

## Research Article

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# Analysis of bridge vibration response for identification of bridge damage using BP neural network

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**Abstract:** In this article, the authors propose a method to identify the bridge damage using a backpropagation (BP) neural network. It uses bridge vibration response to solve the accuracy of bridge damage. A particle swarm optimization algorithm based on chaotic mutation is adopted to perform chaotic mutation operations and make the group jump out of the local optimum. CPSO (particle swarm optimization algorithm based on chaotic variation) algorithm can make up for the BP neural network model, easy to fall into the shortcomings of local optima, so the author will combine the two algorithms and discuss the environmental data of the bridge. Establishing a finite element model of the bridge through actual analysis, through data comparison, comparing the frequencies of the intact stages with the frequencies of the damaged stages, and verifying the neural network with random samples, for the degree of bridge damage, we get the root mean square error  $mse$  and the correlation coefficient  $r$ . The result shows that the root mean square error  $mse = 0.003196$ , and the correlation coefficient  $r = 0.9654$ . There are only a few individual points; it seems that the relative error is relatively large. The rest of the fit is basically the same; it can meet the factors of vibration through the environment and perform damage identification for the structural damage monitoring of the bridge. Using the BP neural network model optimized by chaotic particle swarms, combined with the modal analysis of environmental vibration, it can be used in the monitoring of the health structure of the bridge, plays a certain recognition effect, and provides a new technical idea.

