Research Article

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Geometric nonlinear analysis based on the generalized displacement control method and orthogonal iteration

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Abstract: In response to the shortcomings of insufficient efficiency in calculating geometric nonlinear features and high environmental impact in the current construction, this study explores the control of nonlinear geometric structures based on an updated Lagrangian model function in the construction. In this method, the change in the geometric nonlinear stiffness is analyzed using generalized stiffness parameters and a displacement increment method, and the iteration step size and loading/unloading directions are adjusted in the iteration process to achieve the convergence of the solution. In the simulation experiment, the proposed method took 126 s to calculate the incremental iteration steps 700 times in a conventional environment, which is 28.8% more efficient than the cross-sectional method. In the simulated disaster environment, the model took 1,615 s to calculate the ultimate load of 84 contact elements, which is 43.1% more efficient than the section method and 62.6% more efficient than the discrete analysis method. Experimental results showed that the displacement judgment calculation efficiency of the method proposed in this study is higher than that of other models under different loading and unloading conditions and even in geological disaster states. This method had high environmental adaptability in solving nonlinear building structures and could improve the efficiency of solving general nonlinear building results.

Keywords: incremental iteration, geometric nonlinearity, building structure, displacement control method, orthogonal constraint, generalized stiffness parameter

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1 Introduction

With the advancement of building materials, equipment, and technological processes, the structural design of modern buildings is increasingly demonstrating flexible characteristics. Although the development of flexible structures in modern architecture has improved the environmental adaptability and natural disaster resistance of buildings, geometric nonlinearity issues such as deflection and curvature involved in flexible structures cannot be ignored [1]. The geometric and load parameters of buildings have a significant impact on flexible structures, making it difficult to control the deformation, fracture, and other behaviors of flexible buildings [2]. The solution of geometric nonlinear problems in flexible buildings is beneficial for optimizing the prefabrication and assembly methods of building structures. While reducing the labor costs and improving the construction efficiency, it can also shorten the construction project cycle. The application of geometric nonlinear problems in architecture is highly in line with the current market demand for resource conservation and environmental sustainability and therefore has broad application prospects. In architectural structural design, geometric nonlinear analysis can more accurately predict the actual behavior of the structure under load. Due to the complex geometric shapes and connection methods of building structures, traditional linear analysis may not accurately reflect the actual response of the structure. The method proposed in the study can simulate the buckling and instability phenomena of structures under large deformations through numerical analysis of flexible structures, which is crucial for ensuring the safety of the structure. The contribution of the research lies in improving the efficiency of displacement increment calculations by introducing orthogonal constraints, thereby providing a high-precision and efficient analysis method for the geometric nonlinearity of current flexible structures.

Due to the promising market prospects of geometric nonlinear structures, academic fields and research projects have long been of interest. Touzé *et al.* proposed a geometric nonlinear structural model reduction method based on an invariant manifold theory, emphasizing the difference between nonlinear mapping and linear techniques. On the basis of discussing the discretization of the finite element method and the implicit condensation technique, recent developments allowing direct computation of reduced-order models relying on invariant manifolds theory are detailed [3]. Deng *et al.* proposed a new geometrically nonlinear planar beam element. The proposed method was based on the corotation process and stability function, using the same direction rotation program method and differential method to handle displacement transformation and establishing the overall balance equation and tangent stiffness matrix. This method had been proven to have accuracy and efficiency in instance verification [4]. Ma *et al.* introduced the latest progress of nonlinear electromagnetic response in detecting and controlling quantum phases based on material classification methods using quantum geometry and topological properties and analyzed the application of this method in device architecture [5]. Xu *et al.* investigated the influence of geometric nonlinearity on large-span bridge structures and analyzed the coupled vibration response of wind turbines and bridges using a full process iterative calculation method. When the train speed was 200 km/h and the wind speed was greater than 35 m/s, the wheel unloading index exceeded the safety threshold [6]. Zhao *et al.* used a shooting method to solve the nonlinear control equations of gradient shallow spherical shells, thereby obtaining the influence of geometric parameters, material properties, and other parameters on shell buckling and critical load. In experiments, this method provided numerical values and curves to assist in the design of nonlinear structures [7]. Wen *et al.* reviewed three types of methods for material nonlinearity, geometric nonlinearity, and boundary nonlinearity and explored their numerical accuracy, computational efficiency, and nonlinear dynamics problems in nonlinear continuum topology optimization [8].

Huang *et al.* conducted nonlinear modal analysis on axially functionally graded truncated cone microscale tubes to improve the vibration response of the microstructure. In the experiment, the influence of material combination changes on the frequency was analyzed through the homotopy perturbation method and the generalized differential quadrature method [9]. Yang *et al.* improved the prediction accuracy of interpretable neural networks by using orthogonal constraints and performed parameter estimation through an improved small batch gradient descent method. In the experiment, this method improved the interpretability of the model while also making the predictive performance comparable to those of multiple benchmark models [10]. Wu *et al.* proposed an orthogonal constrained least squares regression model for preserving more discriminative information in feature selection. This method could effectively reduce the feature dimension and improve the classification performance [11]. Maghami and Schillinger proposed an improved truncation error criterion for incremental iterative path tracking step size adaptation in large deformation structural mechanics problems by calculating scalar stiffness parameters. In the experiment, the number of points in the incremental iteration path of the model was significantly reduced, improving the efficiency [12]. Jouneghani and Haghollahi analyzed the seismic requirements and bearing capacity of elliptical braced moment resisting frames using an incremental iteration method and determined the ductility and response correction factors of modern steel braced structural systems. The response correction factors of 9.5 and 6.5 were applicable to the performance of the structural system under different stress states [13].

