Review Article

Hang Zhang, Cong Gao, Haichao Li*, Fuzhen Pang*, Tongda Zou, Hongfu Wang, and Na Wang

Analysis of functionally graded carbon nanotube-reinforced composite structures: A review

https://doi.org/10.1515/ntrev-2020-0110
received November 19, 2020; accepted December 1, 2020

Abstract: Functionally graded carbon nanotube-reinforced composite (FG-CNTRC) is a novel nanomaterial; the mechanical behavior of FG-CNRC has become a hot topic in the Materials Science and Engineering Science recently, thanks to its excellent mechanical and electrical properties after its fusion with matrix. In this paper, the review efforts for research progress on the modeling and analysis of FG-CNTRC structures are carried out. Firstly, the development background of FG-CNRC is presented, as well as some basic theories and main equations for mechanical analysis of FG-CNTRC structure. Then, the mechanical behaviors of FG-CNTRC beams, plates, and shells under loading conditions are comprehensively reviewed, with the emphasis on discussing the bending, buckling, and vibration behaviors of the structures. Finally, the future research orientation of the field is considered and prospected.

Keywords: carbon nanotubes, functionally graded, nanostructures, composite, mechanical behaviors

*** Corresponding author: Haichao Li,** College of Shipbuilding Engineering, Harbin Engineering University, Harbin, 150001, China; Yantai Research Institute and Graduate School of Harbin Engineering University, Yantai, 264000, China, e-mail: lihaichao@hrbeu.edu.cn
*** Corresponding author: Fuzhen Pang,** College of Shipbuilding Engineering, Harbin Engineering University, Harbin, 150001, China; Yantai Research Institute and Graduate School of Harbin Engineering University, Yantai, 264000, China, e-mail: pangfuzhen@hrbeu.edu.cn
Hang Zhang, Cong Gao, Tongda Zou, Hongfu Wang: College of Shipbuilding Engineering, Harbin Engineering University, Harbin, 150001, China
Na Wang: College of Power and Energy Engineering, Harbin Engineering University, Harbin, 150001, China; Yantai Research Institute and Graduate School of Harbin Engineering University, Yantai, 264000, China

1 Introduction

Beams, plates, and shells, as fundamental components, have been applied extensively in various types of engineering structure [1]. They make up most of the products in many industrial fields, such as construction [2], aviation [3], naval architecture [4], etc. Under the practical circumstances, these structural elements are commonly subjected to various forms of dynamic loadings in their service life [5]. To avoid the damage caused by structural decay, it is significant to develop their mechanical analysis and make some judgments about whether they have the ability to work well [6]. At present, many advanced analysis theories and accurate computing methods have been proposed constantly in order to reflect the essential features of the structure and evaluate the mechanical property of structures with the kind of formats or materials [7].

Nanomaterials, due to their particular dimension effect, exhibit unusual physical, chemical, and biological behavior, so they have been applied to various fields [8]. As recognized, metal-matrix composites with ceramic fibers and particulates could prefer higher specific strength and elastic modulus than their monolithic alloys [9]. Using the nanomaterials as particulates and compositing with conventional materials, their mechanical properties in certain conditions would be further increased [10]. For instance, Mohan *et al.* [11] reported that the presence of the high-modulus graphene in a low modulus matrix can lead to significant reinforcement. Wu *et al.* [12] texted matrix composites reinforced by graphene nanoplatelets and found that the microhardness value of nanocomposite increases by 24.51% compared to the matrix. Liu *et al.* [13] focused on the Honeycomb structure and showed the result that with the filling circular tubes into a buckling area, a considerable mass efficiency improvement with respect to deflection resistance can be obtained. Scholars in growing number are exploring how to improve the mechanical properties of traditional structures and trying to find a new kind of material which can make the fundamental components own an excellent property.

Carbon nanotube (CNT), a type of carbonaceous nanomaterial, was first found by Iijima [14]. Its excellent mechanical and electrical properties make it a prevalent topic in nanotechnology [15]. Lau and Hui [16] have reported that the nanotubes own remarkable mechanical properties with theoretical Young's modulus and tensile strength as high as 1 TPa and 200 GPa, respectively. Lu and Hu [17] used the computational simulation for predicting mechanical properties of carbon nanotubes (CNTs) and get the result that the CNTs have a high elastic modulus of 0.349 TPa. So, the study of the carbon nanotube-reinforced composite structure is of great significance and has attracted the attention of scholars [18] (Figure 1).

This article first introduces a new type of nanocomposites, the functionally graded carbon nanotube-reinforced (FG-CNTRC), then reviews the mechanical analysis of the composites used in the beams, plates, and shells in recent years. It mainly focused on the theoretical method and solving results in the process of analysis. Finally, some consideration about study and the direction of future research was prospected.

2 Functionally graded carbon nanotube-reinforced composite structures

Evidence shows that the CNTs' content and arrangement in composite may affect the mechanical property of structures sensitively [20]. How to improve the macro mechanical properties of carbon nanotube-reinforced composite structures in the case of low content initiated the scholars' ponder. Chinese researcher Shen [21] first proposed the concept of functionally graded carbon nanotube-reinforced composite structures, which the volume content of CNTs in the material was distributed in gradient. This has made the composite acquire a satisfying reinforcing agent and designable characteristic, so that the research of FG-CNTRC has become popular in recent decades.

2.1 Background of functionally graded materials

As we know, composite materials have the outstanding material characteristics cause of the complementary and correlation of each component. However, when the composite structures encounter the extreme working conditions, the delamination failure or crack phenomena will be emerged [22]; then the composite materials are subjected to sharp transition of properties at the interface which can lead to component failure by delamination [23].

To overcome this problem, the concept of functionally graded materials (FGM) was firstly proposed by Niino [24]. These materials replace sharp interface with gradient interface which results in smooth transition of properties from one material to the other; then they could eliminate interface problems like stress concentrations and poor adhesion [25]. Therefore, this composite structure can adapt to the special environment and be widely used in aerospace, defense, and so forth [26]. Several kinds of analysis methods have been developed in order to design and evaluate the FG structure better. For instance, Li and Pang [27–31] provide a new semi-analytical method to analyze the free vibration of functionally graded circular cylindrical shells under complex boundary conditions. In addition, their team extended the research object to porous cylindrical shell [32] and axisymmetric doubly curved shells [33]. Karami *et al.* [34] investigated FG nanoplates made of anisotropic material (beryllium crystal as a hexagonal material) and used Galerkin's approach to solve the buckling problem for different boundary conditions.

2.2 The micromechanical models of FG-CNTRC

In fact, the FG-CNTRC is a special application of FGM [35]. Before the structural analysis, it is necessary to predict the effective material properties of carbon nanotube-

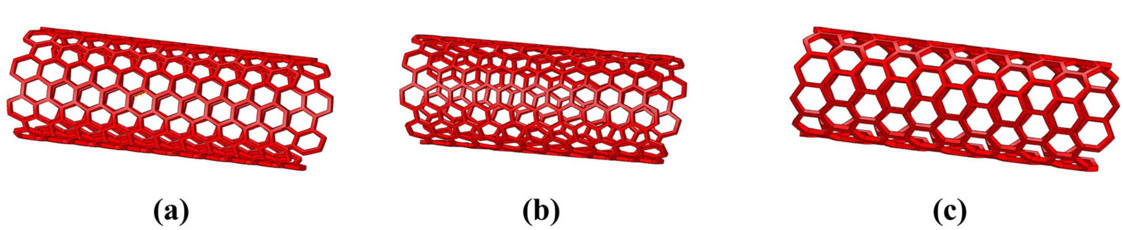

Figure 1: Schematical presentation of SWCNTs [19]. (a) Armchair, (b) chiral, (c) zigzag.

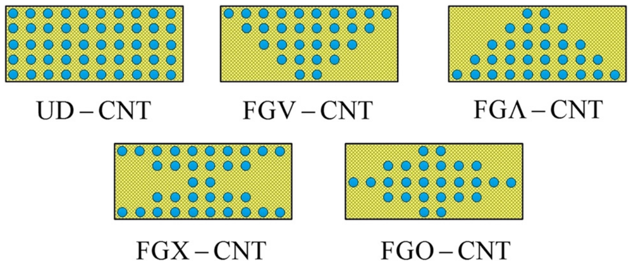

Figure 2: Cross section of UD, FG-Λ, FG-V, FG-O, and FG-X CNTRC [38].

reinforced composites [36]. Due to limitation of manufacturing, the distribution forms of carbon nanotubes are presented in the following types: UD, FG-V, FG-Λ, FG-X, and FG-O [37], which are depicted in Figure 2.

Type UD represents that the CNTs volume fraction was uniformly distributed in Z direction. In type FG-V and FG-Λ, the CNT fiber concentration increases and decreases through the selected direction, respectively. In type X, the CNT's volume fraction decreases gradually from the top surface to the middle and then increases symmetrically until reaching the other side. Type O materials have opposite CNT distribution method by having the maximum CNT concentration in the middle of the composite and the lowest at the surfaces [39]. The respective representation of CNTs' volume fraction can be expressed as follows [40]:

$$\text{UD:}\ \ V_{\text{cnt}} = V_{\text{tcnt}} \tag{1}$$

$$\text{FG-V:}\ \ V_{\text{cnt}} = \left(1 + \frac{2z}{h}\right)V_{\text{tcnt}} \tag{2}$$

$$\text{FG-}\Lambda\text{:}\ \ V_{\text{cnt}} = \left(1 - \frac{2z}{h}\right)V_{\text{tcnt}} \tag{3}$$

$$\text{FG-X:}\ \ V_{\text{cnt}} = 4\frac{|z|}{h}V_{\text{tcnt}} \tag{4}$$

$$\text{FG-O:}\ \ V_{\text{cnt}} = 2 - 4\frac{|z|}{h}V_{\text{tcnt}} \tag{5}$$

where z are the coordinate value along the thickness direction, h are the structure thickness, and V_{tcnt} are the total volume fraction of CNTs which can be written by:

$$V_{\text{tcnt}} = \frac{w_{\text{cnt}}}{w_{\text{cnt}} + \left(\frac{\rho^{\text{cnt}}}{\rho^{\text{m}}}\right) - \left(\frac{\rho^{\text{cnt}}}{\rho^{\text{m}}}\right)w_{\text{cnt}}} = 1 - V_{\text{m}} \tag{6}$$

where w_{cnt} is the mass fraction of CNTs, ρ^{cnt}, ρ^{m} are the mass density of CNTs, and isotropic matrix, V_{m}, is the total volume fraction of matrix.

Based on the above, there are mainly two typical micromechanical models that have been approved by most scholars; one is the extended rule of mixture [41] and the other is the Eshelby–Mori–Tanaka [42], both in order to estimate the effective constitutive law of the elastic isotropic matrix with dispersed elastic inhomogeneities (CNTs).

The rule of mixture is based on Krenchel's model for three-dimensional randomly dispersed short-fiber composites. It introduces the CNTs' efficiency parameters to indicate the size-dependent material properties [43]. After finishing and correction, the effective Young's modulus and shear modulus can be written as [44]:

$$E_y = \eta_1 V_{\text{m}} E_y^{\text{cnt}} + V_{\text{m}} E^{\text{m}} \tag{7}$$

$$\frac{\eta_2}{E_x} = \frac{V_{\text{cnt}}}{E_x^{\text{cnt}}} + \frac{V_{\text{m}}}{E^{\text{m}}} \tag{8}$$

$$\frac{\eta_3}{G_{xy}} = \frac{V_{\text{cnt}}}{G_{xy}^{\text{cnt}}} + \frac{V_{\text{m}}}{G^{\text{m}}} \tag{9}$$

where η_i $(i = 1, 2, 3)$ represents the CNTs' efficiency parameters, which are determined by the results of the molecular dynamics simulation.