**Keywords:** BP neural network, CPSO-BP algorithm, bridge vibration, bridge damage

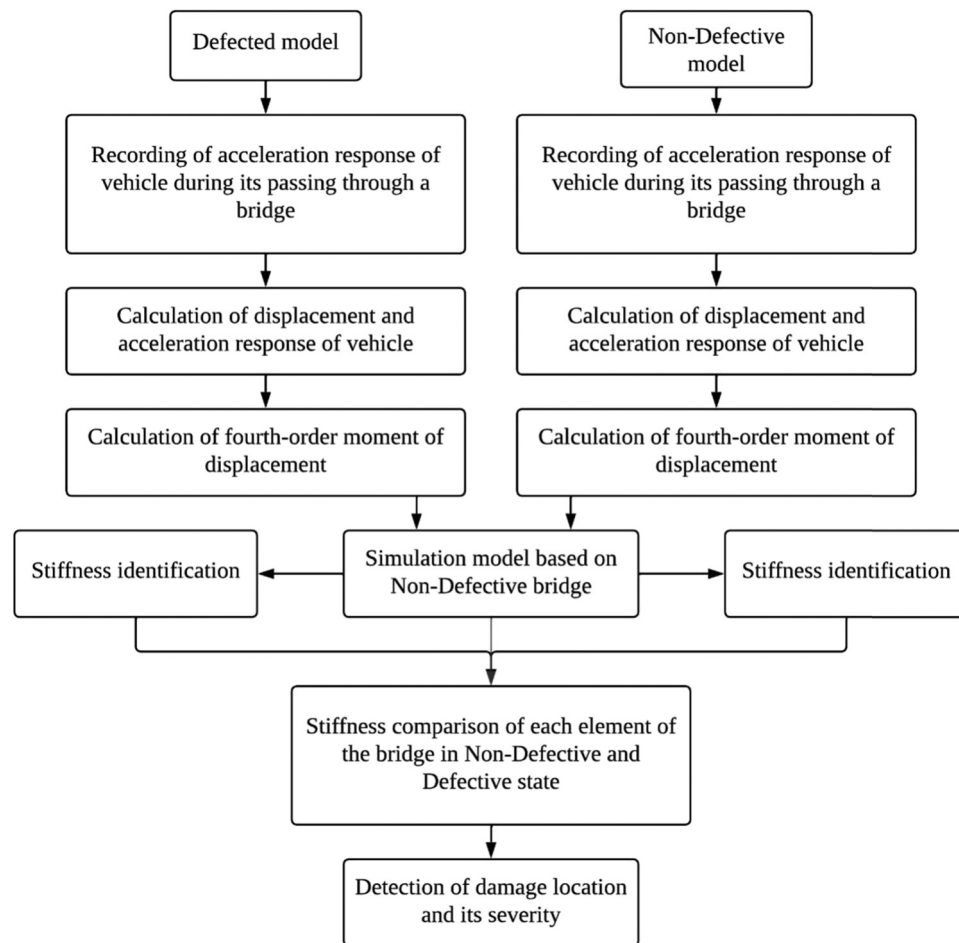
## 1 Introduction

After the bridge structure is damaged, although its dynamic characteristics have changed (potential damage, included in the natural frequency and mode shape of the structure), however, with traditional methods, it is difficult to establish the natural frequency and mode shape of the structure after damage, mapping relationship with structural damage [1]. BP neural network imitates the reaction process of human brain neurons to external stimuli and establishes a multi-layer perception model, using the learning mechanism of signal forward propagation and error reverse adjustment; through multiple iterations of learning, it can build an intelligent network model for processing nonlinear information. It has a complete theoretical system, clear algorithm flow, powerful data recognition and simulation functions, in addressing similar structural damage and natural frequencies, and the non-linear mapping relationship with the mode shape; with obvious advantages, the bridge damage identification process is shown in Figure 1 [2].

Backpropagation (backprop, BP) is a popular approach in machine learning for training feedforward neural networks. Backpropagation can be generalized to various artificial neural networks (ANNs), as well as to functions in general. Backpropagation is the collective term used to refer to all of these groups of algorithms. The physical characteristics of a structure, such as stiffness and mass, are crucial for tracking structural health since changes in these characteristics point to actual damage. The mass of a structure is typically expected to be constant before and after the incidence of damage in the majority of damage detection techniques. As a result, the most important dynamic property for identifying damage is a structure's change in stiffness. In this study, it is assumed that the test vehicle periodically monitors the bridge under inquiry and records the acceleration reaction from the test car during each transit. Throughout the bridge's operation, this comparison process is repeated. The analytical process is summarized in the section below, and the corresponding flowchart is presented in Figure 1.

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**Figure 1:** Bridge damage identification process.

The following procedure, which is regarded as the non-defective state, was used to complete previous monitoring of the bridge of concern. For the current monitoring, which is thought to be the defective state, the test vehicle's acceleration reaction is recorded as it crosses the bridge. Using the vehicle reaction data from the previous and current monitoring runs, the  $n$ th frequency of the bridge's vibration is determined. The bridge's  $n$ th mode shape from the component response's instantaneous amplitude at the  $n$ th frequency is recovered. The stiffness EI for the bridge is determined using the  $n$ th frequency and matching mode shape, which are used to identify structural problems. If no damage is found, the current monitoring is taken to be in a non-defective state, and the damage detection process is repeated for the following monitoring.

The rest of this article is organized as: Section 2 presents the most recent studies related to damage detection and rectification using several techniques. Section 3 presents the methodology for the environmental vibration

test and data pre-processing. Several advantages and disadvantages of the BP neural network are also discussed in the same section. The results and discussion are discussed in Section 4 which is followed by the concluding remarks in Section 5.

## 2 Literature review

In response to this research question, Ye *et al.* [3] according to the special nature of the vibration data spectrum increase define four frequency-related indicators and use them to establish the likelihood function of the existence of structural resonance, and then studied the neural network, the application of only output modal recognition in structural engineering. Mao *et al.* used ANNs in their self-deployment schemes, trained the bridge response data, generate a probabilistic model of bridge behavior, and proposed a way to evaluate bridge response data, in order to identify the

method of structural damage [4]. The mode shape is processed by Zhou *et al.* [5] using parameterization, as the input of BP neural network, a method to identify the damage location of the CA filling layer of the CRTSI slab ballastless track. Pathirage *et al.* [6] used convolutional neural networks to extract structural features to identify damage. Geng *et al.* [7] used BP neural network to identify the substructures such as pylons, main beams, and cables of large cable-stayed bridges. Coppe *et al.* [8] synthesized the combined parameters of the natural frequency and the modal components of a few points, as the input vector of the neural network, to avoid the defect of using a single parameter. Wu *et al.* [9] used structural displacement modal tests, strain modal test parameters, and neural network methods to study the location and extent of the damage. However, the research on the application of BP neural network to the damage identification of medium-span bridges is relatively few. Weinstein *et al.* [10] believe that regarding structural damage recognition is generally divided into two categories: model correction method and fingerprint analysis method. Farahani and Penumadu [11] analyzed the feasibility of neural network method in modal recognition, find out how to judge the damage and cracks of the bridge, it is proposed that the existing neural network is in damage recognition, existing problems, and improvements. Chen *et al.* [12] discuss the principle of statistical feature extraction through ANN, and proposed the corresponding extraction method, research on the safety and durability evaluation health system of cable-stayed bridges. On the basis of current research, the author proposed a bridge damage identification study based on BP neural network for bridge vibration response, a particle swarm optimization algorithm based on chaotic mutation is adopted, performed chaotic mutation operations, and made the group jump out of the local optimum. The results show that only a few points seem to have relatively large errors, and the rest of the fit is basically the same; it can meet the factors of environmental vibration to monitor the structural damage of the bridge, for damage identification, it is believed that, in addition to not including, the damage recognition in the training sample is a bit flawed, and the rest of the damage identification has achieved good results. Therefore, in the subsequent identification process, various damage depths and crack lengths should be passed, increase the training mode of the neural network, and try to include a variety of injuries, in order to achieve the optimal effect on the structural damage identification of the bridge. The modal specification of the structure is used in the current work to offer a BP neural network-based method for estimating structural damages in the joints of truss bridges. Substructural identification is used to get around problems

