

M. Gnaneswara Reddy\*

# Velocity and thermal slip effects on MHD third order blood flow in an irregular channel through a porous medium with homogeneous/heterogeneous reactions

<https://doi.org/10.1515/nleng-2017-0008>

Received December 6, 2016; accepted February 19, 2017.

**Abstract:** This communication presents the transportation of third order hydromagnetic fluid with thermal radiation by peristalsis through an irregular channel configuration filled a porous medium under the low Reynolds number and large wavelength approximations. Joule heating, Hall current and homogeneous-heterogeneous reactions effects are considered in the energy and species equations. The Second-order velocity and energy slip restrictions are invoked. Final dimensionless governing transport equations along the boundary restrictions are resolved numerically with the help of NDSolve in Mathematica package. Impact of involved sundry parameters on the non-dimensional axial velocity, fluid temperature and concentration characteristics have been analyzed via plots and tables. It is manifest that an increasing porosity parameter leads to maximum velocity in the core part of the channel. Fluid velocity boosts near the walls of the channel where as the reverse effect in the central part of the channel for higher values of first order slip. Larger values of thermal radiation parameter  $R$  reduce the fluid temperature field. Also, an increase in heterogeneous reaction parameter  $K_s$  magnifies the concentration profile. The present study has the crucial application of thermal therapy in biomedical engineering.

**Keywords:** Radiation; MHD; Porous medium; Hall current; Second-order velocity slip; Thermal slip; Homogeneous/heterogeneous reactions

## 1 Introduction

Peristalsis is the one kind of physiological fluid motion induced by a periodic continuous wave of proceeds axially over the length of distensible duct containing liquid. The occurrence of peristalsis has many physiologists recognized as the subject of bio-medical engineering and scientific research over the recent years due to wide applications include in female fallopian tube, micro blood vessels, smart heart lung machine, cardiopulmonary bypass surgery, blood hemolysis, transport of spermatozoa in lymphatic ductus, movement of ovum in vessels. The fluid transport on peristalsis is originated by Latham [1]. Shapiro et al. [2] studied the phenomenon of Peristaltic motion with approximations of long wavelengths at small Reynolds number. A mathematical model for the peristaltic flow in a tube was presented by Raju and Devanathan [3]. They obtained the expression for the stream function as a power series. The interaction of two dimensional peristaltic transport in a couple stress fluid has analyzed by Mekheimer [4]. They revealed that the pressure rise enhanced with boosting values of amplitude ratio. Mishra and Ramachandra Rao [5] have addressed the flow analysis in an irregular channel through the peristalsis by considering the suppositions of low Reynolds number along the large wave length. The peristaltic flow in Newtonian fluid and Non-Newtonian fluid over various aspects has been extensive studied many of the investigators [6–15].

The interaction of magnetohydrodynamic (MHD) peristaltic transport flow through a channel is of great importance with certain movement of physiological conductive fluids problems such as blood pumping machines, MHD peristaltic compressor, and magnetic drug process in cancer treatment, bio-magnetic substance in hemodynamics. The influence of applied magnetic field on blood flow was examined by Sud et al. [16]. Effect of Lorentz force on peristaltic couple stress fluid in a non-uniform channel has investigated by Mekheimer [17]. He noted

\*Corresponding Author: M. Gnaneswara Reddy: Department of Mathematics, Acharya Nagarjuna University Campus, Ongole – 523 001, India, E-mail: mgrmaths@gmail.com

that the pressure rise declines for higher values of couple stress parameter. Hayat et al. [18] investigated the blood flow of Johnson–Segalman fluid in the presence of magnetic field. The solution for the axial velocity evaluated in terms of Weissenberg number. The interaction of magnetohydrodynamics and Joule heating on Peristaltic blood flow in a nanofluid have presented by Ganeswara Reddy and Vanugopal Reddy [19]. The hydromagnetic two dimensional peristaltic motion of a Jeffrey fluid in an irregular channel have been examined by Ganeswara Reddy and coauthors [20, 21]. They are utilizing the low Reynolds number and small long wavelength suppositions for the dimensionless governing equations. They found that the axial velocity reduces in the core part of the channel while the reverse behavior in near the walls. Peristaltic transport through the effects of magnetic field and porous medium with heat transfer are investigated many investigators [22–24]. The impacts of Hall current in Newtonian/non-Newtonian fluids are focused recent studies by the peristalsis [25–29].

For the dealing the bio-physiological problems of polishing of artificial heart valves, fluid motion within human body flow problems, thin film, and flow on internal neuronal cavities, rarefied fluid in the presence of slip boundary conditions in the consideration is quite essential in the peristalsis. Chu and Fang [30] have analyzed the slip flow in peristalsis. Effects of wall slip conditions and MHD on the peristaltic motion in a non-uniform channel have presented Ebaid [31]. Ali et al. [32] have examined the velocity slip on the peristaltic MHD fluid flow. Tripathi et al. [33] have investigated the effect of slip restriction on peristaltic transport in a viscoelastic fluid. Mekheimer et al. [34] have discussed velocity slip effect on 2D MHD peristaltic motion in a wall filled a porous medium. It concludes that as increase the partial slip parameter results the inner fluid velocity strengthened. Ganeswara Reddy [35] have discussed the analysis of partial slip and MHD peristaltic flow filled a porous medium with heat and mass transfer. It is revealed that both axial velocity and fluid temperature enhances with an increase in the velocity slip parameter. The study of thermal radiation in the blood flows plays an emerging role in cancer therapy, laser surgery, medicine cryosurgery processes in the human body, prediction of blood flow with high temperature. Ramesh [36] have discussed the analysis of pulsatile motion on the couple stress fluid in an non-uniform channel with convective conditions and partial slip. He concluded that axial velocity near the walls of the channel is boosted for higher velocity slip. Sinha and Shit [37] have investigated the combined effects of thermal radiation and MHD heat transfer blood flow through a capillary. Ganeswara

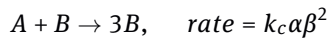
Reddy [38] have reported the impacts of chemical reaction and thermal radiation on hydromagnetic convective boundary layer slip flow. Hayat et al. [39] have addressed the influences of thermal radiation on peristaltic rotation flow in Jeffrey fluid. Exact solutions for velocity, dimensionless temperature are obtained with the considerations of small Reynolds number and large wavelength. They found that the dimensionless temperature function is a diminishing function for the thermal radiation parameter.

With the above discussed analysis, the current article being motivated to investigate the characteristics of the peristaltic third order hydromagnetic flow and heat transfer in an irregular channel filled a porous medium with homogeneous-heterogeneous reactions by considering the effects of hall current and thermal radiation. The present analysis is also revoked the second order velocity and thermal slips. The final dimensionless governing equations are highly non-linear and coupled. Thus it is very conflict to find its exact solution of velocity and temperature. The numerical solution is obtained with the help of NDSolve in Mathematica package. The computed numerical results are displayed graphically and analyzed to the effects of the sundry relevant dimensionless flow quantities. The present study has a useful application in thermal therapy in cancer treatment. The rest of the paper is summarized is as follows. The mathematical modeling of the problem is described in Section 2. Section 3 displays the numerical solution of the final controlled nonlinear coupled ODEs. The numerical results and discussion analyzed in detail with the aid of plots and tables in Section 4. Finally, the important findings of the current study reported in Section 5.

