

Research Article

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Parametric simulation of hybrid nanofluid flow consisting of cobalt ferrite nanoparticles with second-order slip and variable viscosity over an extending surface

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Abstract: This study explores the unsteady hybrid nanofluid (NF) flow consisting of cobalt ferrite (CoFe_2O_4) and copper (Cu) nano particulates with natural convection flow due to an expanding surface implanted in a porous medium. The Cu and CoFe_2O_4 nanoparticles (NPs) are added to the base fluid water to synthesize the hybrid

NF. The effects of second-order velocity slip condition, chemical reaction, heat absorption/generation, temperature-dependent viscosity, and Darcy Forchheimer are also assessed in the present analysis. An ordinary differential equation system is substituted for the modeled equations of the problem. Further computational processing of the differential equations is performed using the parametric continuation method. A validation and accuracy comparison are performed with the Matlab package BVP4C. Physical constraints are used for presenting and reviewing the outcomes. With the increase in second-order velocity slip condition and unsteady viscosity, the rates of heat and mass transition increase significantly with the variation in Cu and Fe_2O_4 NPs. The findings suggest that the uses of Cu and Fe_2O_4 in ordinary fluids might be useful in the aerodynamic extrusion of plastic sheets and extrusion of a polymer sheet from a dye.

Keywords: stretching surface, BVP4C, parametric continuation method, hybrid approach, slip conditions, chemical reaction.

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Nomenclature

$A = (\zeta/u_0)$	unsteadiness coefficient
$\tilde{C}$	concentration of the fluid
$\tilde{C}_\infty$	concentration at infinity
D_{hnf}	mass diffusion
$\text{Fr} = \frac{C_b}{k^{*1/2}}$	Forchheimer number
g	gravitational force
$\kappa = \frac{v_f(1-\zeta t)}{Ku_0}$	permeability constant
k_{hnf}	thermal conductivity
K_r	chemical reaction
K_n	Knudsen number
N_0	thermal slip term
$\text{Pr} = \frac{v_f(\rho C_p)}{k_f}$	Prandtl number

Q_0	heat absorption/generation
R	varying viscosity
$Sc = \frac{\nu_f}{D_f}$	Schmidt number
$\bar{T}$	temperature of the fluid
$\bar{T}_s$	surface temperature
$\bar{T}_\infty$	fluid temperature at infinity
$\tilde{u}$	velocity component on x -axis
$\tilde{u}_{slip}$	velocity slip
ν_{hnf}	kinematic viscosity
$\tilde{v}$	velocity component on y -axis
μ_f	dynamic viscosity of fluid
λ	free path molecular mean
α	momentum accommodation constant
μ_{hnf}	dynamic viscosity
$\beta = N_0 \sqrt{\tilde{u}_0/\nu_f}$	thermal slip
$\gamma = a \sqrt{\tilde{u}_0/\nu_f}$	slip velocity term of 1st-order
$\Gamma = ((1 - \zeta_t)Q_0/\tilde{u}_0(\rho C_p)_f)$	heat source term
μ_0	reference viscosity of carrier fluid
ρ_{hnf}	density
β_{hnf}	volumetric expansion
$(\rho C_p)_{hnf}$	specific heat capacity
$\delta = b(\tilde{u}_0/\nu_f)$	slip velocity term of second-order
$\tau = (g\beta_f(\bar{T}_b - \bar{T}_s)/x\tilde{u}_0^2)$	Grashof number

1 Introduction

Various applications of fluid flow analysis across an extending sheet have been achieved in various sectors, including polymer manufacturing and aerodynamic extrusion of plastics and so on [1]. To simulate the behavior of simple and hybrid nanomaterials, Gul *et al.* [2] used a stretching sheet. Relative to simple nanofluids (NFs), the hybrid nanoliquid is more efficacious in heat conduction. Bilal *et al.* [3] documented the ferrofluid flow on a stretchy substratum with suction and injection influences. Puneeth *et al.* [4] evaluated the effects of mass transfer on the dynamics of a Casson hybrid NF on an asymmetrical non-linear stretchy sheet. In a systematic review, Hussain *et al.* [5] investigated the heat transference properties of magneto-hydro-dynamic (MHD) nanofluid (NF) flow across an expanding surface. Rotation is known to promote heat flux, while lowering skin friction. Shuaib *et al.* [6] observed the fluid dynamics and heat distribution. As compared to a classical model, the fractional model seems to be more accurate. The poroelastic rotatory nanoliquid flow across a linearly increasing surface under magnetic impact was studied by Raza *et al.* [7]. The energy transmission through NF flow over a contracting sheet was considered by Uddin

et al. [8]. Rasool *et al.* [9] theoretically studied the Darcy-Forchheimer MHD NF flow with a non-linear extending sheet. The findings show that when the intensity of heat radiation increases, the thermal layer undergoes considerable changes in the progressive manner. An analysis of the effects of a 3D flow of Casson nano liquid over a Riga plate was published by Upreti *et al.* [10]. An analysis of Cu–Ag/H₂OC₂H₆O₂ hybrid NF flow was conducted by Joshi *et al.* [11]. They examined the considered phenomena in the presence of dissipation of viscosity. Upreti *et al.* [12] evaluated the nanoliquid flow between two parallel surfaces. They analyzed the entropy generation phenomena in the hybrid NF.

Hybrid NF is a revolutionary variant of liquid that surpasses regular fluids at energy transfers. Thermodynamic implications for hybrid NFs include chilling in elevated temperature [13]. In addition to renewable radiation, hybrid nuclear reactors are used in heating systems, heat exchangers, air conditioning units, automobiles, electrical refrigerators, turbines, radioactivity equipment, broadcasters, spacecraft, and biotech. Copper (Cu) and cobalt ferrite (CoFe₂O₄) NPs in the global solvent water are the purposes of this research. Electrodeposition, a “green” chemical approach, may be used to make Cu NPs from plant water extracts. New antitubercular medicines are being carried by Cu NPs [14]. Cu acts as an antiseptic, antibacterial, and antiviral ingredient when introduced to aquatic for varnishes, polymeric, and fibers. Cu-containing dietary supplements have a high absorption rate. Tensile strength is strong in Cu alloys and metals [15]. Metallic compounds encompass iron and cobalt NPs. Iron reduces interstitial endurance, permitting charge/ion mobility on the surface [16]. Imaging methods have shown to be critical in accurately identifying disorders. MRI is the most flexible of them all because it can give you morphological and physiological information. Bi-magnetic granules are used to make it more effective. Bi-magnetic NPs can be utilized for therapeutic agents and photoacoustic imaging, making them interesting candidates for development as enhanced MRI contrast agents. Bi-magnetic NPs can also be applied in therapeutic agents and photoacoustic imaging. Magnetic therapy is employed to cure tumors because cancerous cells are much more susceptible to less heat than normal tissues. Energy elevation produced by magnetic NPs accumulation can prevent cancer while exhibiting little consequences on adjacent tissues [17].