caused by a large number of unknowns. Two numerical example analyses on truss bridge constructions are presented to show the efficacy of the neural network approach. Once a hypothetical simple truss had been used successfully, the same technique was used for an actual bridge truss.

## 3 Methods

### 3.1 Bridge environmental vibration test and data preprocessing

Under environmental excitation, the identification of structural modal parameters, directly from the excitation under environmental vibration, the environmental vibration test utilizes wind load, and natural conditions such as ground motion loads and traffic loads, in order to stimulate the bridge structure; it is a widely used monitoring method in bridge health monitoring technology [13]. It does not require labor costs, it will not affect the normal operation of the bridge engineering structure, and it will take less time [14]. Compared with the traditional method of identifying structural modal parameters using input and output information, the use of environmental vibration has advantages. First of all, the cost of testing is low, convenient, and quick. Parameter identification under environmental excitation, using seismic wind, vehicle, and other excitation response data parameter identification, is more convenient, no manual incentives are required, and expensive incentives are not used. Second, time-saving and high-efficiency, modal parameter identification method under environmental excitation, no need to manually load the test signal, just test the response data obtained, and save loading time and test time [15]. Third, the use of environmental vibration signals will not interrupt the normal use of bridge engineering. In the traditional recognition method, in order to achieve artificial excitation and reduce interference, the use of bridge sections must be suspended, the modal parameter identification method under environmental excitation, and the structural response measured by natural excitation, so there is no need to interrupt the normal operation of the bridge structure. And artificial excitation can only act on the local vibration signal of the bridge, as the test progresses, if the artificially applied excitation signal increases the frequency of the vibration along with the test, it is likely to cause certain damage to the bridge structure; however, this problem can be completely avoided under environmental incentives [16].

### 3.2 Preprocessing method of environmental vibration data

The bridge health monitoring system uses various sensors installed on the bridge structure and collects structural information in real time; then, through the network, the data are transmitted to the database of the monitoring center, which provides follow-up structural early warning and assessment services. But in the data collection of environmental vibration tests, there are inevitably some problems [17]. For example, in environmental vibration testing, since only the response of the structure under environmental vibration is measured; therefore, the input information of the structure cannot be measured; moreover, the structure dynamic response test data under natural environment vibration and some special data processing techniques are needed to identify system parameters and assess the safety of the bridge structure. There will be signal noise such as external interference, sensor failure, and network transmission failure in data collection. Therefore, the data are collected by the monitoring system; when the reliability of the data used for bridge early warning and assessment cannot be guaranteed, the correctness of the early warning and assessment results cannot be guaranteed. Therefore, the vibration signal is excited by the environment of the bridge, to analyze the health of the bridge; attention must be paid to the preprocessing of environmental incentive data [18].

#### 3.2.1 Random decrement method

The random decrement method is used from the random vibration response signal of the structure, the processing method of extracting the structure of the free attenuated vibration signal, through averaging and mathematical statistics; from the environmental stimulus response, the extracted structure is free to dampen the vibration response. It mainly uses the characteristic that the average value of the static random vibration signal is zero and distinguishes between measured vibration response signals including deterministic vibration signals, and the two components of the random signal, deterministic and random signals, are free. After attenuating the vibration response to the letters, the time domain identification method is used to identify the modal parameters [19].