In summary, in the current research on geometric nonlinearity of building structures, most methods are based on finite element analysis. In the application of finite element analysis, the Lagrangian formulation and the section method occupy the majority. However, the cross-sectional method and the single Lagrangian formulation have the characteristics of high computational complexity and complexity. Therefore, to optimize the numerical calculation methods in flexible structural engineering, the study uses orthogonal constraint conditions to optimize the incremental iteration method based on generalized displacement control (GDC). The purpose of this study is to improve the applicability and efficiency of existing numerical analysis methods for flexible structures in dealing with complex structures. This research contributes to improving the accuracy of solving current geometric nonlinear structures and enhancing the computational efficiency. The innovation of the research lies in the use of updated Lagrangian equations to construct stiffness matrices and the adoption of GDC methods to calculate increments.

2 Methods

Aiming at solving the parameter sensitivity problem for the geometric nonlinear characteristics of flexible structures, a displacement control method based on incremental iteration is studied. The GDC method is applied and combined with orthogonal constraint conditions to optimize the calculation method. In the incremental iteration method, the $N + 1$-dimensional method is used to determine the load increment coefficient and the displacement increment of the structural unit.

2.1 GDC method for nonlinear geometric elements

In nonlinear analysis, incremental theory is used to gradually solve the response of structures under nonlinear forces (such as material nonlinearity, geometric nonlinearity, *etc.*). When applying the incremental theory for nonlinear analysis, it is necessary to decompose the loading process of the structure into multiple equilibrium states for step-by-step analysis, as shown in Figure 1.

As shown in Figure 1, the loading process is usually divided into three stages: the initial state, the deformation state calculated in the previous step, and the deformation state currently being solved. The initial state and the deformation state calculated in the previous step are known as equilibrium states, and a new equilibrium state is solved by analyzing the known equilibrium states [14]. Incremental stiffness can be calculated from the four parameters of structural displacement, elastic stiffness, geometric stiffness, and the force that generates displacement. The state changes of non-linear geometric structures are decomposed into linear problems through incremental stiffness, and the convergence of calculation errors is achieved through iterative increments. In the linear iteration process, the incremental equilibrium equation is expressed as:

$$[S_{j-1}^i]\{\Delta D_j^i\} = \{L_j^i\} - \{F_{j-1}^i\}. \tag{1}$$

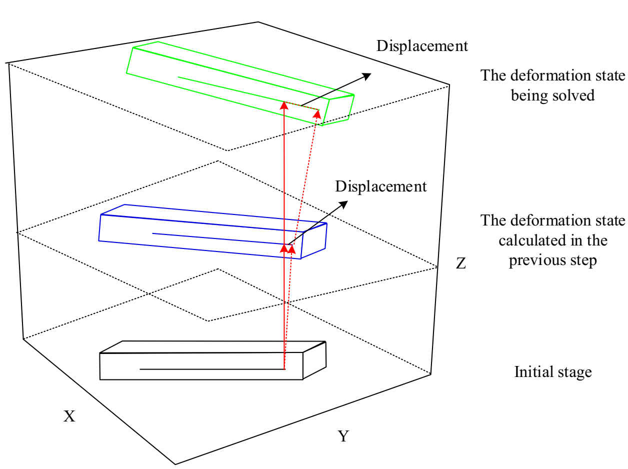

Figure 1: Deformation of building structural units in three-dimensional space.

In Eq. (1), i represents the number of incremental steps; j represents the number of iteration steps. $[S_{j-1}^i]$ represents the overall stiffness matrix of the $j-1$-th incremental step of the structure. ΔD represents the displacement increment in the iteration; L represents the vector formed by the node forces caused by external loads; F represents the vector formed by the node forces caused by internal loads. The parameter L is obtained by calculating the load vector and the load increment coefficient. The unbalanced force represented by $\{L_j^i\} - \{F_{j-1}^i\}$ in Eq. (1) is a limiting condition for the number of incremental iterations. Therefore, Eq. (2) is rewritten as follows:

$$[S_{j-1}^i]\{\Delta D_j^i\} = \lambda_j^i\{\hat{L}\} + \{U_{j-1}^i\}. \tag{2}$$

In Eq. (2), λ represents the load increment coefficient; $\hat{L}$ represents the initially set load vector; U represents the unbalanced force. Under the rigid body criterion, the initial displacement vector of the structure multiplied by the stiffness matrix equals the external load vector, while the displacement increment of the structure multiplied by the stiffness matrix equals the unbalanced force vector. Therefore, the calculation equation for the displacement increment can be derived, and the displacement equation for incremental steps can be obtained by iterating all displacement increments, as shown in Eq. (3)

$$\{D_j^i\} = \{D_{j-1}^i\} + \{\Delta D_j^i\}. \tag{3}$$

According to the structural incremental equilibrium equation of Eq. (2), N equilibrium equations can be constructed, but there are $N + 1$ unknowns in the equations, namely N unknown displacement quantities and one load increment coefficient. Due to the presence of more unknowns than equations, an additional constraint equation needs to be introduced to solve this problem. The general form of the constraint equation is expressed as:

$$\{\chi\}^T\{\Delta D_j\} + \kappa\lambda_j = C_j. \tag{4}$$

In Eq. (4), χ represents the parameter that determines the displacement change of the object; κ represents the parameter that determines the magnitude of the object load; C represents the limiting parameter of the object displacement system. T represents the transposition operation, which is used to swap the rows and columns of a vector or matrix. To obtain all unknowns, the constraint equation of Eq. (4) and the incremental equilibrium equation of Eq. (2) are combined to form the following equation:

$$\begin{Bmatrix} \lambda_j\{\Delta\hat{D}_j\} \\ \lambda_j \end{Bmatrix} = [\hat{S}_{j-1}^i]\begin{Bmatrix} \{0\} \\ C_j \end{Bmatrix} - [\hat{S}_{j-1}^i]^{-1}[\bar{S}_{j-1}]\begin{Bmatrix} \Delta\bar{D}_j \\ 1 \end{Bmatrix}. \tag{5}$$