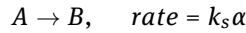
## 2 Modeling of the Problem

A two-dimensional electrical conducting of an incompressible third order peristaltic transport flow in an irregular channel (asymmetric channel) of width  $d_1 + d_2$  filled a homogeneous porous medium is considered. The channel of propagating irregular sinusoidal wave trains unvarying speed  $c$  through flexible walls. The thermal radiation, Hall current, second order velocity slip and energy slip conditions are taken in to consideration in the present study. The co-ordinate system and somatic proposed model is presented in Fig. 1. A uniform magnetic field imposed in opposite direction to the flow. It is consider a simple model for the relationship reaction between bulk (or homogeneous) and surface (or heterogeneous) which incriminate the two chemical species  $A$  and  $B$  (see [40, 41]). The homogeneous

(or bulk) reaction can be given as



and the heterogeneous (or surface) reaction is given by



where  $\alpha$  and  $\beta$  are the species concentrations  $A$  and  $B$  respectively and  $k_c$  and  $k_s$  are the rates of the constants. It is further considered that both process reactions are isothermal.

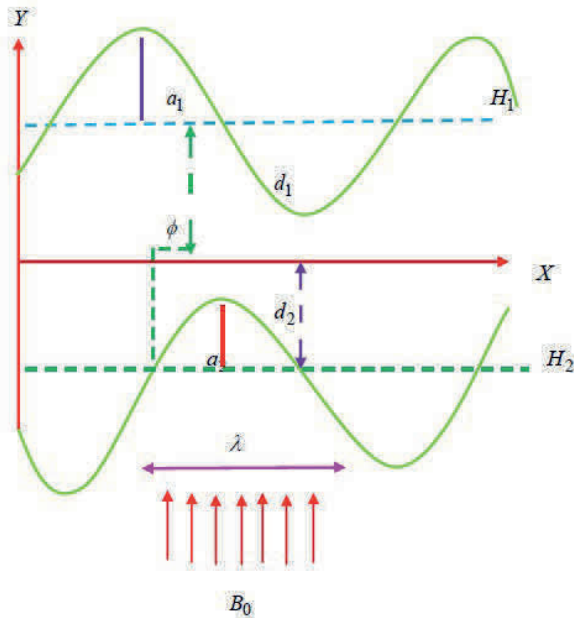


Fig. 1: Physical model of the problem.

The irregular channel of the walls of the flow is defined by

$$H_1(X, t) = d_1 + a_1 \cos \left[ \frac{2\pi}{\lambda} (X - ct) \right], \quad (1)$$

$$H_2(X, t) = -d_2 - a_2 \cos \left[ \frac{2\pi}{\lambda} (X - ct) + \phi \right], \quad (2)$$

in which  $a_1, b_1$  are the amplitudes of the waves,  $d_1 + d_2$  is the width of the channel,  $\lambda$  is the wave length,  $\phi$  is the phase difference which varies in the range  $0 \leq \phi \leq \pi$ .

The governing equation of flow, energy and both reactions for the fluid are

$$\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} = 0, \quad (3)$$

$$\rho_f \left[ \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right] = -\frac{\partial p}{\partial X} + \frac{\partial}{\partial X} (S_{XX}) + \frac{\partial U}{\partial Y} (S_{XY}) - \frac{\sigma B_0^2}{1+m^2} (U - mV) - \frac{\nu}{K} U,$$

$$\rho_f \left[ \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right] = -\frac{\partial p}{\partial Y} + \frac{\partial}{\partial X} (S_{YX}) + \frac{\partial}{\partial Y} (S_{YY}) - \frac{\sigma B_0^2}{1+m^2} (V + mU) - \frac{\nu}{K} V, \quad (4)$$

$$\rho_f c_f \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \right) = K^* \left( \frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2} \right) + \left( S_{XX} \frac{\partial U}{\partial X} + S_{XY} \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) + S_{YY} \frac{\partial V}{\partial Y} \right) - \frac{\partial q_r}{\partial Y} + \frac{(\sigma B_0^2 + \frac{1}{K}) U^2}{(1+m^2)}, \quad (5)$$

$$\frac{d\alpha}{dt} = D_A \left( \frac{\partial^2 \alpha}{\partial X^2} + \frac{\partial^2 \alpha}{\partial Y^2} \right) - k_c \alpha \beta^2, \quad (6)$$

$$\frac{d\beta}{dt} = D_B \left( \frac{\partial^2 \beta}{\partial X^2} + \frac{\partial^2 \beta}{\partial Y^2} \right) + k_c \alpha \beta^2, \quad (7)$$

where  $U$  and  $V$  are the velocity components in the laboratory frame  $(X, Y)$ ,  $p$  is the pressure,  $\sigma$  is the electrical conductivity of the fluid,  $K^*$  is the thermal conductivity,  $\nu$  is the kinematic viscosity,  $K$  is the permeability of the porous medium,  $\rho_f$  is the density of the fluid,  $c_p$  is the specific heat at constant pressure,  $S_{XX}$ ,  $S_{XY}$ ,  $S_{YY}$  are the components of extra stress tensor,  $T$  is the temperature,  $C$  is the concentration,  $T_1$ ,  $T_0$  and  $C_1$ ,  $C_0$  are the temperature and concentration at walls respectively, and  $q_r$  is the radiative heat flux,  $m$  is the Hall parameter,

By utilizing the Rosseland approximation for the thermal radiation, the heat flux for the radiation is given as

$$q_r = -\frac{16\sigma^* T_0^3}{3k^*} \frac{\partial T}{\partial Y} \quad (8)$$

in which  $\sigma^*$  and  $k^*$  are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.

The extra stress tensor for third order fluid [42] is given by

$$S = \mu \bar{A}_1 + \alpha_1 \bar{A}_2 + \alpha_2 \bar{A}_1^2 + \beta_1 \bar{A}_3 + \beta_2 (\bar{A}_2 \bar{A}_1 + \bar{A}_1 \bar{A}_2) + \beta_3 (tr \bar{A}_1^2) \bar{A}_1, \quad (9)$$

where  $\mu$  is the dynamic viscosity,  $\bar{A}_i (i = 1, 2)$  are Rivlin Ericksen tensors and  $\alpha_i (i = 1, 2)$  and  $\beta_i (i = 1, 2, 3)$  are material constants.