Numerous scientists and academics focus on the quantitative perspective to the abovementioned issues and challenges. Bilal *et al.* [18] evaluated the influence of electric and ferromagnetic effects on the hybrid NF flow across two rotary interfaces. The momentum boundary

layer is enhanced by the electric factor. A hybrid NF was assessed with CoFe_2O_4 and Fe_3O_4 in ethylene glycol by Ramesh *et al.* [19]. The results of metallic cushions and nanostructured materials were calculated with an MWCNT- Fe_3O_4 hybrid NF by Wang *et al.* [20]. Using a transparent extendable sheet with heat and momentum transitions, Bilal *et al.* [21] explored the effect of radiant energy on unsteady peristaltic flow. The distribution of microrotations is influenced by the porosity factor. In their study of NF flow, Algehyne *et al.* [22] used motile microbes and nanoparticles on porous moveable substrates. Few of the earlier studies [23–27] include some related literature of NFs and hybrid NFs.

The constant fluid physical parameters have been considered in various NF flow simulations. NFs, however, have variably viscous properties, which are critical to NF flow because they vary greatly with temperature. Internal resistance, for example, generates heat, which raises the temperature and hence influences the viscosity of NFs. To accurately assess the flow characteristics of NFs, it is crucial to take into account the viscosity–temperature discrepancy. Kuttan *et al.* [28] investigated the effects of temperature-dependent viscosity on fluid flow across a smooth surface. Shafiq *et al.* [29] investigated the NF flow on thin slandering needles. The authors dispersed carbon nanotubes in ethylene glycol to make homogeneous suspension. Udawattha *et al.* [30] used the viscoelasticity of solid particle dispersion to determine the effective viscosity of NFs. There is currently a scarcity of data on the flow of $\text{Cu-CoFe}_2\text{O}_4/\text{H}_2\text{O}$ hybrid NFs when temperature-dependent viscosity is considered.

There are many cases whereby a no-slip boundaries condition is not appropriate. For example, polymer melts exhibit some wall slips for distinct non-Newtonian fluids, and these slips tend to follow a monotonous and non-linear relationship between slip velocity and adhesion friction mathematical problems [31]. Fluids with a slip boundary condition are beneficial in a range of technical and biomedical disciplines, including costly lubrication, optical coatings, cooling apparatus, purification of prosthetic heart valves, internal chambers, and other industrial operations. As a result, some researchers have looked into the implication of the slip boundary condition [32]. For instance, Oyelakin *et al.* [33] and Tlili *et al.* [34] investigated the impact of slip conditions on NF flow and energy conversion across a rigid sheet. Khan *et al.* [35] extensively documented the impact of slip conditions on NF flow. They discovered that when the value of second-order velocity slip conditions increases, the thermal allocation and thermal pressure gradient rise. However, nothing is known regarding the effect of second-order slip on the flow of $\text{Cu-CoFe}_2\text{O}_4/\text{H}_2\text{O}$

hybrid NFs overstretched surfaces. One of the goals of this research is to fill up the gaps in the previously mentioned information.

As a result of the abovementioned literature review, we examined the flow and heat transmission of $\text{Cu-CoFe}_2\text{O}_4/\text{H}_2\text{O}$ hybrid NF surfaces over stretching surfaces. In addition to temperature-dependent viscosity, heat source, free convection, Darcy Forchheimer, second-order slip, and chemical reaction are taken into account. A spectral parametric continuation method (PCM) is used to evaluate the governing equations. It is a highly accurate and fast-convergent method. Tables and graphs are also presented to illustrate how controlling factors affect hybrid NF flow velocity and temperature gradients. In biological domains, such as enzymatic biosensors, drug delivery, cancer therapy, and enzymatic biosensors, the proposed model has promising implications. So far, no assessment has been conducted on this topic. In comparison with other studies that have been conducted, this work is unique and innovative.

2 Mathematical formulation

We illustrated a 2D unsteady hybrid NF flow with velocity and thermal slip conditions passing over linearly extending texture embedded in the permeable medium. The Cu and CoFe_2O_4 nanoparticles are equally dispersed in water, to reform hybrid NF. The flow regime is $\gamma > 0$, where x -axis and y -axis are along the flow direction and normal to the flow field, respectively. Initially, the hybrid NF is at rest with a consistent temperature $\tilde{T}_s$ at $t = 0$. But at $t > 0$, the surface starts stretching linearly with velocity $\tilde{u}_w = (\tilde{u}_0/1 - \xi t)x$, where $\tilde{u}_0 < 0$ show surface shrinking and $\tilde{u}_0 > 0$ represent surface stretching. On behalf of the above presumption and Boussinesq approximation, the modeled equations for the fluid flow are calculated as follows [36,37]:

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0, \quad (1)$$

$$\rho_{\text{hnf}} \left(\frac{\partial \tilde{u}}{\partial t} + u \frac{\partial \tilde{u}}{\partial x} + v \frac{\partial \tilde{u}}{\partial y} \right) = \mu_{\text{hnf}} \frac{\partial^2 \tilde{u}}{\partial y^2} + \frac{\partial \tilde{u}}{\partial y} \frac{\partial}{\partial y} (\mu_{\text{hnf}}) - \frac{\mu_{\text{hnf}}}{K} \tilde{u} + g \left(\frac{(\rho\beta_t)_{\text{hnf}}(\tilde{T} - \tilde{T}_s)}{(\rho\beta_c)_{\text{hnf}}(\tilde{C} - \tilde{C}_s)} \right) - \text{Fu}^2, \quad (2)$$

$$\begin{aligned} (\rho C_p)_{\text{hnf}} \left(\frac{\partial \tilde{T}}{\partial t} + \tilde{u} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial y} \right) \\ = k_{\text{hnf}} \frac{\partial^2 \tilde{T}}{\partial y^2} + Q_0(\tilde{T} - \tilde{T}_{\infty}), \end{aligned} \quad (3)$$

$$\left(\frac{\partial \tilde{C}}{\partial t} + \tilde{u} \frac{\partial \tilde{C}}{\partial x} + \tilde{v} \frac{\partial \tilde{C}}{\partial y} \right) = D_{\text{hnf}} \frac{\partial^2 \tilde{C}}{\partial y^2} + \text{Kr}(\tilde{C} - \tilde{C}_{\infty}). \quad (4)$$