The Eshelby–Mori–Tanaka model was established by the theory of Eshelby in micromechanics; it combined the concept of average stress in the matrix from Mori–Tanaka [45]. It can be used to the CNTs as clustered particles randomly distributed in the matrix. Kamarian *et al.* [46] suggests that this model considered some important parameters such as agglomeration effect of CNTs, while the rule of mixture cannot. The effective bulk modulus K and shear modulus G are given by:

$$K = K_m + \frac{V_r(\delta_r - 3K_m\alpha_r)}{3(V_m + V_r\alpha_r)} \tag{10}$$

$$G = G_m + \frac{V_r(\eta_r - 2G_m\beta_r)}{2(V_m + V_r\beta_r)} \tag{11}$$

where the subscript r and m, respectively, represent the parameter of particle and matrix. α, β, and η can be determined by the Hill's elastic moduli of CNT. For SWCNT, the effective Young's modulus E and Poisson's ratio v can be expressed as

$$E = \frac{9KG}{3K + G} \tag{12}$$

$$v = \frac{3K - 2G}{6K + 2G} \tag{13}$$

2.3 The fundamental theory of FG-CNTRC

Essentially, FG-CNTRC is a kind of nonhomogeneous composite materials, so that the conventional linear theory will not be suitable for them. Thanks to the efforts of numerous researchers, the analysis computation on material nonlinear has gradually been developed and improved. There are four approaches used in the previous studies in order to conduct the nonlinear model of FG-CNTRC in the governing equations.

2.3.1 Von-Kármán strain-displacement relation

It has been proposed by Von-Kármán and developed in the 1960s by Stansky and Whitney. It aims to analyze the heterogeneous anisotropic plates and nonlinear structure [47]. As the external load acting on the composite, the ratio of out-of-plane deformation to thickness is no longer a small quantity and the membrane force plays a great role in bending equilibrium. According to these, the strain-displacement relation can be written by:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{14}$$

$$\varepsilon_y = \frac{\partial v}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{15}$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{16}$$

where ε_x, ε_y, and ε_{xy} are the strain on the middle surface; u, v, and w denote the displacements in x, y, and z directions.

2.3.2 First-order shear deformation theory

Since the volume content of reinforcement in FG-CNTRC was distributed in gradient, the equivalent transverse shear modulus of structure is much less than the equivalent elastic modulus in the fiber direction [48], so that the FG-CNTRC is sensitive to the transverse shear deformation and the transverse shearing distortion must be considered. First-order shear deformation theory (FSDT) [49,50] takes account of the effect of transverse shearing distortion and the displacement function can be expressed as:

$$u = u_0(x, y) - z\psi_x \tag{17}$$

$$v = v_0(x, y) - z\psi_y \tag{18}$$

$$w = w_0(x, y) \tag{19}$$

where u, v, and w are the displacements of a generic point in the FG-CNTRC plates and u_0, v_0, and w_0 represent the displacements projections on the mid-plane. ψ_x and ψ_y are the transverse normal which can be given by:

$$\psi_x = \frac{\partial u}{\partial z} \tag{20}$$

$$\psi_y = \frac{\partial v}{\partial z} \tag{21}$$

First-order shear deformation (FSDT) is one of the most popular approaches to establish the governing equations of FG-CNTRC. Wang *et al.* [51] used it to analyze the free and transient vibration behavior of composite laminated open cylindrical shells. Lee and Hwang [52] studied the nonlinear transient behaviors of FG-CNTRC spherical shells based on the FSDT. In the analysis of beam, FSDT is also known as Timoshenko beam theory. It is worthy to note that researchers when applying this theory tend to introduce a shear correction factor in order to balance the shear strain energy changed by the FSDT [53]. In some study [54], the shear correction factor is taken as 5/6 referring to the isotropic materials, but most researchers [55] preferred the other formula which was proposed by Efraim and Eisenberger [56]; this formula considered the change of Poisson's ratio in thickness direction and can be written as

$$K = \frac{5}{(6 - (\nu_m V_m + \nu_{cnt} V_{cnt}))} \tag{22}$$

where V_m and V_{cnt} are the total volume fraction of matrix and CNTs, ν_m and ν_{cnt} are the Poisson's ratio of them.

2.3.3 Higher order shear deformation theory

Actually, the shear correction factor is closely linked not only with the Poisson's ratio, but also with material properties, laying mode, etc. [57]. So, in order to overcome the bad effects caused by it, several high-order shear deformation theories have been developed, and the most commonly used one was suggested by Reddy [58]. The displacement expressions of it can be expressed as:

$$\begin{aligned} u &= u_0 + z\left[\psi_x - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\psi_x + \frac{\partial w}{\partial x}\right)\right] \\ &= u_0 - z\frac{\partial w}{\partial x} + f(z)\left(\psi_x + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{23}$$

$$\begin{aligned} v &= v_0 + z\left[\psi_y - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\psi_y + \frac{\partial w}{\partial y}\right)\right] \\ &= v_0 - z\frac{\partial w}{\partial y} + f(z)\left(\psi_y + \frac{\partial w}{\partial y}\right) \end{aligned} \tag{24}$$

$$w = w_0(x, y) \tag{25}$$

where, u, v, w, u_0, v_0, w_0, z, ψ_x, and φ_y exist in similar significance with FSDT, h is the thickness of structure. After arranging, the shear deformation function $f(z)$ is defined as

$$f(z) = z\left(1 - \frac{4z^2}{3h^2}\right) \tag{26}$$

Based on the Higher order shear deformation theory (HSDT), Shen and Xiang [59] derived the motion equations of a CNTRC beam on an elastic foundation and considered the influence caused by thermo. Phungvan *et al.* [60] considered the effects of carbon nanotube volume fraction and plate width-to-thickness ratio on natural frequencies and deflections of FG-CNTRC. To further simulate the continuity conditions of transverse shear stress at layer interfaces, different higher order polynomial and trigonometric functions already have been tried, as shown in the Table 1.

2.3.4 Three-dimensional elasticity theory

Although using higher order shear deformation theory can get a relatively accurate result about the composite materials structure, it is still a two-dimensional equivalent theory. With the increase of the thickness ratio and the difference of the materials in each layer, the theoretical error increases sharply [67]. Therefore, three-dimensional elasticity theory has attracted the researchers' attention [68], as it abandons the assumption of displacement or stress and consider all the stress and displacement components as well as the interlaminar continuity condition.

Table 1: Shear deformation functions corresponding to different higher order theories

Proposed by	Shear deformation functions	Reference
Reissner (1975)	$f(z) = \frac{5}{4}z\left(1 - \frac{4z^2}{3h^2}\right)$	[61]
Touratier (1991)	$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$	[62]
Soldatos (1992)	$f(z) = h\sinh\left(\frac{z}{h}\right) - z\cosh\left(\frac{1}{2}\right)$	[63]
Karama (2002)	$f(z) = ze^{-2\left(\frac{z}{h}\right)^2}$	[64]
Akavci and Tanrikulu (2008)	$f(z) = \frac{3\pi}{2}h\tanh\left(\frac{z}{h}\right) - \frac{3\pi}{2}z\,\mathrm{sech}^2\left(\frac{1}{2}\right)$	[65]
Mantari (2012)	$f(z) = \sin\left(\frac{\pi z}{h}\right)e^{\frac{1}{2}\cos\left(\frac{\pi z}{\pi}\right)} + \frac{\pi}{2h}z$	[66]

However, during the process of application, a large number of equilibrium equations may lead to the difficulty of solution [69]. So, some exact and more targeted calculation methods have been proposed to analyze specific case in different geometries, boundary conditions, and layers, such as Pagano's classical approach [70], state space approach [71], series expansion [72], perturbation methods [73], and 3-D finite element method [74]. Because of space limitations, these approaches could not be introduced in detail. For FG-CNTRC, Alibeigloo [75] used the state space technique across the thickness direction to study the bending behavior of cylindrical panel. Thomas calculated the dynamic responses of FG-CNTRC shell structure subjected to impulse load by 3-D finite element method [76] (Figure 3).

3 Mechanical analysis of FG-CNTRC

As mentioned above, beams, plates, and shells are the most fundamental components in engineering structure, so that any kind of new materials to be used have to be analyzed for the mechanical response of components under the actions of environmental loads or external load. In this section, the review of mechanical analysis of FG-CNTRC is discussed from four aspects (beams, plates, shell, and component), mainly covering the bending, buckling, and vibration of FG-CNTRC.

3.1 Beams

In the series-study of bending, Wuite and Adali [78] performed a benchmark research to study static responses of CNTRC beams according to the classical beam theory. They found that the different stacking sequences in components would influence material properties. Wattanasakulpong and Ungbhakorn's research [79] proved that this feature also applies to FG-CNTRC. They presented linear bending of FG-CNTRC beams resting on the Pasternak elastic foundation and got the result that, with the distribution of FG-X, the beam can be the strongest with the smallest transverse displacement, followed by the FG-UD, FG-V, and FG-O beams. Shen and Xiang [59] further considered the nonlinear bending behavior of simply supported FG-CNTRC beams resting on the Pasternak elastic foundation as a result of thermal effects. Based

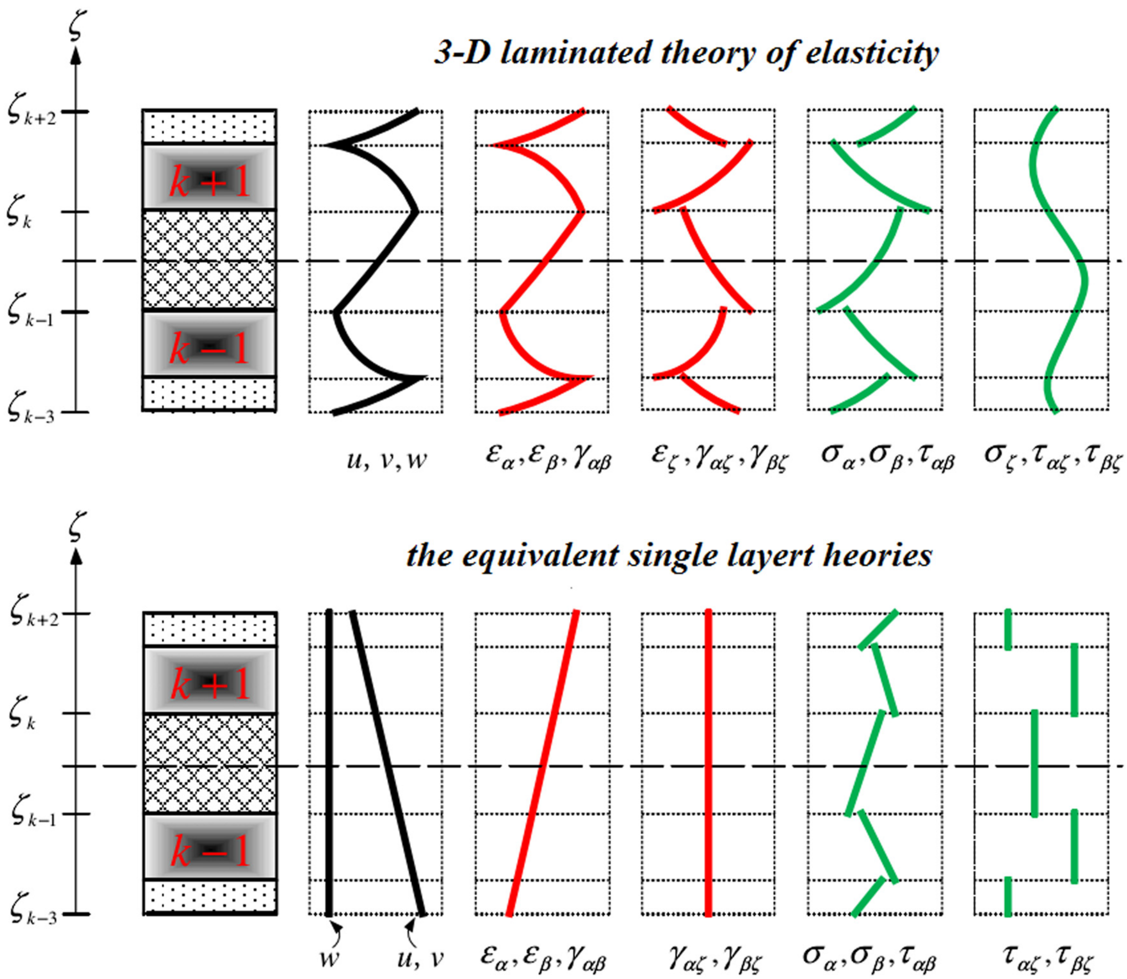

Figure 3: Distributions of displacements, strains, and stresses along ζ of the ESL theories and 3-D laminated theory of elasticity [77].

on the Reddy higher order shear deformation theory and applied the two-step perturbation technique, they obtained the effect of temperature variation on the nonlinear bending behavior of CNTRC beams, which the deflections are increased with increase in temperature.