In order for the iterative algorithm to converge to the correct solution, the constraint equation needs to satisfy the stability and boundedness conditions. To ensure the boundedness of load increment parameters and displacement increments, the determinant of the generalized stiffness matrix must be non-zero to avoid mathematical singularities. The expression for the determinant of the generalized stiffness matrix is shown in the following equation:

$$\det[\hat{S}^{i}_{j-1}] = (\kappa + (\chi)^{T}\{\Delta\hat{D}^{i}_{j}\})\det[S^{i}_{j-1}]. \tag{6}$$

In Eq. (6), κ and $(\chi)^T$ are both sub-terms of the stiffness matrix, so the load increment coefficient can be expressed as:

$$\lambda_j = \frac{1}{\{\chi\}^{T}\{\Delta\bar{D}_j\} + \kappa}(C_j - \{\chi\}^{T}\{\Delta\bar{D}_j\}). \tag{7}$$

In the conventional incremental iteration method, it mainly includes the prediction stage that affects the speed and efficiency of algorithm convergence, as well as the calculation stage that directly determines whether the algorithm can converge to the exact solution, as shown in Figure 2.

In Figure 2, when conducting nonlinear structural analysis, the incremental iterative calculation method needs to meet the requirement of automatically adjusting the step size to adapt to changes in structural stiffness [15]. Meanwhile, it is necessary to ensure stable calculation in areas where extreme values or rebound may occur. Therefore, the GDC method is introduced on the basis of incremental iterative calculation. The method uses the generalized stiffness parameter (GSP) to determine the size of the load increment and adjusts the iteration step size and loading/unloading directions in the iterative process to achieve the convergence of the solution [16]. Compared to other methods, it has advantages in dealing with complex nonlinear problems, dynamic problems, and material nonlinear problems with multiple critical points.

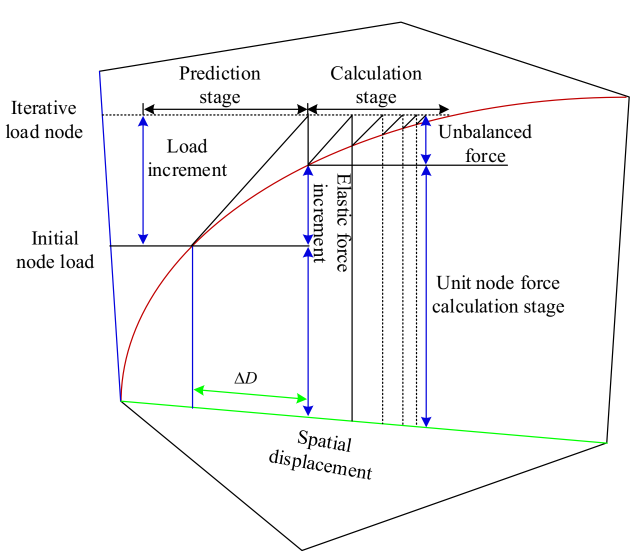

Figure 2: Incremental iterative calculation model.

In the GDC method, the constraint parameters κ is 0, $\{\chi\} = \lambda_1\{\Delta\hat{D}^{i-1}_1\}$, $C_1 = (\lambda^1_1)^2\{\Delta\hat{D}^1_1\}^T\{\Delta D^1_1\}$, $C_{j,j\geq 2} = 0$. The constraint parameters are substituted into Eq. (7) to obtain the expression of the constraint equation, as shown in the following equation:

$$\lambda^{i}_{1}(\Delta\hat{D}^{i-1}_{1})^{T}\{\Delta D^{i}_{j}\} = C_j. \tag{8}$$

From Eq. (8), the calculation equation for the load increment coefficient can be simplified as:

$$\begin{cases} \lambda_1 = \pm\lambda^1_1|\text{PSG}| = \pm\lambda^1_1\left|\dfrac{\{\Delta\hat{D}^1_1\}^T\{\Delta\hat{D}^1_1\}}{\{\Delta\hat{D}^{i-1}_1\}^T\{\Delta\hat{D}_1\}}\right|, j = 1 \\ \lambda_j = -\dfrac{\{\Delta\hat{D}^{i-1}_1\}^T\{\Delta\bar{D}_j\}}{\{\Delta\hat{D}^{i-1}_1\}^T\{\Delta\hat{D}_j\}}, j \geq 2. \end{cases} \tag{9}$$

In Eq. (9), PSG represents the generalized stiffness parameter. In the GDC method, both the stability at the limit point and rebound point, as well as the determination of loading and unloading directions during the iteration process, rely on GSP to achieve. GSP reflects the ratio of structural displacement increments and is numerically bounded and stable. The sign of GSP is related to the angle between the displacement increment [17]. When passing through extreme points, GSP is negative, while in other cases it is positive. The positive and negative values of GSP can be used to determine loading or unloading during the iteration process, helping the iteration process smoothly pass through the limit points. Therefore, the role of generalized stiffness parameters in nonlinear geometric problems is to adjust the value of the load increment parameter, thereby controlling the iteration step size.

When the number of iterations is greater than or equal to 2, the constraint Eq. (8) can be simplified into the following equation:

$$\{\Delta\hat{D}^{i-1}_1\}\{\Delta D^i_j\} = 0. \tag{10}$$

From Eq. (10), it can be seen that in the GDC iterative algorithm, the vector generated in the previous iteration is orthogonal to the vector generated in the current iteration. Therefore, the application of GSP can be used to ensure that the calculation of displacement changes of structures under load is carried out along new and unexplored directions, to avoid getting stuck in local minima or overfitting problems.

