Machireddy Gnaneswara Reddy*

Influence of Lorentz force, Cattaneo-Christov heat flux and viscous dissipation on the flow of micropolar fluid past a nonlinear convective stretching vertical surface

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Abstract: The problem of micropolar fluid flow over a nonlinear stretching convective vertical surface in the presence of Lorentz force and viscous dissipation is investigated. Due to the nature of heat transfer in the flow past vertical surface, Cattaneo-Christov heat flux model effect is properly accommodated in the energy equation. The governing partial differential equations for the flow and heat transfer are converted into a set of ordinary differential equations by employing the acceptable similarity transformations. Runge-Kutta and Newton's methods are utilized to resolve the altered governing nonlinear equations. Obtained numerical results are compared with the available literature and found to be an excellent agreement. The impacts of dimensionless governing flow pertinent parameters on velocity, micropolar velocity and temperature profiles are presented graphically for two cases (linear and nonlinear) and analyzed in detail. Further, the variations of skin friction coefficient and local Nusselt number are reported with the aid of plots for the sundry flow parameters. The temperature and the related boundary enhances enhances with the boosting values of M. It is found that fluid temperature declines for larger thermal relaxation parameter. Also, it is revealed that the Nusselt number declines for the hike values of B_i.

Keywords: Magnetohydrodynamics; micropolar fluid; heat transfer; convective boundary condition; Cattaneo-Christov heat flux

***Corresponding Author: Machireddy Gnaneswara Reddy,** Department of Mathematics, Acharya Nagarjuna University Campus, Ongole -523001, A.P, India, E-mail: mgrmaths@gmail.com

1 Introduction

Micropolar fluids has come into prominence because of the conventional Newtonian fluids cannot accurately represent the features of fluid flow in various engineering applications namely polymeric fluids, paints and colloidal solutions, biology etc. A class of fluids has also considering its microstructure is known as micropolar fluids. Micropolar fluids are physically represent fluids in which the deformation of the particles omitted in the fluids and consisting of a suspension of rigid, spherical particles in a viscous medium. Initially, Eringen [1, 2] has been reported an analysis of theory of micropolar fluids which provided a mathematical model for its non-Newtonian flows in his studies. An analysis of boundary-layer theory for a micropolar fluid has extended by Peddieson and McNitt [3]. Heat transfer flow in a polar fluid towards a non-isothermal stretching surface with suction and blowing have been numerically investigated by Hassanien and Gorla [4]. For the formulation of mathematic model they are utilizing the Erigen theory of micropolar fluids. Rama Subba Reddy [5] have investigated the problem of mixed convection boundary layer on unsteady micropolar flow over a vertical plate and reveals that the friction factor and rate of heat transfer enhances to the improve of buoyancy force. Hady [6] studied an analytical solution of heat transfer in micropolar fluid over a non-isothermal permeable stretching sheet. The flow characteristics of micropolar fluid in moving wedge and flat plate have been analyzed by Ishak et al. [7]. By the same authors [8] have presented to the heat transfer analysis on micropolar fluids in a stretching surface. Mixed convection flow of a micropolar fluid towards a non-linear stretching surface investigated Hayat et al. [9]. The series solution retained by imposing HAM to the transformed coupled non-linear ordinary differential equations. The problem of micropolar fluid filled a porous channel was discussed by Sajid et al. [10]. Heat transfer on micropolar fluid flow over a stretchable melting surface was discussed by Animasaun [11].

Magnetohydrodynamics (MHD) has several momentous applications in the fields of aerospace engineering, medicine and geo and astrophysics. Several equipments including turbulent pumps, bearings, MHD generators and boundary layer control flow are influenced due to the interaction between the electrically conducting liquid and magnetic field. Ashraf and Batool [12] studied the magnetohydrodynamic flow and heat transfer behavior of a micropolar fluid past a stretchable disk. It is predict that the fluid flow of magnetic field strengthen the skin friction coefficient. Mahmoud and Waheed [13] studied heat transfer flow in micropolar fluid through a stretching sheet in the presence of magnetic field. Heat generation effect on steady hydromagnetic flow of micropolar fluid over a moving surface was presented by Gnaneswara Reddy [14]. Viscous dissipation effect for the convection flow of a micropolar fluid towards a stretching surface is examined by El-Aziz [15]. Gnaneswara Reddy and Venugopal Reddy [16] have reported the interaction of Joule heating on magnetohydrodynamic peristaltic transport of a Nanofluid. The interaction of heat transfer and hydromagnetic on a rotating micropolar fluid was recently analyzed by Mehmood et al. [17]. Gnaneswarara Reddy [18, 19] have reported the effect of viscous dissipation on steady MHD flow over stretched surface. The peristaltic transport of Jeffrey nanofluid over an asymmetric channel with magnetic field investigated recently Gnaneswara Reddy and Makinde [20]. Makinde and Animasaun [21] have investigated the heat transfer analysis on MHD bioconvection of nanofluid in paraboloid of revolution. Koriko et al. [22] have studied the effect MHD micropolar fluid flow along a vertical surface.