Define the following dimensionless quantities:

$$\begin{aligned}\bar{x} &= \frac{x}{\lambda}, \bar{y} = \frac{y}{d_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c}, h_1 = \frac{H_1}{a_1}, h_2 = \frac{H_2}{d_1}, \bar{t} = \frac{ct}{\lambda}, \\ \bar{p} &= \frac{d_1^2}{\lambda \mu C} p, \delta = \frac{d_1}{\lambda}, d = \frac{d_2}{d_1}, \\ a &= \frac{a_1}{d_1}, b = \frac{a_2}{d_1}, Re = \frac{\rho c d_1}{\mu}, M = \sqrt{\frac{\sigma}{\mu}} B_0 d_1, K = \frac{K}{d_1^2}, \\ \bar{\psi} &= \frac{\psi}{c d_1}, Pr = \frac{\mu c_p}{k^*}, u = \frac{\partial \psi}{\partial y}, v = -\delta \frac{\partial \psi}{\partial x}, \\ Ec &= \frac{c^2}{c_f (T_1 - T_0)}, \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0}, R = \frac{16 \sigma^* T_0^3}{3 k^* \mu_o c_f}, \\ Br &= Pr Ec, \alpha = \frac{f}{\alpha_0}, \beta = \frac{f}{\alpha_0}, \bar{S} = \frac{a}{\mu c} S, \\ K_2 &= \frac{a^2 k_c \alpha_0^2}{\nu}, \gamma = \frac{D_B}{D_A},\end{aligned}\quad (10)$$

By employ the above quantities and (8) and (9) in the governing transport equations with the assumptions of large wave length and  $Re \rightarrow 0$ , we have the following dimensionless flow, energy and reaction equations

$$\frac{\partial^2}{\partial y^2} \left( \frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 \right) - \frac{1}{1+m^2} \left( Ha^2 + \frac{1}{K} \right) \frac{\partial^2 \psi}{\partial y^2} = 0 \quad (11)$$

$$\begin{aligned}(1 + R Pr) \frac{\partial^2 \theta}{\partial y^2} + Br \frac{\partial^2 \psi}{\partial y^2} \left( \frac{\partial^2 \psi}{\partial y^2} + 2\Gamma \left( \frac{\partial^2 \psi}{\partial y^2} \right)^3 \right) \\ + Br \frac{1}{1+m^2} \left( Ha^2 + \frac{1}{K} \right) \left( \frac{\partial \psi}{\partial y} + 1 \right)^2 = 0\end{aligned}\quad (12)$$

$$\frac{1}{Sc} \frac{\partial^2 f}{\partial y^2} - K_2 f g^2 = 0 \quad (13)$$

$$\frac{\gamma}{Sc} \frac{\partial^2 g}{\partial y^2} + K_2 f g^2 = 0 \quad (14)$$

It is worthy mentioned that the coefficients of diffusion of chemical species  $A$  and  $B$  will not be equal. These diffusion coefficients are assumed to be equal in size and hence  $D_A = D_B$ . Then the equation for reactions lead to the equation [43]:

$$f + g = 1 \quad (15)$$

and thus the single equation for reactions as

$$\frac{\partial^2 f}{\partial y^2} - Sc K_3 f (1 - f)^2 = 0 \quad (16)$$

and the associated boundary conditions are given by

$$\begin{aligned}\psi &= \frac{q}{2}, \frac{\partial \psi}{\partial y} + \beta_1 \frac{\partial^2 \psi}{\partial y^2} + \beta_2 \frac{\partial^3 \psi}{\partial y^3} = -1, \theta + \gamma \frac{\partial \theta}{\partial y} = 0, \\ \frac{\partial f}{\partial y} &= K_s f, \text{ at } y = h_1(x) = 1 + a \cos(2\pi x),\end{aligned}\quad (17)$$

$$\begin{aligned}\psi &= -\frac{q}{2}, \frac{\partial \psi}{\partial y} - \beta_1 \frac{\partial^2 \psi}{\partial y^2} - \beta_2 \frac{\partial^3 \psi}{\partial y^3} = -1, \theta - \gamma \frac{\partial \theta}{\partial y} = 1, \\ f &= 1, \text{ at } y = h_2(x) = -d - b \cos(2\pi x + \phi),\end{aligned}\quad (18)$$

where  $q$  is the flux in the wave frame,  $Ha$  is the Hartmann number,  $K$  is the permeability parameter,  $\Gamma$  is the Deborah number,  $m$  is the Hall parameter,  $Pr$  is the Prandtl number,  $R$  is the radiation parameter,  $Br$  is the Brinkman number,  $Sc$  is the Schmidt number,  $K_1$  is the homogeneous reaction parameter,  $K_s$  is the heterogeneous reaction parameter,  $\beta_1$  and  $\beta_2$  are the first and second order velocity slip parameters,  $\gamma$  is the thermal slip parameter.

The non-dimensional flow rates in wave frame and fixed frame are related by

$$Q = q + 1 + d. \quad (19)$$

It is also worth note that the irregular channel reduce to a symmetric channel by setting  $a = b, d = 1, \phi = 0$ .

Also, the present problem reduced to the published work of Ebaid [31] by taking  $\Gamma = m = \beta_2 = R = \gamma = 0$ .

**Table 1:** Comparison of temperature.

y	Ha	Ebaid [31]	Ganeswara Reddy [35]	Present results
0.0	0.5	0.6129	0.6128	0.613001
	1.0	0.9126	0.9127	0.912690
0.5	0.5	0.5218	0.5219	0.521906
	1.0	0.7315	0.7316	0.731608
1.0	0.5	0.3216	0.3217	0.321609
	1.0	0.4120	0.4120	0.412002

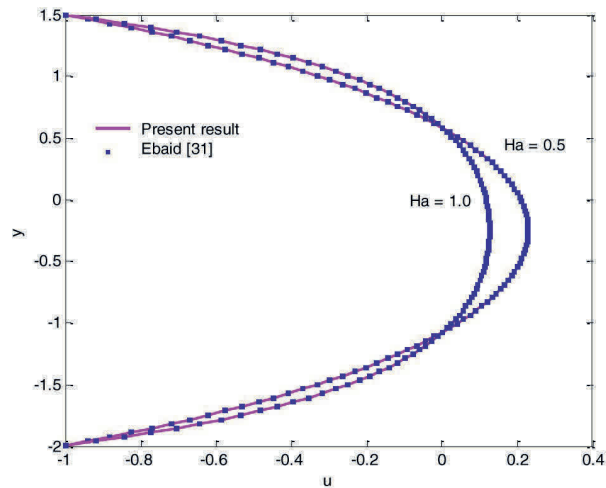
### 3 Results and discussion

The exact solutions of dimensionless governing flow equations are difficult due to highly non-linear and coupled. Hence we can find the numerical solution of non-dimensional equations (11), (12) and (16) along the boundary conditions (17) and (18) with the aid of NDSolve in computational package Mathematica. To check the accuracy of present numerical results with special case those of earlier published work by plots and Table 1. Figs. 2 and 3 displayed two distinct values of  $Ha$  on the comparison of axial velocity and fluid temperature with  $\Gamma = m = \beta_1 = \beta_2 = R = \gamma = 0$ . From these figures we can conclude that our numerical results are in nice agreement with the published literature of Ebaid [31]. A comparison of temperature distribution on the basis of our current study with those of Ebaid [31] and Ganeswara Reddy [35] have been shown

**Table 2:** The computational values of skin friction and heat transfer coefficients for the distinct sundry parameters at upper wall  $h_1$  of the channel.