The boundary conditions are as follows:

$$\left. \begin{aligned} \tilde{u}(x, t) &= \tilde{u}_w + \tilde{u}_{\text{slip}}, \quad \tilde{v} = 0, \quad \tilde{T} = \tilde{T}_b + N_0 \frac{\partial \tilde{T}}{\partial y}, \\ C &= C_b \text{ at } y = 0, \\ \tilde{u}(x, t) &\rightarrow 0, \quad \tilde{v} \rightarrow 0, \quad \tilde{T} \rightarrow \tilde{T}_s, \quad C \rightarrow C_s \text{ as } y \rightarrow \infty \end{aligned} \right\}, \quad (5)$$

where ρ_{hnf} , μ_{hnf} , D_{hnf} , and ν_{hnf} show density, dynamic viscosity, mass diffusivity, and kinematic viscosity of hybrid NF. Here N_0 , Q_0 , β_{hnf} , Kr , and g are the thermal slip term, heat absorption/generation, volumetric expansion of hybrid NF, chemical reaction, and gravitational force, respectively.

$$\mu_f = \mu_0 e^{-R\theta(\zeta)}, \quad (6)$$

$$\tilde{u}_{\text{slip}} = a \frac{\partial \tilde{u}}{\partial y} + b \frac{\partial^2 \tilde{u}}{\partial y^2}, \quad (7)$$

where a and b are expressed as follows:

$$\begin{aligned} a &= \frac{2}{3} \left(\frac{3 - \alpha r^2}{\alpha} - \frac{3}{2} \frac{1 - r^2}{K_n} \right) \lambda, \\ ab &= -\frac{1}{4} \left(r^4 + \frac{2}{K_n^2} (1 - r^2) \right) \lambda^2. \end{aligned} \quad (8)$$

Table 1: Experimental values of Cu, CoFe_2O_4 , and water [38]

	$\rho(\text{kg/m}^3)$	$C_p(\text{J/kg K})$	$k(\text{W/mK})$
Water	997.1	4,179	0.613
Cu	8,933	385	401
CoFe_2O_4	4,907	700	3.7

Here λ is the free path molecular mean and K_n is the Knudsen number. R is the varying viscosity ($R < 0$ for liquid & $R > 0$ for gas), and μ_0 is the reference viscosity of carrier fluid (Tables 1 and 2).

The similarity variables are as follows:

$$\begin{aligned} \tilde{u} &= \frac{\tilde{u}_0}{1 - \zeta t} x f'(\zeta), \quad \tilde{v} = \sqrt{\frac{\tilde{u}_0 \nu_f}{1 - \zeta t}} f(\zeta), \\ \zeta &= \sqrt{\frac{\tilde{u}_0}{\nu_f(1 - \zeta t)}} y, \quad \varphi(\zeta) = \frac{\tilde{C} - \tilde{C}_s}{\tilde{C}_b - \tilde{C}_s}, \quad \theta(\zeta) = \frac{\tilde{T} - \tilde{T}_s}{\tilde{T}_b - \tilde{T}_s}. \end{aligned} \quad (9)$$

Substituting equation (8) in equations (1)–(7), we get

$$f'''(\zeta) + \wp_1 \wp_2 \left(f(\zeta) f''(\zeta) - A \left(f'(\zeta) + \frac{\zeta}{2} f''(\zeta) \right) \right) - f'^2(\zeta) + \wp_3 \tau \theta(\zeta) = 0, \quad (10)$$

$$- R \theta'(\zeta) - \kappa f'(\zeta) \text{Fr} = 0,$$

$$\begin{aligned} \frac{k_{\text{hnf}}}{k_f} \theta''(\zeta) + \wp_4 \text{Pr} \left(f(\zeta) \theta'(\zeta) - A \left(2\theta(\zeta) + \frac{\zeta}{2} \theta'(\zeta) \right) \right) \\ + \text{Pr} \Gamma \theta(\zeta) = 0, \end{aligned} \quad (11)$$

$$\frac{D_{\text{hnf}}}{D_f} \frac{1}{\text{Sc}} \varphi'' + A f \varphi' - K c \varphi = 0. \quad (12)$$

The transform conditions are as follows:

$$\left. \begin{aligned} f(0) &= 0, \quad f'(0) = 1 + \delta f'''(0) + \gamma f''(0), \\ \theta(0) &= 1 + \beta \theta'(0), \quad \varphi(0) = 1, \\ f'(\infty) &\rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \varphi(\infty) \rightarrow 0 \end{aligned} \right\}. \quad (13)$$

Here

$$\wp_1 = \frac{\mu_{\text{hnf}}}{\mu_{\text{bf}}}, \quad \wp_2 = \frac{\rho_{\text{hnf}}}{\rho_{\text{bf}}}, \quad \wp_3 = \frac{k_{\text{hnf}}}{k_{\text{bf}}}, \quad \wp_4 = \frac{(\rho C_p)_{\text{hnf}}}{(\rho C_p)_{\text{bf}}}, \quad (14)$$

Table 2: Mathematical model for hybrid nanoliquid ($\phi_1 = \phi_{\text{Cu}}$, $\phi_2 = \phi_{\text{CoFe}_2\text{O}_4}$) [38]