2.2 Improved GDC method based on orthogonal constraint conditions

The convergence of traditional incremental iteration methods depends on the initial guess value. If the initial conditions are not properly selected, it may lead to divergence or convergence to the wrong solution during the iteration process. Meanwhile, the efficiency and convergence of iterative methods may depend on the size and distribution of load increments. Since the orthogonal relationship between iterative increments has been analyzed in the above equation expression, the study aims to improve the GDC method by analyzing and optimizing the orthogonal constraint conditions. In incremental iteration analysis, the purpose of defining the load increment factor is to provide sufficient known conditions for solving the displacement and load increments in nonlinear equations. In the first iteration, it is assumed that the structure is in equilibrium and there are no unbalanced forces. At this stage, the structure is subjected to external loads, and the resulting displacement increment is calculated based on linear analysis. Starting from the second iteration, the iterative process gradually adjusts and optimizes to eliminate the unbalanced forces in the initial approximate solution, ultimately reaching an equilibrium state that makes the calculation results more accurate and reliable, as shown in Figure 3.

In Figure 3, 0 represents the initial position, and 1, 2, 3, and j represent the number of iterations. In the first iteration, considering the degree of nonlinearity of the structure is crucial for determining the incremental step size. The stiffness change of a structure is an intuitive indicator for determining the degree of nonlinearity. A large change in stiffness means a high degree of nonlinearity, therefore requiring a smaller incremental step size. A small change in stiffness means a low degree of nonlinearity, and a larger incremental step size can be used. Therefore, the expression of the load increment coefficient should be optimized to the following equation:

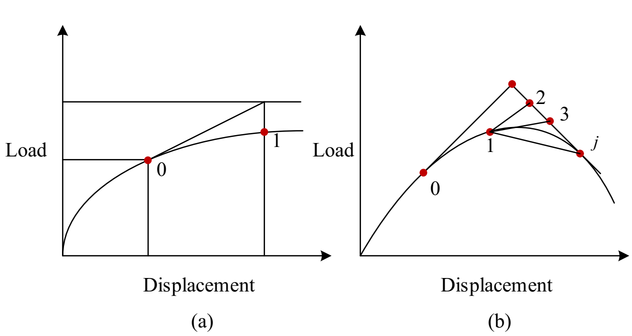

Figure 3: Two-dimensional simplification of traditional (a) incremental iteration and (b) generalized incremental orthogonal iteration.

$$|\lambda_1^i| = \lambda_1^1 f(x). \tag{11}$$

In Eq. (11), $f(x)$ is expressed as the correlation function of structural stiffness. Its expression is shown in the following equation:

$$f(x) = \frac{|\lambda_1^i|}{\lambda_1^1}\sqrt{\frac{\{\Delta\hat{D}_1^1\}^T\{\Delta\hat{D}_1^1\}}{\{\Delta\hat{D}_1^i\}^T\{\Delta\hat{D}_1^I\}}}. \tag{12}$$

In the first iteration of the incremental iteration method, in addition to determining the numerical value of the load increment coefficient, it is also necessary to determine its positive or negative sign. The current stiffness parameters used may not be sufficient to accurately describe the behavior of the structure near the limit point and rebound point, where special measures need to be taken to ensure the safety and stability of the structure. In practical applications, it is necessary to adjust the stiffness parameters or adopt more complex models to better handle these issues [18]. Therefore, the study introduces additional directional parameters to determine that when the dot product of two displacement increments is positive, the sign of the load increment coefficient is the same as that of the return parameter, as shown in the following equation:

$$\{\Delta\hat{D}_1^{i-1}\}\{\Delta\hat{D}_1^i\} > 0, \lambda_1^i = \mathrm{sign}(\lambda_1^{i-1})\lambda_1^1 f(x), \quad i \geq 2. \tag{13}$$

In Eq. (13), sign represents the return parameter of the function, and when the dot product of two displacement increments is negative, it indicates that the sign of the load increment coefficient is opposite to that of the return parameter, as shown in the following equation:

$$\{\Delta\hat{D}_1^{i-1}\}\{\Delta\hat{D}_1^i\} < 0, \lambda_1^i = -\mathrm{sign}(\lambda_1^{i-1})\lambda_1^1 f(x), \quad i \geq 2. \tag{14}$$

In Eq. (14), λ_1^1 represents the preset value of the load increment coefficient, which is determined by the stiffness related function. In the initial iterative calculation, a simplified linear model obtains an approximate structural response of the nonlinear structure. The simplified linear calculation may lead to errors between the calculated results and the actual structural behavior. Therefore, the second part of the GDC method aims to reduce these errors through iteration. During the iteration process, a clear path or strategy needs to be defined to guide how to gradually adjust the load and displacement to approximate the true equilibrium state of the structure. The iteration path needs to be able to simultaneously adjust the load increment applied to the structure and the displacement increment of the structure. This adjustment is to gradually reduce the unbalanced state of the structure, so that the internal and external forces of the structure can reach equilibrium. In addition, the limit point and rebound point are nonlinear characteristic points in structural response. Inappropriate adjustment of the iteration path near these points can lead to divergence or inability to converge to the correct equilibrium state. The constraint equation of the current stiffness parameter method is simplified to Eq. (10), which shows that the current stiffness parameter uses orthogonal constraint conditions as its convergence criterion. Using the displacement increment within the incremental step as a reference direction vector can make the iteration direction closer to the equilibrium path. Therefore, the final GDC iteration path based on orthogonal constraint conditions is represented by the following equation:

$$\begin{cases} \{\Delta\hat{D}_1^{i-1}\}\{\Delta D_j^i\} = 0 \\ \lambda_j^i = -\dfrac{\{\Delta\hat{D}_1^i\}^T\{\Delta\bar{D}_j^i\}}{\{\Delta\hat{D}_1^i\}^T\{\Delta\hat{D}_j^i\}}. \end{cases} \quad (15)$$

Through this orthogonal iteration, the GDC method can effectively handle nonlinear problems in structural analysis. In structural analysis, a key parameter, the generalized stiffness parameter, is introduced to monitor the stiffness changes of the structure during loading. In the first step of iteration, the GDC method will set a step size limit to ensure the stability of the structure during loading and to determine whether to increase or decrease the load. In the subsequent steps of iteration, the GDC method will set a reference direction vector for the structure. This means that the iterative path of displacement increment will remain perpendicular to the reference direction vector, *i.e.*, orthogonal, to ensure the efficiency and accuracy of the iterative process. Through the above iterative process, the goal of the GDC method is to stabilize and converge the calculation results to the equilibrium state of the structure. The research process is shown in Figure 4.