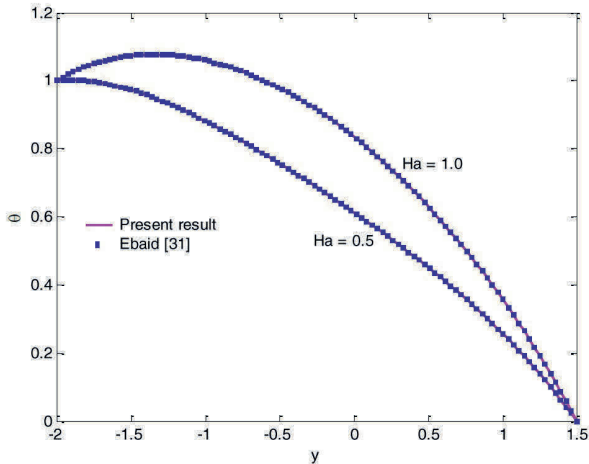
Ha	K	m	$\Gamma$	$\beta_1$	$\beta_2$	R	Br	$\gamma$	$C_f$	Nu
0.5									-1.760794	-0.464241
1.0									-1.815366	-0.553458
2.0									-2.033031	-0.914399
	1.0								-1.796849	-0.523663
	2.0								-1.760794	-0.464241
		1.0							-1.741478	-0.430919
		2.0							-1.723221	-0.397673
			0.5						-2.208634	-0.322706
			1.0						-2.536795	0.040043
				0.1					-1.521579	-0.441977
				0.2					-1.296173	-0.422687
					0.1				-1.571117	-0.451633
					0.2				-1.380642	-0.440300
						1.0			-1.760794	-0.399706
						2.0			-1.760794	-0.348079
							0.5		-1.760797	-0.754647
							1.0		-1.760797	-1.238656
								0.2	-1.760794	-0.450660
								0.5	-1.760797	-0.417170

in Table. It is found that an excellent agreement present results with these published results.



**Fig. 2:** Comparison of axial velocity for two distinct values of  $Ha$ .

The variations of Hartmann number  $Ha$ (ratio of electromagnetic force to the viscous force), porosity parameter  $K$ , Hall parameter  $m$ , Deborah number  $\Gamma$ , first order velocity slip parameter  $\beta_1$ , second order velocity slip parameter  $\beta_2$  on axial velocity ( $u$ ) are depicted in Figs. 4–9. From Fig. 4 revealed that the axial velocity declines in the middle of the irregular channel (due to Lorentz force) with ascending the values of  $Ha$  while the reverse effect



**Fig. 3:** Comparison of temperature distribution for two distinct values of  $Ha$ .

on the near walls of the channel. An enhancing value of Hartmann number strengths the magnetic field gives opposite force to decline the axial velocity. The variations is little more in the core part of the channel as comparing the walls. These physical expectations are coincide with Hayat et al. [27]. Fig. 5 sketched to explore the influence of porosity parameter on velocity. An increasing porosity parameter leads to maximum velocity in the central part of the channel. It is clear that larger porosity parameter  $K$  raises the permeability of the medium and which gives small resistance to the flow and enhance of the axial velocity. It is



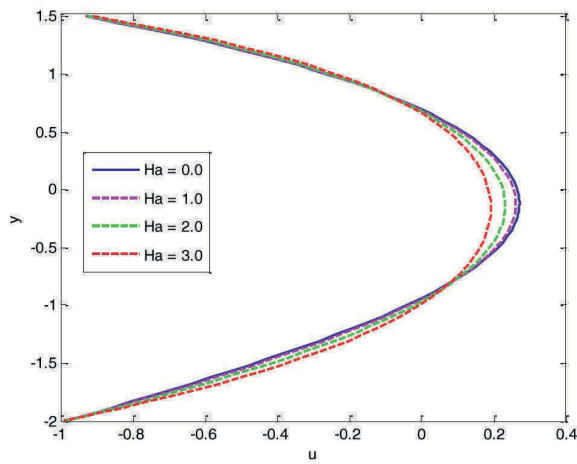


Fig. 4: Impact of Hartmann number  $Ha$  on velocity.

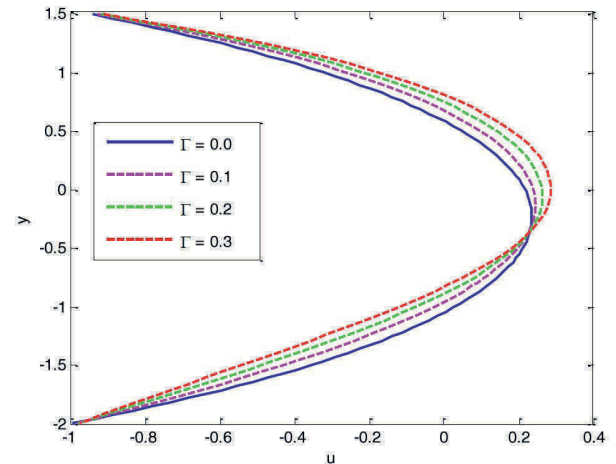


Fig. 7: Impact of Deborah number  $\Gamma$  on velocity.

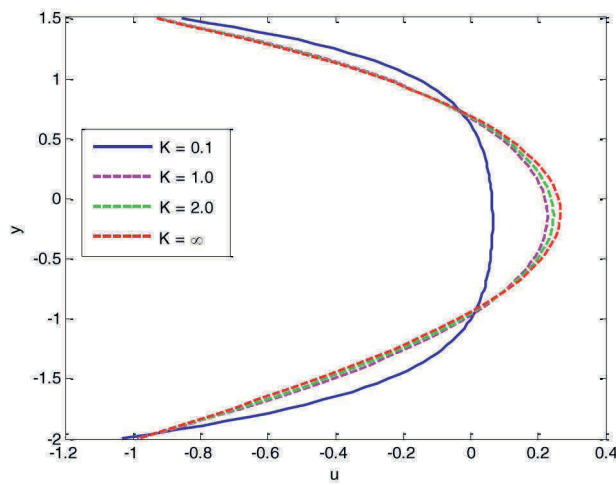


Fig. 5: Impact of porosity parameter  $K$  on velocity.

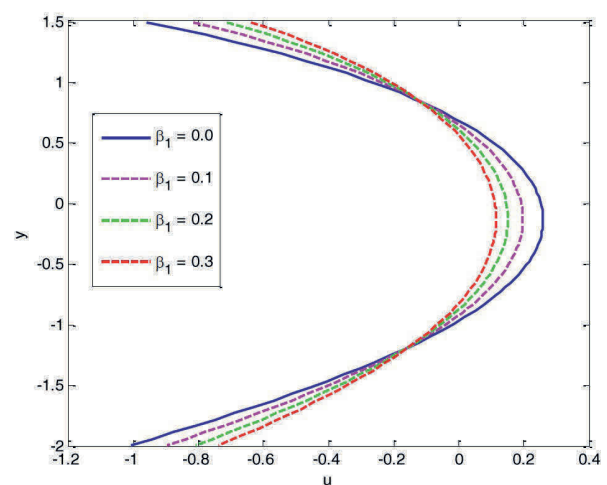


Fig. 8: Impact of first order velocity slip parameter  $\beta_1$  on velocity.

