alpha^2 + (A_{68} + C_{13})\beta^2, \quad a_{46} = (A_{57} + C_{22})\alpha^2 + (A_{67} \\
&\quad + C_{12})\beta^2, \\
a_{55} &= -D_{11}\alpha^2 - D_{21}\beta^2 + E_{11}, \quad a_{56} = -D_{12}\alpha^2 - D_{22}\beta^2 + E_{12}, \\
a_{66} &= -F_{11}\alpha^2 - F_{22}\beta^2 + E_{22}.
\end{aligned}$$