The classical Fourier's [23] heat conduction is amongst the best utilized models in classical physics and which is used for the description of heat transfer transport mechanism in various associated situations. The one of major limitation for parabolic energy equation for temperature is that unique disturbance and simultaneously it dispute the principle of causality throughout whole medium. Fourier's law of heat conduction is modified in context of thermal relaxation by Cattaneo [24]. Hyperbolic type energy equation exists in presence of Cattaneo's expression. Christov [25] improved the heat conduction analysis of Cattaneo [24] by introducing thermal relaxation time along with Oldroyd's upper-convicted derivative, in order to attain the material invariance of the model. Cattaneo–Cheristov heat flux model is the modified Fourier's form which gives thermal relaxation time. Cattaneo-Christov model with influence of thermal convection was studied by Straughan [26]. Uniqueness of Cattaneo-Christov heat flux model for flow of incompressible fluids has been analyzed Tibullo and Zampoli [27]. Han et al. [28] reported the Cattaneo-Christov heat flux model on boundary layer flow of a Maxwell fluid. Mustafa [29] investigated the interaction of Cattaneo–Christov heat flux model for rotating flow of a Maxwell fluid with heat transfer. The problem of Cattaneo-Christov heat flux model on Viscoelastic Flow over an exponentially stretching surface is recently discussed khan et al. [30]. Impact of temperature dependent thermal conductivity fluid and Cattaneo–Christov heat flux flow of over a surface is examined Hayat et al. [31].

In view of the above cited studies and applications, still no study on the Cattaneo-Christov heat flux and MHD convection flow of micropolar fluid towards nonlinear stretched surface. The convective condition is also considered in the energy boundary conditions. The dimensionless nonlinear partial differential equations are transformed into ordinary differential equations by utilizing suitable similarity transformations and are solved numerically. A detailed parametric study is presented graphically to explore the interaction of various controlling flow physical parameters on velocity, micro-rotation and temperature distribution. Also the friction factor and local Nusselt number are plotted and analyzed for emerging dynamical parameters.

2 Mathematical Analysis

Consider a two-dimensional free convection flow of a viscous and an incompressible electrically conducting micropolar fluid induced by a nonlinear stretching surface. The physical model of the present flow problem is shown in Fig. 1. It is also further considered a general power-law surface velocity distribution $u_w = ax^n$ with $a > 0$. There is a non-uniform applied magnetic field of strength $B(x) = B_0 x^{\frac{n-1}{2}}$ in the direction of y-axis which is normal to x-axis. The Cattaneo-Christov heat flux model is considered in the heat transfer analysis. It is assumed that the energy at the place is passively adjusted through heated fluid of temperature T_f below the surface of the wall. Let T_∞ be the temperature outside the thermal boundary layer. The thermal conductivity of fluid is not constant is retained. The effects of viscous dissipation and convective condition are also invoked. It is also assumed that the electric field is absent whereas the induced magnetic field is neglected by assuming low magnetic Reynolds number. Under the above the assumptions and boundary layer approximation, the governing equation of motion and energy [4, 31, 33] are given by

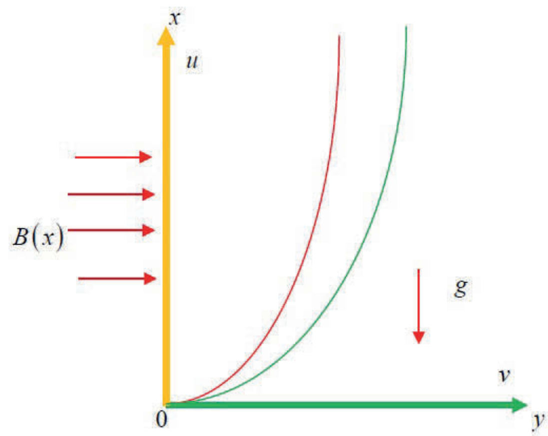

Fig. 1: Schematic diagram of the flow problem.

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(v+\frac{\kappa}{\rho})\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial N}{\partial y}+g\beta(T-T_\infty) -\frac{\alpha B^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\kappa}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_1\left(u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y} +v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}+2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}\right)=\frac{1}{\rho c_p}\frac{\partial^2 T}{\partial y^2} +\frac{1}{c_p}\left(v+\frac{\kappa}{\rho}\right)\left(\frac{\partial u}{\partial y}\right)^2+\frac{\alpha B^2(x)}{\rho c_p} \tag{4}$$

and the relevant boundary conditions for the present model are

$$u=u_W=ax^n,\ v=0,\ N=-m_0\frac{\partial u}{\partial y},$$
$$-k_1\frac{\partial T}{\partial y}=h_f(T_f-T)\text{ at }y=0$$
$$u\to 0,\ N\to 0,\ T\to T_\infty,\ \text{as } y\to\infty \tag{5}$$

where the components u and v are the velocities along x- and y-axis respectively, k is the vortex viscosity, ρ is the fluid density, μ is the dynamic viscosity, g is the gravitational acceleration, β is the thermal expansion coefficient, v is the kinematic viscosity, $\gamma^*=\left(\mu+\frac{k}{2}\right)j$ is the spin gradient viscosity, $\alpha=\frac{k_1}{\rho c_p}$ is the thermal diffusivity, N is the micro rotation velocity, $j=v/a$ is the micro inertia, k is the thermal conductivity, T_f is the convective fluid temperature, c_p is the specific heat at constant pressure, m_0 is the boundary parameter, and h_f is the convective heat transfer coefficient.