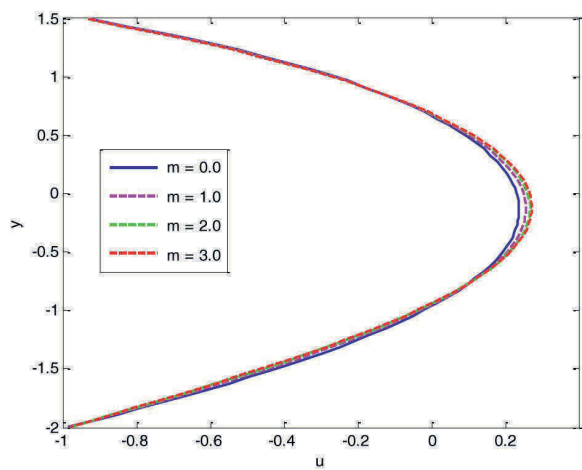


Fig. 6: Impact of Hall parameter  $m$  on velocity.

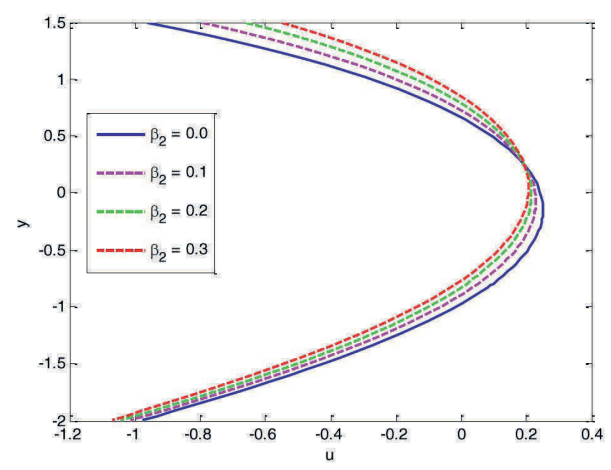
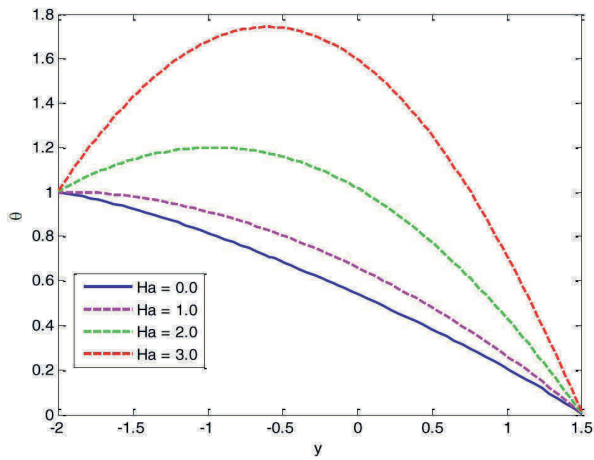
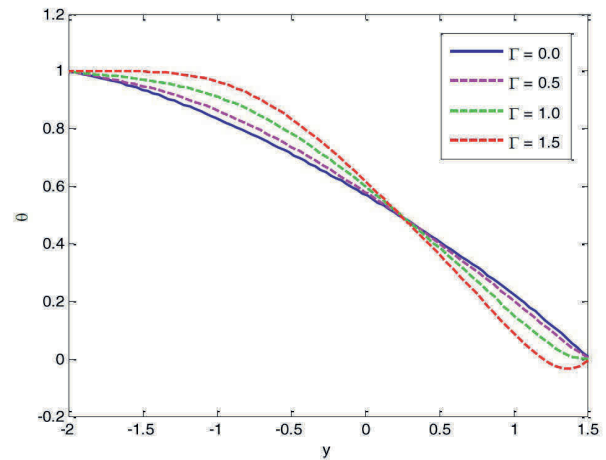
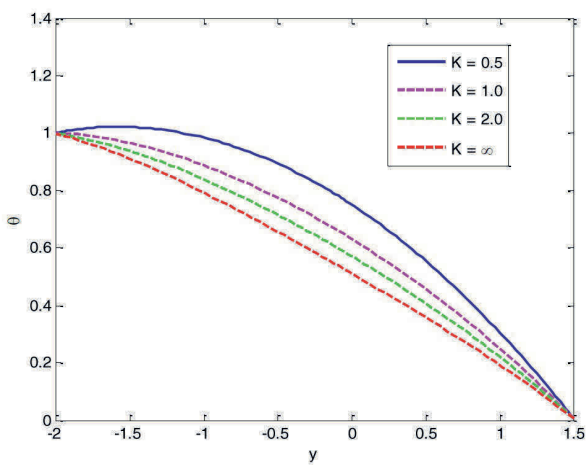
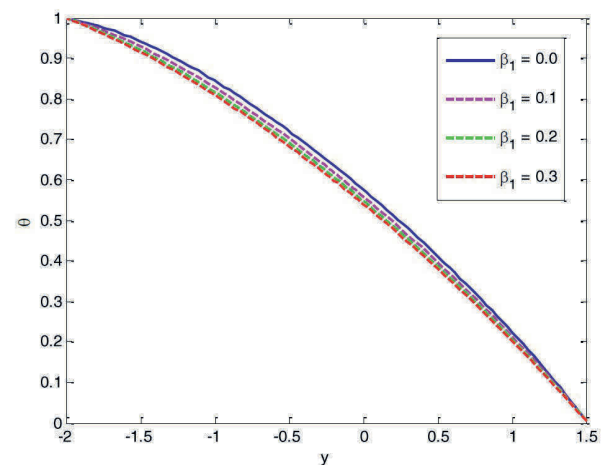
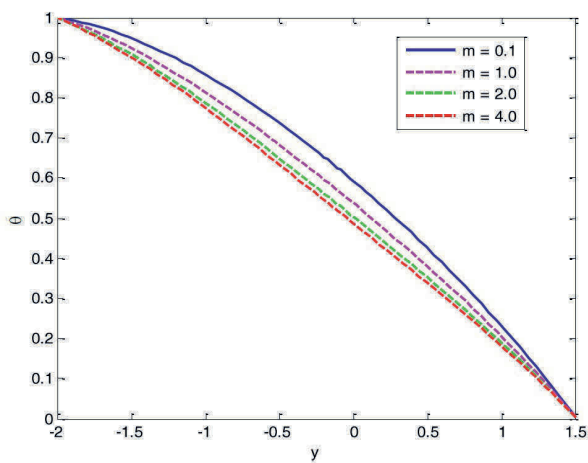
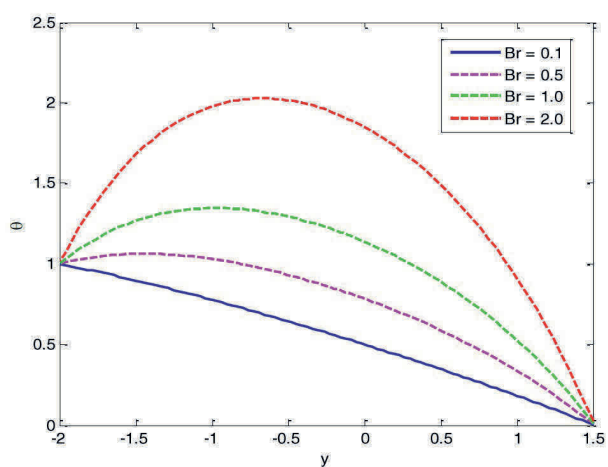
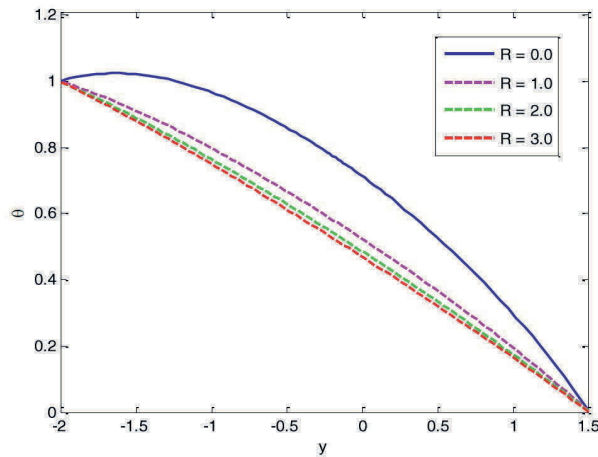
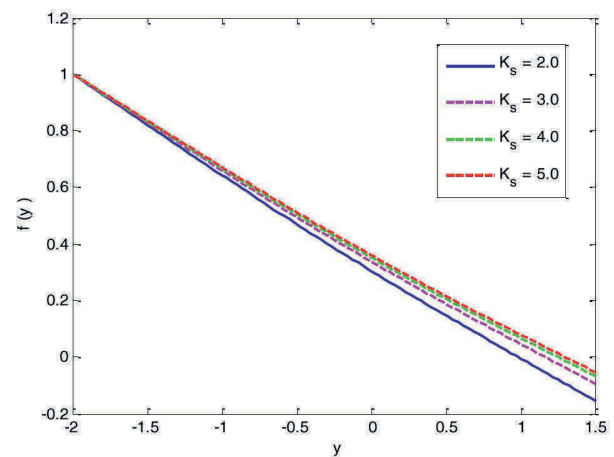
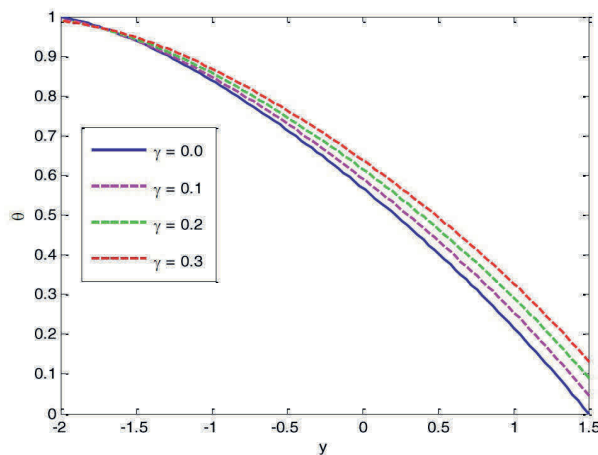
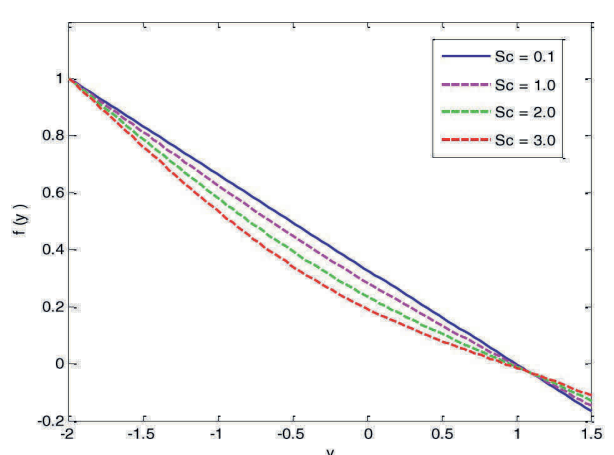
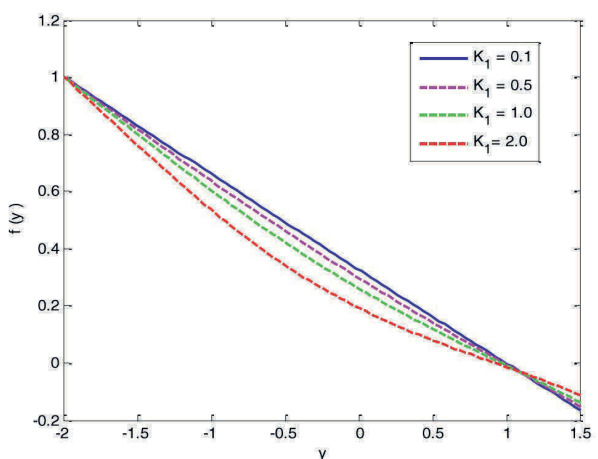


