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Modelling of Imbibition Phenomena in Fluid Flow through Heterogeneous Inclined Porous Media with different porous materials

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Abstract: In this paper, the counter – current imbibition phenomenon is discussed in an inclined heterogeneous porous media with the consideration of two types of porous materials like volcanic sand and fine sand. Adomian decomposition method is applied to find the saturation of wetting phase and the recovery rate of the reservoir. Finally, a simulation result is developed to study the saturation of wetting phase and the optimum recovery rate of reservoir with the choices of some interesting parametric values. This problem has a great importance in the field of oil recovery process.

Keywords: Counter – current imbibition, Heterogeneous porous media, Brooks Corey model Adomian decomposition method

MSC: 76S05, 74R10

1 Introduction

This paper discusses mathematically the phenomenon of imbibition in an inclined heterogeneous porous media with the consideration of capillary pressure and different porous materials. When the reservoir oil (non-wetting phase) comes into contact with water (wetting phase) then there is a spontaneous flow of the wetting phase (Water) into the medium and a counter flow of the resident fluid i.e. non wetting phase (oil) from the medium initiated by imbibition. Due to the difference in viscosities of water and oil, the water saturation on the right side of imbibition face will travel only a small distance 'l' due to capillary pressure effect (without external force) initiated by imbibition. Oil recovery by imbibition process is accomplished by contacting water with the porous solid. The water is then imbibed into the pore matrix, where upon reaching the regions of oil saturation, the water will encroach along the solid surface causing a portion of trapped oil to be displaced. Water imbibition is an alternative enhanced oil production strategy capable of recovering a portion of this trapped oil through a replacement mechanism that exchanges water for oil. The effectiveness of this process depends on several parameters; including matrix block size, rock porosity and permeability, fluid viscosities, interfacial tensions, and rock wettability.

The following empirical function first proposed by Aronofsky et al. (1958) to study the recovery rate of the reservoir:

$$R = R_\infty \left(1 - e^{-yT}\right)$$

Where R is the recovery, R_∞ is the ultimate recovery and y is a constant that best matches the data with a value of approximately 0.5. It was proposed for strongly water-wet media and ignores the effects of wettability and here we have defined the dimensionless time as $T = \frac{K p_d}{\phi \mu_w L^2} t$ for studying the recovery rate with dimensionless time.

Zeybek et al. [21] discussed the capillary imbibition in porous structures and studied the simulation results numerically with the study of the effect of heterogeneity of permeability and wettability on counter – current and co – current imbibition. Zhou et al [22] investigated the effect of wettability and aging time on oil recovery and concluded that wettability has a significant impact on the imbibition rate. Hughes and Blunt [5] found that the pattern of displacement and the rate of imbibition is depends on the relationship between capillary number, contact angle and initial wetting phase saturation. Pooladi–Darvish and Firoozabadi [11] modelled the co – and counter – current imbibition in water–wet rocks. According to their findings, when porous media are partially covered by water, flow is dominated by co–current imbibition and concluded that the oil recovery from co – current imbibition is higher than the one from the counter – current. Shah and Verma [19] studied the multiphase flow through slightly dipping porous media with magnetic fluid. Karpyn et al.

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[7] identified three distinct flow intervals during spontaneous imbibition in layered sandstone with a single longitudinal fracture, where counter–current flow was dominant at early and intermediate times; while both co – current and counter – current flow mechanisms coexisted at late times. Hatiboglu and Babadagli [6] conducted imbibition experiments on aged Berea sandstone and limestone and investigated the effect of different pore structures. Darvishi et al. [3] investigated the effect of permeability on spontaneous imbibition using carbonate cores. They reported that water imbibed more easily in cores with higher permeability, even in oil–wet cores at connate water saturation. Meher et al. [8] discussed the series solution for porous medium equation arising in fingero–imbibition phenomenon during oil recovery process and concluded that at a fix distance saturation of wetting phase is increases with time. Patel et al. [13] discussed the Imbibitions phenomena in a multiphase flow through porous media and also discussed it under special conditions as homogeneous medium with capillary pressure, involving magnetic fluid, homogeneous medium inclined at small angle involving magnetic fluid. Desai et al. [4] discussed this phenomena in a homogeneous porous media by using Homotopy perturbation method and concluded that if permeability and capillary pressure is more, than the saturation of water will advances faster in the porous medium which will result oil recovery in lesser time. Similarly if the viscosity of the fluid and the porosity of the medium are high, the saturation of water will advances slowly into the medium which will result to oil recovery after a longer time. Mishra [9] discussed this phenomena in homogeneous porous media by using Homotopy perturbation transform method and observed that the saturation of water advances with the time and reaches to a constant value after a very long time. Meher et al. [10] discussed this phenomena with capillary pressure mathematically and concluded that the saturation of water be increases exponentially with distance X for any time $T > 0$. Patel et al. [14] studied this phenomena in heterogeneous porous media and concluded that the saturation rate as well as the recovery rate be more in homogeneous porous matrix as compared to heterogeneous porous media. Patel and Meher [15] studied the fingering phenomena in a fluid flow through fracture porous media with inclination and gravitational effect and developed a simulation result of saturation of wetting phase with the consideration of inclination effect for some interesting choices of parametric data and studied the recovery rate of the oil reservoir with dimensionless time and concluded that the saturation of wetting phase be more with zero inclination and in homogeneous porous media but as the inclination of the porous matrix be increases, the saturation rate be less as compared to zero inclination. Patel and Meher [16, 17] studied approximate analytical study of counter – current imbibition phenomenon in a heterogeneous porous media and a study on recovery rate for counter – current imbibition phenomenon with Corey's model arising during oil recovery process. Patel and Meher [18] studied simulation of counter – current imbibition phenomenon with Corey's Model in double phase flow through heterogeneous porous medium with capillary pressure.

To analyse the behavior of saturation of wetting phase in imbibition phenomena, an advance analytical approach i.e. Adomian Decomposition Method is applied here to study the behavior of solution along with its stability analysis in counter current case. The most advantage of Adomian Decomposition Method is that it assumes that the unknown function can be expressed as an infinite series and the non-linear operator can be decomposed in to a special series of polynomials referred as adomian polynomials and it converges faster than other numerical method. We defined a dimensionless time with almost all the parameters considered. These include porosity, permeability, size, shape, boundary conditions, wetting and non-wetting phase, relative permeability, wettability, gravity and different porous materials. The definition of the dimensionless time based on theoretical analysis of the fluid flow mechanisms that governed counter – current imbibition. A general analytical solution to the relation between recovery and saturation rate for counter – current imbibition is derived. The analytical solution predicts a correlation between the imbibition rate and recovery by counter – current imbibition in most fluid/fluid/rock systems.