In Figure 4, for the geometric nonlinearity problem of flexible structures, the GDC method and orthogonal constraints constructed by the research mainly divide the incremental iteration into two processes: the first iteration and the subsequent iteration.

In the first iteration, the stiffness matrix of the structure is determined and the GSP is calculated, and the positive and negative directions of the load are determined through the first iteration. The stiffness matrix of the second iteration is updated, and the load increment and displacement increment are calculated. Whether the displacement increment is less than the safety standard is judged. If yes, output the displacement increment; otherwise, continue iterative calculation. In the calculation and analysis of building structures, although traditional single displacement control methods are favored for their stability, they appear cumbersome when dealing with structural buckling problems. To ensure computational convergence when dealing with paths with significant curvature, it is usually necessary to set the initial load increment factor very small. In order to expand the application scope of displacement control iteration method, orthogonal constraint conditions are introduced to optimize the calculation process of the structure. By comparing the maximum load increment factor at convergence under different constraint conditions, the efficiency of structural analysis and calculation has been increased.

3 Results

To verify the GDC method under orthogonal constraint optimization constructed by the research method, the adaptability of the method in displacement increment control calculation will be explored from the finite element analysis model of the spatial truss structure, and the algorithm control performance under simulated disaster conditions will be compared.

3.1 Applicability experiment of the orthogonal constraint optimization GDC method

The study uses a hexagonal star-shaped spatial truss structure as the experimental simulation object. When constructing a finite element analysis model using the ANSYS platform, LINK8 elements are used to construct the spatial truss structure, as shown in Figure 5.

In Figure 5, in the finite element model constructed using ANSYS, each member is simplified into a single element for simulation. The contact relationship between each particle is simulated as 132 contact units, of which 48 contact points could rotate but are not allowed to translate at the connection. Additionally, 84 contact points allow for a certain degree of translation and rotation. The specific model parameters are shown in Table 1.

In the table, the initial load factor is used to determine the incremental step size and loading direction. The

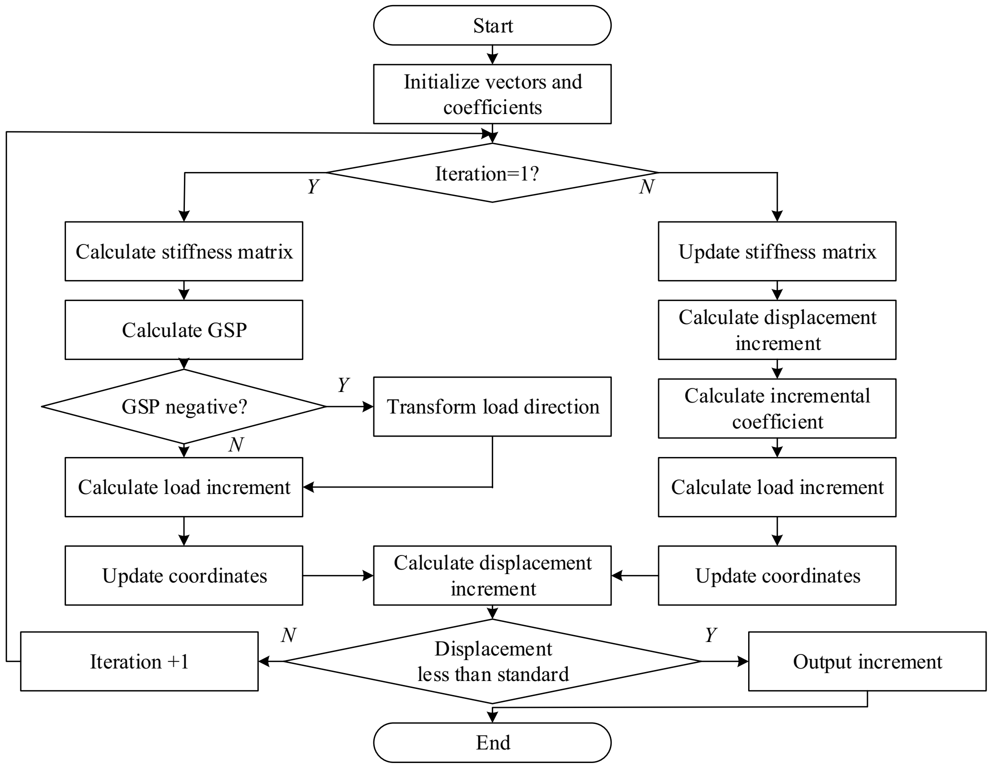

Figure 4: Algorithm flow based on GDC and incremental orthogonal iteration constraints.