Define the following similarity transformations and variable quantities:

$$\Psi=\sqrt{\frac{2}{n+1}vax^{n+1}}f(\eta),\ \eta=\sqrt{\frac{a(n+1)}{2v}}x^{\frac{n-1}{2}}g(\eta),$$
$$N=ax^n\sqrt{\frac{a(n+1)}{2v}}x^{\frac{n-1}{2}}g(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$
$$u=cx^nf'(\eta),\ v=\sqrt{\frac{cv(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right],$$
$$K=\frac{k}{\mu},\ \lambda=\frac{Gr_x}{Re_x^2},\ Pr=\frac{v}{\alpha},\ Re_x=\frac{u_wx}{v},$$
$$Gr_x=\frac{g\beta(T_w-T_\infty)x^3}{v^2},\ M^2=\frac{\sigma B^2(x)}{a\rho x^{n-1}},\ \gamma=\lambda_1 ax^{n-1},$$
$$Ec=\frac{u_w^2}{c_p(T_w-T_\infty)},\ Bi=\frac{h_f}{k_1}\sqrt{\frac{v}{a}}x^{-\left(\frac{n-1}{2}\right)} \tag{6}$$

It is obvious that the continuity equation (1) is automatically satisfied. The remaining governing flow equations (2) – (4) and corresponding boundary conditions (5), which are reduced to the following dimensionless, coupled ordinary differential equations

$$(1+K)f'''+ff''-\frac{2n}{n+1}f'^2+Kg'+\frac{2}{n+1}\lambda\theta -\frac{2}{n+1}M^2f'=0, \tag{7}$$

$$\left(1+\frac{K}{2}\right)g''+fg'-\frac{3n-1}{n+1}f'g-\frac{2K}{m+1}(2g+f'')=0, \tag{8}$$

$$\theta''+Pr\left[\gamma\left(\frac{n-3}{2}ff'\theta'-\frac{n+1}{2}f^2\theta''\right) +\theta\left(\gamma ff''-\frac{2n}{n+1}\gamma f'^2-\frac{2}{n+1}f'\right)+f\theta' +Ec(1+K)f''^2+EcM^2f'^2\right]=0, \tag{9}$$

and the reduced conditions are:

$$f(0)=0,\ f'(0)=1,\ g(0)=-m_0f'(0),\ \theta'(0)=-Bi(1-\theta(0)),$$
$$f'(\eta)\to 0,\ g(\eta)\to 0,\ \theta(\eta)\to 0 \text{ as } \eta\to\infty \tag{10}$$

where primes indicates differentiation with respect to η, K the micro polar parameter, λ the mixed convection parameter, Gr_x the thermal buoyancy parameter, Re_x the local Reynolds number, M the Hartman number, Pr the Prandtl number, γ the thermal relaxation parameter, Ec the Eckert number, and Bi the Biot number.

For physical quantities of interest, the local friction factor coefficient C_f with surface shear stress τ_w and local Nusselt number Nu_x with surface heat flux q_w are given by

$$C_f = \frac{2\tau_w}{\rho u_w^2} \tag{11}$$

$$Nu_x = \frac{x q_w}{k_1(T_f - T_\infty)} \tag{12}$$

where the quantities τ_w and q_w are given by

$$\tau_w = \left((\mu + k)\frac{\partial u}{\partial y} + kN\right)_{y=0} \tag{13}$$

$$q_w = -k_1\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{14}$$

By incorporating the similarity transformations and quantities in to Equations (11) and (12), the reduced dimensionless skin friction coefficient and local Nusselt number are given by

$$C_f Re_x^{1/2} = \sqrt{\frac{n+1}{2}}(1 + (1 - m_0)K)f''(0) \tag{15}$$

$$Nu_x Re_x^{1/2} = -\sqrt{\frac{n+1}{2}}\theta'(0) \tag{16}$$

where $Re_x = \frac{u_w x}{\nu}$ is the local Reynolds number.

3 Numerical Solution

The final dimensionless ordinary differential equations (7) – (9) with the boundary restrictions (10) are not possible to find the exact solutions due to the equations are highly nonlinear and coupled. Hence there equations are resolved numerically with Runge-Kutta based shooting method. Initially, the set of nonlinear ordinary differential equations (7), (8) and (9) converted to first order ordinary differential equations, by using the following procedure:

$$\begin{aligned} &f = y_1,\ f' = y_2,\ f'' = y_3, \\ &f'' = \frac{1}{1+K}\left[-y_1y_3 + \frac{2n}{n+1}y_2^2 - ky_5 - \frac{2}{n+1}\lambda y_6 \right. \\ &\left. \quad +2\frac{2}{n+1}M^2 y_2\right] \end{aligned} \tag{17}$$

$$\begin{aligned} &g = y_4,\ g' = y_5, \\ &g'' = \frac{1}{(1+K/2)}\left[-y_1y_5 + \frac{3n-1}{n+1}y_2y_4 + \frac{2K}{n+1}(2y_4 + y_3)\right] \end{aligned} \tag{18}$$