Kumar and Srinivas [80] perform a numerical analysis on the static and dynamic behaviors of beams made up of functionally graded carbon nanotube reinforced polymer and hybrid laminated composite containing the layers of carbon-reinforced polymer with CNTs. The hybrid laminated composite beam was considered to have a combination of FG-CNTRC and FRC layers, whose material modeling and mathematical formulation for multilayer beam are described by the Timoshenko beam theory. Unlike in ref. [80] where each CNTRC layer is assumed to be linear functionally graded, in Yang study [81] the CNTRC layers are arranged in a piecewise functionally graded (FG) pattern in the thickness direction of the beam. The novelty of their study can be reflected by the identification of the negative Poisson's ratio of CNTRC laminated beams with the functionally graded configurations by performing the nonlinear bending analysis (Figure 4).

Based on the CNTs employing rule of mixture and Timoshenko beam theory, Yas and Samadi [82] investigate the buckling of FG-CNTRC beams on elastic foundations. The governing equations are derived through using Hamilton's principle and then solved by using the generalized differential quadrature method (GDQM). They respectively considered the beam with the different distributions under the four boundary conditions, including hinged–hinged (H–H), clamped–hinged (C–H), clamped–clamped (C–C), and clamped–free (C–F) and obtained the results that FG-X distribution has higher critical buckling load in comparison with other distributions. However, Shen and Xiang [59] proposed that CNTRC beam with intermediate CNT volume fraction does not necessarily have intermediate nonlinear frequencies, buckling

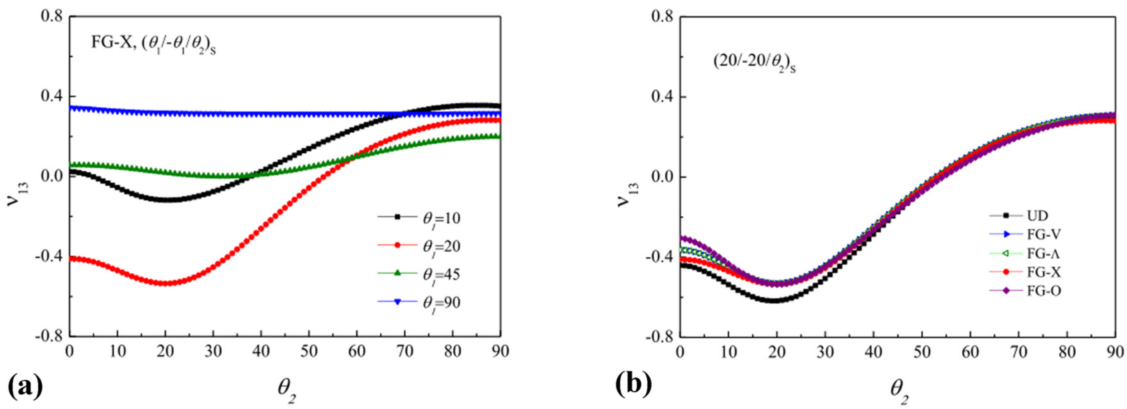

Figure 4: The relationship between the effective Poisson's ratios ν_{13} and the lamination angles θ_2 [81]. (a) Different lamination angles, (b) different distribution forms.

temperatures, and thermal postbuckling strengths; it means that the buckling analysis of FG-Λ by Yas and Samadi may be reconsidered.

Rafiee *et al.* [83] discussed buckling behavior of FG-CNTRC beams with surface-bonded piezoelectric layers by multiple scales method. According to the Euler–Bernoulli beam theory and von-Kármán geometric nonlinearity, this paper found that thermal buckling phenomenon may be delayed by applying the appropriate voltage to the actuator piezoelectric layers, but the solution method is valid only for beams which are clamped on both ends and ignored the shear deformation of FG-CNTRC beams. In fact, the defects of carbon nanotubes may affect their mechanical properties whether in macroscopic or in microscopic case [84]. Wu *et al.* [85] first extended the research to the composite with various geometric imperfections; they suggested that the mechanical behavior of beam structures is sensitive to the presence of a small imperfection, especially the postbuckling behavior. Based on the first-order shear deformation beam theory and modified Newton-Raphson iterative technique, they drew the postbuckling equilibrium paths of imperfect and perfect CNTRC beams, respectively.

In addition, the sandwich beam was studied by Kiani and his coauthors. The research [86] showed that due to the antipathetic lateral loading, there is a snap-through phenomenon when the temperature elevation is large enough and the snap-through intensity will be influenced by the temperature, the thickness ratio, and the volume fraction of CNTs. Based on the first-order shear deformation theory and von Kármán type of geometrical nonlinearity, thermal postbuckling response of a sandwich beam made of a stiff host core and carbon nanotube (CNT)-reinforced face sheets is analyzed though Ritz method [87]. It is shown that graded profile of CNTs, length to thickness ratio, host thickness to face thickness ratio, volume fraction of CNTs, boundary conditions, and temperature dependency are important factors on critical buckling temperature and postbuckling equilibrium path of sandwich beams. Recently, Khosravi *et al.* [88] innovatively change the study object from stationary structures to rotating FG-CNTRC structures, with the beam acquiring a constant angular rotating speed. Prebuckling deformations of the beam are studied carefully to discuss the conditions for thermal and inertial buckling under the simultaneous actions of rotation and heating.

The vibration characteristics of FG-CNTRC beam also have been a concern for scholars. Asadi and his coauthors [89] set the scope of the study in aerospace applications; they analyze the nonlinear dynamic responses of functionally graded carbon nanotube-reinforced composite beams exposed to axial supersonic airflow in thermal environments. With regard to the first-order shear deformation theory and harmonic differential quadrature method, they applied a direct iterative procedure to determine thermal bifurcation points and critical aerodynamic pressure.

As previously mentioned, applying the first-order beam theories may cause errors because of the shear correction factor. Lin and Xiang [90] performed a comparative analysis on free vibrations of an FG-CNTRC beam between the first-order and third-order shear deformable beam theories. The result showed a substantial difference between these two theories as shown in Figure 5, especially for beams with both edges clamped. Jam and Kiani [91] examined the low velocity impact response of FG-CNTRC beams in thermal environment. The solution of the resulting equations is traced in time using the well-known Runge–Kutta method. In fact, the vibrational resonance can also occur at excitation frequencies Ω

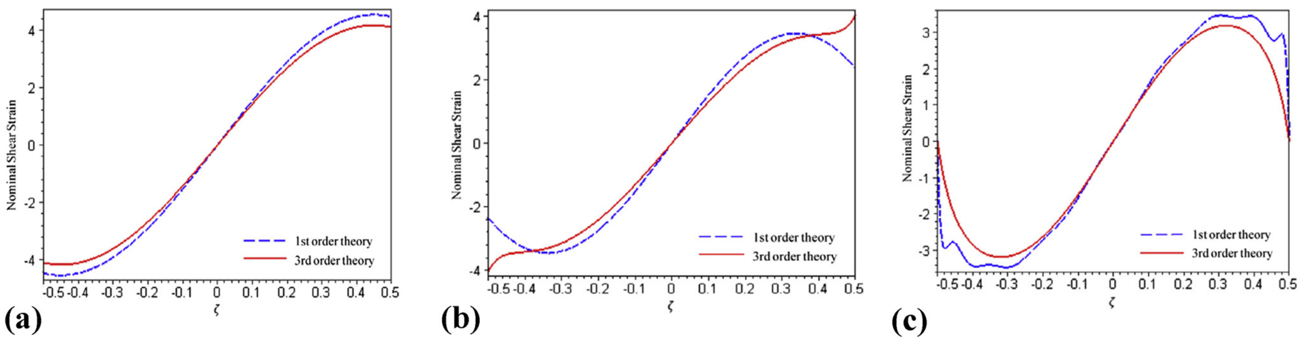

Figure 5: Nominal shear strains ζ along the FG-CNT beam in different boundary with $V_{tcnt} = 0.17$ and $L/h = 10$ [90]. (a) P–P ($U_0 = W_0 = 0$), (b) SC–SC ($U_0 = W_0 = \Phi_\xi = 0$), (c) HC–HC ($U_0 = W_0 = \Phi_\xi = \frac{\partial w}{\partial \xi} = 0$).

which are multiple of the natural frequency ω [92]. As the excitation frequency Ω is near to ω/n or $n\omega$, the super-harmonic and subharmonic resonances would occur, respectively, which bring some effect to the structure. Wu *et al.* [93] investigated the nonlinear primary and super-harmonic resonances of FG-CNTRC beams and their paper first used the incremental harmonic balance (IHB) method to solve the discretized equations. The numerical results showed that super-harmonic resonance exhibits only in the case of FGΛ-CNTRC beam, while does not occur in the case of other three beams reinforced by symmetrically or uniformly distributed CNTs. In addition, they extended the research to subharmonic resonance [94] and obtained a similar conclusion about the effects of material, geometry, and excitation parameters on the responses. Heidari and Arvin [95] discussed the free vibration of rotating FG-CNTRC Timoshenko beam and suggested the fundamental natural frequency of the considered hinged-clamped beam by the augmentation in the rotation speed initially, but after a threshold value it shows a opposite effect because of the induced compressive force by the centrifugal force which tends to destabilize the beam.

Besides the single-layer beams, the sandwich beam and FG-CNTRC beams received much attention in the recent years. Mirzaei and Kiani [96] studied the large amplitude free vibration of temperature-dependent sandwich beams with carbon nanotube-reinforced face sheets, applying the polynomial-based Ritz method into the Hamilton principle.

Kamarian *et al.* [46] used the Eshelby–Mori–Tanaka approach to assume the material properties in order to consider the agglomeration effect of CNTs and the results presented the fact that the free vibrations of sandwich beams are seriously affected by CNTs agglomeration. Vo-Duy and Hohuu [97] established a laminated composite beam analysis model and considered the effect of the numbers of layers, using the finite element method to solve the model under various boundary conditions. As the matrix of the laminated beams is cracked, the dynamic behavior of the structure may be changed evidently. Fan and Wang [98] focused on the matrix-cracked shear deformable laminated beams on elastic foundations in thermal environments and established two kinds of damage models for matrix cracking, namely self-consistent model and elasticity theory model.