Properties	
Viscosity	$\frac{\mu_{\text{hnf}}}{\mu_{\text{bf}}} = \frac{1}{(1 - \phi_{\text{Cu}} - \phi_{\text{CoFe}_2\text{O}_4})^2}$
Density	$\frac{\rho_{\text{hnf}}}{\rho_{\text{bf}}} = \phi_{\text{Cu}} \left(\frac{\rho_{\text{Cu}}}{\rho_{\text{bf}}} \right) + \phi_{\text{CoFe}_2\text{O}_4} \left(\frac{\rho_{\text{CoFe}_2\text{O}_4}}{\rho_{\text{bf}}} \right) + (1 - \phi_{\text{Cu}} - \phi_{\text{CoFe}_2\text{O}_4})$
Thermal capacity	$\frac{(\rho C_p)_{\text{hnf}}}{(\rho C_p)_{\text{bf}}} = \phi_{\text{Cu}} \left(\frac{(\rho C_p)_{\text{Cu}}}{(\rho C_p)_{\text{bf}}} \right) + \phi_{\text{CoFe}_2\text{O}_4} \left(\frac{(\rho C_p)_{\text{CoFe}_2\text{O}_4}}{(\rho C_p)_{\text{bf}}} \right) + (1 - \phi_{\text{Cu}} - \phi_{\text{CoFe}_2\text{O}_4})$
Thermal conductivity	$\frac{k_{\text{hnf}}}{k_{\text{bf}}} = \left[\frac{\phi_{\text{Cu}} k_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} k_{\text{CoFe}_2\text{O}_4}}{\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}} + 2k_{\text{bf}} + 2(\phi_{\text{Cu}} k_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} k_{\text{CoFe}_2\text{O}_4}) - 2(\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}) k_{\text{bf}} \right] \\ \left[\frac{\phi_{\text{Cu}} k_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} k_{\text{CoFe}_2\text{O}_4}}{\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}} + 2k_{\text{bf}} - 2(k_{\text{Cu}} \phi_{\text{Cu}} + k_{\text{CoFe}_2\text{O}_4} \phi_{\text{CoFe}_2\text{O}_4}) + (\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4})^2 k_{\text{bf}} \right]$
Electrical conductivity	$\frac{\sigma_{\text{hnf}}}{\sigma_{\text{bf}}} = \left[\frac{\phi_{\text{Cu}} \sigma_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} \sigma_{\text{CoFe}_2\text{O}_4}}{\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}} + 2\sigma_{\text{bf}} + 2(\phi_{\text{Cu}} \sigma_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} \sigma_{\text{CoFe}_2\text{O}_4}) - 2(\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}) \sigma_{\text{bf}} \right] \\ \left[\frac{\phi_{\text{Cu}} \sigma_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} \sigma_{\text{CoFe}_2\text{O}_4}}{\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}} + 2\sigma_{\text{bf}} - (\phi_{\text{Cu}} \sigma_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4} \sigma_{\text{CoFe}_2\text{O}_4}) + (\phi_{\text{Cu}} + \phi_{\text{CoFe}_2\text{O}_4}) \sigma_{\text{bf}} \right]$

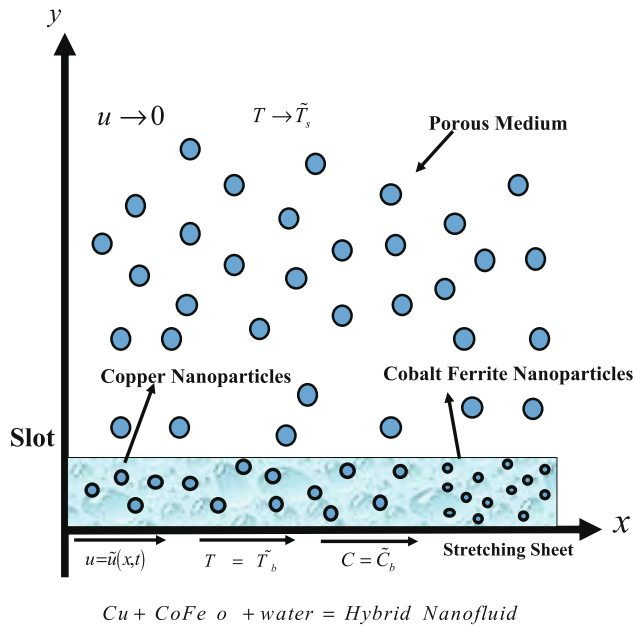


Figure 1: Physical sketch of the hybrid NF flow through stretching sheet.

where $Pr = \frac{\nu_f(\rho C_p)}{k_f}$, is the Prandtl number, $Fr = \frac{C_b}{k^{*1/2}}$ is the Darcy Forchheimer number, $Sc = \frac{\nu_f}{D_f}$ is the Schmidt number, $\beta = N_0\sqrt{\tilde{u}_0/\nu_f}$ is the thermal slip, $\delta = b(\tilde{u}_0/\nu_f)$ is the slip velocity term of second-order, $\gamma = a\sqrt{\tilde{u}_0/\nu_f}$ is the slip velocity term of first-order, $\tau = (g\beta_t(\tilde{T}_b - \tilde{T}_s)/b\tilde{u}_0^2)$ is the Grashof number, $\Gamma = ((1 - \zeta t)Q_0/\tilde{u}_0(\rho C_p)_f)$ is the heat source, $A = (\zeta/u_0)$ is the unsteadiness coefficient, and $\kappa = \frac{\nu_f(1 - \zeta t)}{K\tilde{u}_0}$ is the permeability constant.

The non-dimensional physical interest quantities are as follows:

$$\begin{aligned} Re_x^{1/2} C_{fx} &= \frac{f''(0)}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}}, \\ Re_x^{1/2} Nu_x &= \frac{k_{hnf}}{k_f} \theta'(0), \quad Re_x^{1/2} Sh_x = -\varphi'(0). \end{aligned} \quad (15)$$

3 Numerical solution

The basic steps while using PCM are as follows [39,40].

Step 1: Simplifying the BVP to a 1st system of ordinary differential equations (ODEs)

$$\begin{aligned} h_1 &= f(\zeta), \quad h_2 = f'(\zeta), \quad h_3 = f''(\zeta), \quad h_4 = \theta(\zeta), \\ h_5 &= \theta'(\zeta), \quad h_6 = \varphi(\zeta), \quad h_7 = \varphi'(\zeta). \end{aligned} \quad (16)$$

By substituting equation (9) in equations (1)–(5), we get

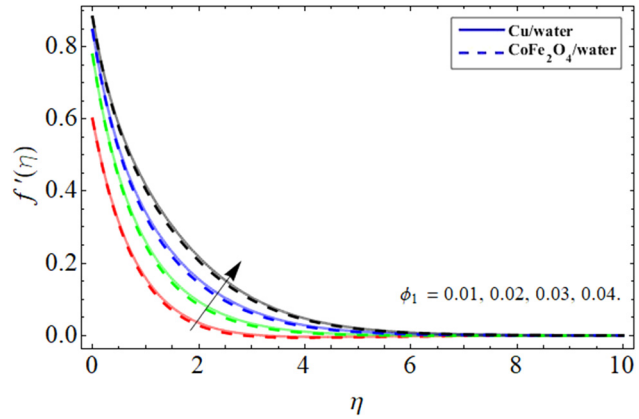


Figure 2: Velocity outlines $f'(\eta)$ vs Cu NPs.