Here we studied the effect of different porous materials like volcanic and fine sand on initial water saturation and the sensitiveness of imbibition phenomena to initial water saturation in an inclined heterogeneous porous media with the consideration of gravitational effect. The purpose of this study is to extend the comprehensive analysis of counter–current imbibition phenomena in a heterogeneous porous media done by Patel et al. [14] to an inclined heterogeneous porous media with the consideration of different types of porous materials like volcanic sand and fine sand and compared the obtained results with homogeneous porous media. Analytical solution for the flow equations is presented for the counter – current imbibition phenomena in an inclined heterogeneous porous matrix by using Adomian decomposition method to study the saturation rate of wetting phase and the simulation result is developed to study the recovery rate as a function of dimensionless time, T of the reservoir. The results obtained here are in perfect agreement with the physical situation.

This can be realized by conducting an experiment with the help of a capillary porous matrix having different porous material filled with oil. The saturation rate for different porous materials and its effect on capillary pressure and relative permeability can be verified from the expression obtained for saturation.it is of great significance in oil recovery, where it can be responsible to increase oil production up to 40%–50% in some cases.

2 Mathematical model

In secondary oil recovery process when water is injected into the oil reservoir through one well, it displaces the oil so that it can be extracted from a neighbouring well since the water is less viscous than the oil and the permeability of the rock is often highly heterogeneous. For the sake of mathematical model: We consider here that a finite cylindrical piece of porous matrix having an oil formatted region having length L and heterogeneous in nature that is completely surrounded by an impermeable surface except for one end (common interface) of the cylinder which is labelled as the imbibition face and this end is exposed to an adjacent formation of 'injected' water and inclined at an certain angle with the base surface. When the reservoir oil (non wetting phase) is come into contact with water (wetting phase) there is a disturbances happens at the interface causing imbibition due to the viscosities differences of both phases and the water saturates on the right side of imbibition face through a small distance 'l' due to the capillary pressure effect(without external force) causes the oil to imbibe on the left side through a small distance initiated by imbibition.The schematic diagram of the phenomena is shown in fig-1

The conservation of mass for multi-phase flow with respect to volume can be formulated as

$$\frac{\partial}{\partial t}\left(\phi(x)S_i\rho_i\right) + \nabla.\ (v_i\rho_i) - \rho_i q_i = 0 \quad (1)$$

Where $i = o, w,\ x \in \Re^3,\ t \geq 0$, $\phi(x)$ denotes the porosity of the porous medium, S_i is the saturation for each phase i, ρ_i is its specific mass, and v_i is its volumetric rate of flow (or, Darcy velocity) which is given by the two phase extension of Darcy's law

$$v_i \ = -K(x)\frac{k_i}{\mu_i}\left(\nabla p_i - \rho_i g \sin\alpha\right) \quad (2)$$

Where $i = o, w,\ x \in \Re^3,\ t \geq 0$, $K(x)[m^2]$ denotes the absolute permeability tensor of the porous medium, p_i is its pressure, k_i is its relative permeability and μ_i is its viscosity of the porous media, α is a inclination angle.

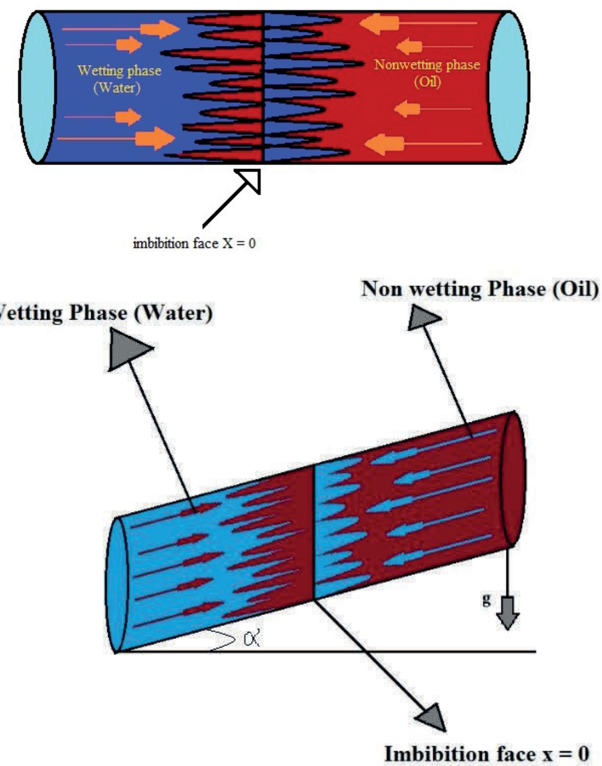

Fig. 1: schematic diagram of Imbibition phenomena

If the compressibility of fluid is neglected, then ρ_i's are constant then the conservation equation becomes:

$$\frac{\partial}{\partial t}\left(\phi(x)\,S_i\right) + \nabla v_i = 0, \quad i = o, w \quad (3)$$

The imbibition condition for counter–current imbibition and capillary pressure can be expressed (Patel et al. [12]) as

$$v_w = -v_o \quad (4)$$

$$p_c = p_w - p_o \quad (5)$$

Since the problem is dealing with the heterogeneous porous media so, the porosity and permeability of heterogeneous porous media can be expressed as

$$\phi(x) = \frac{1}{a(t) - b(t)x}$$

$$K(x) = K_c\phi(x)$$

The most famous $p_c - S_w$ relationships determined experimentally are those of (Brooks and Corey [2]):

$$p_c\,(S_w) = p_d S_e^{-\frac{1}{\lambda}} = p_d\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}} \quad (6)$$

Where p_d is the entry pressure, it represents the minimum pressure needed by a non-wetting fluid to displace the wetting fluid which initially fills in the porous medium, S_e is called effective saturation. The parameter λ describes grain size distribution in the medium.

The relative permeability, k_w of the wetting and non-wetting phases in the domain are governed by the following (Brooks and Corey [2]):

$$k_w = S_e^{\frac{2+3\lambda}{\lambda}} \tag{7}$$

Combining eq. (2), (4) and (5), we get

$$v_w = K(x)\frac{k_o k_w}{k_w\mu_o + k_o\mu_w}\left[\frac{\partial p_c}{\partial x} - (\rho_o - \rho_w)\, g \sin\alpha\right] \tag{8}$$

Hence the conservation eq. (3) with eq. (8) can be written as

$$\phi\frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K(x)\frac{k_o k_w}{k_w\mu_o + k_o\mu_w}\left\{\frac{\partial p_c}{\partial S_w}\frac{\partial S_w}{\partial x} - (\rho_o - \rho_w)\, g\sin\alpha\right\}\right] = 0 \tag{9}$$

By using eq. (6), (7) and (b) with eq. (9), it yields

$$\phi\frac{\partial S_w}{\partial t} + \frac{K_c\, p_d}{\mu_w}\frac{\partial}{\partial x}\left[\phi\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\left\{\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial x} - \frac{(\rho_o - \rho_w)\, g\sin\alpha}{p_d}\right\}\right] = 0 \tag{10}$$

Where $\frac{k_o k_w}{k_w\mu_o + k_o\mu_w} \approx \frac{k_w}{\mu_w}$ (Patel et al. [12])

Simplifying eq. (10) becomes

$$\frac{\partial S_w}{\partial t} + \frac{K_c\, p_d}{\mu_w}\left[\begin{array}{l} \frac{\partial}{\partial x}\left(\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\left\{\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial x}\right\}\right) + \left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\left\{\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial x}\right\}\frac{1}{\phi}\frac{\partial\phi}{\partial x} \\ -B\sin\alpha\left\{\frac{\partial}{\partial x}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}} + \left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{1}{\phi}\frac{\partial\phi}{\partial x}\right\} \end{array}\right] = 0 \tag{11}$$

Using the dimensionless variables $X = \frac{x}{L}$ and $T = \frac{K_c\, p_d}{\mu_w L^2}t$.