3.2 Plates

Shen [21] first investigated the bending behavior of functionally graded carbon nanotube-reinforced composite plates, based on the Von-Kármán strain-displacement relation and Reddy type of higher order shear deformation plate theory. Applying the two-step perturbation technique, the effect of the thermal was considered. Since then, static, dynamic, and buckling behaviors of FG-CNTRC structures have been studied and reported in the literature.

Zhu *et al.* [99] carried out bending analysis of FG-CNTRC plates by FEM based on first-order shear deformation plate theory (FSDT) with some similar conclusions. Phungvan *et al.* [60] proposed an effective formulation to investigate the static behavior of FG-CNTRC plates, using isogeometric elements based on Nonuniform Rational B-Spline (NURBS) basis functions. Compared with the traditional FEM, the IGA easily fulfills the continuity requirements for plate elements stemming from the HSDT, which is the key of this study. Zhang and his coauthors [55] further proposed an element-free IMLS-Ritz method, which used a set of scattered nodes to replace the meshing in the problem domain.

Recently, Sobhy [100] employed a new shape function, based on the higher order shear deformation theory, to analyze the bending response of FG-CNT rectangular plate under the double-layered elastic foundations in thermal environments. In addition, Aakash and his coauthors [101] applied the inverse hyperbolic shear deformation theory to study the similar problem; actually his method also provided a new shear deformation function which used the inverse hyperbolic function to satisfy the zero transverse shear stress conditions at the extreme surfaces of the plate. Keleshteri *et al.* [102] selected the FG-CNTRC annular plates as the research object and investigated the effect of thickness profile in detail. The generalized differential quadrature method is adopted and the nonlinear system of equations is solved via the Newton-Raphson iterative method.

For laminated plates, Natarajan *et al.* [103] investigated the bending of sandwich plates with CNTRC face sheets using QUAD-8 shear flexible element, which accounts for the realistic variation of the displacements through the thickness and the possible discontinuity in slope at the interface. Chavan and Lal [104] developed a 9-node isoparametric element with seven degrees of freedom per node to acquire precise computation of the deflection and stresses of reinforced composite plate. Based on the Reddy's third-order shear deformation plate and two-step perturbation approach, Shen and his coauthors [105] presented the findings on the nonlinear bending behaviors of FG-CNTRC laminated plates with negative Poisson's ratios. They found that the (±22)3T CNT/PmPV and (±70)3T CNT/PmPV laminated plate correspond to the maximum NPR. In fact, besides the theories we mentioned above, there are also some other approaches to derive the strain-displacement relation. Sciuva and Sorrenti [106] presents an application of the extended Refined Zigzag Theory (eRZT) in conjunction with the Ritz method to analyze the mechanical property of FG-CNTRC sandwich plates. He got the numerical results of bending under transverse uniform which were contrasted with FSDT and Reddy's TSDT. It confirms the superior predictive capabilities of the eRZT over the traditional FSDT and TSDT, also for FG-CNTRC sandwich plates.

The buckling behavior of FG-CNTRC plates also has attracted the most attention from scholars. Wang [107] applied a semi-analytical solution and discussed the buckling of FG-CNTRC plates based on the classical plate theory and Galerkin technique. The transverse displacement is expressed in a sum of products of characteristic beam functions in one direction and unknown functions in the other and it can be suitable for arbitrary combinations of boundary conditions.

Shen and Zhang [108] adopted a two-step perturbation technique for the buckling and postbuckling of rectangular plates subjected to uniform or in-plane parabolic temperature loading, but all edges of the plate are considered to be simply supported in this research. Torabi *et al.* [109] developed a unified numerical formulation for the thermal buckling of FG-CNTRC plates in the variational framework. They employed the variational differential quadrature (VDQ) approach to present the governing equations, in which the total potential energy of the structure can be instantly discretized applying two-dimensional GDQ-based differential and integral operators. Further, they [110] improved the solution process and proposed a VDQ-FEM technique in which the space domains of plate are transformed into a number of finite elements in order to overcome the deficiency of VDQ used in structural with concave domains, as shown in Figure 6.

Kiani [111] examined the shear buckling of FG-CNTRC rectangular plates and proposed a Ritz-based solution

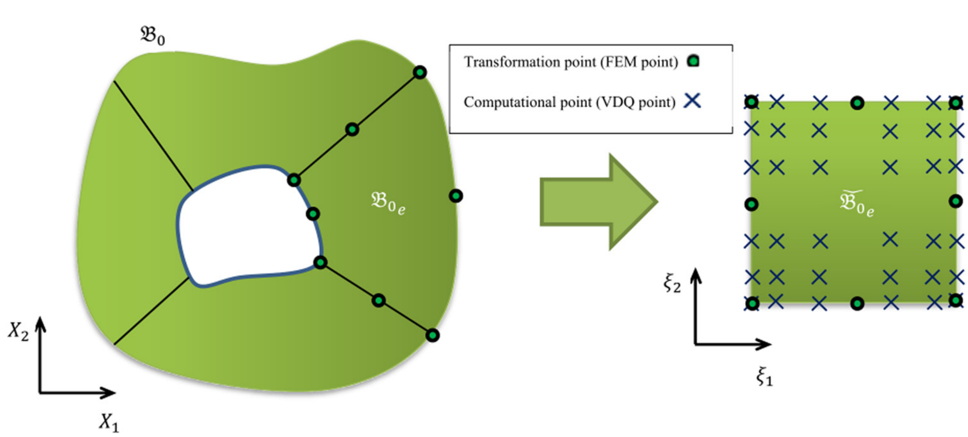

Figure 6: Mapping procedure of physical arbitrary shape domain into regular computational one in VDQ [110] (where the X_2, X_1, ξ_2, ξ_1 are the coordinates in the original coordinate system and the transformed coordinate system, the B_0e is the space domain, and the $\widetilde{B_0e}$ is the natural computational domain).

with Chebyshev polynomials to construct an eigenvalue problem associated with the onset of buckling. The result showed that increasing the volume fraction of CNT can increase the buckling capacity of the plate. On the basis of it, their research group [112] expanded the research object to skew plate through the coordinate changes. Civalek and Jalaei [113] take the FG-CNTRC skew plates as the research subjects. With the help of discrete singular convolution method in geometric transformation, the related governing equations of skew plate buckling and boundary conditions are transformed from the physical domain into a square computational domain and the accuracy has been validated by comparing with other existing literature. Based on the Eshelby–Mori–Tanaka model, Safaei and his coauthors [114] investigated the bucking behaviors of sandwich plates with the consideration of porosity and the formation of CNT clusters. They used amesh-free method to obtain results that porosity considerably improved thermal buckling behavior, but reduced the critical mechanical loads of sandwich plates. Jiao [115] investigates the buckling behaviors of thin rectangular FG-CNTRC plate subjected to the arbitrarily nonuniform load – distributed partial edge compression loads. They transformed the arbitrarily distributed partial edge compression load at one plate edge to the load acting on all internal points of this edge, then obtained the accurate critical buckling loads and buckling modes.

In the series-study of vibration, Zhang and his coauthors [116] employed the element-free IMLS-Ritz method to investigate the free vibration of FG-CNT plates with elastically restrained edges, based on the FSDT. They provided a set of vibration frequencies of structural and found that the FG-X types furnish the highest frequency values of all the cases. Further, using the same method, they analyzed the thick plates which were rested on elastic foundations and compared their solutions with the existing results [117].

Based on the FSDT and variational principle, Zhong *et al.* [118] adopted the modified Fourier series to replace the traditional admissible functions of the Ritz method in order to remove the potential discontinuous of the original rectangular plates unknown and their derivatives at the edges. Thai and his coauthors [119] presented a moving Kriging (MK) mesh-free method combined with HSDT and nonlocal theory for small size effect analysis of FG CNTRC nanoplates. Unlike the other mesh-free method, this method does not require to use the Lagrange multiplier or penalty methods to impose essential boundary condition. García-Macías and his coauthors [120] considered the vary of mechanical properties and the uncertainties inherent in the fabrication technique and focused on the Structural Health Monitoring. This study on the basis of stochastic representation of the grading profiles used the Kriging and High-Dimensional Model Representations (RS-HDMR) to surrogate the finite element model. Based on the Reddy's high-order shear deformation theory, Di *et al.* [121] established the calculation model for the impact system with the use of the weak form quadrature element method. In addition, they employed the nonlinear Hertzian contact law to describe the impact process between a rigid impactor and the rectangular FGCNTRC plate.

Besides the single-layer, the laminated plates, like sandwich plates, also attracted the attention of researchers. Beni [122] first extended the Carrera Unified Formulation for analysis of the free vibration of asymmetric annular sector plates, where all three displacement components of layer k in a layered structure are expressed as a set of thickness functions. The governing equations were obtained employing the Principle of Virtual Displacements and solved with GDQ method (Figure 7). Moradi-Dastjerdi and Momeni-Khabisi [123] investigated the vibrational behavior of sandwich plates resting on Pasternak elastic foundation and subjected to periodic loads. It is worth noting that the researches consider the effects of CNTs aspect ratio and waviness, as the CNT curvature (waviness index) dramatically decreases modulus of elasticity. In order to overcome the limitation that continuum micromechanics equations cannot capture the scale difference between the nano- and micro-levels, Tahouneh Vahid [124] used the Eshelby–Mori–Tanaka approach to determine the effect of CNT agglomeration on the elastic properties of randomly oriented CNTRCs and provide a 3D elasticity solution for sandwich sectorial plates. Fu *et al.* [125] investigated the acoustic radiation behavior of laminated FG-CNTRC plates in thermal environments. The sound pressure and radiation efficiency are calculated through Rayleigh integral, and the results show that the peaks and dips of the structural and acoustic response will increase as the values of CNT volume fraction increase.

Some scholars also pay their attention to FG-CNTRC plates with the piezoelectric layer. For the CNT-reinforced composite plates with piezoelectric layers, Selim *et al.* [126] focused on the active vibration control of FG-CNTRC plate and adopted constant velocity feedback approach to determine two positions of piezoelectric sensor and actuator layers. Keleshteri *et al.* [127] dealt with large amplitude vibration analysis of FG-CNTRC annular sector plates with surface-bonded piezoelectric layers. The nonlinear frequencies of the FG-CNTRC annular sector plate were obtained through the generalized differential quadrature method along with direct iterative method.