$$\begin{aligned} h_3' + \wp_1 \wp_2 \left(h_1 h_3 - A \left(h_2 + \frac{\zeta}{2} h_3 \right) - h_2^2 + \wp_3 \tau h_4 \right) - R h_5 \\ - \kappa h_2 Fr = 0, \end{aligned} \quad (17)$$

$$\frac{k_{hnf}}{k_f} h_5' + \wp_4 Pr \left(h_1 h_5 - A \left(2h_4 + \frac{\zeta}{2} h_5 \right) \right) + Pr \Gamma h_4 = 0, \quad (18)$$

$$\frac{D_{hnf}}{D_f} \frac{1}{Sc} h_7' + Af h_7 - Kch_6 = 0. \quad (19)$$

The transform conditions are as follows:

$$\left. \begin{aligned} h_1(0) &= 0, \quad h_2(0) = 1 + \gamma h_3(0) + \delta h_3'(0), \\ h_4(0) &= 1 + \beta h_5, \quad h_6(0) = 1, \\ h_2(\infty) &\rightarrow 0, \quad h_4(\infty) \rightarrow 0, \quad h_6(\infty) \rightarrow 0. \end{aligned} \right\} \quad (20)$$

Step 2: Introducing parameter p

$$\begin{aligned} h_3' + \wp_1 \wp_2 \left(h_1(h_3 - 1)p - A \left(h_2 + \frac{\zeta}{2} h_3 \right) - h_2^2 + \wp_3 \tau h_4 \right) \\ - R h_5 - \kappa h_2 Fr = 0, \end{aligned} \quad (21)$$

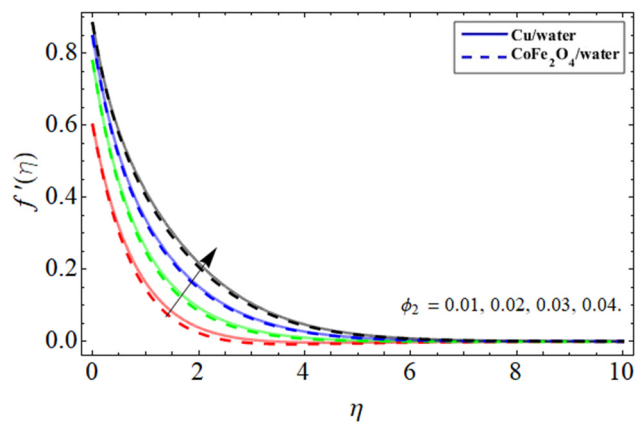
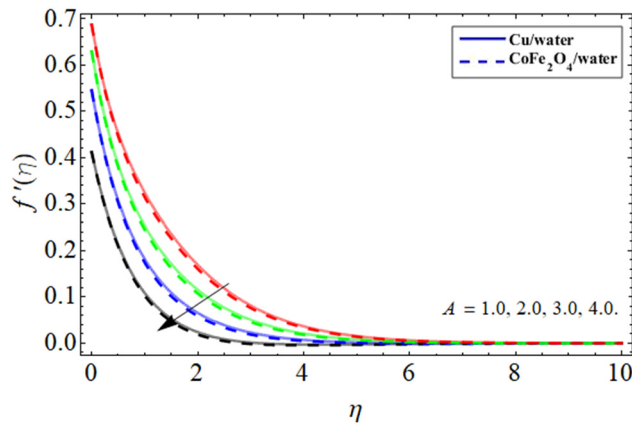
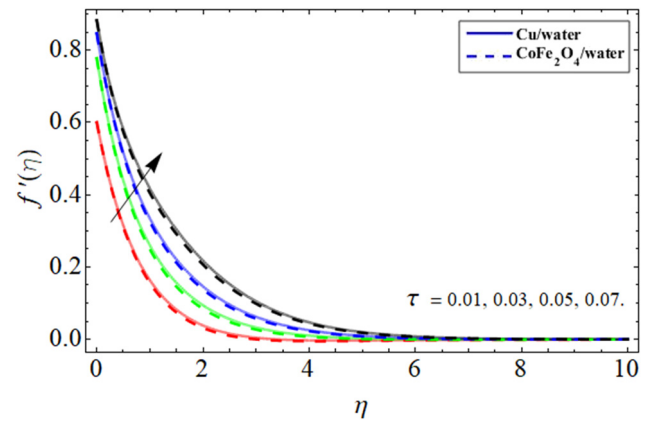
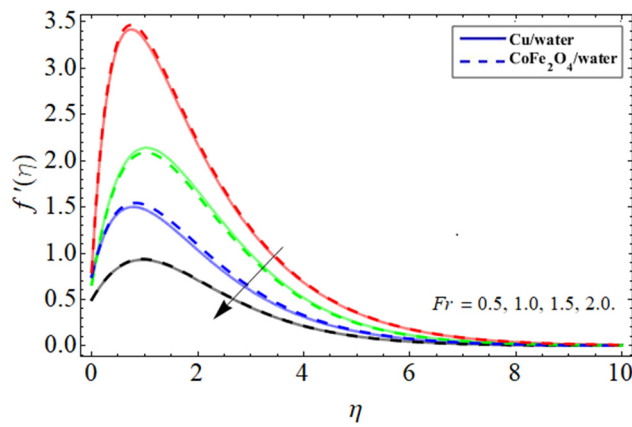
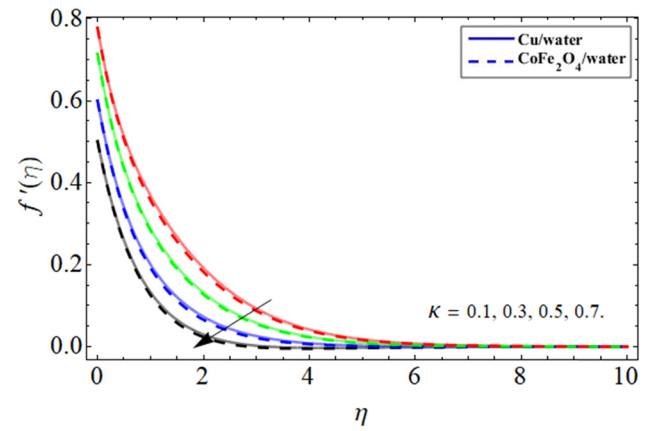


Figure 3: Velocity outlines $f'(\eta)$ vs $CoFe_2O_4$ NPs.