Simplification of $\frac{1}{\phi}\frac{\partial\phi}{\partial X}$ as

$\frac{1}{\phi}\frac{\partial\phi}{\partial X} = \frac{\partial}{\partial X}(\log\,\phi) = \frac{\partial}{\partial X}\left[\frac{b}{a}LX - \log\, a\right]$ (Neglecting higher order term of X) $= \frac{bL}{a}$

The dimensionless forms of eq. (11) can be written as

$$\begin{aligned} &\frac{\partial S_w}{\partial T} + \frac{\partial}{\partial X}\left[\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial X}\right] \\ &+ \frac{bL}{a}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial X} + B\sin\alpha\left[\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}} + \frac{bL}{a}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\right] = 0 \end{aligned} \tag{12}$$

Where $C = \frac{g(\rho_o - \rho_w)L}{\lambda\, p_d}$.

Equation (12) describes the counter–current imbibition phenomena in a heterogeneous porous media with inclination effect for different porous materials.

3 Analysis of the Method

For the purpose of illustration of the Adomian decomposition method, in this study we shall consider eq. (13) in an operator form as

$$L_T S_w(X, T) + L_X(N_1 S_w) + \frac{bL}{a}[N_1 S_w] + B\sin\alpha\,[N_2 S_w] = 0 \tag{13}$$

Where $N_1 S_w = \left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}\frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{-\frac{1}{\lambda}}\frac{\partial S_w}{\partial X}$, $N_2 S_w = \frac{\partial}{\partial S_w}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}} + \frac{bL}{a}\left(\frac{S_w - S_{wr}}{1 - S_{wr}}\right)^{\frac{2+3\lambda}{\lambda}}$ and S_{w0} can be solved subject to the corresponding initial condition $S(X, 0) = f(X) = e^{-X}$.

Following Adomian [1] defined the linear operators $L_T = \frac{\partial}{\partial T}$ and $L_X = \frac{\partial}{\partial X}$ the definite integration inverse operator L_T^{-1} and the nonlinear term as $N_1 S_w$ and $N_2 S_w$.

Operating the inverse operator and following the analysis of Adomian decomposition, we set the recursive relation of the Eq. (13) as

$$\sum_{n=0}^{\infty} S_{wn}(X,T) = e^{-X} + L_T^{-1}\left[L_X\left(\sum_{n=0}^{\infty} A_n\right)\right] + \frac{bL}{a}L_T^{-1}\left[\sum_{n=0}^{\infty} A_n\right] + C\sin\alpha L_T^{-1}\left[\sum_{n=0}^{\infty} B_n\right] \quad (14)$$

Which gives the recurrence relation as

$$S_{w0} = S_w(X,0) = e^{-X}$$
$$S_{w,k+1} = L_T^{-1}\left[L_X(A_k)\right] + \frac{bL}{a}(A_k) + C\sin\alpha\,(B_k),\ \ k \geq 0 \quad (15)$$

and it can be written in the series form up to four terms as

$$S_w(X,T) = S_{w0} + S_{w1} + S_{w2} + \cdots$$
$$S_w(X,T) = e^{-X} + \frac{e^{-X}}{\lambda^3 a(1-S_{wr})}\left(3C\sin\alpha\left(\lambda + \frac{2}{3}\right)((bL+3a)\lambda + 2a)e^{-2X}\left(e^{-X}\right)^{\frac{2}{\lambda}} + \lambda((bL-3a)\lambda - a)e^{-2X}\left(e^{-X}\right)^{\frac{1}{\lambda}}\right)T + \cdots \quad (16)$$

Equation (16) represents the saturation of wetting phase during counter–current imbibition phenomena in a heterogeneous porous media with inclination effect for different porous materials.

Table 1: Model parameters used in simulation

Property	λ	S_{wr}	$p_d\ (N/m^2)$	$K\ (\mu^2)$	ϕ
Volcanic sand	2.29	0.157	16	18	0.351
Fine sand	3.70	0.167	41	2.5	0.377

4 Numerical results and Discussion

4.1 Effect of inclination on initial water saturation for different porous materials

Fig. 2 and 3 discusses the variation of initial water saturation in heterogeneous porous media for volcanic and fine sand at fixed distance. It shows that the initial water saturation rate be more for $\alpha = 0°$ as well as in volcanic sand and less for $\alpha = 0°$ and in fine sand as compared to volcanic sand.

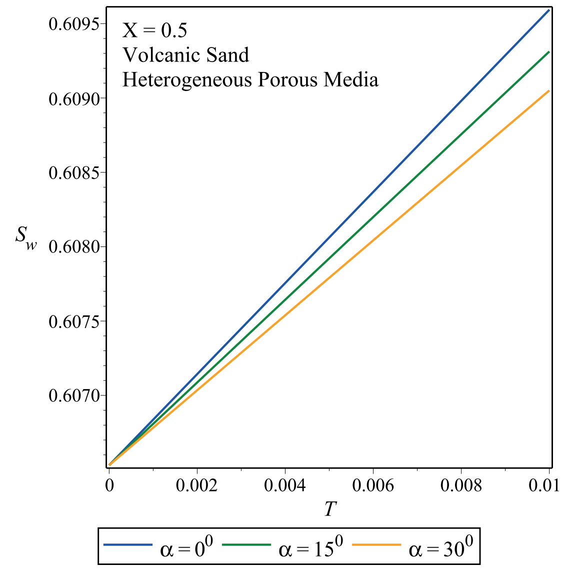

Fig. 2: Comparison of saturation vs. dimensionless times for different inclination $\alpha = 0°, 15°, 30°$ in Volcanic Sand