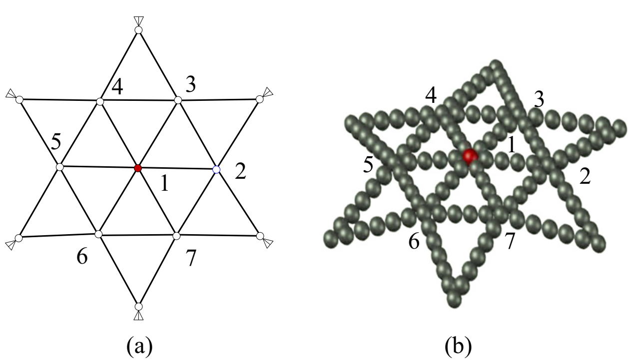

Figure 5: Model diagram and parallel simplification of the truss structure: (a) plan view and (b) discrete model diagram.

incremental step size should be adjusted according to the stiffness change of the structure, and the larger the stiffness, the smaller the incremental step size should be. Therefore, in order to reflect the changes in structural stiffness and adapt to the direction changes of loads beyond the limit point in iterative calculations, this value of 0.00005 is studied as the initial incremental coefficient. First, the load–displacement curve of the model is analyzed in the experiment to verify the rationality of the proposed orthogonal constraint optimization GDC method, as shown in Figure 6.

In Figure 6, ANA represents the finite element analysis method, while GDC represents the analysis and calculation method constructed for the study. The proposed method is

Table 1: Finite element model parameters of the spatial truss structure

Project	Value	Project	Value
Initial load of node	1,000 N	Cross sectional area of the rod	317 mm^2
Initial load increment coefficient	0.00005	Torsional moment of inertia	14,110 mm^4
Elastic modulus	3,030 MPa	Sectional moment of inertia	8,370 mm^4
Shear modulus	1,262 MPa	Number of rods	24
Nodes	13	—	—

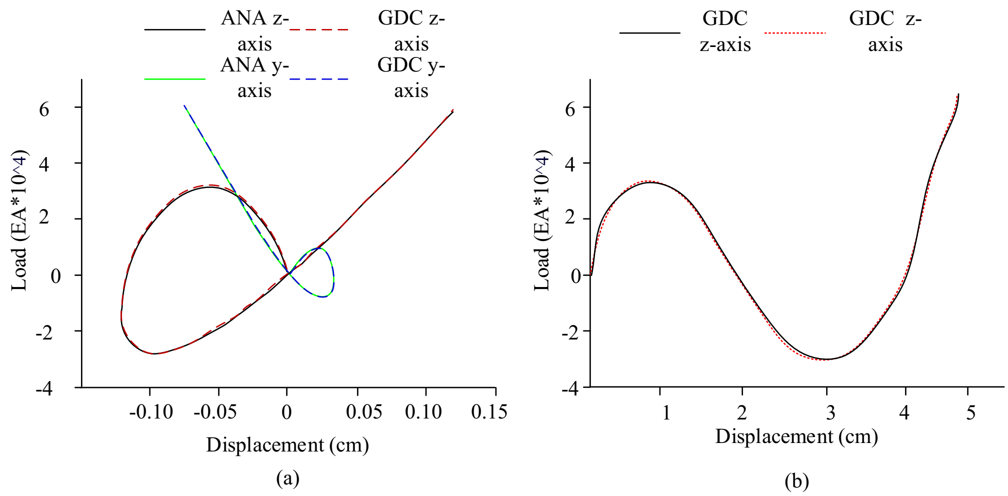

Figure 6: Node load–displacement curves using different methods: (a) Node 2 load displacement curve and (b) Node 1 load displacement curve.

very close to the theoretical exact solution in the load–displacement curves of two nodes, indicating the accuracy of the analysis method. Experiments showed that the model constructed by the study could accurately predict and handle the performance of spatial frame structures when they reached their ultimate load-bearing capacity after introducing a stiffness matrix for analysis. Afterward, the study evaluates the efficiency of different methods, as shown in Figure 7.

In Figure 7, ARC represents the cross-sectional method, while GDC represents the analytical calculation method constructed for the study. The method proposed in the study had similar load–displacement curves and section method curves, with a final time of 126 s in 700 incremental iteration steps, while the section method had a final time of 177 s. The method proposed in the study had optimized the time consumption by 28.8% compared to the cross-sectional method while maintaining the same performance.

3.2 Comparative experiment of the orthogonal constraint optimization GDC method

First, to analyze the computational performance of the model under different load conditions, the study will design different load conditions and analyze the performance of the orthogonal constraint optimized GDC method constructed under different loads in the calculation. First, the load is set to be uniformly applied from node 1. In the first working condition, the load method is to apply

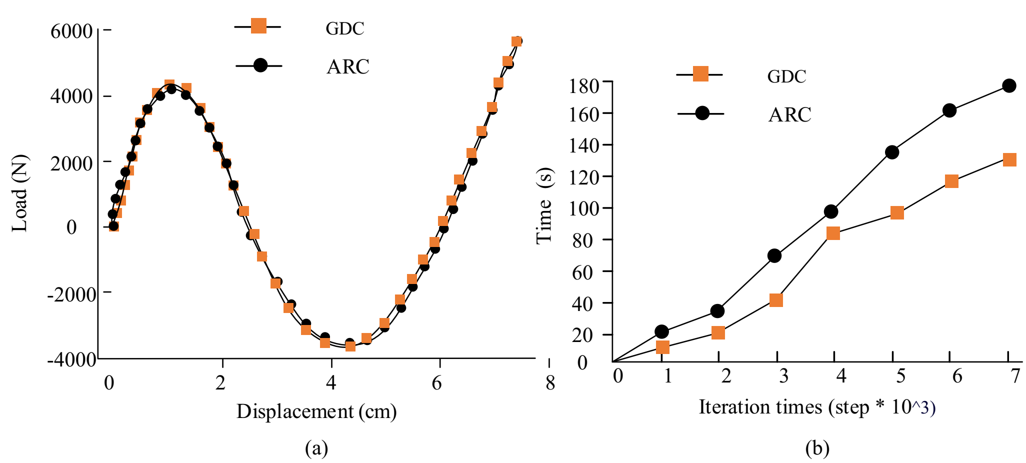

Figure 7: Load–displacement curves and calculation time of different methods: (a) Node 1 load displacement curve and (b) calculation time.