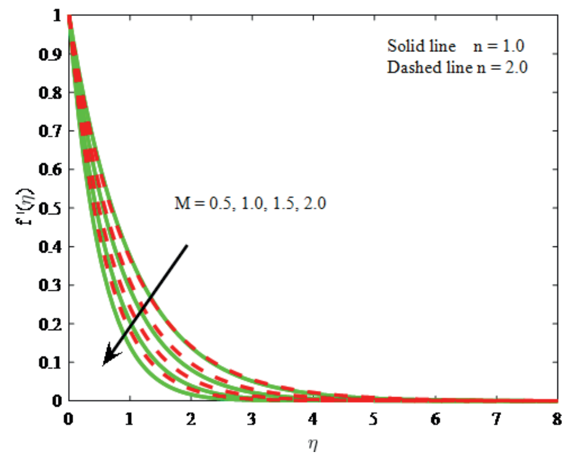

Fig. 2: Impact of M on velocity.

$$\begin{aligned} &\theta = y_6,\ \theta' = y_7,\ \theta'' = \frac{1}{(1 - Pry(\frac{n+1}{2})y_1^2)} \\ &\left[-Pr\left(yy_1y_3 - \frac{2n}{n+1}yy_2^2 - \frac{2}{n+1}y_2\right) - \epsilon y_7^2 \right. \\ &-Pry\left(\frac{n-3}{2}y_1y_2y_7\right) - PrEcM^2y_2^2 - Pry_1y_7 \\ &\left. -PrEc(1+K)y_3^2\right] \end{aligned} \tag{19}$$

with the corresponding boundary conditions

$$y_1 = 0,\ y_2 = 1,\ y_4 = -m_0y_2,\ y_7 = -Bi(1 - y_6) \text{ at } \eta = 0 \tag{20}$$

$$y_2 = 0,\ y_4 = 0,\ y_6 = 0,\ \text{ at } \eta \to \infty \tag{21}$$

First guess the values of undefined initial conditions $y_3(0)$, $y_5(0)$ and $y_7(0)$ which are involved in equations (17) – (19). Once all initial conditions are found then we solve the equations (17) – (19) are integrated by using Runge-Kutta fourth order method with the successive iterative step length is 0.01.

The validation of the present obtained results for $-f''(0)$ by comparing with the earlier published works when $Pr = 1$, $\epsilon = 0$, and $y = 0$ is presented in Table 1. It can conclude a nice agreement of the present results with the existed literature of Cortell [32] and Waqas et al. [33]. This provides the validity of the present obtained results along with the accuracy of the numerical technique we used in the present study. It is seen that the friction factor is an enhancement for the uplifting values of M.

Table 1: Comparative of the present numerical results of $-f''(0)$ for the variations of n and M with the published literature when $Pr = 0.7$, $\epsilon = 0$, and $\gamma = 0$.

		Cotell [32]	Waqas et al. [33]	Present results
1	0.5	0.86595	0.86596	0.865956
	1.0	1.09705	1.09705	1.097058
	2.0	1.75371	1.75372	1.753714
	3.0	2.74458	2.74459	2.744580
2	0.5	0.95029	0.95028	0.950290
	1.0	1.10152	1.10153	1.101523
	2.0	1.57268	1.57268	1.572680
	3.0	2.42941	2.42940	2.429416

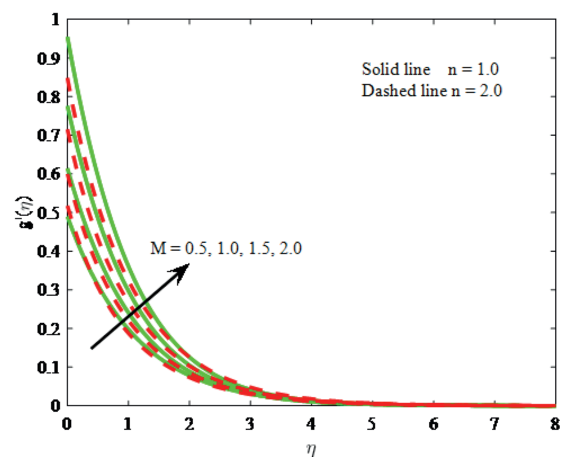

Fig. 3: Impact of M on micro-rotation velocity.

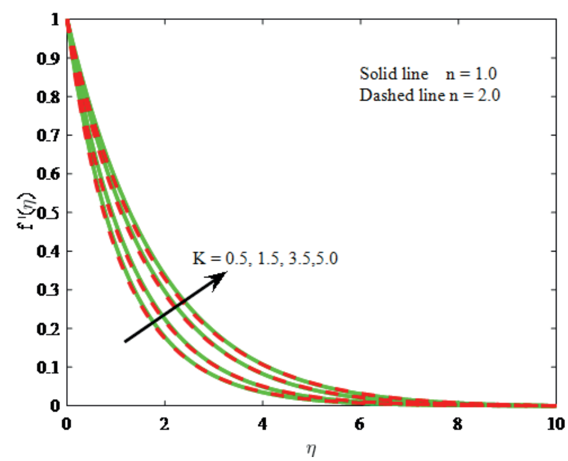

Fig. 5: Impact of K on velocity.