Fig. 9: Impact of second order velocity slip parameter  $\beta_2$  on velocity.

Fig. 10: Impact of Hartmann number  $Ha$  on temperature.Fig. 13: Impact of Deborah number  $\Gamma$  on temperature.Fig. 11: Impact of porosity parameter  $K$  on temperature.Fig. 14: Impact of first order velocity slip parameter  $\beta_1$  on temperature.Fig. 12: Impact of Hall parameter  $m$  on temperature.Fig. 15: Impact of Brinkman number  $Br$  on temperature.

Fig. 16: Impact of radiation parameter  $R$  on temperature.Fig. 19: Impact of heterogeneous reaction parameter  $K_s$  on concentration.Fig. 17: Impact of thermal slip parameter  $\gamma$  on temperature.Fig. 20: Impact of Schmidt number  $Sc$  on concentration.Fig. 18: Impact of homogeneous reaction parameter  $K_1$  on concentration.

noticed from Fig. 6 that the velocity boosts for increasing values of Hall parameter  $m$ . The velocity of flow enhances in the upper wall of the irregular channel while the opposite trend for the lower wall of the channel (see Fig. 7). Further, the velocity of non-Newtonian fluid ( $\Gamma \neq 0$ ) is larger with the comparison with Newtonian fluid ( $\Gamma = 0$ ) in the half part of the channel (near the upper wall). It is found from Fig. 8 that an enhancing of  $\beta_1$  the velocity boosts near the walls of the channel where as the reverse effect in the central part of the channel. This behavior match well that obtained by Ebaid [31]. From Fig. 9 observed that the velocity is opposite behavior near the channel walls with maximum values of  $\beta_2$ .