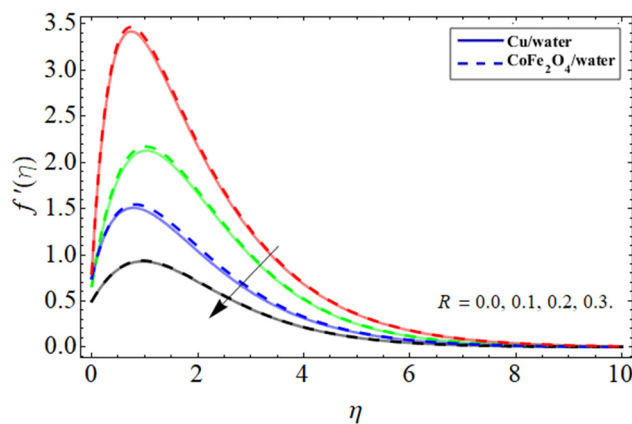
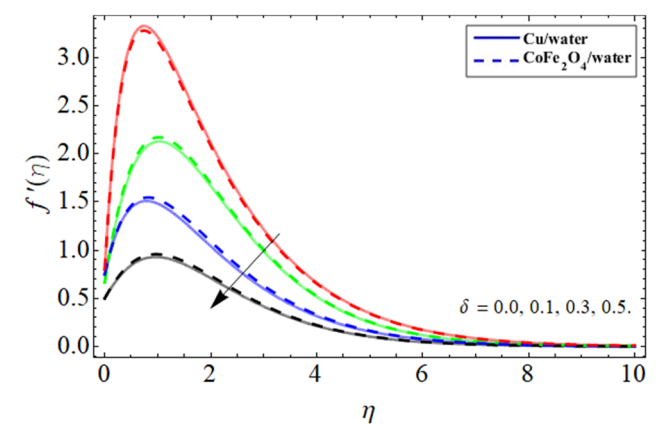
Figure 4: Velocity outlines $f'(\eta)$ vs unsteadiness constraint.Figure 7: Velocity outlines $f'(\eta)$ vs Grashof number.Figure 5: Velocity outlines $f'(\eta)$ vs Darcy Forchheimer number.Figure 8: Velocity outlines $f'(\eta)$ vs permeability term.

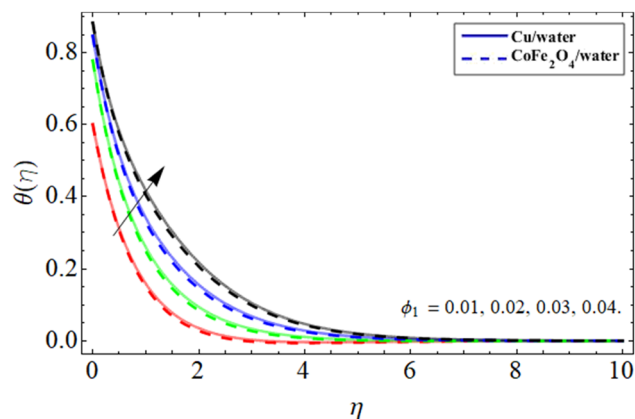
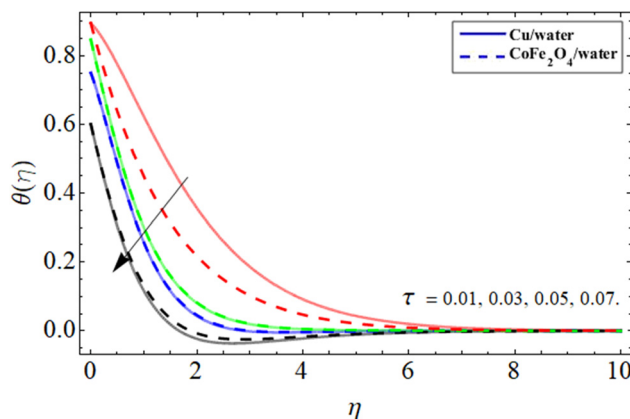
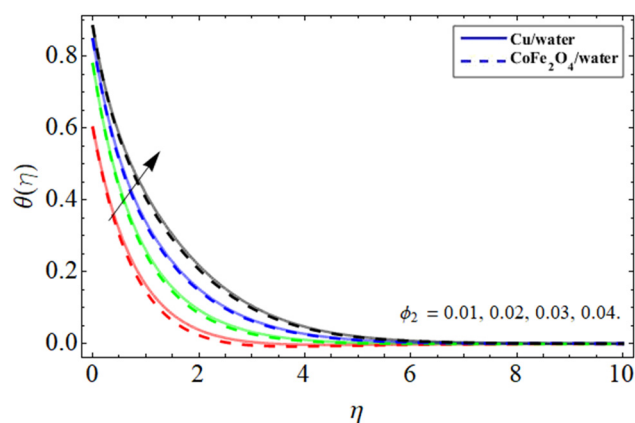
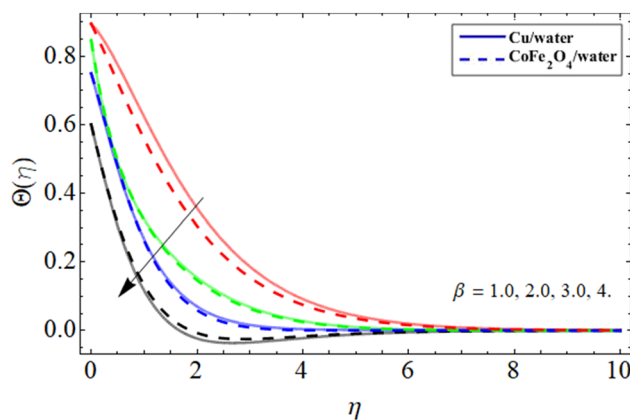
$$\frac{k_{hnf}}{k_f} \tilde{h}'_5 + \wp_4 \text{Pr} \left(\tilde{h}_1(\tilde{h}_5 - 1)p - A \left(2\tilde{h}_4 + \frac{\zeta}{2} \tilde{h}_5 \right) \right) + \text{Pr} \Gamma \tilde{h}_4 = 0, \quad (22)$$

$$\frac{D_{hnf}}{D_f} \frac{1}{\text{Sc}} \tilde{h}'_7 + A f(\tilde{h}_7 - 1)p - K c \tilde{h}_6 = 0. \quad (23)$$

Step 3: Applying Cauchy principal

Equations (21)–(23) are further discretized and manipulated through PCM Matlab code.