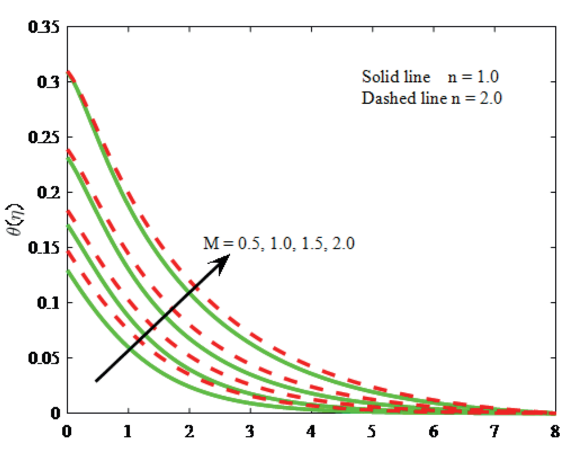

Fig. 4: Impact of M on temperature.

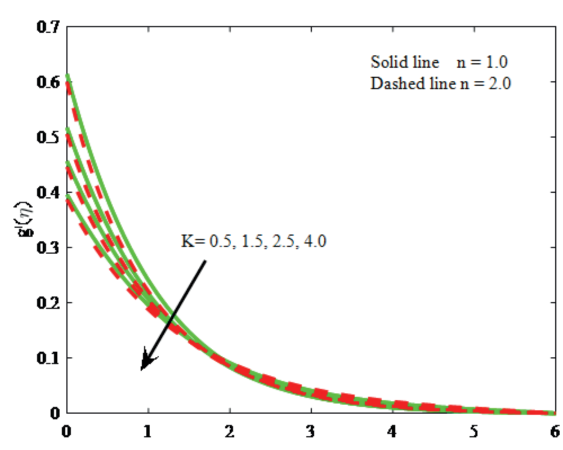

Fig. 6: Impact of K on micro-rotation velocity.

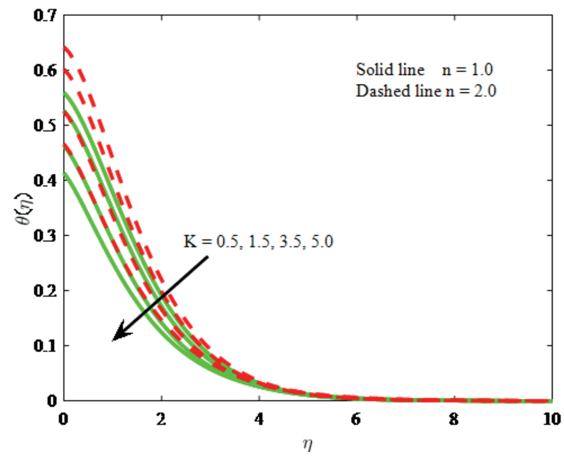

Fig. 7: Impact of K on temperature.

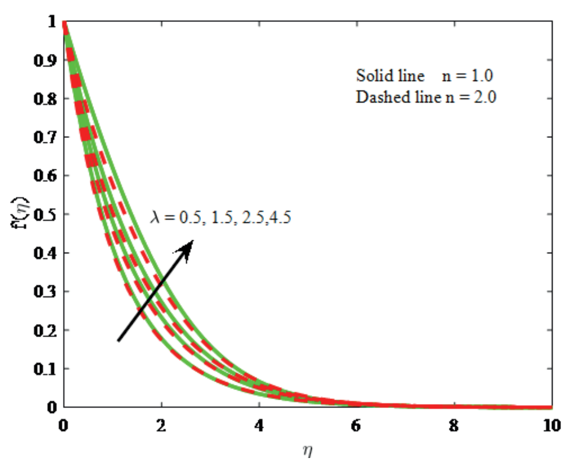

Fig. 8: Impact of λ on velocity.

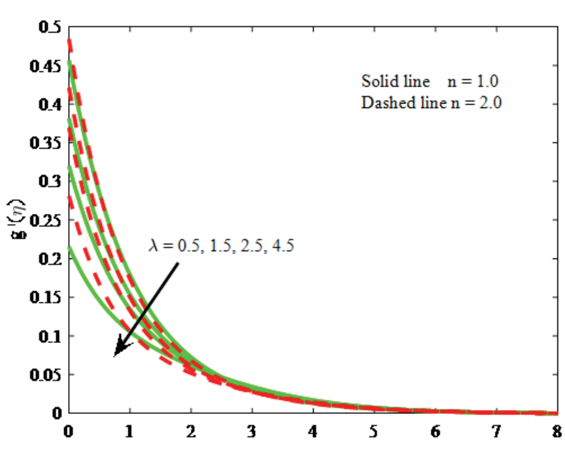

Fig. 9: Impact of λ on micro-rotation velocity.