#### 3.2.2 Moving average method

The moving average method is a commonly used data processing method. This method advances from the data

item by item, calculates the average of  $N$  items, and uses the moving average to replace the original value, thereby eliminating random fluctuations and obtaining data trends. When the moving average distance  $N$  is larger, the processing effect is better, but more items are lost [20]. Therefore, when it is necessary to ensure the effect of eliminating random interference, and when the minimum amount of data reduction is required, the size of  $N$  must be carefully selected [21].

### 3.3 Structure of BP neural network

**Multi-layer structure:** As can be seen from the BP neural network structure diagram, the level of BP neural network design, it can make the transmission and processing of information better and the ability to solve practical problems through network training [22].

**The transfer function of the BP network must be separable:** The transfer between BP neural network layers and layer routes is handled by functions; BP neural network, the gradient descent rule adopted, requires the transfer function to be negligible [23].

**Backpropagation algorithm:** In the learning and training process of the BP network, the input data of the network are transmitted backward through each layer, and the principle of minimizing the target error is adjusted when adjusting the weight. Then, the weight is corrected forward through the opposite layer [24].

### 3.4 Advantages and disadvantages of BP neural network

BP neural network is in the deformation prediction of engineering construction, has a good effect, and is highly sought after by researchers. It can be said, the research on it has not stopped, and there are countless research results, and related research is relatively mature. BP neural network has the following advantages in deformation prediction.

**Strong applicability:** Whether it is settlement prediction or horizontal displacement prediction, BP neural network can be applied to large samples or small samples. This is determined by the training mechanism of the BP neural network itself.

**Good fitting and prediction accuracy:** The deformation of engineering buildings generally presents a nonlinear deformation law, BP neural network's powerful nonlinear mapping ability, it is unmatched by general forecasting models [25].

BP neural network also has certain shortcomings, which are mentioned below.

**Strong sample dependence:** Although the BP neural network, for small samples with strong regularity, also has good mapping ability, if the sample is too small or the regularity is not strong, BP network cannot learn enough knowledge from it to realize the accuracy of prediction. In addition, when the BP neural network processes more contradictory samples, or more redundant samples, the model approximation ability is greatly reduced.

**The system's own defects:** Slow convergence, sensitive to initial weights, and easy to fall into local extremes.

### 3.5 Classic BP neural network algorithm

Classic BP neural network, using gradient descent method, as the direction adjustment of the network weight threshold. The construction of the BP network includes input layer, hidden layer, and output layer [26].

Three-layer BP neural network  $\xi_i$  represents the expected output, adjusting the threshold  $w$  can minimize the error between the network output  $o_i$  and the desired output  $\xi_i$ . Suppose we have  $n_u$  training modes  $u = 1, 2, \dots, n_u$ , then the input for the hidden layer unit  $j$  is presented in Eq. (1).

$$h_j^u = \sum_k w_{jk} \xi_k^u. \quad (1)$$

The corresponding output of the hidden layer unit  $j$  is calculated using Eq. (2).

$$v_j^u = f(h_j^u) = f\left(\sum_k w_{jk} \xi_k^u\right). \quad (2)$$

Then the input of output layer unit  $i$  is calculated using Eq. (3).

$$h_j^u = \sum_j w_{jk} v_j^u = \sum_j w_{ij} f\left(\sum_k w_{jk} \xi_k^u\right). \quad (3)$$

Then the final output of  $i$  is calculated using Eq. (4).

$$o_i^u = f(h_i^u) = f\left(\sum_j w_{ij} v_j^u\right) = f\left(\sum_j w_{ij} f\left(\sum_k w_{jk} \xi_k^u\right)\right). \quad (4)$$

If the initial weight of the network is set arbitrarily, then for each input mode  $u$ , there will be an error in half of the network output and the expected output, and the error function is defined as expressed in Eq. (5), and the result is obtained using Eq. (6).

$$E(w) = \frac{1}{2} \sum_u (\xi_i^u - o_i^u)^2. \quad (5)$$

$$E(w) = \frac{1}{2} \sum_u \left[ \xi_i^u - f\left(\sum_j w_{ij} f\left(\sum_k w_{jk} \xi_k^u\right)\right) \right]^2, \quad (6)$$

where  $E$  is the continuously differentiable function of the weights of each layer, the steepest gradient descent method is used to adjust the weights, and the multi-layer forward neural network learning is realized using Eqs. (7) and (8).

$$\Delta w_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} = \eta \sum_u (\xi_i^u - o_i^u) f'(h_i^u) v_j^u = \eta \sum_u \delta_i^u v_j^u, \quad (7)$$

$$\delta_i^u = f'(h_i^u) (\xi_i^u - o_i^u). \quad (8)$$

$\delta_i^u$  represents the error correction amount of the implicit unit, 8" through weighting and all units  $j$  output the error correction amount  $\delta_i^u$  of the previous layer connected. The upper layer is represented by  $a$ , and  $z$  is the lower layer, which is connected in the training mode.