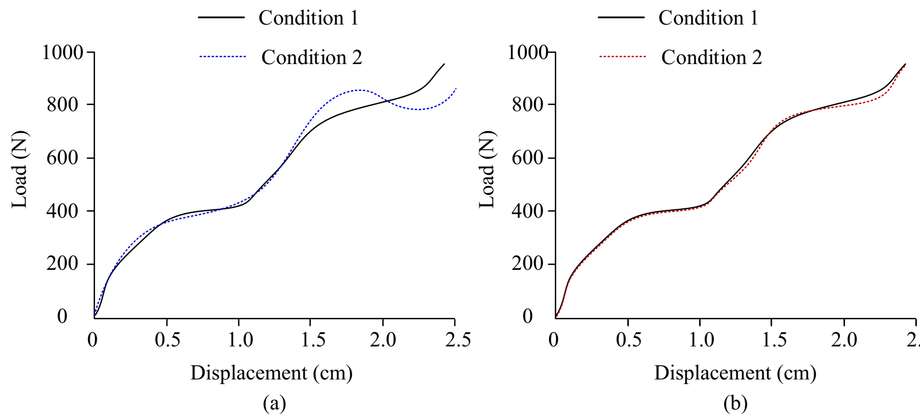

Figure 8: Load–displacement curves under different working conditions: (a) ARC load displacement curve and (b) GDC load displacement curve.

external loads directly, while in the second condition, the load is applied in a form greater than the external load. When the load of the second working condition exceeds that of the first working condition excessively, it will cause significant differences in nonlinear behavior. Therefore, the load of the second working condition in the study area is 1.01 times that of the first working condition. The load–displacement curves under two operating conditions are shown in Figure 8.

In Figure 8, under different operating conditions, the load–displacement curve of the cross-sectional method varied greatly, while the model constructed by the study is less affected by the change in operating conditions. From Figure 8(a), it can be seen that under the section method analysis, when the displacement of the contact element reaches 2.5 cm, the calculated load difference is as high as 97 N. In Figure 8(b), under the method of research construction, after the displacement of the method reaches 2.5 in the two working conditions, the load difference between different working conditions is only 4 N. To analyze and study the role of the proposed optimized GDC method in nonlinear structural deformation impact, a seismic wave with a peak acceleration of 1.2 g is input for 14 s in the simulation experiment. In the simulated disaster environment, the horizontal displacement of the simulated spatial truss structure is shown in Figure 9.

The DEM in Figure 9 represents the discrete element analysis method. In Figure 9, the horizontal displacement results of the optimized GDC structure used in the study had a high similarity in waveform with the horizontal acceleration of seismic waves. The experiment showed that the $N + 1$ dimension theory used in the research method could effectively respond to and adjust the scope of application of the calculation method. The consistency and accuracy of calculation and analysis could also be maintained under the influence of geological disasters. The study analyzes the ultimate load fluctuations of contact elements in spatial truss structures under the

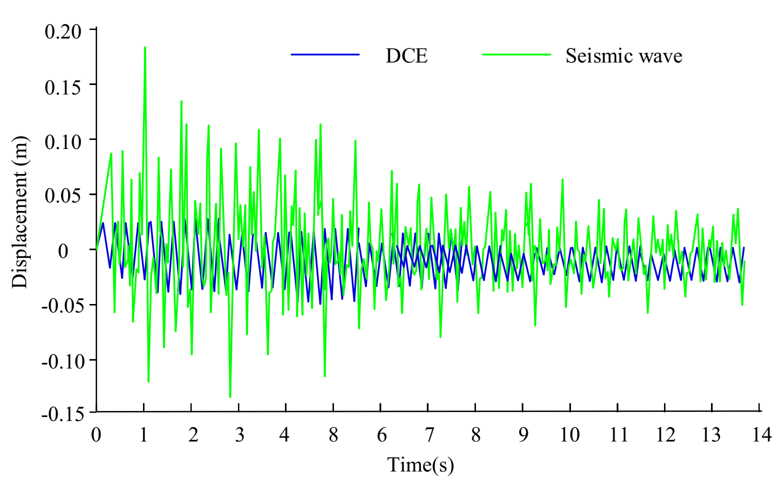

Figure 9: Horizontal displacement of spatial truss structures in simulated disaster environments.

influence of geological disasters, and the specific results are shown in Figure 10.

As shown in Figure 10, the difference in the ultimate load calculation of the contact element of the spatial truss structure between the algorithm models proposed by the three algorithms and the section method is relatively small, while the ultimate load obtained by the discrete element analysis method is larger compared to the other two methods. The average load calculation result of the discrete element analysis method for 84 contact elements is 112.6 kN, while the calculation results of the section method and the GDC method constructed in the study are 113.9 and 114.7 kN, respectively. In Figure 10(b), the constructed method took 1,615 s to calculate the ultimate load of 84 contact elements, while the cross-sectional method took 2,840 s to calculate all contact elements. The discrete element analysis method performed the worst in terms of computational efficiency, with a final computation time of 4,322 s. The time complexity of different algorithm models is shown in Figure 11.

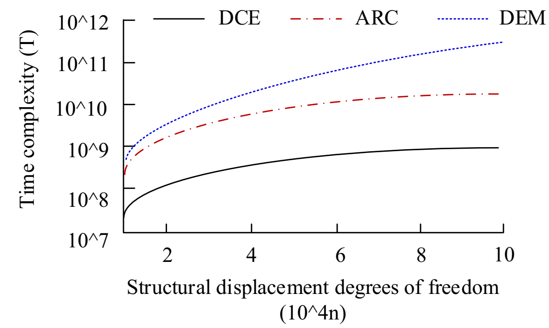

Figure 11: Time complexity of different algorithm models.