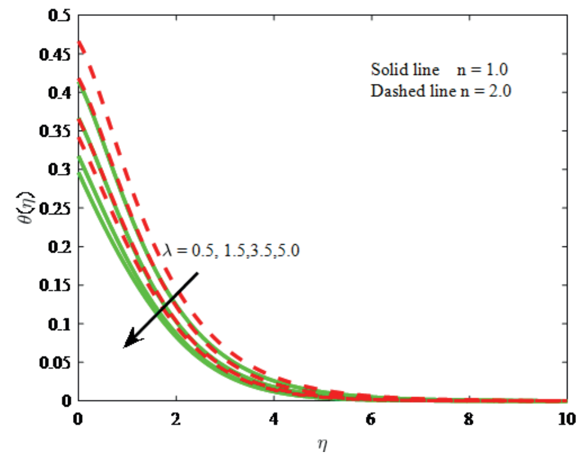

Fig. 10: Impact of λ on temperature.

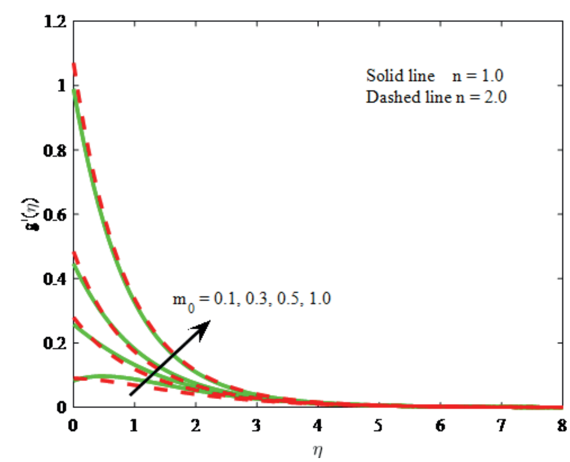

Fig. 11: Impact of m_0 on micro-rotation velocity.

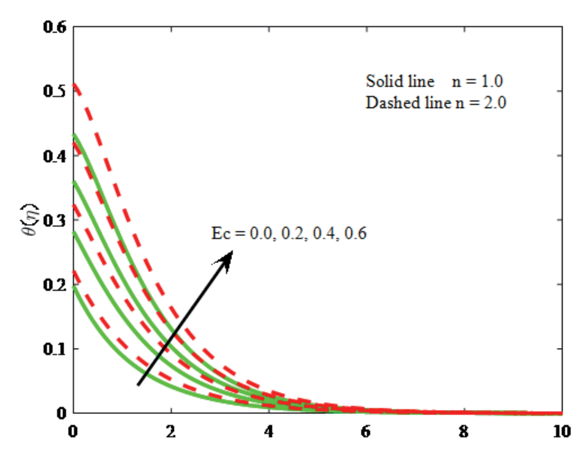

Fig. 12: Impact of Ec on temperature.

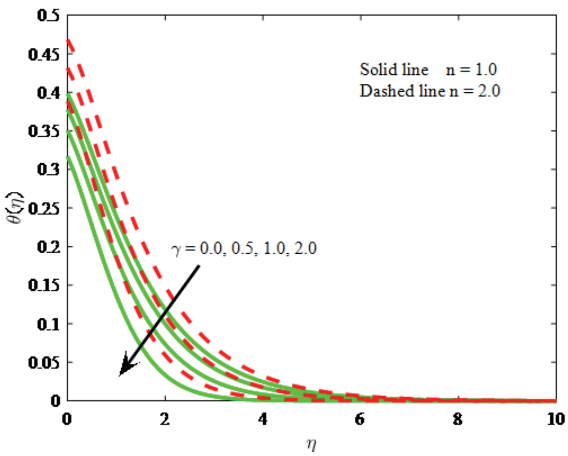

Fig. 13: Impact of γ on temperature.

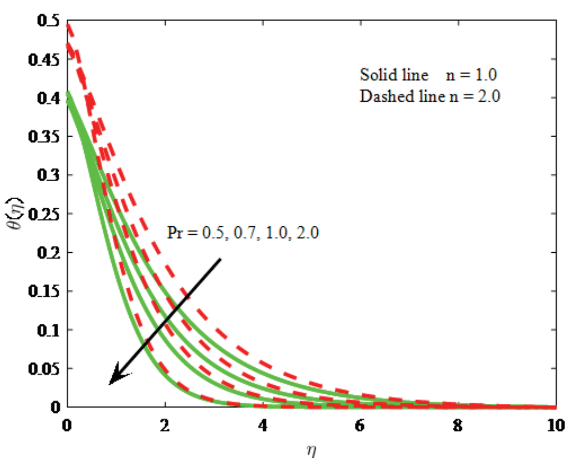

Fig. 14: Impact of Pr on temperature.

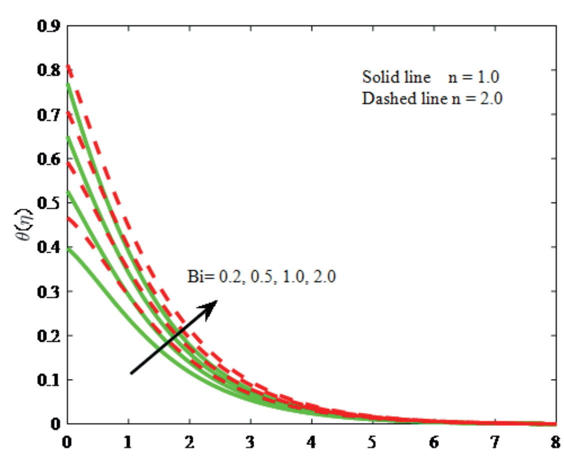

Fig. 15: Impact of Bi on temperature.