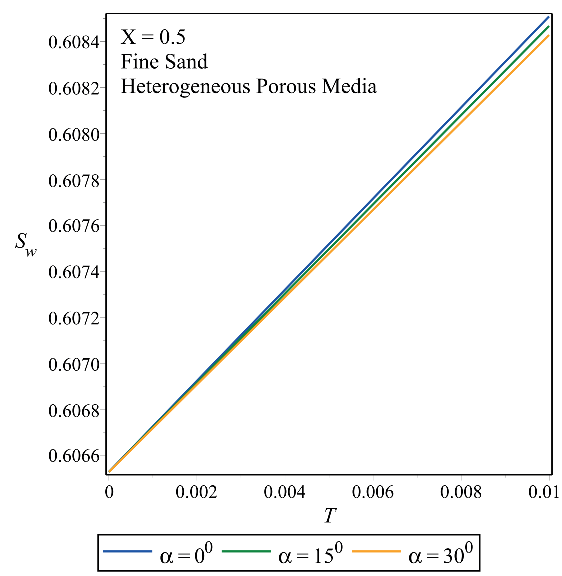

Fig. 3: Comparison of saturation vs. dimensionless times for different inclination $\alpha = 0°, 15°, 30°$ in Fine Sand

Table 2: Saturation of water in Volcanic Sand and in Fine sand for Heterogeneous inclined Porous Media

X = 0.5						
	Volcanic sand			Fine sand		
	$\alpha = 0°$	$\alpha = 15°$	$\alpha = 30°$	$\alpha = 0°$	$\alpha = 15°$	$\alpha = 30°$
T = 0.001	0.6068369601	0.6068088623	0.6067826753	0.6067286320	0.6067244237	0.6067205015
T = 0.002	0.6071432609	0.6070870653	0.6070346913	0.6069266046	0.6069181878	0.6069103434
T = 0.003	0.6074495616	0.6073652682	0.6072867072	0.6071245771	0.6071119520	0.6071001854
T = 0.004	0.6077558624	0.6076434711	0.6075387232	0.6073225496	0.6073057161	0.6072900274
T = 0.005	0.6080621631	0.6079216741	0.6077907391	0.6075205222	0.6074994803	0.6074798694
T = 0.006	0.6083684639	0.6081998770	0.6080427551	0.6077184947	0.6076932444	0.6076697113
T = 0.007	0.6086747646	0.6084780799	0.6082947710	0.6079164672	0.6078870086	0.6078595533
T = 0.008	0.6089810654	0.6087562828	0.6085467869	0.6081144398	0.6080807728	0.6080493953
T = 0.009	0.6092873661	0.6090344858	0.6087988029	0.6083124123	0.6082745369	0.6082392373
T = 0.010	0.6095936668	0.6093126887	0.6090508188	0.6085103849	0.6084683011	0.6084290792

4.2 Effect of heterogeneity on initial water saturation in volcanic sand

Figure 4 discusses the variation of initial water saturation in homogeneous as well as in heterogeneous porous media in volcanic sand for different inclined plane $\alpha = 0°$, $\alpha = 15°$ and $\alpha = 30°$. It shows that the initial water saturation rate be more in homogeneous porous media and for $\alpha = 0°$ as compared to heterogeneous porous media in volcanic sand.

4.3 Effect of heterogeneity on initial water saturation in fine sand

Figure 5 discusses the variation of initial water saturation in homogeneous as well as in heterogeneous porous media for fine sand for different inclined plane $\alpha = 0°$, $\alpha = 15°$ and $\alpha = 30°$. It shows that the initial water saturation rate be more in homogeneous porous media and for $\alpha = 0°$ as compared to heterogeneous porous media in fine sand.

4.4 Effect of Capillary pressure on initial water saturation in heterogeneous inclined porous matrix in volcanic sand at different inclination

Figure 6 discusses the variation of capillary pressure with initial water saturation in heterogeneous porous media in volcanic sand at different inclined plane. It shows that the capillary pressure be more in inclined plane having $\alpha = 30°$ as compared to zero inclined plane in volcanic sand.

4.5 Effect of Capillary pressure on initial water saturation in heterogeneous inclined porous matrix in Fine sand at different inclination

Figure 7 discusses the variation of capillary pressure with initial water saturation in heterogeneous porous media in fine sand at different inclined plane. It shows that the capillary pressure be more in fine sand and in inclined plane having $\alpha = 30°$ as compared to zero inclined plane and in fine sand.

4.6 Effect of Relative permeability on initial water saturation in heterogeneous inclined porous matrix in Volcanic sand at different inclination

Figure 8 discusses the variation of Relative permeability with initial water saturation in heterogeneous porous media in volcanic sand at different inclined plane. It shows that the value of relative permeability be more at zero inclined plane implies saturation rate be more as compared to the inclined plane having $\alpha = 30°$ in volcanic sand.

4.7 Effect of Relative permeability on initial water saturation in heterogeneous inclined porous matrix in Fine sand at different inclination

Figure 9 discusses the variation of Relative permeability with initial water saturation in heterogeneous porous media in fine sand at different inclined plane. It shows that

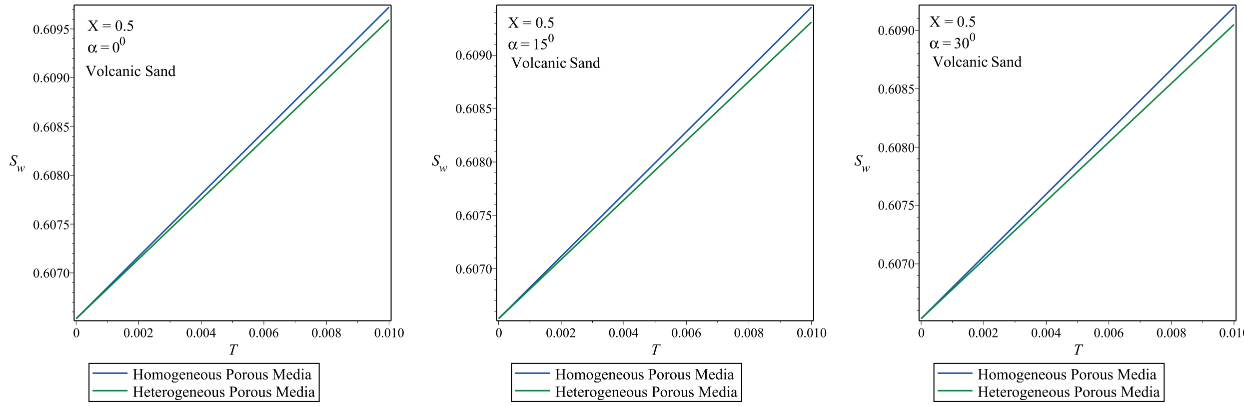

Fig. 4: comparison of saturation vs. dimensionless times for homogeneous and heterogeneous porous media in Volcanic Sand for different inclined plane $\alpha = 0°, 15°, 30°$

Table 3: Saturation of water in Volcanic Sand for Homogeneous and Heterogeneous Inclined Porous Media