From Figure 11, it can be seen that with the increase of structural degrees of freedom, the time complexity of all three models shows an upward trend. However, the model constructed by the research has the smallest increase in magnitude, with a final time complexity of less than 10^9 T, while the time service reading of the discrete element model is greater than 10^{11} T, and the time complexity of the cross-sectional method is close to 10^{10} T. Afterward, the horizontal displacement and deformation degree of 13 nodes are compared in the study, and the specific results are shown in Table 2.

As observed in Table 2, among the 13 nodes, the maximum error of the constructed method in displacement calculation is only 0.06 cm, and the minimum error is 0 cm. Meanwhile, in all 13 nodes, the deformation judgment results calculated by the proposed method are correct. Therefore, the model constructed in the study can still accurately determine the displacement and deformation properties of building structures even after simulating earthquake disasters. Finally, the study will compare the differences between the GDC method, multi-point displacement control, and traditional LDL decomposition method constructed under different load conditions. In the comparative experiment, the three operating conditions are set as static overturning load, continuously increasing load, and reciprocating load. The specific results are shown in Table 3.

From Table 3, it can be seen that under the static overturning load conditions, the proposed algorithm has a time complexity of 0.52×10^7 T, which is lower than other algorithm models. At the same time, under dynamic working conditions such as continuously increasing loads and reciprocating loads, the proposed algorithm has time complexities of 0.32×10^8 T and 0.52×10^7 T, respectively, which are lower than other algorithm models.

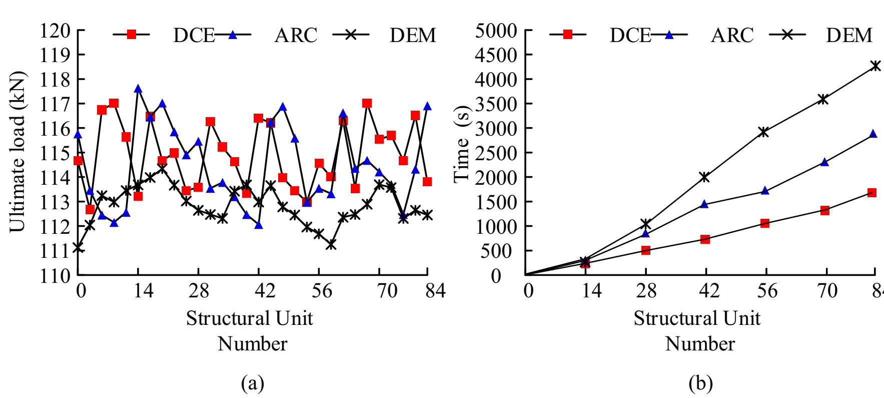

Figure 10: Ultimate load calculation and time consumption of contact elements in spatial truss structures: (a) structural critical load of different algorithms and (b) calculation time of different algorithms.

Table 2: Displacement and deformation of spatial truss structure nodes

Node	Prediction displacement (cm)	Theory displacement	Deformation prediction	Theory deformation
1	0.62	0.57	Undeformed	Undeformed
2	0.88	0.85	Undeformed	Undeformed
3	0.91	0.92	Deformation	Deformation
4	0.86	0.85	Undeformed	Undeformed
5	0.81	0.86	Undeformed	Undeformed
6	0.86	0.86	Undeformed	Undeformed
7	0.91	0.93	Deformation	Deformation
8	0.25	0.19	Undeformed	Undeformed
9	0.15	0.16	Undeformed	Undeformed
10	0.21	0.18	Undeformed	Undeformed
11	0.22	0.21	Undeformed	Undeformed
12	0.23	0.26	Undeformed	Undeformed
13	0.22	0.24	Undeformed	Undeformed

Table 3: Load calculation and time complexity of different algorithm models under different operation conditions

Working conditions Algorithm	Static overturning load		Continuously increasing the load		Cyclic loading	
	Average load (kN)	Time complexity (T)	Average load (kN)	Time complexity (T)	Average load (kN)	Time complexity (T)
LDL	114.2	0.55×10^7	116.3	0.16×10^9	113.9	0.72×10^8
MDC	115.4	0.93×10^8	117.1	1.4×10^{10}	114.4	1.1×10^{10}
DCE	114.7	0.52×10^7	115.6	0.32×10^8	114.6	0.52×10^7

4 Conclusions

To verify the performance of the improved incremental iteration method constructed in geometric nonlinear calculations, this study explored the advantages of the method from its adaptability and performance comparison. First, in the 700 incremental iteration steps of simulating the hexagonal star-shaped spatial truss structure, the final time consumption was 126 s, while the cross-sectional method took 177 s. The method proposed in the study optimized the time consumption by 28.8% compared to the cross-sectional method while maintaining the same performance. Meanwhile, under the influence of simulated seismic waves, the method constructed in the study took 1,615 s to calculate the ultimate load of 84 contact elements, while the section method took 2,840 s to calculate all contact elements. The discrete element analysis method performed the worst in computational efficiency, with a final calculation time of 4,322 s. Finally, among the 13 nodes of the spatial model, the maximum error of the constructed method in displacement calculation was only 0.06 cm, and the minimum error was 0 cm, while the deformation judgment results obtained by the method were all correct. Experimental results have shown that the model constructed by the study has good computational accuracy and efficiency in conventional environments. Meanwhile, it can maintain a good displacement increment calculation performance even under changes in load application methods and seismic effects. The limitation of the study is that only one type of structure was used for simulation in the experiment. In future research, additional building structures can be considered for exploration.

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