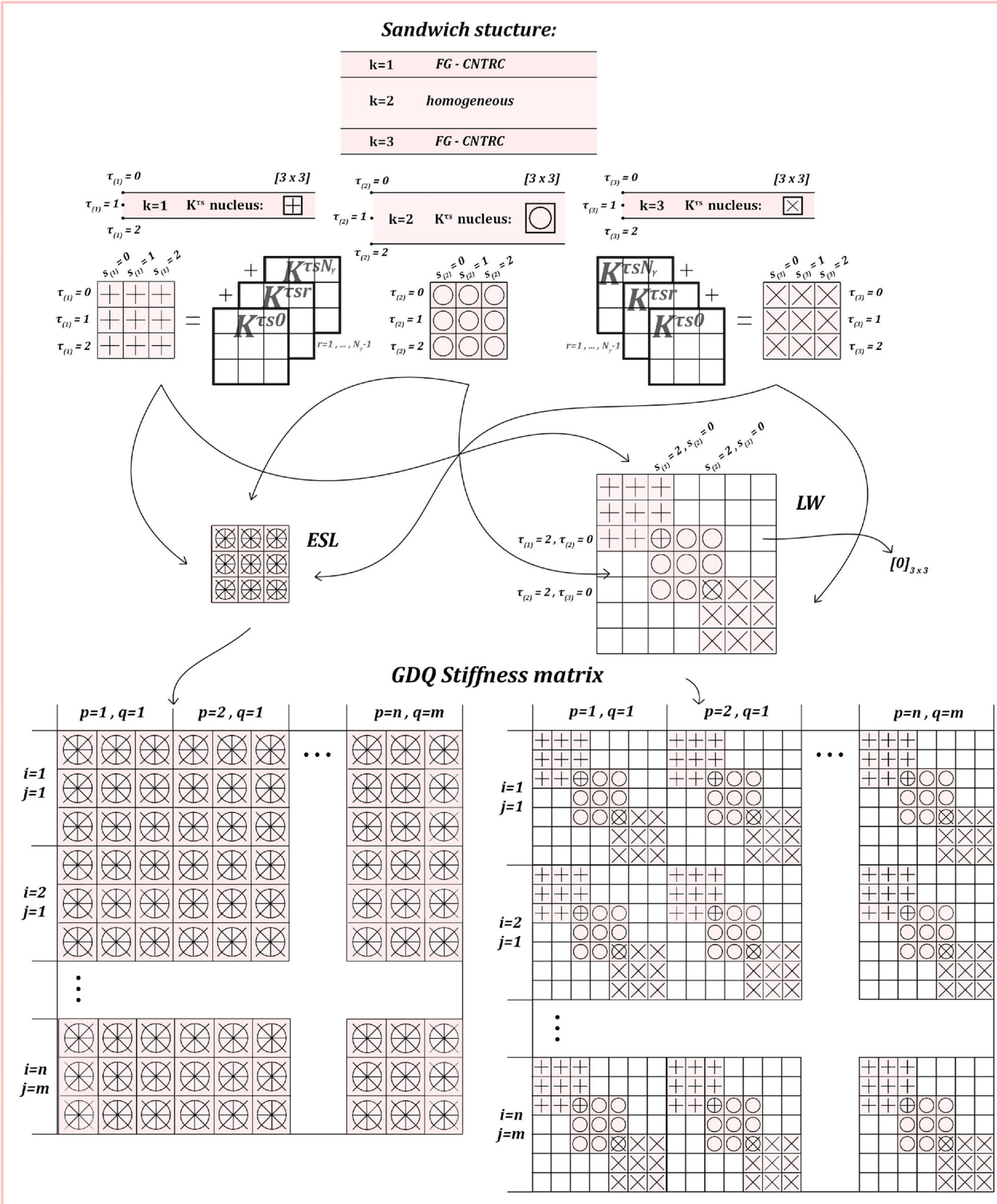

Figure 7: Fundamental nucleus assembly procedure for a sandwich structure with FG-CNTRC face sheets via GDQ grid points [122] (the order of expansion through the thickness $N = 2$ and $n \times m$ GDQ grid points).

Nguyen-Quang *et al.* [128] investigated the dynamic response of PFG-CNTRC plates by using the extended isogeometric method with nonuniform rational B-spline and the HSDT.

3.3 Shells

Because of the high load-carrying ability, the shell structures are largely employed for practical applications. To further enhance the physical properties of it, the research of FG-CNTRC shells has been increasingly developed by scholars. In static analysis, Zghal *et al.* [129] used a discrete double-directors shell element to derive the governing equations of FG-CNTRC cylindrical panel and investigate the bending behavior under the uniformly distributed load. The obtained results were compared with those given by Zhang *et al.* [130], which employed the mesh-free kp-Ritz and validated the effectivity of this model. In addition, the Kirchhoff shell theory was used to analyze large displacements and rotations of thin FG-CNTRC shell structures by Zghal *et al.* [131], which can assure notably the compromise between good accuracy and low computational costs. They carried out using four node and three node finite elements, overcoming the locking problems during the analysis. Ansari and Kumar [132] developed a finite element (FE) coding for the functionally graded CNT-reinforced doubly curved singly ruled truncated rhombic cone by using a C^0 nine noded element.

Alibeigloo [133] assumed that the effective thermo-elastic constants are independent of temperature and obtained the temperature distribution in three dimensions by solving heat conduction differential equation with variable coefficient. Then the paper applied Fourier series expansion to acquire the stress and displacement fields along the axial and circumferential direction and state space technique along the radial direction. In addition, Alibeigloo and Zanoosi [134] considered thermo-electro-elastic deformations of FG-CNTRC cylindrical shell integrated with piezoelectric layers and come to a conclusion that the sign of stresses and displacements distribution in mechanical loading is opposite to the electric voltage loading so that adjusting the load voltage can be used to control bending behavior of FG-CNTRC layer.

The researches on buckling of FG-CNTRC shells mainly focus on different structural forms, numerical solution, and loading they are subjected. Mehri *et al.* [135] analyzed the bifurcation of a composite truncated conical shell with embedded single-walled carbon nanotubes subjected to combined axial compressive load and hydrostatic pressure. They proposed a semi-analytical solution on the basis of the trigonometric expansion through the circumferential direction along with the harmonic differential quadrature discretization in the meridional direction.

Based on the first-order shear deformation shell theory, Nguyen and his coauthors [136] first used nonuniform rational B-Spline basis functions to perform postbuckling analysis of FG-CNTRC shells. It is an advanced numerical method integrated idea between CAD and FEM, in which the geometric data from computer-aided design (CAD) can be used directly for numerical simulation. Based on the three-dimensional elasticity theory, Liew and Alibeigloo [137] studied the buckling behavior of FG-CNTRC simply supported cylindrical panel with initial normal axial and circumferential stresses. Hajlaoui and his coauthors [138] innovatively used the enhanced solid-shell elements with a parabolic shear strain distribution imposed on the compatible strain part to investigate the buckling behavior of cylindrical shell under external pressure and axial compression. They employed the assumed natural strain (ANS) method and the enhanced assumed strain (EAS) method to overcome the locking phenomena in traditional three-dimensional finite element modeling. Safarpour *et al.* [139] considered the effects of critical voltage and CNT

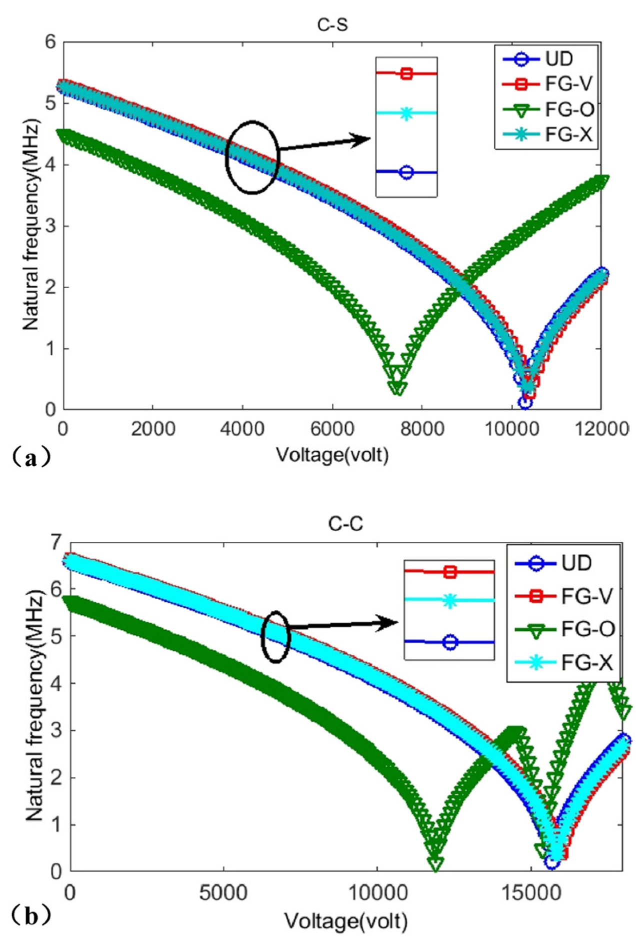

Figure 8: The effect of CNT distribution types on natural frequency in different boundary where $L/R = 0.1$, $h/R = 0.1$, and $\Omega = 1$ MHz [139]. (a) Clamped–simply, (b) clamped–clamped.

reinforcement on the piezoelectric rotating cylindrical CNTRC shell. The results show that FG-O has the lowest critical voltage and the FG-V pattern has high stability area in comparison with other patterns, as shown in Figure 7. Mehar *et al.* [140] focused on the graded CNT-reinforced composite sandwich shell structure; they derived the eigenvalue type of buckling equation by the variational technique, considering the geometrical distortion due to temperature loading, and solved numerically via isoparametric displacement controlled FEM (Figure 8).

For the free vibration analysis of carbon nanotube-reinforced functionally graded composite shell, Qin and his coauthors [141] take the rotating FG-CNTRC cylindrical shells as the research subjects. They considered the Coriolis and centrifugal effects on the strain and kinetic energy of shell due to rotating and applied the Ritz method to derive the motion equations where the displacement fields of the shell are expressed by Chebyshev polynomials.

Kiani [142] carried out the research deals with the free vibration response of FG-CNTRC spherical panel. The solution method is based on the Ritz method whose shape functions are estimated according to the Gram-Schmidt process. With the similar method, Kiania and his coauthors [143] focused on the FG-CNTRC conical panels. They concluded that boundary conditions and angles of embrace of the conical shell play an important role on the fundamental frequencies of the structure. Based on the FSDT and Ritz-variational energy method, Wang *et al.* [144] used a semi-analytical method for vibration analysis of FG-CNTRC doubly curved panels and shells of revolution in which the translation and rotation displacements are expressed as the superposition of a standard cosine Fourier series and several auxiliary functions; some selected mode shapes of the shells are given in Figure 9. Hamilton's principle and the assumed mode method are used to formulate the equation of motion of the CNT-reinforced functionally graded closed cylindrical shell by Song and his coauthors [145]. They investigated the effects of natural frequency on the parameter. On the basis of free vibration analysis, Kiani [146] utilized the Newmark time marching scheme to trace the resulting dynamic equations in time and explore the influences of load velocity. Frikha and his coauthors [147] also adopted the same method in their prior research which