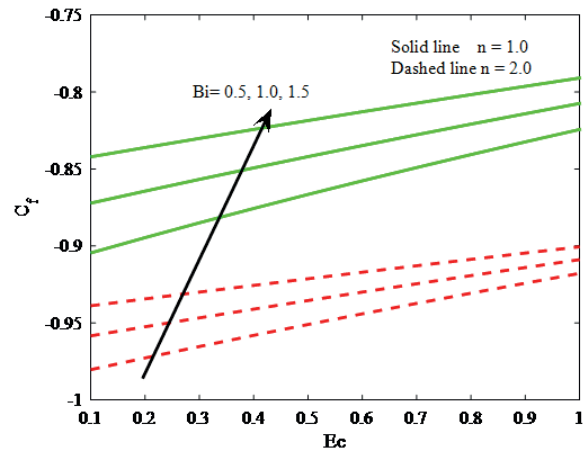

Fig. 16: Impacts of Ec and Bi on friction factor.

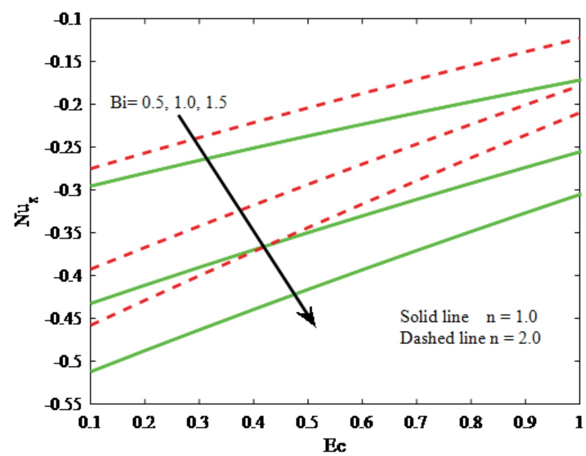

Fig. 17: Impacts of Ec and Bi on Nusselt number.

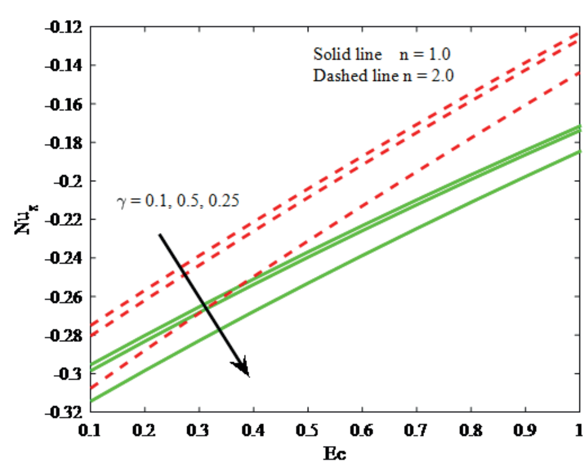

Fig. 18: Impacts of Ec and γ on Nusselt number.

4 Results and Discussion

The dimensionless velocity, micro-rotation velocity and temperature distributions are presented graphically in Figs. 2 – 14 to the effect of various pertinent dynamical parameters such as magnetic field parameter M, material parameter K, mixed convection parameter λ, boundary parameter m_0, thermal relaxation parameter y, Prandtl number Pr, Eckert number Ec, and Biot number Bi. For numerical computations acquiring, we have utilize fixed values of the dimensionless numbers for this study: $M = 1.0$, $K = 0.5$, $\lambda = 0.3$, $Pr = 2.0$, $Ec = 0.1$, $y = 0.5$, $m_0 = 0.5$, $Bi = 0.5$. The solid (Green color) and dashed (Red color) lines signify the profiles of linear and non-linear stretching surface respectively.

The influence of magnetic field parameter on velocity, mirco-rotation and dimensionless temperature are described in Figs. 2 – 4 respectively. It is revealed from Fig. 2 that for escalating values of magnetic field parameter decreases the magnitude of velocity distribution and reduced the related boundary layer thickness. This is due to an increase in magnetic field on the fluid flow produces a resistance fore and which is called the Lorentz force. Generally, escalating the values of M generates Lorentz force in the flow, which is capable of declining the flow velocity by acting in the opposite direction of the flow. From Fig. 3 we can found that the mirco-rotation velocity distribution is rises with boosting values of M. The fluid temperature enhances with the boosting values of M and this is evident from Fig. 4. As a consequence, the heat will be generated in the flow field, due to which a hike in the fluid temperature is also perceived and simultaneously increased the thickness thermal boundary layer. Furthermore, the impact of magnetic field parameter on mirco-rotation and temperature is similar trend but the opposite for the velocity. It is further noticed that thermal and micro-rotation velocity boundary layers of linear stretching are low by the Lorentz force when compared with the non-linear stretching surface.