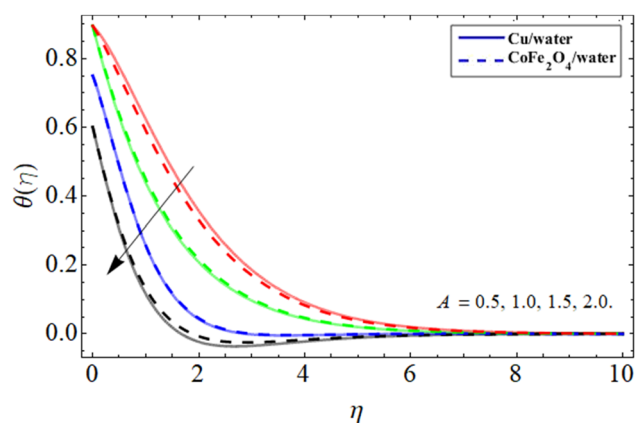
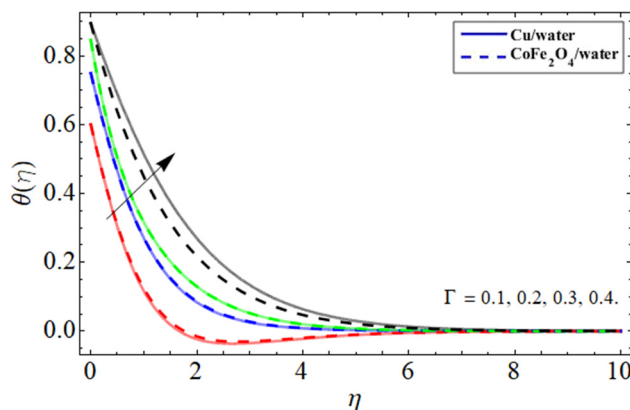
Figure 6: Velocity outlines $f'(\eta)$ vs variable viscosity term.Figure 9: Velocity outlines $f'(\eta)$ vs second-order slip constraint.

Figure 10: Energy outlines $\theta(\eta)$ vs Cu NPs.Figure 13: Energy outlines $\theta(\eta)$ vs Grashof number.Figure 11: Energy outlines $\theta(\eta)$ vs CoFe_2O_4 NPs.Figure 14: Energy outlines $\theta(\eta)$ vs thermal slip term.

4 Results and discussion

The purpose of this section is to explain the physics behind the graphical results and tables. Results are displayed using figures. Cu NF is represented by solid lines,

while CoFe_2O_4 NF is represented by dashes. The mechanical process of hybrid NF flow across a permeable elongating sheet is described in Figure 1. The

Figure 12: Energy outlines $\theta(\eta)$ vs unsteadiness term.Figure 15: Energy outlines $\theta(\eta)$ vs heat absorption/generation constraint.

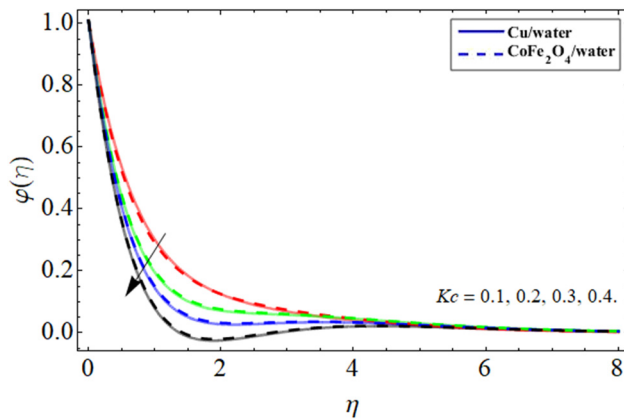
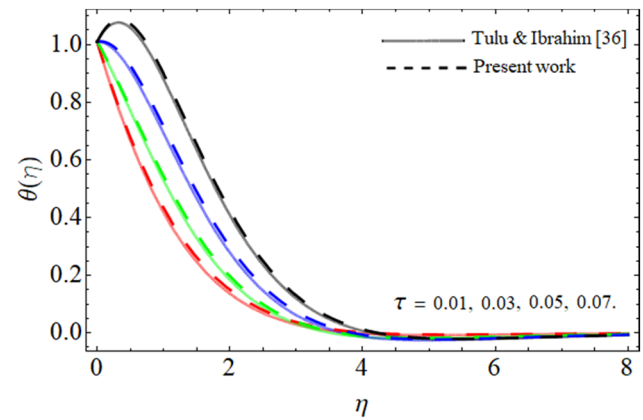
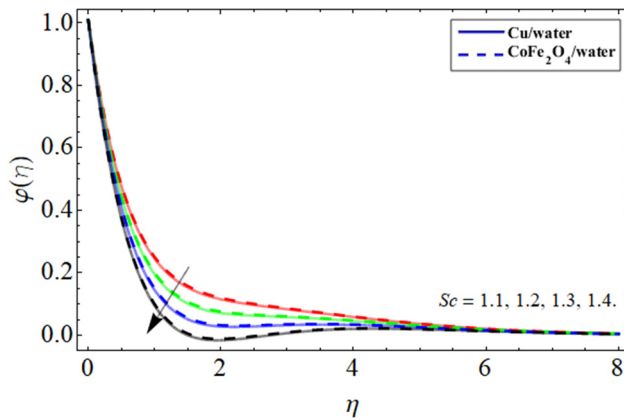
Figure 16: Energy outlines $\theta(\eta)$ vs chemical reaction.

Figure 18: Validation of the present outcome with the published work.

Figure 17: Energy outlines $\theta(\eta)$ vs Schmidt number.

default values of parameters are taken as $A = 1.0$, $\phi_1 = \phi_2 = 0.01$, $Fr = 0.5$, $R = 0.1$, $\tau = 0.01$, $\kappa = \delta = 0.1$, $\beta = 0.2$, $\Gamma = 0.1$.

Figures 2–9 highlight the behavior of velocity $f'(\eta)$ outlines vs Cu NPs, CoFe_2O_4 NPs, unsteadiness term A , Forchheimer number Fr , variable viscosity R , Grashof

Table 3: Statistical assessments of Sherwood and Nusselt numbers

η	PCM		BVP4C	
	$\theta'(0)$	$\varphi'(0)$	$\theta'(0)$	$\varphi'(0)$
0.00	1.4854	1.5953	1.4781	1.5934
0.03	1.3562	1.5361	1.3232	1.5315
0.06	1.2482	1.4481	1.2221	1.4217
0.09	1.0114	1.2010	2.0073	1.1032

number τ , permeability factor κ , and second-order slip δ , respectively. Figures 2 and 3 reveal that the velocity contour enhances with the inclusion of ϕ_1 and ϕ_2 NPs. Because water has a significantly higher specific heat capability than Cu and CoFe_2O_4 NPs, while having a lower thermal efficiency, the addition of hybrid NPs, particularly Cu, diminishes its average heat-absorbing efficiency, resulting in an increase in velocity field. Figures 4 and 5 reveal that the velocity outlines decline with the upshot of unsteadiness term A and Forchheimer number Fr . Figures 6 and 7 describe that the velocity curve