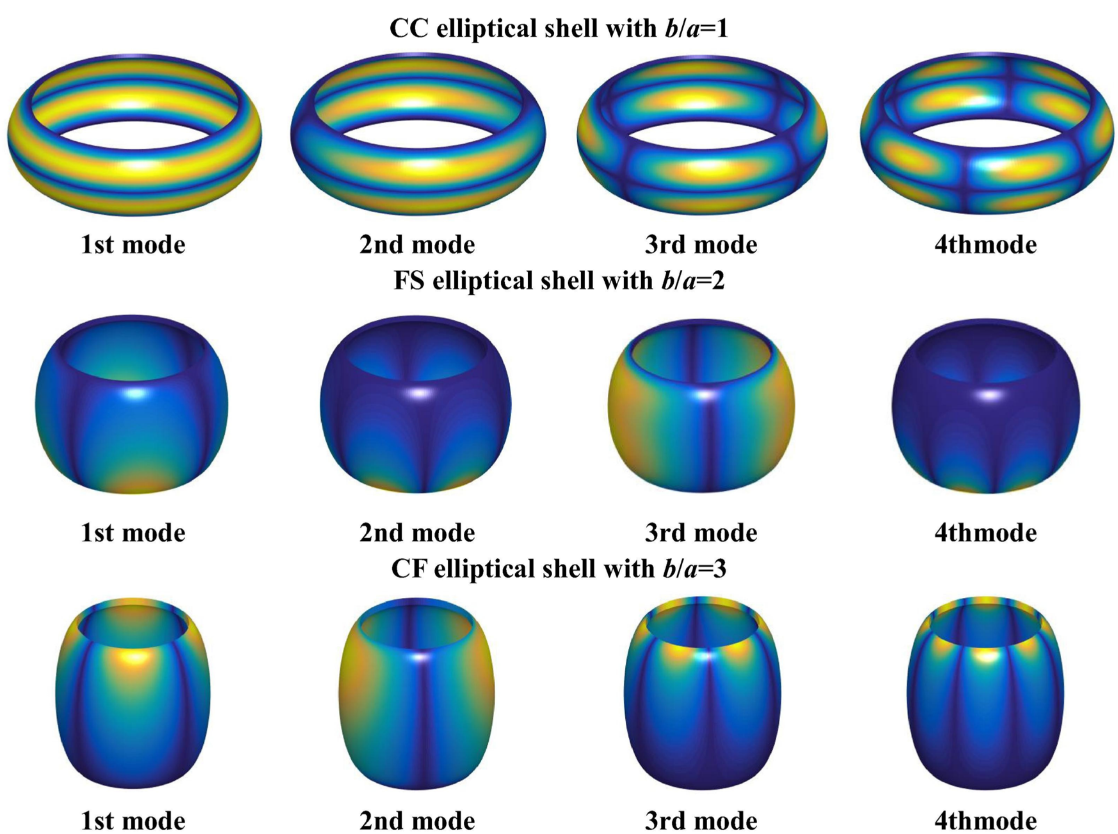

Figure 9: The first four lowest mode shapes of UD-CNTRC elliptical shell with various b/a [144]; a and b are characteristic parameters of the hyperbolic meridian.

used a double-directors finite shell element and temporal responses were drawn using the Newmark time scheme.

For the structures with an initial imperfection, based on the classical thin shell theory, Foroutan [148] studied the nonlinear vibration analysis of imperfect FG-CNTRC cylindrical panels subjected to the external pressure in the thermal environment. They presented the influence of parameters on results, like initial imperfection, temperature, and CNTs distributions. What's more, Nguyen *et al.* [149] used the Reddy's TSDT to analyze the imperfect thick FG-CNTRC double curved shallow shells subjected to the combination of blast load and temperature. The shell was assumed to rest on elastic foundations and solved through the Galerkin method and fourth-order Runge–Kutta method.

4 Conclusion

A comprehensive review is given in the present paper, in which the mechanical analysis of functionally graded carbon nanotube-reinforced composite beams, plates, and shells has been discussed. The review of various investigations on the bending, buckling, and vibration behavior of nanostructures including beams, plates, and nanoshells has been carried out. Based on the literature research, some considerations about study and the direction of future research are as follows:

(1) It can be confirmed that the volume rate change of CNT has a significant effect on the bending, buckling and vibration characteristics of the functionally graded carbon nanotube-reinforced composite beams plates and shells structure.
(2) Currently, the main solution methods for mechanical research of FG-CNTRC structure mainly include variational method, finite element method, generalized differential quadrature, two perturbation method, etc.
(3) There are few evaluation methods for the material parameters and CNT performance parameters of FG-CNTRC, and the two existing mainstream analysis methods have their own limitations.
(4) The research on the monomer structure of functionally graded carbon nanotube-reinforced composites has been relatively complete, but there are relatively few studies on combined structures, such as spherical-cylindrical combined structures.
(5) With the development of the composite material manufacturing process, more experiments on the mechanical properties of the FG-CNTRC structure need to be carried out in the future.
(6) Based on the research of mechanical properties, there can be more extensive research on the structure of FG-CNTRC, such as the investigation of vibro-acoustic response.

Acknowledgments: This study was funded by National Natural Science Foundation of China (U2006229), National key Research and Development program (2016YFC0303406), Key Research and Development program of Shandong Province (2019JZZY010125), National Key Laboratory on Ship Vibration & Noise Fund of China (6142204190207), and Ph.D. Student Research and Innovation Fund of the Fundamental Research Funds for the Central Universities (HEUGIP201902).

Conflict of interest: The authors declare no conflict of interest regarding the publication of this paper.

References

[1] Chandel VS, Wang G, Talha M. Advances in modelling and analysis of nano structures: a review. Nanotechnol Rev. 2020;9(1):230–58.
[2] Meng T, Zhang J, Wei H, Shen J. Effect of nano-strengthening on the properties and microstructure of recycled concrete. Nanotechnol Rev. 2020;9(1):79–92.
[3] Li H, Pang F, Wang X, Du Y, Chen H. Free vibration analysis of uniform and stepped combined paraboloidal, cylindrical and spherical shells with arbitrary boundary conditions. Int J Mech Sci. 2018;145:64–82.
[4] Li H, Pang F, Wang X, Du Y, Chen H. Free vibration analysis for composite laminated doubly-curved shells of revolution by a semi analytical method. Compos Struct. 2018;201:86–111.
[5] Li H, Pang F, Miao X, Gao S, Liu F. A semi analytical method for free vibration analysis of composite laminated cylindrical and spherical shells with complex boundary conditions. Thin Walled Struct. 2019;136(3):200–20.
[6] Sofiyev AH, Hui D. On the vibration and stability of FGM cylindrical shells under external pressures with mixed boundary conditions by using FOSDT. Thin Walled Struct. 2019;134:419–27.
[7] Alhijazi M, Zeeshan Q, Qin Z, Safaei B, Asmael M. Finite element analysis of natural fibers composites: a review. Nanotechnol Rev. 2020;9(1):853–75.
[8] Bisheh H, Wu N, Hui D. Polarization effects on wave propagation characteristics of piezoelectric coupled laminated fiber-reinforced composite cylindrical shells. Int J Mech Sci. 2019;161–162:105028.

[9] Lau K, Hung P, Zhu MH, Hui D. Properties of natural fibre composites for structural engineering applications. Compos Part B Eng. 2018;136:222–33.
[10] Kumar R, Singh R, Hui D, Feo L, Fraternali F. Graphene as biomedical sensing element: state of art review and potential engineering applications. Compos Part B Eng. 2018;134:193–206.
[11] Mohan VB, Lau KT, Hui D, Bhattacharyya D. Graphene-based materials and their composites: a review on production, applications and product limitations. Composites Part B Engineering. 2018;142:200–20.
[12] Wu L, Wu R, Hou L, Zhang J, Zhang M. Microstructure, mechanical properties and wear performance of AZ31 matrix composites reinforced by graphene nanoplatelets(GNPs). J Alloy Compd. 2018;750:530–6.
[13] Liu J, Wang Z, Hui D. Blast resistance and parametric study of sandwich structure consisting of honeycomb core filled with circular metallic tubes. Compos Part B Eng. 2018;145:261–9.
[14] Iijima S. Helical microtubules of graphitic carbon. Nature. 1991;354(6348):56–8.
[15] Bosi S, Fabbro A, Ballerini L, Prato M. Carbon nanotubes: a promise for nerve tissue engineering? Nanotechnol Rev. 2013;2(1):47–57.
[16] Lau KT, Hui D. The revolutionary creation of new advanced materials – carbon nanotube composites. Compos Part B Engineering. 2002;33(4):263–77.
[17] Lu X, Hu Z. Mechanical property evaluation of single-walled carbon nanotubes by finite element modeling. Compos Part B. 2012;43(4):1902–13.
[18] Raji K, Sobhan CB. Simulation and modeling of carbon nanotube synthesis: current trends and investigations. Nanotechnol Rev. 2013;2(1):73–105.
[19] Jena SK, Chakraverty S, Malikan M, Tornabene F. Stability analysis of single-walled carbon nanotubes embedded in winkler foundation placed in a thermal environment considering the surface effect using a new refined beam theory. Mech Base Des Struct Mach. 2019;1–15.
[20] Lau KT, Lu M, Hui D. Coiled carbon nanotubes: synthesis and their potential applications in advanced composite structures. Compos Part B Eng. 2006;37(6):437–48.
[21] Shen H. Nonlinear bending of functionally graded carbon nanotube-reinforced composite plates in thermal environments. Compos Struct. 2009;91(1):9–19.
[22] Li H, Pang F, Miao X, Li Y. Jacobi–Ritz method for free vibration analysis of uniform and stepped circular cylindrical shells with arbitrary boundary conditions: a unified formulation. Comput Math Appl. 2019;77(2):427–40.
[23] Pang F, Li H, Cui J, Du Y, Gao C. Application of flügge thin shell theory to the solution of free vibration behaviors for spherical-cylindrical-spherical shell: a unified formulation. Eur J Mech A Solid. 2019;74:381–93.
[24] Niino M, Maeda S. Recent development status of functionally gradient materials. ISIJ INT. 1990;30:699–703.
[25] Bharti I, Gupta N, Gupta KM. Novel applications of functionally graded nano, optoelectronic and thermoelectric materials. Int J Mater Mech Manuf. 2013;1(3):221–4.
[26] Behdinan K, Moradi-Dastjerdi R, Safaei B, Qin Z, Chu F, Hui D. Graphene and CNT impact on heat transfer response of nanocomposite cylinders. Nanotechnol Rev. 2020;9(1):41–52.
[27] Pang F, Li H, Wang X, Miao X, Li S. A semi analytical method for the free vibration of doubly-curved shells of revolution. Compu Math Appl. 2018;75(9):3249–68.
[28] Li C, Ke L, He J, Chen Z, Jiao Y. Free vibration analysis of uniform and stepped functionally graded circular cylindrical shells. Steel Compos Struct. 2019;33(2):163–80.
[29] Li H, Pang F, Chen H. A semi-analytical approach to analyze vibration characteristics of uniform and stepped annular-spherical shells with general boundary conditions. Eur J Mech A Solid. 2019;74:48–65.
[30] Li H, Cong G, Li L, Pang F, Lang J. A semi analytical solution for free vibration analysis of combined spherical and cylindrical shells with non-uniform thickness based on Ritz method. Thin Walled Struct. 2019;145:106443.
[31] Li H, Pang F, Miao X, Du Y, Tian H. A semi-analytical method for vibration analysis of stepped doubly-curved shells of revolution with arbitrary boundary conditions. Thin Walled Struct. 2018;129:125–44.
[32] Li H, Pang F, Chen H, Du Y. Vibration analysis of functionally graded porous cylindrical shell with arbitrary boundary restraints by using a semi analytical method. Compos Part B Eng. 2019;164:249–64.
[33] Li H, Pang F, Gong Q, Teng Y. Free vibration analysis of axisymmetric functionally graded doubly-curved shells with un-uniform thickness distribution based on Ritz method. Compos Struct. 2019;225(10):111145.
[34] Karami B, Janghorban M, Tounsi A. Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions. Eng Comput. 2019;35(4):1297–316.
[35] Pang F, Li H, Chen H, Shan Y. Free vibration analysis of combined composite laminated cylindrical and spherical shells with arbitrary boundary conditions. Mech Adv Mater Struct. 2019;1–18.
[36] Hui D, Chipara M, Sankar J, Lau KT. Mechanical properties of carbon nanotubes composites. J Comput Theor Nanosci. 2004;1(2):204–15.
[37] Shen HS. Functionally graded materials: nonlinear analysis of plates and shells. Boca Raton, FL: CRC Press; 2016. p. 1–257.
[38] Shi Z, Yao X, Pang F, Wang Q. An exact solution for the free-vibration analysis of functionally graded carbon-nanotube-reinforced composite beams with arbitrary boundary conditions. Sci Rep. 2017;7(1):18–25.
[39] Khaniki HB, Ghayesh MH. A review on the mechanics of carbon nanotube strengthened deformable structures – science direct. Eng Struct. 2020;220:29–32.
[40] Pang F, Li H, Du Y, Shan Y, Ji F. Free vibration of functionally graded carbon nanotube reinforced composite annular sector plate with general boundary supports. Curv Layer Struct. 2018;5(1):49–67.
[41] Kolahchi R, Zarei MS, Hajmohammad MH, Nouri A. Wave propagation of embedded viscoelastic FG-CNT-reinforced sandwich plates integrated with sensor and actuator based on refined zigzag theory. Int J Mech Sci. 2017;130:534–45.
[42] Sobhani Aragh B, Nasrollah Barati AH, Hedayati H. Eshelby–Mori–Tanaka approach for vibrational behavior of