Volcanic sand						
	X = 0.5					
	$\alpha = 0°$		$\alpha = 15°$		$\alpha = 30°$	
	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous
T =0.001	0.6068499680	0.6068369601	0.6068228503	0.6068088623	0.6067975768	0.6067826753
T =0.002	0.6071692764	0.6071432609	0.6071150411	0.6070870653	0.6070644941	0.6070346913
T =0.003	0.6074885848	0.6074495616	0.6074072318	0.6073652682	0.6073314113	0.6072867072
T =0.004	0.6078078932	0.6077558624	0.6076994226	0.6076434711	0.6075983286	0.6075387232
T =0.005	0.6081272016	0.6080621631	0.6079916133	0.6079216741	0.6078652458	0.6077907391
T =0.006	0.6084465100	0.6083684639	0.6082838040	0.6081998770	0.6081321631	0.6080427551
T =0.007	0.6087658184	0.6086747646	0.6085759948	0.6084780799	0.6083990803	0.6082947710
T =0.008	0.6090851268	0.6089810654	0.6088681855	0.6087562828	0.6086659976	0.6085467869
T =0.009	0.6094044352	0.6092873661	0.6091603763	0.6090344858	0.6089329148	0.6087988029
T =0.010	0.6097237436	0.6095936668	0.6094525670	0.6093126887	0.6091998320	0.6090508188

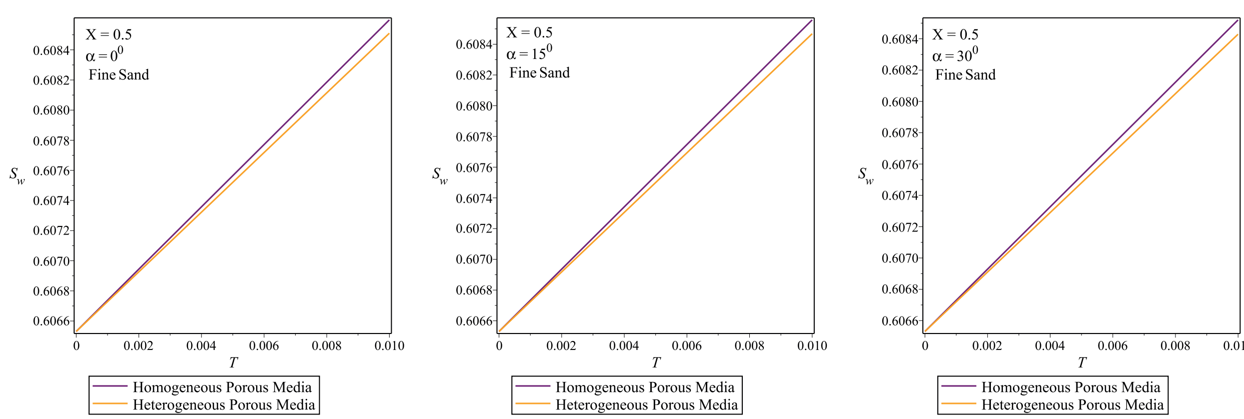

Fig. 5: comparison of saturation vs. dimensionless times for homogeneous and heterogeneous porous media in Fine Sand for different inclined plane $\alpha = 0°, 15°, 30°$

Table 4: Saturation of water in Fine Sand for Homogeneous and Heterogeneous Inclined Porous Media

Fine sand X = 0.5						
	$\alpha = 0°$		$\alpha = 15°$		$\alpha = 30°$	
	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous	Homogeneous	Heterogeneous
T = 0.001	0.6067374863	0.6067286320	0.6067334380	0.6067244237	0.6067296650	0.6067205015
T = 0.002	0.6069443130	0.6069266046	0.6069362164	0.6069181878	0.6069286705	0.6069103434
T = 0.003	0.6071511398	0.6071245771	0.6071389949	0.6071119520	0.6071276759	0.6071001854
T = 0.004	0.6073579666	0.6073225496	0.6073417734	0.6073057161	0.6073266814	0.6072900274
T = 0.005	0.6075647934	0.6075205222	0.6075445519	0.6074994803	0.6075256869	0.6074798694
T = 0.006	0.6077716201	0.6077184947	0.6077473303	0.6076932444	0.6077246924	0.6076697113
T = 0.007	0.6079784469	0.6079164672	0.6079501088	0.6078870086	0.6079236979	0.6078595533
T = 0.008	0.6081852737	0.6081144398	0.6081528873	0.6080807728	0.6081227034	0.6080493953
T = 0.009	0.6083921004	0.6083124123	0.6083556657	0.6082745369	0.6083217088	0.6082392373
T = 0.010	0.6085989272	0.6085103849	0.6085584442	0.6084683011	0.6085207143	0.6084290792

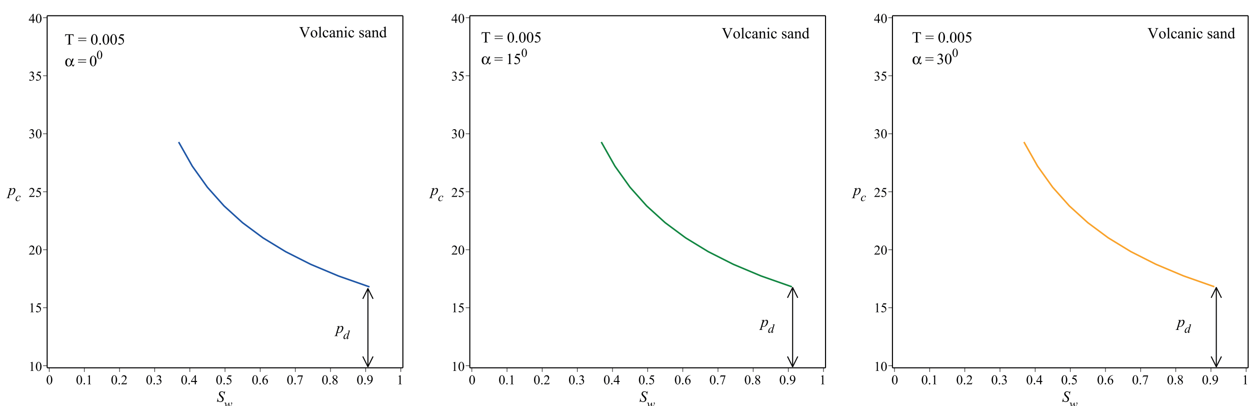

Fig. 6: Capillary pressure vs. Saturation in Heterogeneous Porous Media for Volcanic Sand at different inclined plane $\alpha = 0°,\ 15°,\ 30°$.