Therefore, the weight correction formula of the BP neural network algorithm is expressed as follows;  $t$  means  $t$  adjustments have been made using Eq. (9).

$$w_{az}(t+1) = w_{az}(t) + \eta \sum_u \delta_p^u v_q^u. \quad (9)$$

In practical applications,  $\eta$  represents the learning factor, the smaller the value of the learning factor, the better, it means that the learning efficiency is very high, the greater the value of  $\eta$ , the more drastic the weight change, causing the learning process to oscillate; in order for  $\eta$  to be large enough without causing the weight to oscillate, a momentum item is usually formed using Eq. (10).

$$w_{az}(t+1) = w_{az}(t) + \eta \sum_u \delta_p^u v_q^u + \alpha [w_{az}(t) - w_{az}(t-1)]. \quad (10)$$

$\alpha$  is a constant, called the situation factor, which determines the degree of influence of the last learning weight update. Usually, it takes many iterations of learning cycles; only then can the iteration of the actual output model be achieved.

### 3.6 Establishment of analysis model

Establishing a finite element model of the bridge through actual analysis, in the construction of the bridge, the rectangle is the main shape structure of the bridge. It is easier to reflect the real model of the bridge, built with steel material, realizes the identification of damaged structures, and uses a neural network to realize the identification, positioning, and damage degree calibration of structural damage.



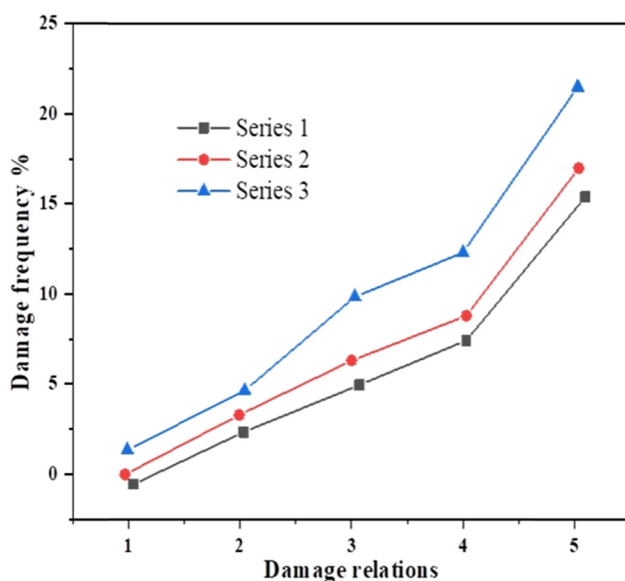
## 4 Results and analysis

The material used by the author is 45# steel, the elastic modulus  $E$  is  $2.1 \times 1,011$  Pa, the shear modulus  $G$  is  $7.65 \times 1,011$  Pa. Poisson's ratio  $\mu$  is 0.28, and mass density  $\rho$  is  $7,800$  KG/m<sup>3</sup>. The established bridge model is a steel structure bridge with piers laterally. It is assumed that the bridge is 150 m long and 20 m wide, the height of the bridge is 2 m, of which the base is 15 m high, 20 m long and 35 m wide, and the height of the base is 25 m. When building the model, the same scale is reduced to millimeters. The bridge is simulated and modeled by hyper mesh finite element software.

### 4.1 Bridge damage identification

Established a finite element model of the bridge, through data comparison, the frequencies of the intact stages and the frequencies of the damaged stages are compared, performing control  $\Delta\phi_i = \phi_{ui} - \phi_{di}$  ( $i = 1, 2, 3, 4, 5$ ), Figure 2 shows the relationship between damage frequency and damage at 450 mm.

From the changes in Figure 2, it can be seen that the series 1 to 3 respectively represent the first-order to the third-order changes; after the rectangular structure of the bridge is damaged, the modal frequency changes to a lower trend; it can be seen from the change of the absolute value of the vibration mode frequency that, the sensitivity of mode frequency to damage is unequal,



**Figure 2:** 450 mm Damage frequency and damage relationship diagram.

from the relative change value of the modal frequency, it can be seen that the frequency transformation caused by the damage, with the degree of damage, the larger the wound, the non-linear frequency drift increases; it shows that frequency changes are more sensitive to larger damage. So, with the change in bridge structure modal, it is easier to identify the damage that has occurred.