continuously graded carbon nanotube-reinforced cylindrical panels. Compos Part B Eng. 2012;43(4):1943–54.
[43] Bakshi SR, Agarwal A. An analysis of the factors affecting strengthening in carbon nanotube reinforced aluminum composites. Carbon. 2011;49(2):533–44.
[44] Roy S, Petrova RS, Mitra S. Effect of carbon nanotube (CNT) functionalization in epoxy-CNT composites. Nanotechnol Rev. 2018;7(6):475–85.
[45] Tornabene F, Bacciocchi M, Fantuzzi N, Reddy JN. Multiscale approach for three-phase CNT/polymer/fiber laminated nanocomposite structures. Polym Compos. 2019;40:102–26.
[46] Kamarian S, Shakeri M, Yas M, Bodaghi M, Pourasghar A. Free vibration analysis of functionally graded nanocomposite sandwich beams resting on Pasternak foundation by considering the agglomeration effect of CNTs. J Sandw Struct Mater. 2015;17(6):632–65.
[47] Shen HS, Xiang Y, Fan Y. Large amplitude vibration of doubly curved FG-GRC laminated panels in thermal environments. Nanotechnol Rev. 2019;8(1):467–83.
[48] Mindlin RD. Influence of rotary inertia and shear on flexural motions of isotropic elastic plates. J Appl Mech. 1951;18(1):31–8.
[49] Pang F, Li H, Jing F, Du Y. Application of first-order shear deformation theory on vibration analysis of stepped functionally graded paraboloidal shell with general edge constraints. Materials. 2019;12(1):21–30.
[50] Li H, Pang F, Li Y, Gao C. Application of first-order shear deformation theory for the vibration analysis of functionally graded doubly-curved shells of revolution. Compos Struct. 2019;212:22–42.
[51] Wang Q, Shao D, Qin B. A simple first-order shear deformation shell theory for vibration analysis of composite laminated open cylindrical shells with general boundary conditions. Compos Struct. 2018;184:211–32.
[52] Lee SY, Hwang JG. Finite element nonlinear transient modelling of carbon nanotubes reinforced fiber/polymer composite spherical shells with a cutout. Nanotechnol Rev. 2019;8(1):444–51.
[53] Gruttmann F, Wagner W. Shear correction factors for layered plates and shells. Comput Mech. 2017;59(1):129–46.
[54] Li H, Pang F, Ren Y, Miao X, Ye K. Free vibration characteristics of functionally graded porous spherical shell with general boundary conditions by using first-order shear deformation theory. Thin Walled Struct. 2019;144:16–22.
[55] Zhang LW, Song Z, Liew KM. Nonlinear bending analysis of FG-CNT reinforced composite thick plates resting on Pasternak foundations using the element-free IMLS-Ritz method. Compos Struct. 2015;128:165–75.
[56] Efraim E, Eisenberger M. Exact vibration analysis of variable thickness thick annular isotropic and FGM plates. J Sound Vib. 2007;299(4–5):720–38.
[57] Madabhusi-Raman, Davalos JF. Static shear correction factor for laminated rectangular beams. Compos Part B Eng. 1996;27(3–4):285–93.
[58] Reddy JN. A simple higher-order theory for laminated composite plates. J Appl Mech. 1984;51(4):745–52.
[59] Shen HS, Xiang Y. Nonlinear analysis of nanotube-reinforced composite beams resting on elastic foundations in thermal environments. Eng Struct. 2013;56(Nov):698–708.
[60] Phungvan P, Abdel-Wahab M, Liew KM, Bordas SPA, Nguyen-Xuan H. Isogeometric analysis of functionally graded carbon nanotube-reinforced composite plates using higher-order shear deformation theory. Compos Struct. 2015;123:137–49.
[61] Reissner E. On transverse bending of plates, including the effect of transverse shear deformation. Int J Solids Struct. 1975;11(5):569–73.
[62] Touratier M. An efficient standard plate theory. Int J Eng Sci. 1991;29(8):901–16.
[63] Soldatos KP. A transverse shear deformation theory for homogeneous monoclinic plates. Acta Mech. 1992;94(3–4):195–220.
[64] Karama M, Afaq KS, Mistou S. Mechanical behaviour of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity. Int J Solids Struct. 2003;40(6):1525–46.
[65] Akavci SS, Tanrikulu AH. Buckling and free vibration analyses of laminated composite plates by using two new hyperbolic shear-deformation theories. Mech Compos Mater. 2008;44(2):145–50.
[66] Mantari JL, Oktem AS, Soares CG. A new higher order shear deformation theory for sandwich and composite laminated plates. Compos Part B. 2012;43(3):1489–99.
[67] Ye T, Jin G, Su Z. Three-dimensional vibration analysis of laminated functionally graded spherical shells with general boundary conditions. Compos Struct. 2014;116(9):571–88.
[68] Safarpour M, Rahimi AR, Alibeigloo A. Static and free vibration analysis of graphene platelets reinforced composite truncated conical shell, cylindrical shell, and annular plate using theory of elasticity and DQM. Mech Base Des Struct Mach. 2020;48(4):496–524.
[69] Dassi F, Mascotto L. Exploring high-order three dimensional virtual elements: bases and stabilizations. Comput Math Appl. 2018;75(9):3379–401.
[70] Pagano N. Exact solutions for composite laminates in cylindrical bending. Mech Compos Mater. 1994;4:72–85.
[71] Wu CP, Liu KY. A state space approach for the analysis of doubly curved functionally graded elastic and piezoelectric shells. Comput Mater Contin. 2007;6:177–99.
[72] Li H, Pang F, Miao X, Du Y, Tian H. A semi-analytical method for vibration analysis of stepped doubly-curved shells of revolution with arbitrary boundary conditions. Thin Walled Struct. 2018;129(Aug):125–44.
[73] Guo XY, Zhang W. Nonlinear vibrations of a reinforced composite plate with carbon nanotubes. Compos Struct. 2016;135:96–108.
[74] Wu CP, Syu YS, Lo JY. Three-dimensional solutions of multilayered piezoelectric hollow cylinders by an asymptotic approach. Int J Mech Sci. 2007;49(6):669–89.
[75] Alibeigloo A. Elasticity solution of functionally graded carbon-nanotube-reinforced composite cylindrical panel with piezoelectric sensor and actuator layers. Smart Mater Struct. 2013;22(7):11–20.
[76] Thomas B, Roy T. Vibration analysis of functionally graded carbon nanotube-reinforced composite shell structures. Acta Mech. 2016;227:581–99.
[77] Tiangui Y. Research on modeling theories and computation methods for the vibration of multilayered structures. Harbin: Harbin Engineering University; 2017.

[78] Wuite J, Adali S. Deflection and stress behaviour of nanocomposite reinforced beams using a multiscale analysis. Compos Struct. 2005;71(3–4):388–96.
[79] Wattanasakulpong N, Ungbhakorn V. Analytical solutions for bending, buckling and vibration responses of carbon nanotube-reinforced composite beams resting on elastic foundation. Comput Mater Sci. 2013;71(Complete):201–8.
[80] Kumar, Srinivas J. Free vibration, bending and buckling of a FG-CNT reinforced composite beam comparative analysis with hybrid laminated composite beam. Multidiscip Model Mater Struct. 2017;13(4):590–611.
[81] Yang J, Huang XH, Shen HS. Nonlinear flexural behavior of temperature-dependent FG-CNTRC laminated beams with negative Poisson's ratio resting on the Pasternak foundation. Eng Struct. 2020;207:21–30.
[82] Yas MH, Samadi N. Free vibrations and buckling analysis of carbon nanotube-reinforced composite Timoshenko beams on elastic foundation. Int J Press Vessel Pip. 2012;98:119–28.
[83] Rafiee M, Yang J, Kitipornchai S. Thermal bifurcation buckling of piezoelectric carbon nanotube reinforced composite beams. Comput Math Appl. 2013;66(7):1147–60.
[84] Lin XT, Han Q, Huang JZ. Effect of defects on the motion of carbon nanotube thermal actuator. Nanotechnol Rev. 2019;8(1):79–89.
[85] Wu HL, Yang J, Kitipornchai S. Imperfection sensitivity of postbuckling behaviour of functionally graded carbon nanotube-reinforced composite beams. Thin Walled Struct. 2016;108:225–33.
[86] Mirzaei M, Kiani Y. Snap-through phenomenon in a thermally postbuckled temperature dependent sandwich beam with FG-CNTRC face sheets. Compos Struct. 2015;134(Dec):1004–13.
[87] Kiani Y. Thermal postbuckling of temperature-dependent sandwich beams with carbon nanotube-reinforced face sheets. J Therm Stresses. 2016;39(7–9):1098–110.
[88] Khosravi S, Arvin H, Kiani Y. Interactive thermal and inertial buckling of rotating temperature-dependent FG-CNT reinforced composite beams. Composites. 2019;175(10):1–10.
[89] Asadi H, Amin Rabiei B. On the nonlinear dynamic responses of FG-CNTRC beams exposed to aerothermal loads using third-order piston theory. Acta Mech. 2018;229(6):2413–30.
[90] Lin F, Xiang Y. Vibration of carbon nanotube reinforced composite beams based on the first and third order beam theories. Appl Math Model. 2014;38(15):3741–54.
[91] Jam JE, Kiani Y. Low velocity impact response of functionally graded carbon nanotube reinforced composite beams in thermal environment. Compos Struct. 2015;132:35–43.
[92] Yang J, Sanjuan MAF, Liu H. Vibrational subharmonic and superharmonic resonances. Commun Nonlinear Sci Numer Simul. 2016;30(1):362–72.
[93] Wu Z, Zhang Y, Yao G, Yang Z. Nonlinear primary and superharmonic resonances of functionally graded carbon nanotube reinforced composite beams (vol 153, p 321, 2019). Int J Mech Sci. 2019;159:502.
[94] Wu Z, Zhang Y, Yao G. 3/2 superharmonic resonance and 1/2 subharmonic resonance of functionally graded carbon nanotube reinforced composite beams. Compos Struct. 2020;241:12–25.
[95] Heidari M, Arvin H. Nonlinear free vibration analysis of functionally graded rotating composite Timoshenko beams reinforced by carbon nanotubes. J Vib Control. 2019;25(14):2063–78.
[96] Mirzaei M, Kiani Y. Nonlinear free vibration of temperature-dependent sandwich beams with carbon nanotube-reinforced face sheets. Acta Mech. 2016;227(7):1869–84.
[97] Voduy T, Hohuu V, Nguyenthoi T. Free vibration analysis of laminated FG-CNT reinforced composite beams using finite element method. Front Struct Civ Eng. 2019;13(2):324–36.
[98] Fan Y, Wang H. The effects of matrix cracks on the nonlinear vibration characteristics of shear deformable laminated beams containing carbon nanotube reinforced composite layers. Int J Mech Sci. 2017;124:216–28.
[99] Zhu P, Lei ZX, Liew KM. Static and free vibration analyses of carbon nanotube-reinforced composite plates using finite element method with first order shear deformation plate theory. Compos Struct. 2012;94(4):1450–60.
[100] Sobhy M. Levy solution for bending response of FG carbon nanotube reinforced plates under uniform, linear, sinusoidal and exponential distributed loadings. Eng Struct. 2019;182:198–212.
[101] Soni A, Grover N, Bhardwaj G, Singh BN. Non-polynomial framework for static analysis of functionally graded carbon nano-tube reinforced plates. Compos Struct. 2020;233:13–20.
[102] Keleshteri MM, Asadi H, Aghdam MM. Nonlinear bending analysis of FG-CNTRC annular plates with variable thickness on elastic foundation. Thin Walled Struct. 2019;135:453–62.
[103] Natarajan S, Haboussi M, Manickam G. Application of higher-order structural theory to bending and free vibration analysis of sandwich plates with CNT reinforced composite facesheets. Compos Struct. 2014;113:197–207.
[104] Chavan SG, Lal A. Bending analysis of laminated SWCNT reinforced functionally graded plate using FEM. Curv Layer Struct. 2017;4(1):134–45.
[105] Shen H, Huang X, Yang J. Nonlinear bending of temperature-dependent FG-CNTRC laminated plates with negative Poisson's ratio. Mech Adv Mater Struct. 2020;27:1–13.
[106] Sciuva MD, Sorrenti M. Bending, free vibration and buckling of functionally graded carbon nanotube-reinforced sandwich plates, using the extended refined zigzag theory. Compos Struct. 2019;227:111324.
[107] Wang M, Li Z, Qiao P. Semi-analytical solutions to buckling and free vibration analysis of carbon nanotube-reinforced composite thin plates. Compos Struct. 2016;144:33–43.
[108] Shen H, Zhang C. Thermal buckling and postbuckling behavior of functionally graded carbon nanotube-reinforced composite plates. Mater Des. 2010;31(7):3403–11.
[109] Torabi J, Ansari R, Hassani R. Numerical study on the thermal buckling analysis of CNT-reinforced composite plates with different shapes based on the higher-order shear deformation theory. Eur J Mech A Solids. 2019;73:144–60.
[110] Ansari R, Hassani R, Gholami R, Rouhi H. Thermal postbuckling analysis of FG-CNTRC plates with various shapes and temperature-dependent properties using the VDQ-FEM technique. Aerosp Sci Technol. 2020;106:35–40.
[111] Kiani Y. Shear buckling of FG-CNT reinforced composite plates using Chebyshev-Ritz method. Compos Part B Eng. 2016;105(Nov):176–87.