Table 5: Comparison of the numerical values for saturation vs. capillary pressure in heterogeneous porous media for volcanic sand at different inclined plane $\alpha = 0°,\ 15°,\ 30°$

Volcanic sand $T = 0.005$					
$\alpha = 0°$		$\alpha = 15°$		$\alpha = 30°$	
S_w	p_c	S_w	p_c	S_w	p_c
0.9108926490	16.79990829	0.9102311724	16.80634925	0.9096146803	16.81235950
0.8230249064	17.73408046	0.8225758545	17.73930430	0.8221573410	17.74417744
0.7438634804	18.74156210	0.7435586358	18.74581488	0.7432745225	18.74978131
0.6724796353	19.83362538	0.6722726876	19.83710347	0.6720798137	19.84034683
0.6080621631	21.02416467	0.6079216741	21.02702482	0.6077907391	21.02969160
0.5498977237	22.33062075	0.5498023509	22.33298822	0.5497134640	22.33519544
0.4973555182	23.77532357	0.4972907732	23.77729883	0.4972304312	23.77914024
0.4498751726	25.38750314	0.4498312196	25.38916707	0.4497902557	25.39071818
0.4069570112	27.20640659	0.4069271732	27.20782493	0.4068993643	27.20914702
0.3681541370	29.28633002	0.3681338811	29.28755693	0.3681150027	29.28870054

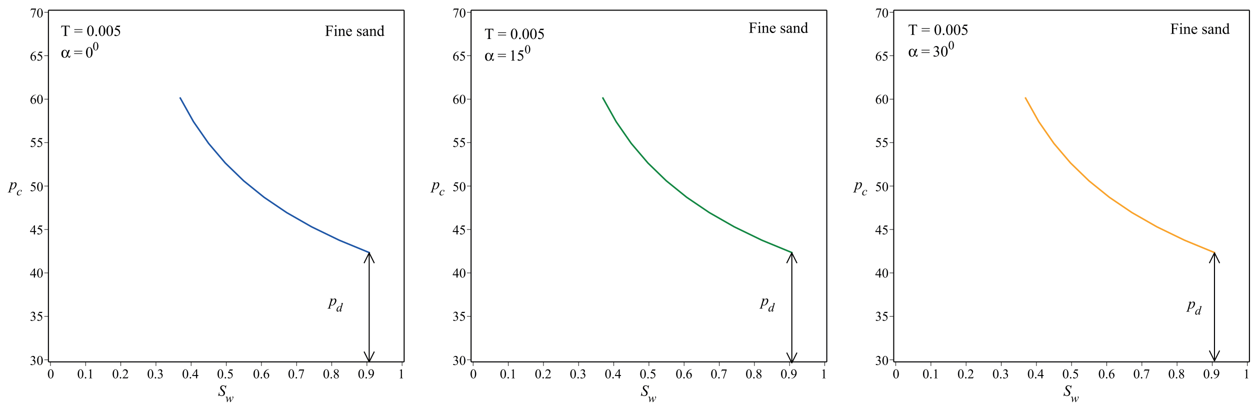

Fig. 7: Capillary pressure vs. Saturation in Heterogeneous Porous Media for Fine Sand at different inclined plane $\alpha = 0^\circ,\ 15^\circ,\ 30^\circ$.

Table 6: Comparison of the numerical values for saturation vs. capillary pressure in heterogeneous porous media for fine sand at different inclined plane $\alpha = 0^\circ,\ 15^\circ,\ 30^\circ$

Fine sand $T = 0.005$					
$\alpha = 0^\circ$		$\alpha = 15^\circ$		$\alpha = 30^\circ$	
S_w	p_c	S_w	p_c	S_w	p_c
0.9084990868	42.30988038	0.9084123627	42.31121792	0.9083315364	42.31246465
0.8213710515	43.76368019	0.8213101854	43.76478047	0.8212534585	43.76580600
0.7427220454	45.30476634	0.7426793272	45.30567494	0.7426395142	45.30652180
0.6716928259	46.94609761	0.6716628448	46.94685136	0.6716349025	46.94755393
0.6075205222	48.70369668	0.6074994803	48.70432546	0.6074798694	48.70491147
0.5495253906	50.59773572	0.5495106226	50.59826367	0.5494968590	50.59875575
0.4970999666	52.65412208	0.4970896019	52.65456890	0.4970799421	52.65498538
0.4497000690	54.90689098	0.4496927947	54.90727286	0.4496860151	54.90762874
0.4068372502	57.40195160	0.4068321448	57.40228186	0.4068273866	57.40258964
0.3680723911	60.20321490	0.3680688079	60.20350485	0.3680654685	60.20377508

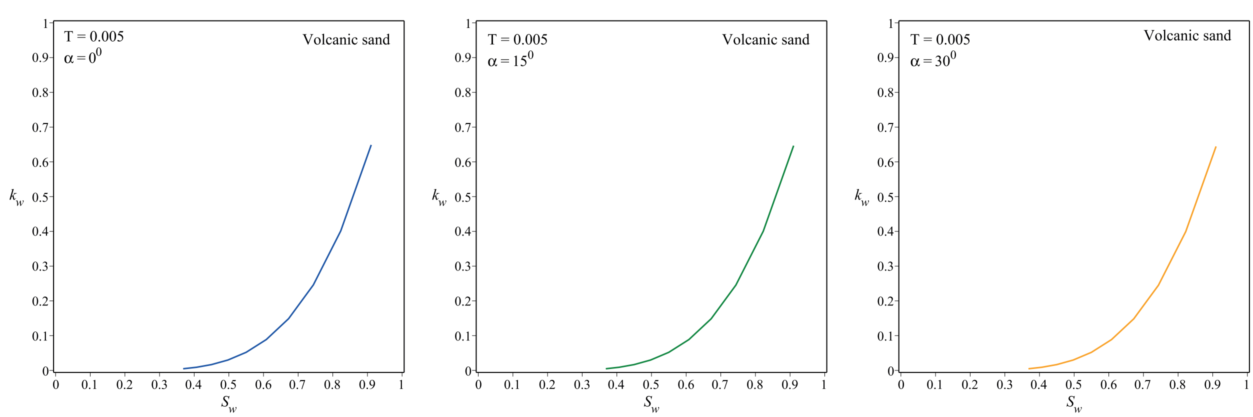

Fig. 8: Relative permeability vs. Saturation in Heterogeneous Porous Media for Volcanic Sand at different inclined plane $\alpha = 0^\circ,\ 15^\circ,\ 30^\circ$.