Figs. 5 – 7 depict the flow, micro-rotation velocity and temperature fields for different values of K. It is manifest that velocity distribution increased for higher values micropolar parameter. Because larger values of micropolar parameter which corresponds to the low viscosity and enhance the velocity. It can be found that initially angular velocity reduces by increasing of K while the opposite behavior to the free stream velocity. Temperature distribution is diminishes with the larger values of micropolar parameter. Furthermore, the micro-rotation velocity and energy boundary layers of linear stretching are opposite for the micropolar parameter when compared with the non-linear stretching surface.

Figs. 8 – 10 illustrate the velocity, micro-rotation velocity and thermal distribution respectively for various values convection parameter λ. From Fig. 8, the velocity boosts with increasing values of mixed convection parameter λ. Because increase in mixed convection parameter to enhance the buoyancy force and this is evident in the progressive increase in the flow velocity. The angular velocity is decreases there is rise in the buoyancy parameter. It is noticed that the temperature diminishes with enhancing the mixed convection parameter λ. The impacts of micropolar parameter K and mixed convection parameter λ have similar on the three common profiles. The effect of boundary parameter m_0 on the angular velocity is portrays in Fig. 11. The angular velocity is enhanced for larger boundary parameter m_0.

Fig. 12 is drawn to observe the variation in temperature distribution under the action of Eckert number Ec. The temperature increases for larger values Eckert number. The increase in the buoyancy force due to an increase in the viscous dissipation parameter rise the temperature. Also, at this effect the fluid temperature is small for the linear while compared to the non-linear. Fig. 13 demonstrates the effect of thermal relaxation parameter (y) on dimensionless fluid temperature profile. It can be seen that the fluid temperature and energy boundary layer diminished for larger values of thermal relaxation parameter y. This is due to reason that for larger thermal relaxation parameter the particles of the material require more opportunity to transfer heat to its adjacent particles. It is found that the heat transfers quickly throughout the objects for $y = 0$. Also, the temperature is lower for Cattaneo-Christov heat flux model when compared with Fourier's law. In addition, the energy boundary layer thickness is higher for the non-linear stretching surface. Fig. 14 is sketched to study the impact of Prandtl number Pr on dimensionless temperature field. It is concluding that both the thermal boundary layer thickness and temperature diminish to the enhancing values of Prandtl number ($Pr = 0.5, 0.7, 1.0, 2.0$). It is also seen that accumulation perception temperature is considerable larger at $Pr = 0.5$ when compared with $Pr = 2.0$. This is due to reason that thermal diffusivity increases for diminishing values of Pr. Fig. 15 is plotted for the variations of Biot number Bi on temperature profiles. Increasing values of Biot number boosts the dimensionless temperature. Because the Biot number contains the heat transfer coefficient and which enhances for larger values of Biot number. It is further revealed that the temperature is high for the non-linear stretching surface.

Finally, the physical quantities of interest such as the skin friction coefficient and Nusselt number for sundry values of dimensionless governing flow field parameters are plotted in Figs. 16 – 18 respectively. Impacts of Ec and Bi on skin friction coefficient is depicted in Fig. 16. It is found that raising the values of Biot number and Eckert number enhances the friction factor. Also, the friction factor is more for the linear stretching surface. Fig. 17 displays the impact of Ec and Bi on the Nusselt number. It is revealed that the Nusselt number declines for the hike values of Bi. Fig. 18 illustrates the variation in Nusselt number for the influence of Ec and γ. It is manifest that boosting values of diminishes the reduced Nusselt number.

5 Conclusions

A Numerical investigation has been carried out for analyzing MHD flow of a micropolar fluid past a nonlinear stretched sheet with Cattaneo-Christov heat flux and viscous dissipation. The dimensionless governing equations are solved numerically using Runge–Kutta based shooting technique. The main findings of the present study are:

1. Temperature increases with the hike values of M.
2. Thermal and micro-rotation velocity boundary layers of linear stretching are low by the Lorentz force when compared with the non-linear stretching surface.
3. Fluid velocity enhanced for escalating values of K.
4. Angular velocity is enhanced for larger boundary parameter m_0.
5. Temperature and energy boundary layer diminished for larger values of thermal relaxation parameter γ.
6. Nusselt number declines for the hike values of Bi.

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Nomenclature

Bi	Biot number
C_f	local friction factor co-efficient
c_p	specific heat at constant pressure
Ec	Eckert number
Gr_x	thermal buoyancy parameter
g	Gravitational acceleration
h_f	convective heat transfer coefficient
j	Micro inertia
K	micropolar parameter
κ	vortex viscosity
k_∞	thermal conductivity of ambient fluid
M	Hartman number
m_0	boundary parameter
N	micro-rotation velocity
Nu_x	Nusselt number
Pr	Prandtl number
q_w	surface heat flux
Re_x	local Reynolds number
T	temperature of the fluid
T_f	convective fluid temperature
T_w	stretching sheet temperature
T_∞	temperature far away from the stretching sheet
u	velocity of the fluid along the x-axis
v	velocity of the fluid along the y-axis
ρ	fluid density
μ	dynamic viscosity
β	thermal expansion coefficient
ν	kinematic viscosity
$\gamma^\star$	spin gradient viscosity
α	thermal diffusivity
λ	mixed convection parameter or buoyancy parameter
γ	thermal relaxation parameter
τ_w	surface shear stress

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