[112] Kiani Y, Mirzaei M. Rectangular and skew shear buckling of FG-CNT reinforced composite skew plates using Ritz method. Aerosp Sci Technol. 2018;77(Jun):388–98.
[113] Civalek O, Jalaei MH. Shear buckling analysis of functionally graded (FG) carbon nanotube reinforced skew plates with different boundary conditions. Aerosp Sci Technol. 2020;99(4):1–19.
[114] Safaei B, Moradi-Dastjerdi R, Behdinan K, Chu F. Critical buckling temperature and force in porous sandwich plates with CNT-reinforced nanocomposite layers. Aerosp Sci Technol. 2019;91:175–85.
[115] Jiao P. Buckling analysis of thin rectangular FG-CNTRC plate subjected to arbitrarily distributed partial edge compression loads based on differential quadrature method. Thin Walled Struct. 2019;145:106417.
[116] Zhang LW, Cui WC, Liew KM. Vibration analysis of functionally graded carbon nanotube reinforced composite thick plates with elastically restrained edges. Int J Mech Sci. 2015;103(24):9–21.
[117] Zhang LW, Lei ZX, Liew KM. Computation of vibration solution for functionally graded carbon nanotube-reinforced composite thick plates resting on elastic foundations using the element-free IMLS-Ritz method. Appl Math Comput. 2015;256:488–504.
[118] Zhong R, Wang Q, Tang J, Shuai C, Liang Q. Vibration characteristics of functionally graded carbon nanotube reinforced composite rectangular plates on Pasternak foundation with arbitrary boundary conditions and internal line supports. Curv Layer Struct. 2018;5:1–34.
[119] Thai CH, Tran TD, Phung-Van P. A size-dependent moving Kriging meshfree model for deformation and free vibration analysis of functionally graded carbon nanotube-reinforced composite nanoplates. Eng Anal Bound Elem. 2020;115:52–63.
[120] García-Macías E, Castro-Triguero R, Friswell MI, Adhikari S, Sáez A. Metamodel-based approach for stochastic free vibration analysis of functionally graded carbon nanotube reinforced plates. Compos Struct. 2016;152:183–98.
[121] Di B, Hu Q, Shen Z, Zhang W, Wang C. Low velocity impact analysis of high-order rectangular FG-CNTRC plates using the weak form QEM. IOP Conf. 2020;758:012097.
[122] Beni NN. Free vibration analysis of annular sector sandwich plates with FG-CNT reinforced composite face-sheets based on the Carrera's unified formulation. Compos Struct. 2019;214(April):269–92.
[123] Moradi-Dastjerdi R, Momeni-Khabisi H. Vibrational behavior of sandwich plates with functionally graded wavy carbon nanotube-reinforced face sheets resting on Pasternak elastic foundation. J Vib Control. 2017;24(11):2327–43.
[124] Vahid T. Vibrational analysis of sandwich sectorial plates with functionally graded sheets reinforced by aggregated carbon nanotube. J Sandw Struct Mater. 2018;22(5):1496–541.
[125] Fu T, Chen Z, Yu H, Hao Q, Zhao Y. Vibratory response and acoustic radiation behavior of laminated functionally graded composite plates in thermal environments. J Sandw Struct Mater. 2019;22(5):1681–706.
[126] Selim BA, Zhang LW, Liew KM. Active vibration control of CNT-reinforced composite plates with piezoelectric layers based on Reddy's higher-order shear deformation theory. Compos Struct. 2017;163(3):350–64.
[127] Mohammadzadeh-Keleshteri M, Asadi H, Aghdam MM. Geometrical nonlinear free vibration responses of FG-CNT reinforced composite annular sector plates integrated with piezoelectric layers. Compos Struct. 2017;171(7):100–12.
[128] Nguyen-Quang K, Vo-Duy T, Dang-Trung H, Nguyen-Thoi T. An isogeometric approach for dynamic response of laminated FG-CNT reinforced composite plates integrated with piezoelectric layers. Comput Methods Appl Mech Eng. 2018;332(4):25–46.
[129] Zghal S, Frikha A, Dammak F. Static analysis of functionally graded carbon nanotube-reinforced plate and shell structures. Compos Struct. 2017;176(9):1107–23.
[130] Zhang LW, Lei ZX, Liew KM, Yu JL. Static and dynamic of carbon nanotube reinforced functionally graded cylindrical panels. Compos Struct. 2014;111(5):205–12.
[131] Frikha A, Zghal S, Dammak F. Finite rotation three and four nodes shell elements for functionally graded carbon nanotubes-reinforced thin composite shells analysis. Comput Methods Appl Mech Eng. 2018;329(2):289–311.
[132] Ansari MI, Kumar A. Bending analysis of functionally graded CNT reinforced doubly curved singly ruled truncated rhombic cone. J Struct Mech. 2019;47(1):67–86.
[133] Alibeigloo A. Elasticity solution of functionally graded carbon nanotube-reinforced composite cylindrical panel subjected to thermo mechanical load. Compos Part B Eng. 2016;87:214–26.
[134] Alibeigloo A, Zanoosi AAP. Thermo-electro-elasticity solution of functionally graded carbon nanotube reinforced composite cylindrical shell embedded in piezoelectric layers. Compos Struct. 2017;173:268–80.
[135] Mehri M, Asadi H, Wang Q. Buckling and vibration analysis of a pressurized CNT reinforced functionally graded truncated conical shell under an axial compression using HDQ method. Comput Methods Appl Mech Eng. 2016;303:75–100.
[136] Nguyen TN, Thai CH, Luu AT, Nguyen-Xuan H, Lee J. NURBS-based postbuckling analysis of functionally graded carbon nanotube-reinforced composite shells. Comput Methods Appl Mech Eng. 2019;347(4):983–1003.
[137] Liew KM, Alibeigloo A. Predicting bucking and vibration behaviors of functionally graded carbon nanotube reinforced composite cylindrical panels with three-dimensional flexibilities. Compos Struct. 2020;256:159–68.
[138] Hajlaoui A, Chebbi E, Dammak F. Buckling analysis of carbon nanotube reinforced FG shells using an efficient solid-shell element based on a modified FSDT. Thin Walled Struct. 2019;144(11):1–12.
[139] Safarpour H, Ghanbari B, Ghadiri M. Buckling and free vibration analysis of high speed rotating carbon nanotube reinforced cylindrical piezoelectric shell. Appl Math Model. 2019;65(1):428–42.
[140] Mehar K, Kumar Panda S, Devarajan Y, Choubey G. Numerical buckling analysis of graded CNT-reinforced composite sandwich shell structure under thermal loading. Compos Struct. 2019;216(5):406–14.
[141] Qin Z, Pang X, Safaei B, Chu F. Free vibration analysis of rotating functionally graded CNT reinforced composite

cylindrical shells with arbitrary boundary conditions. Compos Struct. 2019;220(7):847–60.
[142] Kiani Y. Free vibration of FG-CNT reinforced composite spherical shell panels using Gram-Schmidt shape functions. Compos Struct. 2017;159:368–81.
[143] Kiani Y, Dimitri R, Tornabene F. Free vibration study of composite conical panels reinforced with FG-CNTs. Eng Struct. 2018;172(10):472–82.
[144] Wang Q, Qin B, Shi D, Liang Q. A semi-analytical method for vibration analysis of functionally graded carbon nanotube reinforced composite doubly-curved panels and shells of revolution. Compos Struct. 2017;174:87–109.
[145] Song ZG, Zhang LW, Liew KM. Vibration analysis of CNT-reinforced functionally graded composite cylindrical shells in thermal environments. Int J Mech Sci. 2016;115–116:339–47.
[146] Kiani Y. Dynamics of FG-CNT reinforced composite cylindrical panel subjected to moving load. Thin Walled Struct. 2017;111:48–57.
[147] Frikha A, Zghal S, Dammak F. Dynamic analysis of functionally graded carbon nanotubes-reinforced plate and shell structures using a double directors finite shell element. Aerosp Sci Technol. 2018;78:438–51.
[148] Foroutan K, Ahmadi H, Carrera E. Nonlinear vibration of imperfect FG-CNTRC cylindrical panels under external pressure in the thermal environment. Compos Struct. 2019;227:135–48.
[149] Nguyen DD, Tran Q, Nguyen D. New approach to investigate nonlinear dynamic response and vibration of imperfect functionally graded carbon nanotube reinforced composite double curved shallow shells subjected to blast load and temperature. Aerosp Sci Technol. 2017;71:360–72.