Table 7: Comparison of the numerical values for saturation vs. permeability in heterogeneous porous media for volcanic sand at different inclined plane $\alpha = 0^\circ,\ 15^\circ,\ 30^\circ$

Volcanic sand $T = 0.005$					
$\alpha = 0^\circ$		$\alpha = 15^\circ$		$\alpha = 30^\circ$	
S_w	k_w	S_w	k_w	S_w	k_w
0.9108926490	0.6487418955	0.9102311724	0.6465398941	0.9096146803	0.6444926391
0.8230249064	0.4014313656	0.8225758545	0.4003840327	0.8221573410	0.3994097512
0.7438634804	0.2458990914	0.7435586358	0.2454047098	0.7432745225	0.2449446132
0.6724796353	0.1487947362	0.6722726876	0.1485634901	0.6720798137	0.1483482105
0.6080621631	0.0887215125	0.6079216741	0.0886145261	0.6077907391	0.0885149015
0.5498977237	0.0519749629	0.5498023509	0.0519261116	0.5497134640	0.0518806132
0.4973555182	0.0298059356	0.4972907732	0.0297839799	0.4972304312	0.0297635282
0.4498751726	0.0166557784	0.4498312196	0.0166460986	0.4497902557	0.0166370809
0.4069570112	0.0090159365	0.4069271732	0.0090117685	0.4068993643	0.0090078852
0.3681541370	0.0046905754	0.3681338811	0.0046888328	0.3681150027	0.0046872091

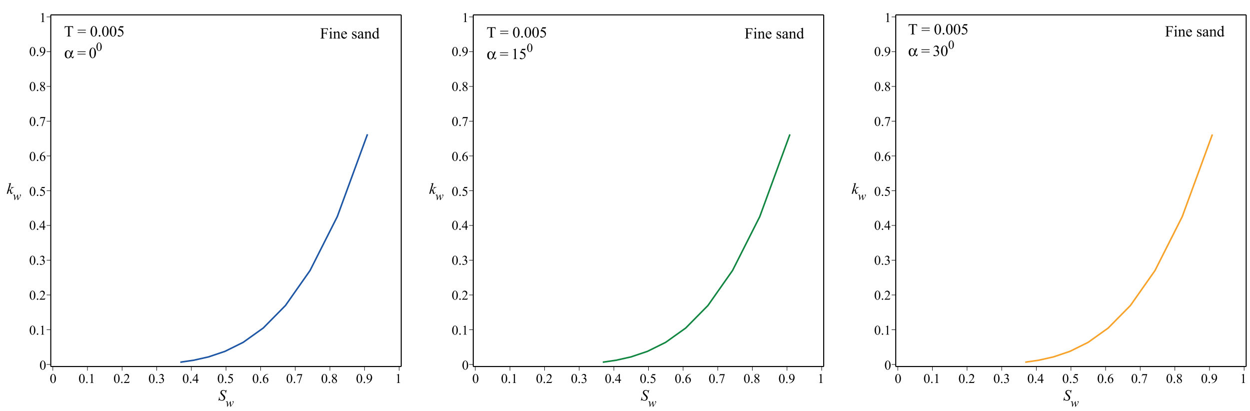

Fig. 9: Relative permeability vs. Saturation in Heterogeneous Porous Media for Fine Sand at different inclined plane $\alpha = 0^\circ,\ 15^\circ,\ 30^\circ$.

Table 8: Comparison of the numerical values for saturation vs. permeability in heterogeneous porous media for fine sand at different inclined plane $\alpha = 0^\circ,\ 15^\circ,\ 30^\circ$

Fine sand $T = 0.005$					
$\alpha = 0^\circ$		$\alpha = 15^\circ$		$\alpha = 30^\circ$	
S_w	k_w	S_w	k_w	S_w	k_w
0.9084990868	0.6623399876	0.9084123627	0.6620657577	0.9083315364	0.6618102502
0.8213710515	0.4254781604	0.8213101854	0.4253380575	0.8212534585	0.4252075122
0.7427220454	0.2703855623	0.7426793272	0.2703145372	0.7426395142	0.2702483544
0.6716928259	0.1696348616	0.6716628448	0.1695991859	0.6716349025	0.1695659411
0.6075205222	0.1048111035	0.6074994803	0.1047933792	0.6074798694	0.1047768622
0.5495253906	0.0635845126	0.5495106226	0.0635758218	0.5494968590	0.0635677228
0.4970999666	0.0377315285	0.4970896019	0.0377273341	0.4970799421	0.0377234253
0.4497000690	0.0217949936	0.4496927947	0.0217949936	0.4496860151	0.0217911577
0.4068372502	0.0121766915	0.4068321448	0.0121757738	0.4068273866	0.0121749185
0.3680723911	0.0065230494	0.3680688079	0.0065226379	0.3680654685	0.0065222543

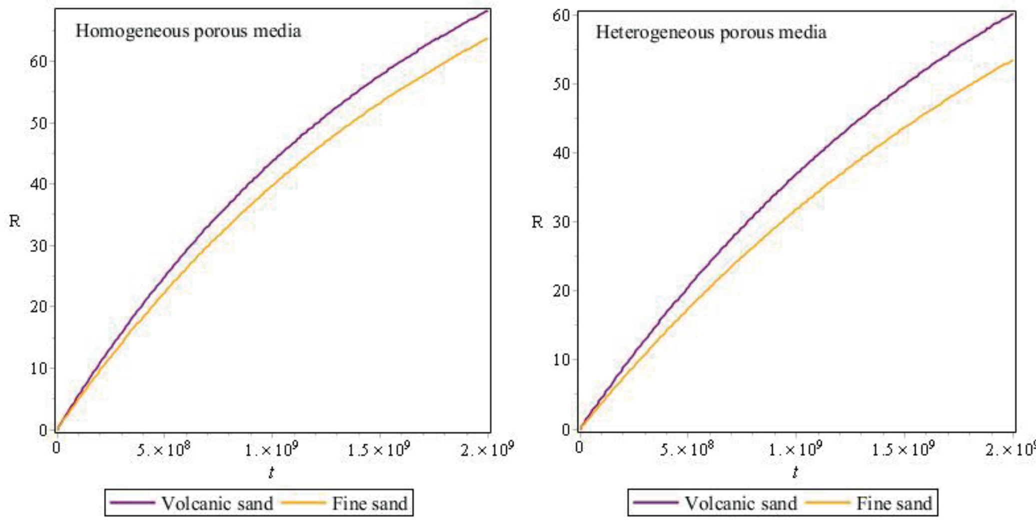

Fig. 10: Comparison of recovery rate for Volcanic sand and Fine sand in Homogeneous and Heterogeneous porous media

Table 9: Comparison of Recovery rate for Volcanic Sand and Fine Sand in Fracture Porous Media

	Recovery Rate			
	Homogeneous porous media		Heterogeneous porous media	
Dimension Time t (s)	Volcanic sand	Fine sand	Volcanic sand	Fine sand
1.089×10^8	6.06	5.37	4.89	4.08
2.179×10^8	11.75	10.46	9.54	7.99
3.268×10^8	17.09	15.27	13.96	11.74
4.358×10^8	22.12	19.82	18.17	15.34
5.447×10^8	26.84	24.13	22.17	18.79
6.537×10^8	31.27	28.20	25.98	22.11
7.626×10^8	35.43	32.06	29.60	25.28
8.716×10^8	39.35	35.71	33.04	28.33
9.806×10^8	43.02	39.17	36.32	31.25
1.089×10^9	46.46	42.42	39.42	34.04

the value of relative permeability be more in fine sand at zero inclined plane implies the saturation rate be more as compared to the inclined plane having $\alpha = 30^\circ$ but less saturation rate as compared to volcanic sand.