Figs. 10–17 are sketched to explore the characteristics of relevant dynamical parameters on the temperature field  $\theta(y)$ . The fluid temperature boosts to the magnifying values of  $Ha$ . Further, the temperature is more strengthen in



hydro-magnetic case as compared to the viscous dynamical flow ( $Ha = 0$ ) and which is evident from Fig. 10. It is revealed that the dimensionless temperature function diminishes for enhancing values of  $K$ . The energy loss is little for the higher permeability of the porous medium. This is due to fact that heat created through friction between porosity medium and fluid to the motion of the fluid. This fact can be observed from Fig. 11. It can be seen from Fig. 12 that the temperature profiles diminishes with an increase in  $m$ . The impact of Deborah number  $\Gamma$  on fluid temperature having opposite trend the near the walls (see Fig. 13). The impact of thermal slip parameter  $\beta$  on the temperature is presented in Fig. 14. The fluid temperature diminishes when the larger values of  $\beta$ . Further, the temperature is more in the case no thermal slip as compare to the thermal slip. From Fig. 15 that, the temperature field  $\theta(y)$  increases with an increase in the Brinkman number  $Br$ . Brinkman number is the product of Eckert number and Prandtl number ( $Br = Pr Ec$ ). An increase in the dissipation which gives more energy produces and thus strengthens the fluid temperature. It is observed from Fig. 16, that larger values of thermal radiation parameter  $R$  will be reduce the temperature distribution  $\theta(y)$ . It can also see that the variation in the presence of radiation is small on the fluid temperature. The similar trend exists in Ganeswara Reddy et al. [20] for the dimensionless radiation number  $R$ . Enhance in fluid temperature when thermal slip parameter  $\gamma$  increase and it can found in Fig. 17. Also, the variations on temperature is more in one wall of the channel than the other wall with higher of  $\gamma$ .

The dimensionless concentration profile diminishes to the higher homogeneous reaction parameter  $K_1$  in the region  $y \in [-1.995, 1.1502]$  (see Fig. 18). It can be revealed from Fig. 19 that an increase in heterogeneous reaction parameter  $K_s$  boosts the concentration field  $f(y)$  through the channel. From Fig. 20 conclude that the dimensionless concentration decays with enhancing values of the Schmidt number  $Sc$ .

The numerical values of the skin-friction, heat transfer coefficients for the distinct emerging numbers at upper wall  $h_1$  of the channel have been displayed in Table 2. From Table 2 that the skin-friction coefficient is a diminishing function for  $K, m, \beta_1, \beta_2$  while the opposite behavior for  $Ha, \Gamma$ . The heat transfer coefficient reduces for the boosting values of  $K, m, \Gamma, \beta_1, \beta_2, R, \gamma$  where as it is enhanced for  $Ha, Br$ . Furthermore, the impact of Hartmann number and porous parameter have opposite effect on friction factor and rate of heat transfer where as the velocity slip parameters have similar trend on these coefficients.

## 4 Conclusions

Thermal radiation, Hall current and MHD homogeneous-heterogeneous reactions on the third order peristaltic flow over an asymmetric channel filled a porous medium with velocity and thermal slip is discussed. The key points of present study are:

1. An increasing porosity parameter  $K$  leads to maximum velocity in the core part of the channel while it decreases with grow values of  $Ha$ .
2. Axial velocity boosts for enhancing values of  $m$ .
3. An enhancing of  $\beta_1$  the velocity boosts near the walls of the channel where as the reverse effect in the middle of the channel.
4. Fluid temperature is reduced with enhancing values of radiation parameter  $R$ .
5. The dimensionless temperature diminishes when the larger values of  $\beta$ .
6. Dimensionless concentration magnifies heterogeneous reaction parameter  $K_s$ .
7. Skin-friction coefficient is a diminishing function for  $K, m, \beta_1, \beta_2$  while the opposite behavior for  $Ha, \Gamma$ .

## Nomenclature

$a_1, b_1$	amplitudes of the waves
$\bar{A}_i (i = 1, 2)$	Rivlin Ericksen tensors
$d_1 + d_2$	Width of the channel
$\lambda$	Wave length
$\phi$	Phase difference
$q$	Flux in the wave frame
$Ha$	Hartmann number
$K$	Porosity parameter
$K_2$	the strength of homogeneous reaction
$\Gamma$	Deborah number
$m$	Hall parameter
$Pr$	Prandtl number
$R$	Radiation parameter
$Br$	Brinkman number
$Sc$	Schmidt number
$K_1$	Homogeneous reaction parameter
$K_s$	Heterogeneous reaction parameter
$\beta_1$	First order velocity slip parameter
$\beta_2$	Second order velocity slip parameter
$\mu$	dynamic viscosity
$\gamma$	Thermal slip parameter