### 4.2 Judgment of the degree of damage to the bridge

After the training of the neural network, the damage location of the bridge can be identified, in the case where the damage location is determined, the degree of structural damage is a single factor function of modal frequency, and the specific feature parameter structure is that  $[\phi_1, \phi_2, \phi_3, \phi_4, \phi_5]$  uses these five feature parameters.

The input of the training sample is shown in the first column of the Table, the ideal output model of neural network training, and the output model of the actual damage, represented by the second column, and the difference between the two indicate the error range of the judgment; the formula  $\alpha = \frac{\rho_d}{\rho_h} \times 100\%$  is used to define the degree of damage, where  $\rho_d$  represents the depth of damage,  $\rho_h$  represents the height of the rectangular beam, the actual damage depth, through inverse transformation, obtains the crack length of the bridge,  $\chi = \rho_h \alpha$ .

Verifying the neural network with random samples, at 150, 240, 350, 480, 570 mm, etc., randomly simulating a total of 60 samples with damage depths of 3, 5, 8, 11, 14, 18, 23, 25, 28, 30, 33, and 35 mm, and determining the training situation of the test through the above formula, combined with CPSO-BP, the damage fit degree determined is shown in Figures 3–6.

From the above degree of damage to the bridge, we get the root mean square error MSE and the correlation coefficient  $r$ , as shown in Table 1.

From the figure and the statistics in Table 1, we can see that it can be obtained that the unknown root mean square error (mse) of the damage simulated everywhere is relatively small, and the correlation coefficients are all above 95%, explaining the established neural network, the degree of damage to the bridge, the length of the crack, and have a good recognition effect and can be sure; through the above identification methods, the degree of damage to the bridge can be judged.

Through the above training mode, a random group of data verification is performed on the overall situation of the bridge and, on the whole bridge, randomly find 60

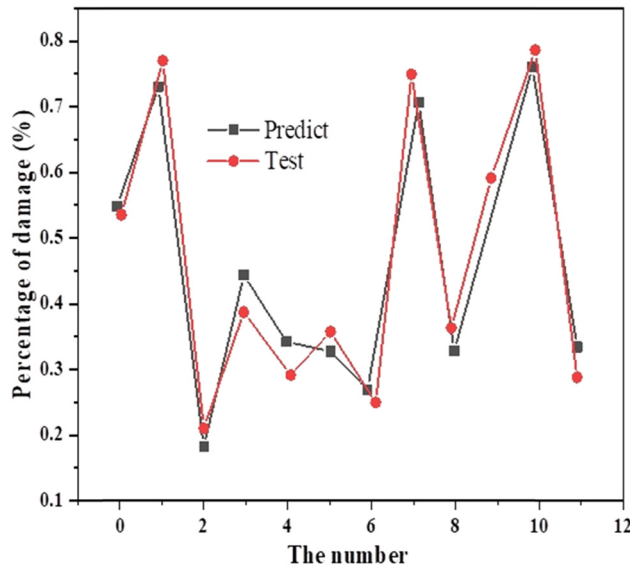


Figure 3: Fitting degree of bridge damage at 150 mm.

different positions and different damage situations, letting the trained model perform self-verification of the damaging effect (Figure 7).

From the above test and identification of the degree of damage to the bridge, it can be seen that, root mean square error  $mse = 0.003196$ , correlation coefficient  $r = 0.9654$ . There are only a few points, it seems that the relative error is relatively large, and the rest of the fit is basically the same; it can meet the factors of vibration through the environment and perform damage identification on the structural damage monitoring of the bridge. It is believed that, except for the damage recognition that is