5 Recovery rate

Figure 10 discusses the variation of saturation rate in homogeneous and heterogeneous porous media with different porous materials which shows that the recovery rate be more in volcanic sand as compared to heterogeneous porous media and in find sand implies the recovery rate be optimum in the presence of volcanic sand in heterogeneous inclined porous media as compared to fine sand.

It is found here that the dependence of the type of porous matrix, effect of inclination and gravitational effect on saturation rate rendered the problem highly nonlinear. The significant part of this study is to study the advantage of the proposed mathematical expression in the determination of saturation of wetting phase and the recovery rate of this phenomenon with the inclusion of inclined plane and different porous materials with suitable choices of parametric values. It is found that there is an impact of inclination, heterogeneity and types of porous materials on saturation of wetting phase in counter – current imbibition phenomena and it shows that the saturation rate be more in case of homogeneous and for zero inclination as well as in volcanic sand as compared to heterogeneous inclined plane porous media in fine sand. The saturation rate be increases with time provided the recovery rate be more in case volcanic sand as compared to fine sand as shown in Fig. 10.

6 Conclusion

Here we studied the saturation rate as well as the recovery rate for a counter – current imbibition phenomenon in a heterogeneous inclined porous media for two types of porous materials like volcanic sand and fine sand. The simulation results for the saturation of wetting phase with time is shown in Table 2, 3 and 4 and recovery rate is shown in table 9 with the choices of suitable parametric values which shows in table 1 that the saturation rate be maximum in homogeneous porous matrix with zero inclination as compared to heterogeneous inclined porous matrix implies recovery rate of oil reservoir be maximum and around 45% in homogeneous porous matrix as well as in volcanic sand as compared to heterogeneous inclined porous matrix and fine sand which is physically consistent with the real world phenomena.

Nomenclature

Symbols	Parameters
R	Recovery
R_∞	Ultimate Recovery
S_i	Saturation of each phase i
v_i	Darcy velocity
ϕ	Porosity
k_i	Relative permeability of each phase i
μ_i	Viscosity of each phase i
p_d	Entry pressure
S_e	Effective saturation
λ	Grain size distribution
α	Inclination angle
S_{wr}	Wetting phase residual saturation
K	Permeability
P_c	Capillary Pressure

References

[1] Adomian, G. (1994): Solving Frontier Problems of Physics: The Decomposition Method, Kluwer, Boston.

[2] Brooks, R. H., & Corey, A. T. (1964): Hydraulic properties of porous media and their relation to drainage design, Transactions of the ASAE, 7(1), 26-0028.

[3] Darvishi, H., Goodarznia, I., & Esmaeilzadeh, F. (2010): Effects of Rock Permeability on Capillary Imbibition Oil Recovery from Carbonate Cores.

[4] Desai, K. R., Pradhan, V. H., Daga, A. R., & Mistry, P. R. (2015): Approximate Analytical Solution of Non-linear Equation in one Dimensional Imbibitions Phenomenon in Homogeneous Porous Media by Homotopy Perturbation Method, Proceeding Engineering, 127, 994–1001.

[5] Hughes, R. G., & Blunt, M. J. (2000): Pore scale modeling of rate effects in imbibition, Transport in Porous Media, 40(3), 295–322.

[6] Hatiboglu, C. U., & Babadagli, T. (2007): Oil recovery by counter–current spontaneous imbibition: Effects of matrix shape factor, gravity, IFT, oil viscosity, wettability, and rock type, Journal of Petroleum Science and Engineering, 59(1), 106–122.

[7] Karpyn, Z. T., Li, G., Grader, A. S., & Halleck, P. M. (2006): Experimental conditions favoring the formation of fluid banks during counter–current flow in porous media, Transport in porous media, 62(1), 109–124.

[8] Meher, R., & Meher, S. K. (2013): Series solution for porous medium equation arising in Fingero–imbibition phenomenon during oil recovery process, Applied Mathematics and Information Science Letters.

[9] Mishra, D. B. (2013): Analytical Solution of One Dimensional Counter–current Imbibition in Porous Media, International Journal of Physical and Mathematical Sciences, 4(1).

[10] Meher, R., Mehta, M. N., & Meher, S. K. (2011): A new approach to B(a)cklund transformations for longitudinal dispersion of miscible fluid flow through porous media in oil reservoir during secondary recovery process, Theoretical and Applied Mechanics, 38, 1–16.

[11] Pooladi-Darvish, M., & Firoozabadi, A. (2000): Co–current and counter–current imbibition in a water-wet matrix block, Spe Journal, 5(01), 3–11.

[12] Patel, K. R., Mehta, M. N., & Patel, T. R. (2013): A mathematical model of imbibition phenomenon in heterogeneous porous media during secondary oil recovery process, Applied Mathematical Modelling, 37(5), 2933–2942.

[13] Patel, A. B., Ratahva, G. A., & Bhathawala P. H.(2015): Imbibitions in Double Phase Flow through Porous Media, International Journal of Modern Science and Engineering Technology, 2(9), 46–51.

[14] Patel, K. K., Mehta, M. N., & Singh, T. R. (2016): A homotopy series solution to a nonlinear partial differential equation arising from a mathematical model of the counter–current imbibition phenomenon in a heterogeneous porous medium, European Journal of Mechanics-B/Fluids, 60, 119-126.

[15] Patel, H. S. & Meher, R. (2016): Fingering Phenomena in Fluid Flow Through Fracture Porous Media with Inclination and Gravitational Effect, Journal of Applied Fluid Mechanics, 9(6), 3135-3145.

[16] Patel, H. S., & Meher, R. (2016): Approximate Analytical Study of Counter – Current Imbibition Phenomenon in a Heterogeneous Porous Media. Applied Mathematical Sciences, 10(14), 673–681.

[17] Patel, H. S., & Meher, R. (2016): A Study on Recovery Rate for Counter - Current Imbibition Phenomenon with Corey's Model Arising during Oil Recovery Process, Applied Mathematics & Information Sciences, 10(5), 1877–1884.

[18] Patel, H. S., & Meher, R. (2016):Simulation of Counter - Current Imbibition Phenomenon with Corey's Model in Double Phase Flow Through Heterogeneous Porous Medium with Capillary Pressure, International Journal of Applied and Computational Mathematics, DOI: 10.1007/s40819-017-0329-1.

[19] Shah, R. C., & Verma, A. P. (2000): Multiphase flow through slightly dipping porous media with magnetic fluid, Indian Journal of Pure & Applied Physics, 38, 166–169.

[20] Verma, A. P. (1970): Fingero–Imbibition in Slightly Dipping Porous Medium, Journal of Applied Physics, 41(9), 3638–3641.
[21] Zeybek, M., Gurakin, G., Donmez, A., & Onur, M. (1995): Effects of capillary heterogeneities on spontaneous imbibition, In SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers.
[22] Zhou, X. (1996): The effect of aging time and initial water saturation on oil recovery by spontaneous imbibition and waterflooding.