## References

- [1] T.W. Latham, Fluid Motion in a Peristaltic Pump, MS. Thesis Massachusetts Institute of Technology, Cambridge, 1966.
- [2] A.H. Shapiro, M.Y. Jaffrin, S.L. Weinberg, Peristaltic pumping with long wavelengths at low Reynolds number, *J. Fluid Mech.* 37 (1969) 799–825.
- [3] K.K. Raju, R. Devanathan, Peristaltic motion of a non-Newtonian fluid, *Rheol. Acta* 11 (1972) 170–179.
- [4] Kh.S. Mekheimer, Peristaltic transport of a couple-stress fluid in a uniform and non-uniform channels, *Biorheology* 39 (2002) 755–765.
- [5] M. Mishra, A. Ramachandra Rao, Peristaltic transport of a Newtonian fluid in an asymmetric channel, *ZAMP* 54 (2003) 532–550.
- [6] S. Srinivas, G. Natarajan, A. Rami Reddy, T.V.K. Iyengar, Peristaltic transport of a Viscoelastic fluid with heat transfer, *Far East J. Appl. Math.* 8 (2002) 203–216.
- [7] M. Mishra, A.R. Rao, Peristaltic transport of a Newtonian fluid in an asymmetric channel, *Z. Angew. Math. Phys.* 54 (2004) 532–550.
- [8] T. Hayat, M. Khan, S. Asghar, A.M. Siddiqui, A mathematical model of peristalsis in tubes through a porous medium, *J. Porous Media* 9 (2006) 55–67.
- [9] M.H. Haroun, Non-linear peristaltic flow of a fourth grade fluid in an inclined asymmetric channel, *Comput. Mater. Sci.* 39 (2007) 324–333.
- [10] N. Ali, T. Hayat, Peristaltic flow of a micropolar fluid in an asymmetric channel, *Computers and Mathematics with Applications* 55 (2008) 589–608.
- [11] P. Muthu, B.V. Rathish Kumar, P. Chandra, Peristaltic motion of micropolar fluid in circular cylindrical tubes: effect of wall properties, *Appl. Math. Modelling* 32 (2008) 2019–2033.
- [12] M. Kothandapani, S. Srinivas, Nonlinear peristaltic transport of a Newtonian fluid in an inclined channel through a porous medium, *Phys. Lett. A* 372 (2008) 1265–1276.
- [13] T. Hayat, N. Ali, S. Asghar, An analysis of peristaltic transport for flow of a Jeffrey fluid, *Acta Mech.* 193(1) (2007) 101–112.
- [14] S. Maiti, J.C. Misra, Non-Newtonian characteristics of peristaltic flow of blood in micro-vessels, *Commun. Nonlinear Sci. Numer. Simul.* 18 (4) (2013) 1970–1988.
- [15] N.S. Akbar, S. Nadeem, Z.H. Khan, Numerical simulation of peristaltic flow of a Carreau nanofluid in an asymmetric channel, *Alex. Eng. J.* 53 (2014) 191–197.
- [16] V.K. Sud, G.S. Sekhon, R.K. Mishra, Pumping action on blood by a magnetic field, *Bull. Math. Biol.* 39 (1977) 385–390.
- [17] Kh.S. Mekheimer, Peristaltic flow of blood under effect of a magnetic field in a non uniform channels, *Appl. Math. Comput.* 153 (2004) 763–777.
- [18] T. Hayat, F.M. Mahomed, S. Asghar, Peristaltic flow of magnetohydrodynamic Johnson–Segalman fluid, *Nonlinear Dyn.* 40 (2005) 375–385.
- [19] M. Ganeswara Reddy and K. Venugopal Reddy, Influence of Joule Heating on MHD Peristaltic Flow of a Nanofluid with Compliant Walls, *Procedia Engineering* 127 (2015) 1002–1009.
- [20] M. Ganeswara Reddy, O.D. Makinde, Magnetohydrodynamic peristaltic transport of Jeffrey nanofluid in an asymmetric channel, *Journal of Molecular Liquids* 223 (2016) 1242–1248.
- [21] M. Ganeswara Reddy, K. Venugopal Reddy and O.D. Makinde, Heat transfer on MHD Peristaltic rotating flow of a Jeffrey fluid in an asymmetric channel, *Int. J. Appl. Comput. Math* (2016) doi:10.1007/s40819-016-0293-1.
- [22] D. Tripathi, S.K. Pandey, O. Anwar Beg, Mathematical modelling of heat transfer effects on swallowing dynamics of viscoelastic food bolus through the human oesophagus. *Int. J. Therm. Sci.* 70 (2013) 41–53.
- [23] T. Hayat, N. Ali, S. Asghar, Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. *Phys. Lett. A* 363 (2007) 397–403.
- [24] M. Ganeswara Reddy, K. Venugopal Reddy, O.D. Makinde, Hydromagnetic peristaltic motion of a reacting and radiating couple stress fluid in an inclined asymmetric channel filled with a porous medium, *Alexandria Engineering Journal* 55 (2016) 1841–1853.
- [25] N.S. Gad, Effect of Hall currents on interaction of pulsatile and peristaltic transport induced flows of a particle–fluid suspension, *Applied Mathematics and Computation*, 217(5), (2011) 4313–4320.
- [26] K. Nowar, Peristaltic flow of a nanofluid under the effect of Hall current and porous medium, *Math. Prob. Engin.* 2014 (2014) 15.
- [27] T. Hayat, M. Iqbal, H. Yasmin, F. Alsaadi, Hall effects on peristaltic flow of couple stress fluid in an inclined asymmetric channel, *Int. J. Biomath.* 7 (2014) 34.
- [28] N.S. Gad, Effects of hall currents on peristaltic transport with compliant walls, *Applied Mathematics and Computation*, 235 (2014), 546–554.
- [29] F.M. Abbasi, T. Hayat, A. Alsaedi, Numerical analysis for MHD peristaltic transport of Carreau–Yasuda fluid in a curved channel with Hall effects, *J. Magn. Magn. Mater.* 382 (2015) 104–110.
- [30] W.K.H. Chu, J. Fang, Peristaltic transport in a slip flow, *EPJ B.* 16 (3) (2000) 543–547.
- [31] A. Ebaid, Effect of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel, *Physics Letters A* 372 (2008) 4493–4499.
- [32] N. Ali, Q. Hussain, T. Hayat, S. Asghar, Slip effects on the peristaltic transport of MHD fluid with variable viscosity, *Phys. Lett. A* 372 (5) (2008) 1477–1489.
- [33] D. Tripathi, P.K. Gupta, S. Das, Influence of slip condition on peristaltic transport of a viscoelastic fluid with fractional Burger’s model, *Therm. Sci.* 15 (2) (2011) 501–515.
- [34] Kh S. Mekheimer, A.M. Salem, A.Z. Zaher, Peristaltically induced MHD slip flow in a porous medium due to a surface acoustic wavy wall. *J. Egypt. Math. Soc.* 22 (2014) 1431–1451.
- [35] M. Ganeswara Reddy, Heat and mass transfer on magnetohydrodynamic peristaltic flow in a porous medium with partial slip, *Alexandria Engineering Journal* 55 (2016) 1225–1234.
- [36] K. Ramesh, Effects of slip and convective conditions on the peristaltic flow of couple stress fluid in an asymmetric channel through porous medium, *Computer Methods and Programs in Biomedicine*, 135 (2016), 1–14.
- [37] A. Sinha, G.C. Shit, Electromagnetohydrodynamic flow of blood and heat transfer in a capillary with thermal radiation, *Journal of Magnetism and Magnetic Materials* 378 (2015) 143–151.
- [38] M. Ganeswara Reddy, Thermal radiation and chemical reaction effects on MHD mixed convective boundary layer slip flow

- in a porous medium with heat source and Ohmic heating, *Eur. Phys. J. Plus* (2014) 129:41.
- [39] T. Hayat, Maimona Rafiq and B. Ahmad, Combined effects of rotation and thermal radiation on peristaltic transport of Jeffrey fluid, *International Journal of Biomathematics* Vol. 8, No. 5 (2015) 1550061 (21 pages), DOI: 10.1142/S1793524515500618.
- [40] M.A. Chaudhary and J.H. Merkin, A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow. I Equal diffusivities, *Fluid Dyn. Res.* 16 (1995) 311-333.
- [41] Yang Zhang, Jinian Shu, Yuanxun Zhang, Bo Yang, Homogeneous and heterogeneous reactions of anthracene with selected atmospheric oxidants, *Journal of Environmental Sciences*, Vol. 25 (5), (2013), 1817–1823.
- [42] N. Ali, T. Hayat, Y. Wang, MHD peristaltic flow of a third order fluid in an asymmetric channel, *International Journal for Numerical Methods in Fluids*, 64(5) (2010), 992–1013.
- [43] M. Ramzan, M. Bilal, Jae Dong Chung, MHD stagnation point Cattaneo–Christov heat flux in Williamson fluid flow with homogeneous–heterogeneous reactions and convective boundary condition - A numerical approach, *Journal of Molecular Liquids*, 225 (2017), 856–862.