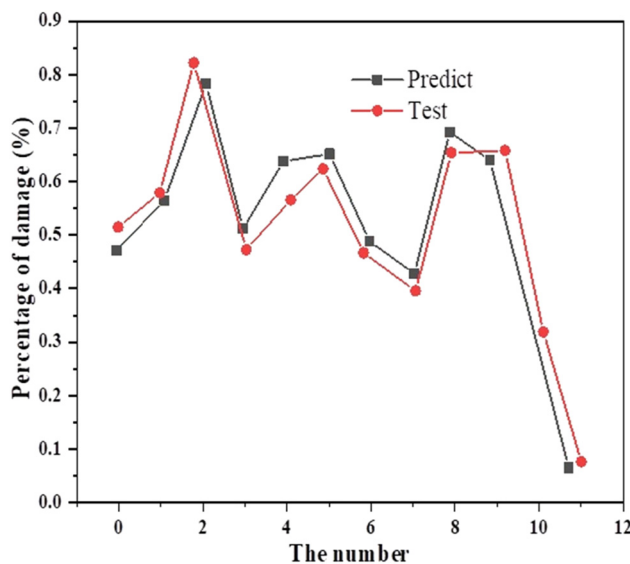


Figure 4: Fitting degree of bridge damage at 240 mm.

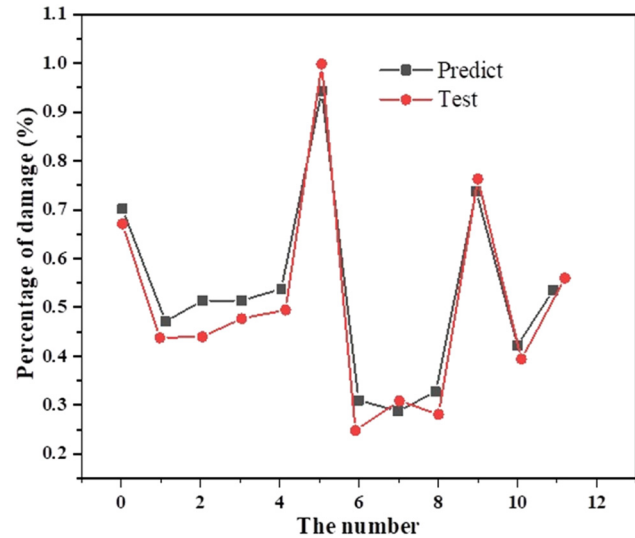


Figure 5: Bridge damage fitting degree at 350 mm.

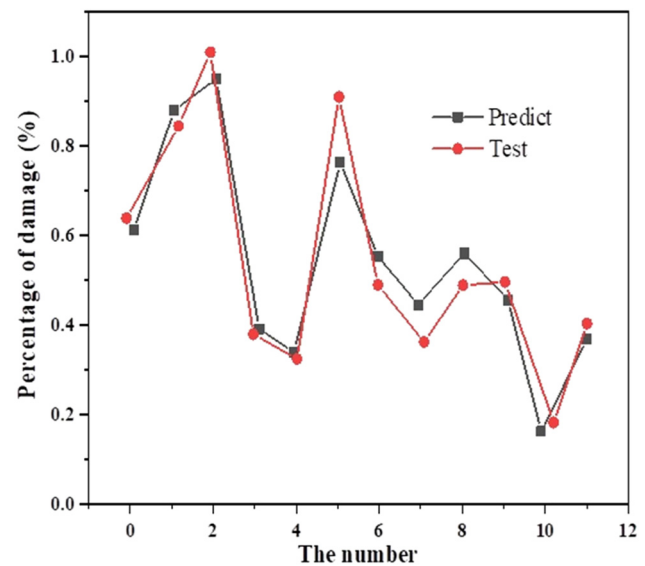
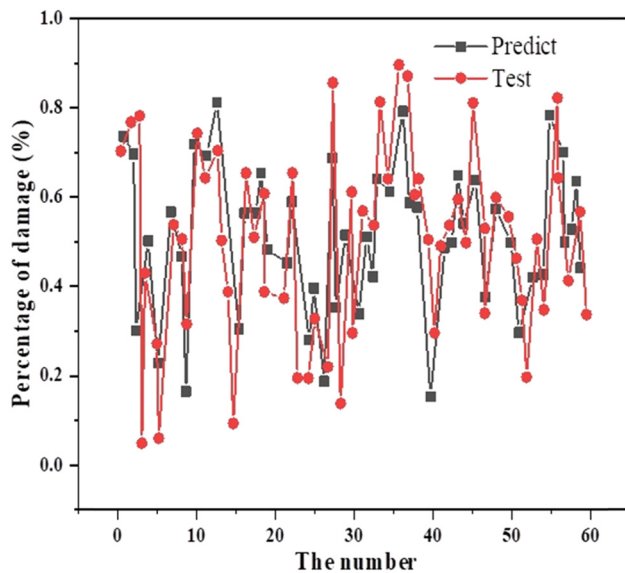