Table 4: Relative comparison of the present outcomes vs the published literature

Parameters		Hayat <i>et al.</i> [41]	Khashi'ie <i>et al.</i> [42]	Present work	Hayat <i>et al.</i> [41]	Khashi'ie <i>et al.</i> [42]	Present work
M	S	$f''(0)$			$f''(1)$		
0.0	0.5	−7.412153	−7.4121525	−7.4121531	4.714303	4.7143028	4.7143032
1.0		−7.592618	−7.5926177	−7.5926182	4.738017	4.7380165	4.7380173
4.0		−8.111334	−8.1113342	−8.1113361	4.821251	4.8212511	4.8212543
9.0		−8.911096	−8.9110956	−8.9110971	4.965870	4.9658698	4.9658723
	0.0	−4.588891	−4.5888911	−4.5888942	1.843447	1.8434469	1.8434483
	0.3	−6.666662	−6.6666620	−6.6666647	3.654695	3.6546948	3.6546974
	0.6	−8.852444	−8.8524442	−8.8524462	5.392248	5.3922475	5.3922488
	1.0	−11.949584	−11.9495843	−11.9495873	7.594426	7.5944262	7.5944291

Table 5: Numerical values for Nusselt number $\theta'(0)$

Parameters					NF	Hybrid
ϕ_1	ϕ_1	τ	β	Pr	CoFe ₂ O ₃	Cu–CoFe ₂ O ₃
0.01	0.01	0.2	0.1	6.2	2.1327595	2.3327595
0.02					2.0074208	2.2074204
0.03					1.4196705	1.5196702
0.04	0.01				2.8247303	2.9247305
	0.02				2.5824944	2.5824976
	0.03				1.1319745	1.2319784
	0.04	0.2			1.9454874	1.9454974
		0.3			2.5824944	2.6825692
		0.4			2.1300553	2.3302493
		0.5	0.1		2.0984188	2.2984354
			0.3		2.5824944	2.7826235
			0.5		1.5330137	1.6335359
			0.7	6.2	1.1675688	1.4697365
				6.4	0.8303698	1.9342547
				6.6	1.6068036	2.6268491
				6.8	2.4090316	3.4509078

declines with the upshot of variable viscosity R , while enhancing Gr. Over time the viscosity of fluid develops, which produces resistance opposite to the flow field, that's why this phenomenon happened as shown in Figure 6. Physically, the heat generation capability of the fluid improves with the effect of Grashof number, which results in the enhancement of fluid velocity. Figures 8 and 9 display that the velocity field lessens under the impact of permeability factor κ and second-order slip δ . The second-order slip component improves fluid motion resistance, reducing the fluid flow field and the thickness

Table 6: Numerical values for Sherwood number $\varphi'(0)$

Parameters					NF	Hybrid
ϕ_1	ϕ_1	Kc	Sc	Pr	CoFe ₂ O ₃	Cu – CoFe ₂ O ₃
0.01	0.01	0.2	0.1	6.2	3.327573	2.3275956
0.02					3.07428	2.074205
0.03					2.196723	1.196704
	0.01				3.247323	2.347306
	0.02				3.824943	2.824977
	0.03				2.319746	1.319785
		0.1			2.454875	1.454975
		0.4			3.824945	2.825693
		0.7			3.300554	2.302494
			0.1		3.984189	2.984355
			0.2		3.824945	2.826236
			0.3		2.330138	1.335378
				6.2	2.675689	1.697376
				6.4	1.303699	1.342564
				6.6	2.068038	2.268582

of the momentum boundary layer. Furthermore, a higher value δ improves the energy transference.

Figures 10–15 report the compartment of energy contour against the variation in Cu, CoFe₂O₄, unsteadiness term A , Grashof number τ , thermal slip β , heat source term Γ , respectively. Figures 10 and 11 depict that the variation in Cu and CoFe₂O₄ boosts the energy transference rate. The addition of Cu and CoFe₂O₄ reduces the average heat capacity of hybrid NF, which causes an advancement of the energy outline. Figures 12 and 13 emphasize that the energy curve decreases with the upshot of unsteadiness term A and Grashof number. Free convection flows are continuously carried from the stretched surface to the free stream and a rise in τ shows that free convection is progressing currents of convection. Figures 14 and 15 report that the energy curve diminishes with the outcome of thermal slip term β and enhances with the action of the heat source term. The impact of heat source term encourages fluid particles and increases their kinetic energy, which results in the elevation of the energy field.

Figures 16 and 17 reveal the mass transmission trend vs the variation in chemical reaction Kr and Schmidt number Sc. It has been noticed from Figures 16 and 17 that the mass propagation profile significantly declines with the variation in Kr and Sc. It is believed that the growing values of Sc and Kr reduce the mass outline. Figure 18 ensures the accuracy of the present results with the published work, while using the value of $\beta = 0$ for no slip condition.

Tables 1 and 2 show Cu and CoFe₂O₄ NPs and their thermochemical properties, respectively. To confirm the validity of the current report, Table 3 presents the quantitative evaluation of PCM and BVP4C approaches. In the calculation, the heat and momentum domains are related. Table 4 expresses the relative comparison between the present results and the published literature. It can be observed that the present outcomes are accurate and reliable. Tables 5 and 6 illustrate the numerical outputs for Nusselt number and Sherwood number vs the variation in physical parameters.

5 Conclusion

An extended surface of Cu–CoFe₂O₄/H₂O hybrid NFs has been studied to determine its flow and heat transmission characteristics. Reliability substitutions are used to summarize the modeled equations of the problem. The obtained set of differential equations is further processed

computationally by employing the PCM approach. This methodology is highly accurate and fast converging. The proposed model has promising implications in biological domains such as cancer therapy, enzymatic biosensors, and drug delivery. The key conclusions of this study are as follows:

- The energy and velocity contour significantly enhances with the variation in Grashof number, Cu nanomaterials $\phi_1 = \phi_{\text{Cu}}$ and CoFe_2O_4 $\phi_2 = \phi_{\text{CoFe}_2\text{O}_4}$ nano particulates.
- The influence of permeability factor κ and second-order slip δ diminishes the velocity field.
- The velocity field declines with the effect of unsteadiness term A , variable viscosity R , and Forchheimer number Fr .
- The energy profile decreases with the upshot of unsteadiness term A , Grashof number, thermal slip term β , and enhances with the action of the heat source.
- The mass propagation profile significantly declines with the variation in a chemical reaction and Schmidt number.
- The inclusion of Cu and CoFe_2O_4 nanosize crystals has a huge tendency to boost the energy transmission for the biomedical and industrial purposes.
- In future, the present model may be modified to non-Newtonian fluid and can be solved with the other numerical, analytical, and fractional techniques.

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