Figure 6: Bridge damage fit degree at 480 mm.

not included in the training sample, it is a bit flawed, the rest of the damage identification has achieved good results; therefore, in the subsequent identification process, various

Table 1: Statistics of fitness of various damages

Damage location	Root mean square error, mse	Correlation coefficient, $r$
150 mm	0.001098	0.9873
240 mm	0.001074	0.9888
350 mm	0.001504	0.9905
480 mm	0.002957	0.9795



**Figure 7:** Fitting degree of structural damage based on CPSO-BP training.

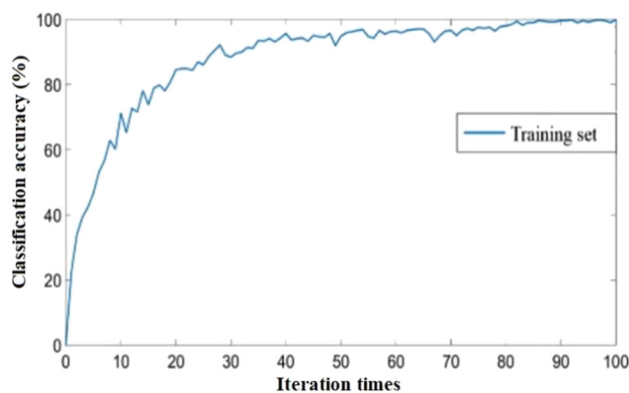
damage depths and crack lengths should be passed, increasing the training mode of the neural network and trying to include all kinds of damage, in order to identify the structural damage of the bridge and to achieve the optimal effect.

Figure 8 displays the classification accuracy rates for the training set and test set, respectively. The outcomes demonstrate that after 82 training sessions, the BP network model's training accuracy rate approaches 94%, and the model steadily converges. As training sessions increase in number, the categorization accuracy rate ultimately makes it to 98. The study of the results reveals that the BP neural network has a 100% accuracy rate in correctly identifying the site of bridge damage. The classification accuracy

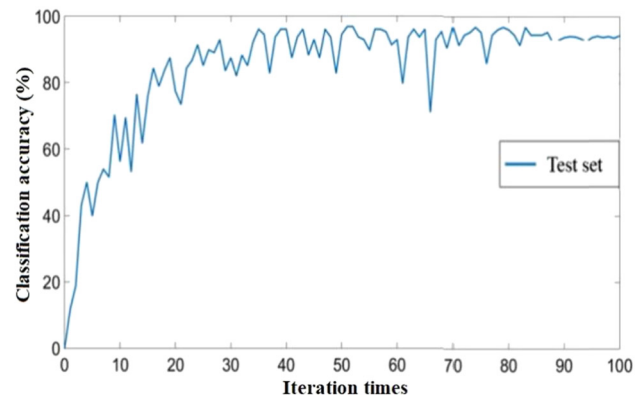
reveals the quantitative identification effect of the BP and confirms that the intelligent identification approach based on BP, and the recurrence graph has an excellent ability because the error only occurs on the damage degree identification.

## 5 Conclusion

The author proposes research on bridge damage identification based on BP neural network's bridge vibration response. The bridge structural health monitoring is considered as the research background. The past traditional manual monitoring on the basis of the transmission of wired bridge structure health monitoring, and using new Internet of Things technology, for the remote real-time transmission of the measured bridge environment excitation data. In the remote real-time monitoring of the health of the bridge structure, the signal of environmental excitation vibration is used for monitoring. According to the incentive effect of the environment, the damage identification of the bridge is discussed. Through a new optimized processing method, the structural damage of the bridge can be better identified more accurately. How to use BP neural network in the future, accurately identify the damage degree of medium-span bridge structure, is still a problem and needs to actively explore. It is suitable for the sensitive parameters of medium-span bridge structure damage and identification methods and is applied to engineering practice. With imperfect measurements of the mode forms in mind, the natural frequencies and mode shapes were employed as input parameters to the neural network for damage diagnosis. In order to



(a)



(b)

**Figure 8:** (a) Classification analysis of BP neural network on training set; (b) classification analysis of BP neural network on the test set.



illustrate the precision and effectiveness of the suggested method, numerical example analyses on truss bridges are presented.

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**Conflict of interest:** The authors declare that they have no competing interest.

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